

A NEW JUNCTION PARAMETERS DETERMINATION USING THE DOUBLE EXPONENTIAL MODEL

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(Received 15 November 2001; In final form 3 December 2001)

A double exponential model is used to characterize the junction properties of microelectronic devices. A method is developed to extract the physical junction parameters from the current-voltage characteristics. The influence of the start values used for the computing processes are considered and the accuracy is discussed from results of simulated extractions. An experimental test considers the emitter-base junction of bipolar transistors.

Keywords: Characteristic; Junction; Parameters; Extraction; Modeling

1 INTRODUCTION

The p-n junction current-voltage (I - V) characteristics have been extensively studied and many mathematical models, based on the physics of semiconductors, have been developed. The I - V characteristic of a junction can be described [1–8] by an implicit equation:

$$I = I_{o1} \left(\exp \left(\frac{V - R_s I}{V_T} \right) - 1 \right) + I_{o2} \left(\exp \left(\frac{V - R_s I}{A V_T} \right) - 1 \right) + \frac{V - R_s I}{R_{sh}} \quad (1)$$

known as the double exponential model. $V_T = kT/q$ is the thermal voltage. A is the junction quality factor: it is commonly considered to be related to the recombination processes, mainly in the space-charge region of the junction [1, 3, 6]. A usually varies between 1 and 2. R_s and R_{sh} are the series and the shunt resistances in the equivalent circuit. I_{o1} and I_{o2} are the components of the diode reverse saturation current (respectively the diffusion current in the bulk of the junction I_{o1} and the recombination I_{o2} current [1, 3, 6]).

The five parameters A , R_s , R_{sh} , I_{o1} , I_{o2} are related to internal properties of the electronic device and have been used in modeling studies. They are of interest for the description of carrier transport phenomena and power losses in operating junctions, and for quantifying the quality of diodes and transistors structures.

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This work presents a new method to extract the junction parameters A , R_s , R_{sh} , I_{o1} , I_{o2} related to the double exponential model, from the experimental (I - V) characteristic. New parameters are introduced for computing processes and a least square fitting of the I - V characteristic is performed, using the high voltage region of the curve. The accuracy of the method is discussed at length, both with “theoretical” characteristics and experimental emitter-base junction characteristics of transistors. It is pointed out that the method is accurate and easy to implant in any technological process for quality control.

2 THEORETICAL METHOD

The method consists in firstly determining R_{sh} and R_s , then in replacing the unknown parameters, A , I_{o1} and I_{o2} by some quantities allowing to use the least square method.

The determination of the series and shunt resistances is performed from the junction I - V characteristics obtained under the form of N experimental couples $(I(k), V(k))_{1 \leq k \leq N}$ by using any acquisition system.

Since, near the origin, each $\exp((V - R_s I)/V_T)$ term is very close to 1, R_{sh} may be determined, for low V and I values, by the approximation :

$$\frac{dI}{dV} = \frac{1}{R_{sh}} \quad (2)$$

A first value ρ_s of R_s can be easily determined by the single diode model [4] and used in our process.

From Eq. (1), the theoretical approach considers I expressed as $I = I_1 + I_2 + I_3$ and introduces the quantities: $I_1 = I_{o1} \exp((V - R_s I)/V_T)$, $I_2 = I_{o2} \exp((V - R_s I)/AV_T)$ and $I_3 = (V - R_s I)/R_{sh}$.

These expressions obviously yield:

$$\frac{V - R_s I}{V_T} = \ln\left(\frac{I_1}{I_{o1}}\right) = \ln\left[\left(\frac{I_2}{I_{o2}}\right)^A\right] \quad (3)$$

then:

$$\frac{I_1}{I_2^A} = \frac{I_{o1}}{I_{o2}^A}. \quad (4)$$

Let us then consider the (unknown) parameter D defined by the following relation:

$$D = \frac{I_1}{I_2^A} = \frac{I_{o1}}{I_{o2}^A}. \quad (5)$$

Using Eq. (5) and the equality : $I - I_3 = I_1 + I_2$, I_2 appears as a solution of the easily solvable (by the Newton’s method) equation:

$$I - I_3 = I_2 + D \cdot I_2^A. \quad (6)$$

Since $V_T \cdot \ln I_2^A = V_T \cdot \ln I_{o2}^A + V - R_s I$, the function $V' = V - V_T \cdot \ln(I_2^A)$ appears as a linear function of I :

$$V' = aI + b \tag{7}$$

$$\text{where : } a = R_s \text{ and } b = -V_T \cdot \ln(I_{o2}^A). \tag{8}$$

This function $V'(I)$ is linear when both A and the deduced parameter D are exactly related to Eq. (1). Section 3 details the use of the least square method, applied to the N experimental couples $(I(k), V(k))$ and to the corresponding I_3, I_2 and I_1 values in order to find out A and D .

Once the actual values of A and D have been determined through the least square method (that determines both a and b , Eq. (8)), Eqs. (5) and (8) give:

$$I_{o2} = \exp\left(\frac{-b}{V_T A}\right) \text{ and } I_{o1} = D I_{o2}^A. \tag{9}$$

3 LEAST SQUARE METHOD APPLIED TO THE EXPERIMENTAL I - V CHARACTERISTICS

Our computing process consists in adjusting the values of A and D so that the experimental representation of Eq. (7) is as linear as possible. For that purpose, a reference table of 15625 values of (A, D) is built as follows. A is arithmetically varied from 1 to 2.25 by step of 0.01, and D is geometrically varied from 10^{-3} to 10^{12} by step of 1.318 (see Appendix 1 for the boundary values of A and D).

The program picks up any value of (A, D) in this reference table. For a given (A, D) value, the least square method is applied as follows. The experimental I - V characteristics is associated to the initial table $(I(k), V(k))$, $1 \leq k \leq N$. The two resistance values can be calculated from section 2.3. Then, the N corresponding values of both $I - I_3$ and I_2 are calculated. Finally, the N values of $V' = V - V_T \cdot \ln(I_2^A)$ are computed.

Starting from an original (experimental) I - V characteristic, our method successively extracts the (estimated) values of $R_s, R_{sh}, A, D, I_{o1}$ and I_{o2} . Then, Eq. (1) allows to build an estimated I - V characteristic.

The well-known function Q [5, 10], defined as $Q = [1/N \sum_{k=1}^{k=N} d(k)^2]^{1/2}$, where $d(k)$ is the distance between the points of the calculated characteristic and the experimental points, in relative units [5], has been used to compare the two characteristics. A calculated value of Q lower than 5×10^{-3} corresponds to a good fitting. In order to provide a faster estimation of A and D , Q is replaced by the function $LQ = \ln(1/Q)$. The choice of the best Q value (close to 0) determines the sought values of A and D , called A_m and D_m .

4 CRITICAL STUDY OF THE METHOD

The accuracy of the different steps of our method depends on the accuracy of the calculus of R_s, R_{sh} and I_2 (section 2).

4.1 Influence of R_s

The series resistance R_s is calculated through a two step process. A first determination ρ_s of the series resistance R_s is obtained from the single diode model [4]. This value ρ_s is then used in the least square method providing a statistically estimated value of the parameter “ a ” that is then assimilated to R_s .

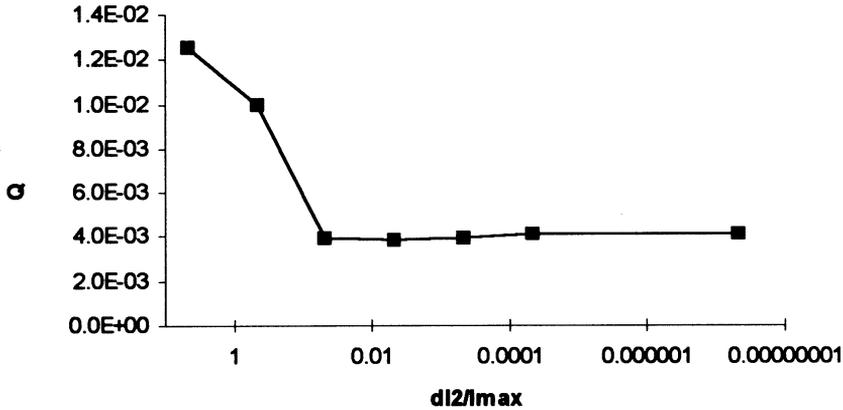


FIGURE 1 Q function versus the ratio dI_2/I_{max} for the emitter-base junction characteristic of a 2N2904A transistor.

In order to test the influence of this choice of ρ_s on the accuracy of the fit between the two characteristics, this method has been performed with a theoretical $I-V$ characteristic for various R_{sh} values. The ratio ρ_s/R_s is mostly lower than 1.4 and very close to 1 as soon as $R_{sh} > 300$ ohm. Q stays lower than 5×10^{-3} for a large range of the shunt resistance value. Besides, the fit (related to low Q values) is all the better as the shunt resistance value is high. Furthermore, when the shunt resistance is beyond 50,000 ohm, the influence of the ratio ρ_s/R_s on the Q variations is negligible: thus, in this case, the choice of ρ_s as initial value in our process leads to a good accuracy.

4.2 Influence of I_2

In our computed process, I_2 was calculated, by Newton's method, through Eq. (6), with a precision lower than 10^{-9} . But, since V' depends on $I_2 (dV' = (-AV_T/I_2)dI_2)$ and because of possible very low values of I_2 , potential discrepancies could appear in the computation of V' .

In the method, the choice of dI_2 depends on the maximum value I_{max} of the current I on the (theoretical or experimental) characteristic : $dI_2 = I_{max} \cdot 10^{-5}$. In our different trials, Q stayed lower than 5×10^{-3} for $dI_2 < 0.05 \cdot I_{max}$ (Fig. 1).

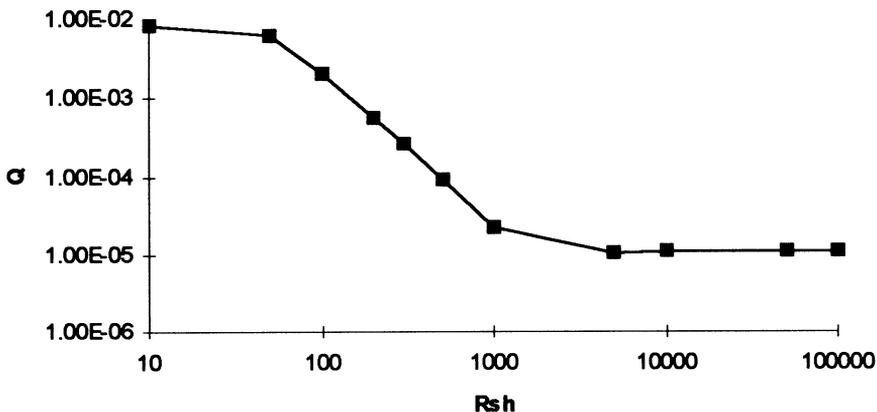
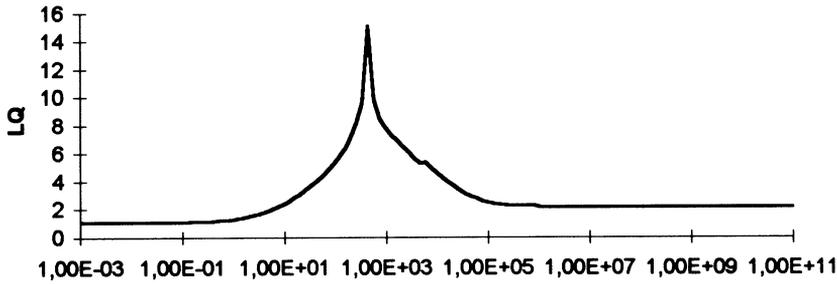


FIGURE 2 Q function versus the shunt resistance value R_{sh} for a theoretical characteristic ($R_s = 5 \Omega$; $A = 1.7$; $I_{o1} = 10^{-12}$ A; $I_{o2} = 10^{-9}$ A).



D

FIGURE 3 Zoom on LQ variations versus D , while D varies around D_m for a theoretical characteristic ($R_{sh} = 60,000 \Omega$; $R_s = 1.5 \Omega$; $A = 1.65$; $I_{o1} = 2 \times 10^{-13} \text{ A}$; $I_{o2} = 5 \times 10^{-10} \text{ A}$).

4.3 Influence of R_{sh}

The determination of R_{sh} implies the $I-V$ characteristic in the vicinity of the origin ($V < 0.5 \text{ V}$). The accuracy of the $I-V$ measurement influences the accuracy dR_{sh} on R_{sh} . When $R_{sh} > 200 \text{ ohm}$ (usual values for good quality junctions), $Q < 5 \times 10^{-3}$ (Fig. 2).

5 THEORETICAL TEST

As an example, the method has been applied to a theoretical $I-V$ characteristic obtained from Eq. (1) with given arbitrary values of the parameters R_s , R_{sh} , A , I_{o1} and I_{o2} . ($R_{sh} = 60,000 \Omega$; $R_s = 1.5 \Omega$; $A = 1.65$; $I_{o1} = 2 \times 10^{-13} \text{ A}$; $I_{o2} = 5 \times 10^{-10} \text{ A}$).

The program yields $Am = 1.65$ and $Dm = 440$. Figure 3 displays LQ zoom around Am and Dm values. Value of LQ about 12 correspond to $Q < 7 \times 10^{-6}$!

Several similar trials have been performed with different initial values of the parameters R_s , R_{sh} , A , I_{o1} and I_{o2} and led to the same quasi perfect fit between the two characteristics.

Table I shows the results obtained with the same initial parameter values, with three different R_{sh} values. This table clearly points out the influence of R_{sh} . Furthermore, the accuracy of the method strongly depends on the fact that R_{sh} is great enough. Practically, most of the junctions are characterized by a shunt resistance R_{sh} greater than 300 ohms which yields a good accuracy of the results.

TABLE I Estimated Parameter Values Extracted with our Method from a Theoretical Characteristic and for Three Shunt Resistance R_{sh} Values.

		A	$R_s(\Omega)$	$I_{o1}(A)$	$I_{o2}(A)$	$R_{sh}(\Omega)$
Characteristic 1 ($R_{sh} = 10^5 \Omega$)	Initial values	1.6	0.1	5×10^{-15}	5×10^{-11}	10^5
	Extracted values	1.61	0.0998	5.07×10^{-15}	7.88×10^{-11}	99982
Characteristic 2 ($R_{sh} = 1000 \Omega$)	Initial values	1.6	0.1	5×10^{-15}	5×10^{-11}	10^5
	Extracted values	1.62	0.104	5.06×10^{-15}	5.00×10^{-11}	1000.1
Characteristic 3 ($R_{sh} = 200 \Omega$)	Initial values	1.6	0.1	5×10^{-15}	5×10^{-11}	10^5
	Extracted values	1.74	0.105	5.23×10^{-15}	1.71×10^{-10}	200.10

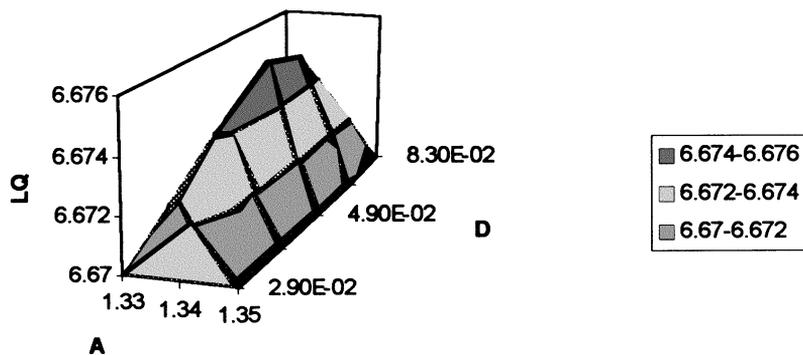


FIGURE 4 LQ variations versus A and D for an experimental characteristic (base-emitter junction of a 2N2904A transistor).

TABLE II Mean and Standard Deviation of the Results Obtained by Successively using 10 Times the Q Criterion with an Experimental Characteristic (Base-Emitter Junction of a 2N2904A Transistor).

		A	$I_{o1}(A)$	$I_{o2}(A)$	$R_s(\Omega)$	$R_{sh}(\Omega)$	Q
Minimal Q criterion	Mean	1.30	8.10×10^{-17}	3.4×10^{-12}	0.516	24838	1.12×10^{-3}
	Standard deviation	0.007	4.31×10^{-17}	0.63×10^{-12}	0.050	4692	0.05×10^{-3}

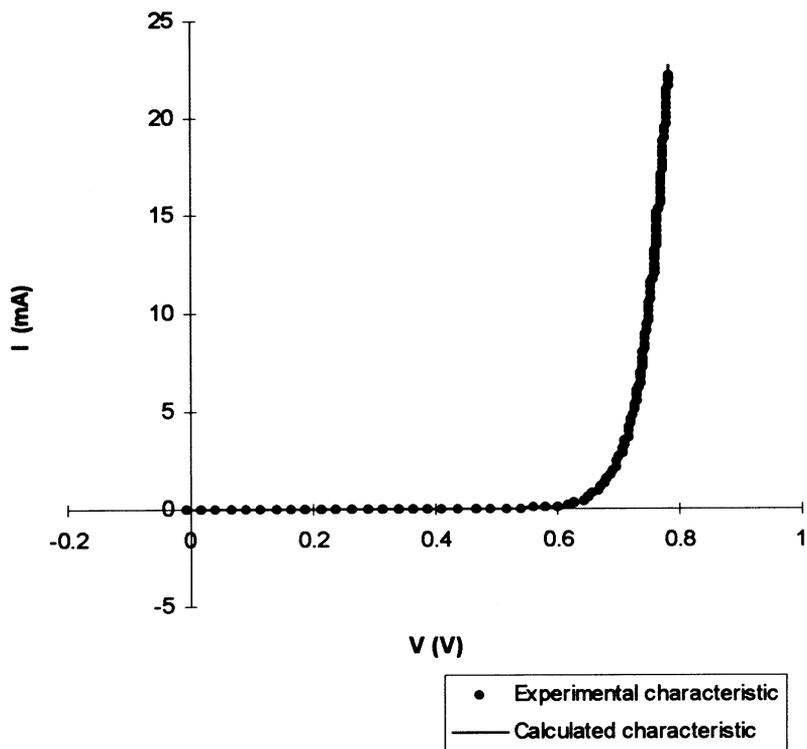


FIGURE 5 Experimental characteristic of the emitter-base junction of a 2N2904A transistor (dots) and its fit by our method (line).

6 EXPERIMENTAL TEST

The method has been applied to a physical junction (the base-emitter junction of a transistor 2N2904A). The maximum value of $Q(A, D)$ was obtained for ($A = 1, 34; D = 0.049$) (Fig. 4).

Table II points out a very low standard deviation value. Figure 5 represents the experimental I - V characteristic and the calculated characteristic obtained with the estimated values of the parameters R_s, R_{sh}, A, I_{o1} and I_{o2} through Eq. (1): the two curves are quasi similar.

CONCLUSION

This paper presents a new method to extract the p-n junction parameters from a description of the current-voltage characteristic with a two exponential model. The influence of the start values used for the computing processes are considered and the accuracy is discussed from results of simulated extractions. An experimental test considers the emitter-base junction of bipolar transistors. The accuracy, the rapidity of the procedure and the simplicity of implanting the calculation process in any program, make it useful for characterization of microelectronic devices and to quantify the quality of any junction.

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APPENDIX 1: DETERMINATION OF THE EXTREME VALUES OF D AND A .

A first approach considers R_{sh} quasi-infinite (greater than 10^3 ohm) : in this case, I_3 is negligible compared to I along the I - V characteristic.

The parameter D Eq. (5) can theoretically reach any real value. Due to the physical constraints, D can be limited between two values D_{min} and D_{max} (see below).

When the value of the ratio $\gamma = I_1/I_2$ is high, I_2 can be neglected compared to I_1 and Eq. (1) corresponds to the one-exponential model with A equal to 1 (ideal junction). On the other hand, when γ value is low, I_1 can be neglected compared to I_2 and Eq. (1) corresponds to the one exponential model with A much greater than 1 (significant recombination processes).

Then, since our study considers a description with the double exponential model, we are concerned with values of parameter γ varying in the range (10^{-2} – 10^2): these values yield respective boundaries for D and A .

From Eq. (5) and the equality $I = I_1 + I_2 = I_2(\gamma + 1)$, we obtain:

$$D = \frac{\gamma(\gamma + 1)^{A-1}}{I^{A-1}}. \quad (10)$$

$$\text{When } \gamma = 10^2, D = \frac{\gamma^A}{I^{A-1}} = \frac{10^{2A}}{I^{A-1}} \quad (11)$$

This latter expression is maximal when both $A = A_{\max} = 2$ and $I = I_{\min}$. In this study, I_{\min} is chosen (Appendix 2) to 10^{-7} A. Then, $D_{\max} = 10^{11}$. In our calculations, the value 10^{12} for D_{\max} was used.

$$\text{When } \gamma = 10^{-2}, D = \frac{10^{-2}}{I^{A-1}} \quad (12)$$

This latter expression is minimal when $A = A_{\min} = 1$ and when $I = I_{\max}$. In this study, I_{\max} is chosen to 1 A. Then, $D_{\min} = 10^{-3}$.

