HARMONIC AND INTERMODULATION PERFORMANCE OF ENVELOPE DETECTORS

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Analytical expressions are obtained for predicting the amplitudes of the harmonics and intermodulation products at the output of an envelope detector excited by a signal formed of a carrier plus a number of sidebands. These expressions are in terms of the ordinary Bessel functions with arguments dependent on the modulations indices. Comparison between results obtained using the proposed technique and previously published results is performed to establish the accuracy of the proposed technique.

Keywords: Envelope detectors; intermodulation; harmonic distortion

INTRODUCTION

Envelope detectors are widely used in electronic systems [1, 2]. While envelope detector circuits are generally simple, their analysis is not. This is attributed to the nonlinear characteristics of the diode(s) and the reactive components involved [3–6]. Of particular interest here is the prediction of the harmonic and intermodulation performance of envelope detectors [7–10]. This problem is of special interest in color-television reception where a vestigial sideband signal is demodulated by an envelope detector [8–10].

Using numerical methods, the nonlinear differential equation of a simple envelope detector can be solved in the time-domain. Then, applying FFT the output spectrum resulting from an input formed of a SSB plus carrier can be obtained [7]. The method can be easily

extended to predict the intermodulation performance of an envelope detector excited by a SSB plus carrier plus interfering signals.

On the other hand, through direct integration of the envelope detector output, closed-form analytical expressions, in terms of the complete elliptic integrals of the first and second kinds, can be obtained for the dc and harmonic contents [8]. Alternatively, through the repeated use of power series expansions, approximate power series expressions can be obtained for the dc and harmonic contents of the envelope detector output [10]. In both cases the input signal to the envelope detector is formed of a SSB plus carrier.

While the use of closed-form expressions, for evaluating the output spectrum of the envelope detector, is more attractive than numerically-based methods, it appears that the expressions obtained in [8] and [10] cannot be used for evaluating the intermodulation performance of the envelope detector.

Thus, there is a need for a new technique for predicting the output spectrum of an envelope detector excited by a carrier plus a number of sidebands. The technique would be attractive if it can yield closed-form analytical expressions for the output dc, harmonics and intermodulation components. It is the major intention of this paper to present such a technique.

ANALYSIS

Figure 1 shows the phasor diagram of an input signal formed of a carrier plus a number of sidebands. At the input of the envelope detector, this signal can be represented by

$$v_i(t) = V \left(\sin \omega_o t + \sum_{k=1}^K m_k \sin (\omega_o - \omega_k) t \right) \quad (1)$$

where ω_o is the carrier frequency, $\omega_o - \omega_k$ is the frequency of the k th sideband and V is the amplitude of the carrier. The output of the

FIGURE 1 Carrier and sidebands phasors at the input.

envelope detector can be expressed as

$$V_o = V \left(1 + \sum_{k=1}^K m_k^2 + 2 \sum_{k=1}^K m_k \cos \theta_k + \sum_{i=1, j=1, j>i}^K 2 m_i m_j \cos (\theta_i - \theta_j) \right)^{1/2} \quad (2)$$

where $\theta_k = \omega_k t$.

Eq. (2) can be rewritten in the form

$$\frac{V_o}{V} \frac{1}{\left(1 + \sum_{k=1}^K m_k^2 \right)^{1/2}} = \left(1 + \sum_{k=1}^K a_k \cos \theta_k + \sum_{i=1, j=1, j>i}^K b_{ij} \cos \Psi_{ij} \right)^{1/2} \quad (3)$$

where

$$a_k = \frac{2 m_k}{\left(1 + \sum_{k=1}^K m_k^2 \right)}$$

$$b_{ij} = \frac{2 m_i m_j}{\left(1 + \sum_{k=1}^K m_k^2 \right)}$$

and

$$\Psi_{ij} = \theta_i - \theta_j$$

Eq. (3) is in the form

$$y = (1 + x)^{1/2} \quad (4)$$

Eq. (4) can be approximated by a Fourier-series of the form

$$y = \alpha_o + \sum_{n=1}^N \left(\alpha_n \cos \left(\frac{2n\pi}{T} x \right) + \beta_n \sin \left(\frac{2n\pi}{T} x \right) \right) \quad (5)$$

where the parameters α_o , α_n and β_n can be obtained using FFT algorithms or curve-fitting techniques. Alternatively, following the procedure described in [11] and [12], first the offset at $x = 0$ is removed and the resulting function is mirror imaged to obtain a complete period of the periodic function $f(x) = y - 1$. Secondly, a number of data points is chosen and connected using straight line segments joined end to end as shown in Figure 2. Denoting the slope of each segment by γ_l , the parameters α_n and β_n can be expressed as [11, 12]

FIGURE 2 The function of Eq. (4) after removing the offset at $x = 0$ and mirror imaging.

$$\alpha_n = \frac{-T}{2(n\pi)^2} \left(\gamma_1 - \gamma_{L-1} + \sum_{l=1}^{L-2} (\gamma_{l+1} - \gamma_l) \cos\left(\frac{2n\pi}{T} x_{l+1}\right) \right) \quad (6)$$

$$\beta_n = \frac{-T}{2(n\pi)^2} \left(\sum_{l=1}^{L-2} (\gamma_{l+1} - \gamma_l) \sin\left(\frac{2n\pi}{T} x_{l+1}\right) \right) \quad (7)$$

where T is the period of the periodic function. The parameter γ_o can be easily obtained by calculating the area under the curve in Figure 2 using any numerical integration method. Thus, γ_o can be expressed as

$$\begin{aligned} \gamma_o = & \frac{1}{T} \left(\frac{1}{2} x_2 f(x_2) + \frac{1}{2} (x_L - x_{L-1}) f(x_{L-1}) \right. \\ & \left. + \sum_{l=2}^{L-2} \left((x_{l+1} - x_l) f(x_{l+1}) - \frac{1}{2} (x_{l+1} - x_l) (f(x_{l+1}) - f(x_l)) \right) \right) \end{aligned} \quad (8)$$

From (6) and (7) one can see that calculation of the parameters α_n and β_n requires only simple mathematical operations, without recourse to sophisticated algorithms for FFT or curve-fitting techniques. Also, inspection of (6) and (7) suggests that as n becomes infinite, the Fourier-series parameters α_n and β_n always approach zero. In fact, the number of terms in (5) can be increased until the inclusion of the next term is seen to make a negligible contribution towards a best fit criterion; for example the minimum relative mean-square (RRMS) error. Table I shows the parameters of the first 21 terms of (5) used for approximating (4).

Using the parameters of Table I and Eq. (5) calculations were made and are summarized in Table II which shows the change of the RRMS error with the number of terms of the Fourier-series. It is obvious that increasing the number of terms improves the accuracy of the Fourier-series approximation.

HARMONIC AND INTERMODULATION PRODUCTS

One of the potential application of the proposed model of (5) is in the prediction of the amplitudes of the harmonics and intermodulations

TABLE I First 21 terms of (5), $\alpha_o = -0.0576$

n	α_n	β_n
1	0.1357E-05	0.4553
2	0.7806E-01	-0.7813E-06
3	-0.7549E-06	-0.7781E-01
4	-0.3161E-01	0.5864E-06
5	0.5748E-06	0.3491E-01
6	0.1841E-01	-0.4608E-06
7	-0.5553E-06	-0.2082E-01
8	-0.1253E-01	0.4290E-06
9	0.3807E-06	0.1425E-01
10	0.9319E-02	-0.3820E-06
11	-0.4410E-06	-0.1059E-01
12	-0.7332E-02	0.3551E-06
13	0.3552E-06	0.8301E-02
14	0.6001E-02	-0.3706E-06
15	-0.3707E-06	-0.6763E-02
16	-0.5054E-02	0.3276E-06
17	0.3153E-06	0.670E-02
18	0.4352E-02	-0.2822E-06
19	-0.3336E-06	-0.4859E-02
20	-0.3812E-02	0.3021E-06
21	0.3075E-06	0.4237E-02

TABLE II Variation of the RRMS with the number of terms N

N	$RRMS\ error$
401	0.5966E-03
31	0.2159E-01
21	0.3363E-01
11	0.7249E-01

products at the output of the envelope detector. Thus, combining (4) and (5) and using the trigonometric identities

$$\sin(z \cos \phi) = 2(J_1(z) \cos \phi - J_3(z) \cos 3\phi + \dots)$$

and

$$\cos(z \cos \phi) = J_0(z) - 2J_2(z) \cos 2\phi + 2J_4(z) \cos 4\phi - \dots$$

where $J_k(z)$ is the Bessel function of order k , and after simple mathematical manipulations it is easy to show that the amplitude of an output product of frequency $\sum_{k=1}^K \delta_k \omega_k + \sum_{i=1, j=1, j>i}^K \delta_{ij} \Psi_{ij}$ and order $\sum_{k=1}^K |\delta_k| + \sum_{i=1, j=1, j>i}^K |\delta_{ij}|$, where δ_k and δ_{ij} are positive, negative

integers or zeros, will be given by

$$\frac{V_{T(\delta_1, \dots, \delta_K, \dots, \delta_{2K})}}{V \left(1 + \sum_{k=1}^K m_k^2\right)^{1/2}} = 2 \sum_{n=1}^N \alpha_n \prod_{k=1}^K J_{|\delta_k|} \left(\frac{2n\pi}{T} a_k \right) \prod_{i=1, j=1, j>i}^K J_{|\delta_{ij}|} \left(\frac{2n\pi}{T} b_{ij} \right) \quad (9)$$

for $\sum_{k=1}^K |\delta_k| + \sum_{i=1, j=1, j>i}^K |\delta_{ij}| = \text{even}$

$$\frac{V_{T(\delta_1, \dots, \delta_K, \dots, \delta_{2K})}}{V \left(1 + \sum_{k=1}^K m_k^2\right)^{1/2}} = 2 \sum_{n=1}^N \beta_n \prod_{k=1}^K J_{|\delta_k|} \left(\frac{2n\pi}{T} a_k \right) \prod_{i=1, j=1, j>i}^K J_{|\delta_{ij}|} \left(\frac{2n\pi}{T} b_{ij} \right) \quad (10)$$

for $\sum_{k=1}^K |\delta_k| + \sum_{i=1, j=1, j>i}^K |\delta_{ij}| = \text{odd}$

and the dc component can be expressed as

$$\frac{V_{dc}}{V \left(1 + \sum_{k=1}^K m_k^2\right)^{1/2}} = 1 + \alpha_o + \sum_{n=1}^N \alpha_n \prod_{k=1}^K J_{|\delta_k|} \left(\frac{2n\pi}{T} a_k \right) \prod_{i=1, j=1, j>i}^K J_{|\delta_{ij}|} \left(\frac{2n\pi}{T} b_{ij} \right) \quad (11)$$

Note that a unity is introduced into (11) to restore the removed offset at $x = 0$. Eqs. (9) and (10) can be used for calculating the amplitudes of the harmonics and intermodulation products at the output of an envelope detector excited by a carrier plus a number of sidebands.

SPECIAL CASE

In this section the special case of an input signal formed of a carrier plus a single sideband will be considered in detail and the results will be compared with previously published results to establish the accuracy of the proposed technique. Under these conditions, the dc output component will be

$$\frac{V_{dc}}{V(1 + m^2)^{1/2}} = 1 + \alpha_o + \sum_{n=1}^N \alpha_n J_o \left(\frac{2n\pi}{T} a_1 \right) \quad (12)$$

the amplitude of the output fundamental component will be

$$\frac{V_{T1}}{V(1+m^2)^{1/2}} = 2 \sum_{n=1}^N \beta_n J_1 \left(\frac{2n\pi}{T} a_1 \right) \quad (13)$$

and the amplitude of the output second-harmonic component will be

$$\frac{V_{T2}}{V(1+m^2)^{1/2}} = 2 \sum_{n=1}^N \alpha_n J_2 \left(\frac{2n\pi}{T} a_1 \right) \quad (14)$$

For sufficiently small values of a_1 , the Bessel functions can be approximated by

$$J_0(z) \cong 1, \quad J_n(z) \cong \frac{(z/2)^n}{n!}$$

and Eqs. (12)–(14) can be approximated by

$$V_{dc} = V(1+m^2)^{1/2} \left(1 + \alpha_o + \sum_{n=1}^N \alpha_n \right) \quad (15)$$

$$V_{T1} = m V \frac{4\pi}{T} \frac{1}{(1+m^2)^{1/2}} \sum_{n=1}^N n \beta_n \quad (16)$$

and

$$V_{T2} = m^2 V \left(\frac{2\pi}{T} \right)^2 \frac{4}{(1+m^2)^{3/2}} \sum_{n=1}^N n^2 \alpha_n \quad (17)$$

From (15) one can see that the dc output component is proportional to the amplitude of the input carrier. From (16) one can see that the amplitude of the fundamental output component is proportional to the product of the input carrier amplitude and the modulation index; that is the amplitude of the input sideband. Finally, from (17) one can see that the amplitude of the output second-harmonic component is proportional to the product of the amplitude of the input carrier and the square of the modulation index. These trends are in excellent agreement with the results reported in [10].

TABLE III Comparison between previously published and present results

	Ref. [8]	Ref. [10]	$N = 31$	$N = 21$	$N = 11$
V_{dc}/V	1.273	1.266	1.274	1.275	1.280
V_{T1}/V	0.849	0.859	0.847	0.843	0.833
V_{T2}/V	–	0.188	0.168	0.164	0.155

Using (12)–(14) the output dc, fundamental and second-harmonic components can be calculated for any value of m . The results obtained for $m = 1$, together with the results obtained using the techniques proposed in [8] and [10] are shown in Table III. It appears from Table III that the results obtained using the present technique are in excellent agreement with the results obtained using previously published techniques. This establishes the accuracy of the present technique.

CONCLUSION

By approximating the envelope detector characteristic of Eq. (2), using a Fourier-series, analytical expressions can be obtained for the harmonic and intermodulation performance of an envelope detector excited by a signal formed of a carrier plus a number of sidebands. The Fourier-series coefficients and the amplitudes of the output components can be evaluated using simple calculations without recourse to numerical integration or FFT. Comparison with previously published results confirms the validity of the proposed technique.

References

- [1] Clarke, K. K. and Hess, D. T. (1971). *Communication Circuits: Analysis and Design*, Reading, MA: Addison-Wesley, pp. 461–466.
- [2] Krauss, H. L., Bostian, C. W. and Raab, F. H. (1980). *Solid State Radio Engineering*, New York: Wiley.
- [3] Simonov, Y. L., Simonov, A. Y. and Zhmurov, V. V. (1986). Analysis of an amplitude detector based on a DC isolated diode, *Telecommunication and Radio Engineering*, **40/41**(3), 152–154.
- [4] Simonov, A. Y., Simonov, Y. L. and Zhmurov, V. V. (1987). Analysis of a square-law amplitude detector, *Telecommunication and Radio Engineering*, **41/42**(8), 39–42.
- [5] Khisham, A. K. (1989). Selection of the time constant of a diode detector, *Telecommunication and Radio Engineering*, **44**(1), 121–123.
- [6] Cvetkovic, Z. and Markovic, A. (1989). Polynomial economization of envelope detector static characteristics, *IEEE Transactions on Consumer Electronics*, **35**, 876–881.

- [7] Naoum, R. I. (1973). A computer aided method for the evaluation of harmonic and intermodulation distortion in simple envelope detectors, *International Journal of Electronics*, **35**, 141–144.
- [8] Crant, P. S. and Townsend, G. B. (1961). *Colour Television, 1: Principles and Practice*, London: Iliffe Books Ltd, 423–429.
- [9] Bluestein, L. I. (1974). Envelope detection of vestigial sideband signals with application to television reception, *IEEE Transactions on Consumer Electronics*, **CE-21**, 369–379.
- [10] Loughlin, B. D. (1982). Color signal distortions in envelope type second detectors, *IEEE Transactions on Consumer Electronics*, **CE-28**, 19–31.
- [11] Abuelma'atti, M. T. (1994). Simple method for calculating Fourier coefficients of experimentally obtained waveforms, *IEEE Proceedings, Science, Measurement and Technology*, **141**, 177–178.
- [12] Abuelma'atti, M. T. (1993). A Simple algorithm for fitting measured data to Fourier-series models, *International Journal of Mathematical Education for Science and Technology*, **24**, 107–112.