Research Article

Fractional Order Memristor No Equilibrium Chaotic System with Its Adaptive Sliding Mode Synchronization and Genetically Optimized Fractional Order PID Synchronization

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This paper introduces a fractional order memristor no equilibrium (FOMNE) chaotic system and investigates its adaptive sliding mode synchronization. Firstly the dynamic properties of the integer order memristor no equilibrium system are analyzed. The fractional order memristor no equilibrium system is then derived from the integer order model. Lyapunov exponents and bifurcation with fractional order are investigated. An adaptive sliding mode control algorithm is derived to globally synchronize the identical fractional order memristor systems and genetically optimized fractional order PID controllers are designed and used to synchronize the FOMNE systems. Finally the fractional order memristor no equilibrium system is realized using FPGA.

1. Introduction

Chaotic systems are a special case of nonlinear systems which can be categorized as chaotic if the system possesses at least one positive Lyapunov exponent and hyperchaotic if the system possesses two or more positive Lyapunov exponents. Lyapunov exponents and fractal dimension studies are important in defining the complexity of chaotic and hyperchaotic systems [1, 2]. Chaotic systems found significant importance after the discovery of a 3D weather model by Lorenz [3].

By early 21st century many researchers have announced different chaotic systems such as Chen system [4] Liu system [5], Sundarapandian system [6], Sundarapandian system [7], and Pham system [8]. Chaotic systems with no equilibrium are of great interest in chaos literature. If sum of all the Lyapunov exponents is zero then the system is a conservative system [9].

Memristors popularly known as the fourth fundamental circuit element were characterized as a nonlinear and low power device was proposed by Chua [10, 11]. In 2008

Hewlett-Packard [HP] engineers announced the first physical realization of memristors. In memristor literatures several models had been presented such as linear and nonlinear ion drift model and threshold adaptive memristor model [12–14]. A Murali-Lakshmanan-Chua's circuit with a piecewise linear active flux controlled memristors with hyperchaotic behavior was investigated by Ishaq Ahamed and Lakshmanan [15]. A memristor based hyperchaotic complex Lu system and its adaptive synchronization were studied by Wang et al. [16].

Synchronization of chaotic systems is of great importance when one chaotic oscillator drives the other. Because of high sensitivity to initial conditions two identical chaotic systems may have exponentially diverging state trajectories. Many methods have been proposed in the literature such as active control method [17, 18], adaptive control method [19, 20], extended back stepping control [21, 22], sliding mode control [23, 24], and adaptive sliding mode [25–27].

Fractional calculus [28–31] has fully emerged into a mathematical field with applications in nonlinear controls, electrical and mechanical controls, and so forth. Fraction

FIGURE 1: 3D state portrait of the novel memristor no equilibrium system (XYZ plane; YZW plane).

chaotic systems have been investigated by many researchers [28–35]. Fractional order controllers [24–27, 36–38] are more effective compared to its integer order models especially in chaos control and synchronization. Fractional order systems with no equilibrium are announced and investigated by Li and Chen [39]. For the numerical simulation of fractional order chaotic system Ivo [40] explained a methodology with a register memory component.

For chaos to exist there should be at least one unstable equilibrium point. Recently many researchers have announced chaotic systems with no equilibrium or one equilibrium [32–34]. Chaotic systems with no equilibrium exhibit hidden attractions as the orbit of attractions does not intersect with any equilibrium points [35, 41]. An algorithmic search methodology was developed by Jafari et al. [42] to discover chaotic flows with no equilibrium.

PID controllers are objects of steady effort for improvements of their quality and robustness. One of the possibilities to improve PID controllers is to use the fractional order controllers (FOPID) with noninteger derivation and integration parts; they generalize the integer order PID controller, used and verified their effectiveness in [38, 40, 43–45], and add more flexibility to control design with accuracy for the real world processes. But finding appropriate parameters values for the FOPID controller is still a difficult task, so in practice control engineers still often use trial and error for the tuning process.

Motivated by the above, in this paper we announce a novel integer order memristor no equilibrium chaotic system. We then derive a fractional order model of the proposed novel system. The dynamic properties of both integer order and fractional order novel systems are investigated. A fractional order adaptive sliding mode control and genetically optimized fractional order PID are proposed to synchronize the identical novel fraction order systems. Finally the proposed fractional order system is implemented in FPGA.

2. Memristor No Equilibrium Chaotic System

In this section we introduce a novel 4D memristor no equilibrium chaotic system (MNECS) with three parameters derived from the Sundarapandian system [46] by including a fourth state which is a combination of state feedback and the flux controlled memristor [47, 48] as described by

$$\begin{aligned}\dot{x} &= az + xy \\ \dot{y} &= 1 - x^2 - z^2 - w^2 \\ \dot{z} &= -bx + yz \\ \dot{w} &= cz - w - w(x)z\end{aligned}\quad (1)$$

and the parameter values are $a = 10$, $b = 9$, and $c = 8.7$ and $w(\cdot)$ is the memductance of a flux controlled memristor characterized by the cubic nonlinearity $w(x) = a_1 + b_1 x^2$, $a_1 = 4$, and $b_1 = 0.01$ [47].

Figure 1 shows the state portrait of system (1).

For numerical simulations, we take the initial values of the hyperchaotic system (1) as

$$\begin{aligned}x(0) &= 1, \\ y(0) &= 1, \\ z(0) &= 1, \\ w(0) &= 1.\end{aligned}\quad (2)$$

3. Properties of the 4D Memristor No Equilibrium Chaotic System

3.1. Equilibrium Points. The equilibrium points of system (1) can be found by solving

$$\begin{aligned}0 &= az + xy \\ 0 &= 1 - x^2 - z^2 - w^2 \\ 0 &= -bx + yz \\ 0 &= cz - w - w(x)z.\end{aligned}\quad (3)$$

FIGURE 2: (a) Bifurcation plot versus a . (b) Bifurcation plot versus b . (c) Bifurcation plot versus c .

From (3) it can be clearly seen that $az^2 + bx^2 = 0$. As $a > 0$ and $b > 0$ the only solution is $x = z = 0$. It can be seen that $cz - w - w(x)z = 0$ and $x = z = 0$ make $w = 0$; we conclude that $y = 0$. Hence system (1) has no equilibrium points.

3.2. Lyapunov Exponents and Kaplan-Yorke Dimension. The Jacobian matrix of the novel system (1) is calculated as

$$J(X) = \begin{bmatrix} y & x & a & 0 \\ -2x & 0 & -2z & -2w \\ -b & z & y & 0 \\ 0.02xz & 0 & c - 4 - 0.01x^2 & -1 \end{bmatrix}. \quad (4)$$

The eigenvalues of the Jacobian matrix at equilibrium are $\lambda_1 = -1.0000$, $\lambda_2 = 9.4868i$, $\lambda_3 = -9.4868i$, and $\lambda_4 = 0$. The Lyapunov exponents of the system are $L_1 = 0.033426$, $L_2 = 0$, $L_3 = -0.028964$, and $L_4 = -1.005153$. The existence of positive Lyapunov exponents confirms the chaotic behavior of system (1). The sum of all the Lyapunov exponents is negative confirming that the system is dissipative.

The Kaplan-Yorke dimension of a chaotic system is defined as

$$D_{KY} = j + \sum_{i=1}^j \frac{L_i}{L_{j+1}}, \quad (5)$$

where j is the maximum integer such that $\sum L_i < 0$.

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.004439. \quad (6)$$

3.3. Bifurcation, Bicoherence, and Poincare Map. In order to understand the dynamical behavior of the MNECS, the bifurcation plots are derived for three cases as follows.

Case 1. Fix $b = 9$, $c = 8.7$, $a_1 = 4$, and $b_1 = 0.01$ and vary a between $[0 \ 20]$.

Case 2. Fix $a = 10$, $c = 8.7$, $a_1 = 4$, and $b_1 = 0.01$ and vary b between $[0 \ 20]$.

Case 3. Fix $a = 10$, $b = 9$, $a_1 = 4$, and $b_1 = 0.01$ and vary c between $[0 \ 20]$.

As discussed in [49], the transient behaviors occurring in memristor based nonlinear systems may result in longer simulation times to reach steady states. Hence we used the ode45 solver for numerical simulations. Figures 2(a), 2(b), and 2(c) show the bifurcation plots of the system for the parameters a , b , and c , respectively. The variation in Lyapunov exponents with reference to the parameters a , b , and c is also derived and compared with the bifurcation plots. Figure 3 shows the variation of the Lyapunov exponents with the parameters. As it is observed from Figures 2 and 3, for the values of $8.5 \leq a, b, c \leq 10.5$, the bifurcation plots show

FIGURE 3: Positive Lyapunov exponent for variation in parameters (a, b, c) .

denser points confirming the existence of a stable positive Lyapunov exponent.

The regular power spectra investigations cannot be used to analyze nonlinear systems as it does not have the required phase information. Hence researchers have adopted bicoherence or bispectrum analysis to analyze the period doubling sequence occurring in chaotic systems. Bicoherence contour of the novel system (1) as shown in Figure 4 shows the coupling between Fourier modes and is centered about the dominant frequencies composing the limit cycles [50]. As the system exhibits positive Lyapunov exponents, the chaotic behavior of the system increases resulting in period doubling and motions at additional frequencies because of the quadratic interactions between the dominant frequencies and itself. In Figure 4, the dominant frequencies are shown in a circled area which contributes to the bispectrum as indicated.

The autobispectrum with the Fourier coefficients can be derived as

$$G(\omega_1, \omega_2) = E[F(\omega_1)F(\omega_2)F^*(\omega_1 + \omega_2)], \quad (7)$$

where ω_n is the radian frequency and F is the Fourier coefficients of the time series. The normalized magnitude spectrum of the bispectrum known as the squared bicoherence is given by

$$g(\omega_1, \omega_2) = \frac{|G(\omega_1, \omega_2)|^2}{P(\omega_1)P(\omega_2)P(\omega_1 + \omega_2)}, \quad (8)$$

where $P(\omega_1)$ and $P(\omega_2)$ are the power spectra at f_1 and f_2 .

The Poincare map is usually constructed by plotting the value of the state variable for every time it passes through a particular plane. Plotting the value every time it passes through the $x = 0$ plane where x is changing from negative to positive is the normal practice done when studying most

FIGURE 4: Contour of bicoherence for the novel system (1).

of the chaotic attractors. In the case of the proposed novel system (1), the regular method of plotting zero crossing of the x plane is of no interest because of the nature of the system. Hence we plotted the zero crossing of system (1) with reference to the z plane. Figure 5 shows the Poincare sections in the 3D state portrait of system (1). $\Sigma+$ indicates the negative to positive crossing and $\Sigma-$ indicates the positive to negative crossing in the z plane.

4. Fractional Order Memristor No Equilibrium Chaotic (FOMNE) System

In this section we derive the fractional order model of the memristor no equilibrium system (1). There are three

FIGURE 5: Poincare sections in the 3D state portrait of system (1) (XYZ plane).

commonly used definitions of the fractional order differential operator, Grunwald-Letnikov, Riemann-Liouville, and Caputo [28–31].

In this section, we will study the dynamical behavior of fractional order system derived from novel system (1) with the Grunwald-Letnikov (GL) definition, which is defined as

$${}_a D_t^q f(t) = \lim_{h \rightarrow 0} \left\{ \frac{1}{h^q} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{q}{j} f(t - jh) \right\} \quad (9)$$

$$= \lim_{h \rightarrow 0} \left\{ \frac{1}{h^q} \Delta_h^q f(t) \right\},$$

where a and t are limits of the fractional order equation, $\Delta_h^q f(t)$ is generalized difference, h is the step size, and q is the fractional order of the differential equation.

For numerical calculations the above equation is modified as

$$({}_{t-L}) D_t^q f(t) = \lim_{h \rightarrow 0} \left\{ h^{-q} \sum_{j=0}^{N(t)} b_j (f(t - jh)) \right\}. \quad (10)$$

Theoretically fractional order differential equations use infinite memory. Hence when we want to numerically calculate or simulate the fractional order equations we have to use finite memory principal, where L is the memory length and h is the time sampling.

$$N(t) = \min \left\{ \left\lceil \frac{t}{h} \right\rceil, \left\lceil \frac{L}{h} \right\rceil \right\}. \quad (11)$$

The binomial coefficients required for the numerical simulation are calculated as

$$b_j = \left(1 - \frac{a + \alpha}{j} \right) b_{j-1}. \quad (12)$$

Using (10), the fractional order model of system (1) is derived as

$$\begin{aligned} D^{q_x} x &= ax + xy \\ D^{q_y} y &= 1 - x^2 - z^2 - w^2 \\ D^{q_z} z &= -bx + yz \\ D^{q_w} w &= cz - w - zw(x), \end{aligned} \quad (13)$$

where $a = 10$, $b = 9$, $c = 8.7$, $a_1 = 4$, $b_1 = 0.01$, q_x, q_y, q_z , and q_w are the fractional orders of the system, and $w(\cdot)$ is the memductance of a flux controller memristor characterized by the nonlinearity $w(x) = a_1 + b_1 x^2$, $a_1 = 4$, and $b_1 = 0.01$ [46]. For numerical simulations, we take the initial values of the hyperchaotic system (1) as $x(0) = 1$, $y(0) = 1$, $z(0) = 1$, and $w(0) = 1$. Figure 6 shows the 3D state portrait of the fractional order system (13) for $q_x = q_y = q_z = q_w = 0.99$.

5. Stability Analysis of FOMNE System

As the FOMNE system has no defined equilibrium points, direct calculation of eigenvalues is impossible and hence we consider the origin $[0, 0, 0, 0]$ as the only time bound trivial solution and use it for stability analysis of the system.

5.1. Commensurate Order. For commensurate FOMNE system of order q , the system is stable and exhibits chaotic oscillations if $|\arg(\text{eig}(J_E))| = |\arg(\lambda_i)| > q\pi/2$, where J_E is the Jacobian matrix at the equilibrium E and λ_i are the eigenvalues of the FOMNE system where $i = 1, 2, 3, 4$. As seen from the FOMNE system, the eigenvalues should remain in the unstable region and the necessary condition for the

FIGURE 6: 3D state portrait of the FOMNE system (XYZ plane; YZW plane).

FOMNE system to be stable is $q > (2/\pi)\tan^{-1}(|\text{Im } \lambda|/\text{Re } \lambda)$. As the eigenvalues of the system are $\lambda_1 = -1.0000$, $\lambda_2 = 9.4868i$, $\lambda_3 = -9.4868i$, and $\lambda_4 = 0$, it is clearly seen that the value λ_2 is a saddle point and remains in the unstable region contributing to the existence of chaotic oscillations.

5.2. Incommensurate Order. The necessary condition for the FOMNE system to exhibit chaotic oscillations in the incommensurate case is $\pi/2M - \min_i(|\arg(\lambda_i)|) > 0$, where M is the LCM of the fractional orders. If $q_x = 0.9$, $q_y = 0.9$, $q_z = 0.8$, and $q_w = 0.8$, then $M = 10$. The characteristic equation of the system evaluated at the equilibrium is $\det(\text{diag}[\lambda^{Mq_x}, \lambda^{Mq_y}, \lambda^{Mq_z}, \lambda^{Mq_w}] - J_E) = 0$ and by substituting the values of M and the fractional orders $\det(\text{diag}[\lambda^9, \lambda^9, \lambda^8, \lambda^8] - J_E) = 0$, the characteristic equation is $\lambda^{34} - 2\lambda^{27} - 3\lambda^{26} + \lambda^{20} + 5\lambda^{19} + \lambda^{18} + 90\lambda^{17} - 2\lambda^{12} - 4\lambda^{11} - 91\lambda^{10} - 180\lambda^9 + \lambda^4 + \lambda^3 + 90\lambda^2 + 90\lambda$. The approximated solution of the characteristic equation is $\lambda_{34} = 1.414$ whose argument is zero which is the minimum argument and hence the stability necessary condition becomes $\pi/20 - 0 > 0$ which solves for $0.0785 > 0$ and hence the FOMNE system is stable and chaos exists in the incommensurate system.

6. Dynamic Analysis of the Fractional Order Memristor No Equilibrium System

Most of the dynamic properties of the integer order system (1) like the Lyapunov exponents, eigenvalues, and bifurcation with parameters are preserved in the fractional order if $q_i > 0.90$, where $i = x, y, z, w$. The most important analysis of interest when investigating a fractional order system is the bifurcation with fractional order. The resistance of the memristor increases from the initial value until it reaches its maximum R_{off} in a certain time period which is called the saturation time t_{sat} [51]. The fractional order parameter can be used to control the saturation time from a part of a second up to several minutes under the same input voltage [51, 52]. The saturation time can be controlled through the fractional order where it can be less than 1 sec when $\alpha < 0.5$

up to higher values when $\alpha > 0.5$. This specific character of the fractional order memristor is useful in determining the largest positive Lyapunov exponent of the system. Figure 7 shows the 3D state portrait (XYZ plane) of the FOMNE system for various fractional orders $q_x = q_y = q_z = q_w = q$. The largest positive Lyapunov exponent ($L_1 = 0.038174$) of the FOMNE system appears when $q = 0.99$ against its largest integer order Lyapunov exponent ($L_1 = 0.033426$). Hence fractional order chaos suppression/control and synchronization prove efficient than the integer order controls as the systems show the largest positive Lyapunov exponent in fractional order close to 1. It can also be seen that, as the fractional order q decreases, the FOMNE system starts losing its largest positive Lyapunov exponent. When $q \leq 0.9$ the only positive Lyapunov exponent of the system becomes negative and thus the chaotic oscillations in the system disappear.

7. Adaptive Fractional Order Sliding Mode Synchronization

In this section we investigate the adaptive fractional order sliding mode synchronization of identical systems. Let us define a generalized fractional order master system as

$$D^{q_m} x_m = f(x_m) + F(x_m)a, \quad (14)$$

where q_m is the fractional order of the master system and generalized slave system with adaptive controller as

$$D^{q_s} x_s = g(x_s) + G(x_s)b + u(t), \quad (15)$$

where $u(t)$ is the adaptive controller for synchronizing the nonidentical systems and q_s is the fractional order of the slave system.

Let us define the synchronization error as

$$e = x_s - x_m. \quad (16)$$

FIGURE 7: 3D state portrait (XYZ plane) of FOMNE system for various fractional orders.

The sliding surface for the integral sliding mode control [26, 27, 36] is defined as

$$s(e) = e + k \int e(\tau) d\tau. \quad (17)$$

The fractional first derivative of the sliding surface is derived as

$$D^q s = D^q e + ke. \quad (18)$$

The fractional order error dynamics are defined as

$$D^q e = D^{q_s} x_s - D^{q_m} x_m. \quad (19)$$

Using (14) and (15) in (19),

$$D^q e = g(x_s) + G(x_s)b + u(t) - f(x_m) - F(x_m)a. \quad (20)$$

Let us define the adaptive controller as

$$u(t) = -g(x_s) - G(x_s)\hat{b} + f(x_m) - F(x_m)\hat{a} - ke - \eta \operatorname{sgn}(s) - \rho s, \quad (21)$$

where k , η , and ρ are positive gain values, $\hat{a}$ and $\hat{b}$ are parameter estimates of master and slave systems, and s is the sliding surface.

Using (21) in (20), the error dynamics simplify to

$$D^q e = G(x_s) [b - \hat{b}] - F(x_m) [a - \hat{a}] - \eta \operatorname{sgn}(s) - \rho s. \quad (22)$$

The Lyapunov candidate function to analyze the stability of the controller is defined as

$$V = \frac{1}{2} s^2 + \frac{1}{2} (b - \hat{b})^2 + \frac{1}{2} (a - \hat{a})^2. \quad (23)$$

The Lyapunov first derivate is derived as

$$\dot{V} = s \cdot \dot{s} + (b - \hat{b}) \left(-\dot{\hat{b}} \right) + (a - \hat{a}) \left(-\dot{\hat{a}} \right). \quad (24)$$

By definition of fractional calculus [29, 30],

$$\dot{x}(t) = D_t^{1-q} \cdot D_t^q x(t). \quad (25)$$

Using (25) in (24),

$$\begin{aligned} \dot{V} = s \cdot D_t^{1-q} \cdot D_t^q s - (b - \hat{b}) \left(D_t^{1-q} \cdot D_t^q \hat{b} \right) \\ - (a - \hat{a}) \left(D_t^{1-q} \cdot D_t^q \hat{a} \right). \end{aligned} \quad (26)$$

Finding the sign of the Lyapunov first derivative using (26) seems difficult and hence we use the modified fractional order Lyapunov method defined by Rajagopal et al. [37] as

$$\frac{1}{2} D_t^q x^2(t) \leq x(t) \frac{1}{2} D_t^q x(t), \quad q \in (0, 1). \quad (27)$$

Using (27), (20), and (18) in (24),

$$\begin{aligned} \dot{V} \leq ks \left[G(x_s) (b - \hat{b}) - F(x_m) (a - \hat{a}) - \eta \operatorname{sgn}(s) \right. \\ \left. - \rho s \right] - (b - \hat{b}) \left(\dot{\hat{b}} \right) - (a - \hat{a}) \left(\dot{\hat{a}} \right). \end{aligned} \quad (28)$$

Let us define the parameter estimate laws as

$$\begin{aligned} D^q \hat{b} &= ks \cdot G(x_s) \\ D^q \hat{a} &= ks \cdot F(x_m). \end{aligned} \quad (29)$$

Using (29) in (28)

$$\dot{V} \leq -\eta |s_w| - \rho s^2 \quad (30)$$

as η and ρ are all positive and $\dot{V}$ is negative definite. Using Barbalat's lemma [53], we conclude that $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$.

7.1. Adaptive Sliding Mode Synchronization of Identical FOMNE Systems. For synchronization let us define the master system as

$$\begin{aligned} D^{q_x} x_m &= az_m + x_m y_m \\ D^{q_y} y_m &= 1 - x_m^2 - z_m^2 - w_m^2 \\ D^{q_z} z_m &= -bx_m + y_m z_m \\ D^{q_w} w_m &= cz_m - w_m - z_m w(x_m) \end{aligned} \quad (31)$$

and the slave system with the adaptive controller is defined as

$$\begin{aligned} D^{q_x} x_s &= \hat{a} z_s + x_s y_s + u_x \\ D^{q_y} y_s &= 1 - x_s^2 - z_s^2 - w_s^2 + u_y \\ D^{q_z} z_s &= -\hat{b} x_s + y_s z_s + u_z \\ D^{q_w} w_s &= \hat{c} z_s - w_s - z_s w(x_s) + u_w, \end{aligned} \quad (32)$$

where $\hat{a}$, $\hat{b}$, and $\hat{c}$ are the unknown system parameters and u_x , u_y , u_z , and u_w are the adaptive controllers.

The synchronization error dynamics are defined as

$$\begin{aligned} D^{q_x} e_x &= D^{q_x} x_s - D^{q_x} x_m \\ D^{q_y} e_y &= D^{q_y} y_s - D^{q_y} y_m \\ D^{q_z} e_z &= D^{q_z} z_s - D^{q_z} z_m \\ D^{q_w} e_w &= D^{q_w} w_s - D^{q_w} w_m, \end{aligned} \quad (33)$$

where e_x , e_y , e_z , and e_w are the synchronization errors given by

$$e_i = i_s - i_m \quad (34)$$

and $i = x, y, z, w, s$, and m are slave and master systems, respectively.

Using (31) and (32) in (33), we simplify error dynamics as

$$\begin{aligned} D^{q_x} e_x &= \hat{a} z_s + x_s y_s - az_m - x_m y_m + u_x \\ D^{q_y} e_y &= -x_s^2 - z_s^2 - w_s^2 + x_m^2 + z_m^2 + w_m^2 + u_y \\ D^{q_z} e_z &= -\hat{b} x_s + y_s z_s + bx_m - y_m z_m + u_z \\ D^{q_w} e_w &= \hat{c} z_s - w_s - z_s w(x_s) - cz_m + w_m \\ &\quad + z_m w(x_m) + u_w. \end{aligned} \quad (35)$$

The parameter estimation errors are defined as

$$\begin{aligned} e_a &= a - \hat{a}; \\ e_b &= b - \hat{b}; \\ e_c &= c - \hat{c}. \end{aligned} \quad (36)$$

The first derivatives of (36) are

$$\begin{aligned} D^q e_a &= -D^q \hat{a} \\ D^q e_b &= -D^q \hat{b} \\ D^q e_c &= -D^q \hat{c}. \end{aligned} \quad (37)$$

Let us define the integral sliding surface as

$$s_i = e_i + k_i \int e_i(\tau) d\tau, \quad (38)$$

where $i = x, y, z, w$ are the states, $k_i > 0$, s_i is the sliding surface, and e_i is the synchronization errors. The first derivative of the sliding surface (38) can be derived as

$$D^{q_i} s_i = D^{q_i} e_i + k_i e_i. \quad (39)$$

Let us define the Lyapunov candidate function as

$$V = \frac{1}{2} [s_x^2 + s_y^2 + s_z^2 + s_w^2 + e_a^2 + e_b^2 + e_c^2]. \quad (40)$$

The first derivative of the Lyapunov candidate function (40) is

$$\dot{V} = s_x \dot{s}_x + s_y \dot{s}_y + s_z \dot{s}_z + s_w \dot{s}_w + e_a \dot{e}_a + e_b \dot{e}_b + e_c \dot{e}_c. \quad (41)$$

By definition of fractional calculus [29, 30],

$$\dot{x}(t) = D_t^{1-q} \cdot D_t^q x(t). \quad (42)$$

Applying (42) in (41)

$$\begin{aligned} \dot{V} = & s_x D_t^{1-q} \cdot D_t^q s_x + s_y D_t^{1-q} \cdot D_t^q s_y + s_z D_t^{1-q} \cdot D_t^q s_z \\ & + s_w D_t^{1-q} \cdot D_t^q s_w + e_a D_t^{1-q} \cdot D_t^q e_a + e_b D_t^{1-q} \cdot D_t^q e_b \\ & + e_c D_t^{1-q} \cdot D_t^q e_c. \end{aligned} \quad (43)$$

As observed in (43), analysis of the Lyapunov first derivative is difficult; hence we use the modified fractional order Lyapunov method defined by Rajagopal et al. [37] as

$$\frac{1}{2} D_t^q x^2(t) \leq x(t) \frac{1}{2} D_t^q x(t), \quad q \in (0, 1). \quad (44)$$

Applying (44) in (41),

$$\begin{aligned} \dot{V} \leq & s_x D^{q_x} s_x + s_y D^{q_y} s_y + s_z D^{q_z} s_z + s_w D^{q_w} s_w \\ & + e_a D^{q_a} e_a + e_b D^{q_b} e_b + e_c D^{q_c} e_c \\ \dot{V} \leq & s_x [\hat{a} z_s + x_s y_s - a z_m - x_m y_m + u_x + k_x e_x] \\ & + s_y [-x_s^2 - z_s^2 - w_s^2 + x_m^2 + z_m^2 + w_m^2 + u_y \\ & + k_y e_y] + s_z [-\hat{b} x_s + y_s z_s + b x_m - y_m z_m + u_z \\ & + k_z e_z] + s_w [\hat{c} z_s - w_s - z_s w(x_s) - c z_m + w_m \\ & + z_m w(x_m) + u_w + k_w e_w] - e_a D^{q_a} \hat{a} - e_b D^{q_b} \hat{b} \\ & + e_c D^{q_c} \hat{c}. \end{aligned} \quad (45)$$

After some mathematical simplifications,

$$\begin{aligned} \dot{V} \leq & s_x [\hat{a} e_z + x_s y_s - x_m y_m + u_x + k_x e_x] + s_y [-x_s^2 \\ & - z_s^2 - w_s^2 + x_m^2 + z_m^2 + w_m^2 + u_y + k_y e_y] \\ & + s_z [-\hat{b} e_x + y_s z_s - y_m z_m + u_z + k_z e_z] + s_w [\hat{c} e_z \\ & - w_s - z_s w(x_s) + w_m + z_m w(x_m) + u_w + k_w e_w] \\ & - s_x z_m e_a + s_z x_m e_b - s_w z_m e_c - e_a D^{q_a} \hat{a} - e_b D^{q_b} \hat{b} \\ & + e_c D^{q_c} \hat{c}. \end{aligned} \quad (46)$$

Let us define the adaptive controllers as

$$\begin{aligned} u_x = & -\hat{a} e_z - x_s y_s + x_m y_m - k_x e_x - \eta_x \operatorname{sgn}(s_x) \\ & - \rho_x s_x \\ u_y = & x_s^2 + z_s^2 + w_s^2 - x_m^2 - z_m^2 - w_m^2 - k_y e_y \\ & - \eta_y \operatorname{sgn}(s_y) - \rho_y s_y \\ u_z = & \hat{b} e_x - y_s z_s + y_m z_m - k_z e_z - \eta_z \operatorname{sgn}(s_z) - \rho_z s_z \\ u_w = & -\hat{c} e_z + w_s + z_s w(x_s) - w_m - z_m w(x_m) - k_w e_w \\ & - \eta_w \operatorname{sgn}(s_w) - \rho_w s_w. \end{aligned} \quad (47)$$

And parameter estimation adaptive laws can be defined as

$$\begin{aligned} D^{q_a} \hat{a} = & -s_x z_m \\ D^{q_b} \hat{b} = & -s_z x_m \\ D^{q_c} \hat{c} = & -s_w z_m. \end{aligned} \quad (48)$$

Using the adaptive controllers (47) and the parameter estimation laws (48) in (46),

$$\begin{aligned} \dot{V} \leq & -\eta_x |s_x| - \eta_y |s_y| - \eta_z |s_z| - \eta_w |s_w| - \rho_x s_x^2 \\ & - \rho_y s_y^2 - \rho_z s_z^2 - \rho_w s_w^2 \end{aligned} \quad (49)$$

as η_x, η_y, η_z , and η_w and ρ_x, ρ_y, ρ_z , and ρ_w are all positive and $\dot{V}$ is negative definite. Using Barbalat's lemma [53], we conclude that $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$. For numerical simulations we take the initial conditions as $x_m(0) = 1$, $y_m(0) = -1$, $z_m(0) = -1$, $w_m(0) = 1$, $x_s(0) = -2$, $y_s(0) = 3$, $z_s(0) = -2$, $w_s(0) = 4$, and $\hat{a} = 1$, $\hat{b} = 4$, and $\hat{c} = 13$. Figure 8 shows the time history of the synchronization errors. Figure 9 shows the adapted parameters.

8. Genetically Optimized Fractional Order PID (FOPID) Controller for the Synchronization of FOMNE Systems

In this section we investigate the synchronization of FOMNE systems using genetically optimized (GA) FOPID controllers (u_x, u_y, u_z, u_w) [38, 44, 45] implemented in feedback loops given by

$$u_i = K_p e_i + K_I \int_0^t e_i d\tau^\beta + K_D \frac{d^\delta e_i}{dt^\delta}, \quad i = x, y, z, w, \quad (50)$$

where u_i is the fractional order PID action control for $i = x, y, z, w$ and δ, β are the fractional order differential and integral operators, e_i is the error signal, and K_p, K_I , and K_D are the proportional, integral, and the derivative gains to synchronize the fractional order model of the memristor no equilibrium system (1). We use genetic algorithm [54, 55] to optimize the controller gains such that the error e_i is minimized. Matlab tools are used for numerical simulations with the following constraints defined.

FIGURE 8: Time history of synchronization errors (e_x, e_y, e_z, e_w). FIGURE 9: Estimated Parameters (a, b, c).

Variable bounds matrix of the proportional, integral, and the derivative gains = $[-0.001, 0.001]$; population size = 80, GA. Genetically, bigger population size give better approximation; number of generations = 100; selection function = stochastic uniform; crossover fraction = 0.8; mutation function = Gaussian; stopping criteria = error performance criterion; length of the chromosome = 12 (decimal codage).

Tuning of PID controllers involves the selection of K_p, K_I , and K_D gains for better control performance which is defined with reference to the required performance index. There are

two important performance indices ISE (Integrated Squared Error) and IAE (Integrated Absolute Error) as given in the following:

$$\begin{aligned} \text{ISE} &= \int_0^{\infty} e_i^2(\tau) d\tau, \\ \text{IAE} &= \int_0^{\infty} |e_i(\tau)| d\tau. \end{aligned} \quad (51)$$

In this paper we use IAE as the objective function and the fitness functions is as given below

$$\text{fitness function} = \frac{1}{\sum \text{abs}(e_i)}, \quad (52)$$

$$e_i = e_x + e_y + e_z + e_w.$$

We synchronize the master system (31) and the slave system (32) using the GA optimized PID controllers with parameters known (Case 1) and parameters unknown (Case 2).

Case 1 (parameters known). In this section we assume that the parameters of the slave system are known. We take the same master system defined by (31) and the slave system (32) is modified with FOPID controllers as given below.

$$\begin{aligned} D^{q_x} x_s &= az_s + x_s y_s + u_x \\ D^{q_y} y_s &= 1 - x_s^2 - z_s^2 - w_s^2 + u_y \\ D^{q_z} z_s &= -bx_s + y_s z_s + u_z \\ D^{q_w} w_s &= cz_s - w_s - z_s W(x_s) + u_w. \end{aligned} \quad (53)$$

The initial conditions and the parameters are taken as

$$\begin{aligned} x_m(0) &= 1, \\ y_m(0) &= -1, \\ z_m(0) &= -1, \\ w_m(0) &= 1, \\ x_s(0) &= -2, \\ y_s(0) &= 3, \\ z_s(0) &= -2, \\ w_s(0) &= 4, \\ a &= 10, \\ b &= 9, \\ c &= 8.7, \\ a_1 &= 4, \\ b_1 &= 0.01. \end{aligned} \quad (54)$$

The synchronization error dynamics are defined as,

$$\begin{aligned} D^{q_x} e_x &= ae_z + x_s y_s - x_m y_m + u_x \\ D^{q_y} e_x &= x_m^2 - x_s^2 + z_m^2 - z_s^2 + w_m^2 - w_s^2 + u_y \\ D^{q_z} e_x &= -be_x + y_s z_s - y_m z_m + u_z \\ D^{q_w} e_x &= ce_z - e_w - z_s W(x_s) + z_m W_m(x_m) + u_w. \end{aligned} \quad (55)$$

Table 1 shows the FOPID gain values of the controllers achieved using genetic algorithm for known parameter values. We get the best solutions tracked over generations for the complete synchronization of the FOMNE systems.

Figure 10 shows the time history of the synchronization errors using genetically optimized fractional order PID controllers; Figure 11 shows the synchronized states and Figure 12 shows the time history of the fractional order PID controllers.

Case 2 (parameters unknown). In this section we assume that the parameters of the slave system are unknown. We take the same master system defined by (31) and the slave system (32) is modified with FOPID controllers as given below.

$$\begin{aligned} D^{q_x} x_s &= \hat{a}z_s + x_s y_s + u_x \\ D^{q_y} y_s &= 1 - x_s^2 - z_s^2 - w_s^2 + u_y \\ D^{q_z} z_s &= -\hat{b}x_s + y_s z_s + u_z \\ D^{q_w} w_s &= \hat{c}z_s - w_s - z_s W(x_s) + u_w, \end{aligned} \quad (56)$$

where $\hat{a}$, $\hat{b}$, and $\hat{c}$ are the unknown system parameters of the slave system. The initial conditions are taken as

$$\begin{aligned} x_m(0) &= 1, \\ y_m(0) &= -1, \\ z_m(0) &= -1, \\ w_m(0) &= 1, \\ x_s(0) &= -2, \\ y_s(0) &= 3, \\ z_s(0) &= -2, \\ w_s(0) &= 4, \\ \hat{a} &= 1, \\ \hat{b} &= 4, \\ \hat{c} &= 13. \end{aligned} \quad (57)$$

The synchronization error dynamics are defined as

$$\begin{aligned} D^{q_x} e_x &= \hat{a}z_s + x_s y_s - az_m - x_m y_m + u_x \\ D^{q_y} e_y &= -x_s^2 - z_s^2 - w_s^2 + x_m^2 + z_m^2 + w_m^2 + u_y \\ D^{q_z} e_z &= -\hat{b}x_s + y_s z_s + bx_m - y_m z_m + u_z \\ D^{q_w} e_w &= \hat{c}z_s - w_s - z_s W(x_s) - cz_m + w_m \\ &\quad + z_m W(x_m) + u_w \end{aligned} \quad (58)$$

and the parameter update laws are defined as

$$\begin{aligned} D^q \hat{a} &= -e_x z_m \\ D^q \hat{b} &= -e_z x_m \\ D^q \hat{c} &= -e_w z_m. \end{aligned} \quad (59)$$

Table 2 shows the FOPID gain values achieved using the genetic algorithm optimization technique for unknown parameters.

TABLE 1: FOPID controller gain values optimized with GA.

FOPID controller	K_p	K_I	K_D
u_x	-0.0271	-0.0095	0.0015
u_y	-0.0469	-0.0809	-0.0029
u_z	-0.0072	0.0011	0.0146
u_w	-0.0199	-0.0063	0.0124

TABLE 2: FOPID controller gain values optimized with GA.

FOPID controller	K_p	K_I	K_D
u_x	-0.0826	-0.1885	-0.0836
u_y	-0.2150	-0.1586	0.0027
u_z	0.1637	-0.1444	0.0888
u_w	-0.2177	-0.0121	0.0304

FIGURE 10: Time history of synchronization errors (e_x, e_y, e_z, e_w). FIGURE 11: State synchronization (x_m, x_s).

FIGURE 12: Time history of PID controllers u_x , u_y , u_z , and u_w .

FIGURE 13: Time history of synchronization errors (e_x , e_y , e_z , e_w).

Figure 13 shows the time history of the synchronization errors using genetically optimized fractional order PID controllers, Figure 14 shows the synchronized states, and Figure 15 shows the time history of fractional order PID controllers. Figure 16 shows the time history of the parameter estimates.

As can be seen from Figures 8, 10, and 13, adaptive GA optimized PID control converges much faster ($t = 40$ ms) than adaptive sliding mode control ($t = 190$ ms). Figure 17 shows the comparison of synchronization errors using adaptive sliding mode control (ASMC) and GA optimized PID control (GAPID).

9. FPGA Implementation of the FOMNE System

In this section we discuss the implementation of the novel hyperchaotic system in FPGA using the Xilinx (Vivado)

system generator toolbox in Simulink. Firstly we configure the available built-in blocks of the system generator toolbox. The Add/Sub blocks are configured with zero latency and 32/16-bit fixed point settings. The output of the block is configured to rounded quantization in order to reduce the bit latency. For the multiplier block a latency of 3 is configured and the other settings are the same as in Add/Sub block. Next we will have to design the fractional order integrator which is not a readily available block in the system generator. Hence we implement the integrators using the mathematical relation discussed in (11) and (12) and the value of h is taken as 0.001 and the initial conditions are fed into the forward register. Figure 18 shows the Xilinx schematics of the FOMNE system (13) implemented in Kintex 7 (device = 7k160t; package = fbg484 S) and Figure 19 shows the Xilinx Kintex 7 schematics of the fractional order integrator. Figure 20(a) shows the power consumed by the FOMNE system and Figure 20(b) shows the power consumption for variation in fractional

FIGURE 14: State synchronization (x_m, x_s). FIGURE 15: Time history of PID controllers u_x, u_y, u_z , and u_w .

FIGURE 16: Time history of parameter estimates.

orders. As it can be seen from Figure 20(b), the FOMNE system consumes maximum power when its fractional order $q = 0.99$ compared to the integer order 1 thus confirming that the fractional order memristor system exhibits largest Lyapunov exponent compared to the integer order. Figure 21 shows the 2D state portraits of the FOMNE system using Xilinx system generator.

10. Conclusion

In this paper we have announced a novel 4D no equilibrium memristor chaotic system. The dynamic properties of the proposed system are investigated to prove the chaotic behavior of the system. The fractional order model of the 4D no equilibrium memristor chaotic system is derived

from its integer model. Fractional order bifurcation property of the FOMNE system is investigated and it is seen that largest Lyapunov exponent of the system exists when the fractional order is close to 1. The identical FOMNE systems are synchronized using adaptive sliding mode controllers and genetically optimized PID controllers. Numerical simulations are done to validate the theoretical results. Finally the proposed FOMNE system is implemented in FPGA to show that the system is hardware realizable.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

FIGURE 17: Comparison of synchronization errors (ASMC and GAFOPID).

FIGURE 18: Xilinx schematics of the FOMNE system implemented in Kintex 7 (device = 7k160t; package = fbg484 S).

FIGURE 19: Xilinx schematics of the Fractional order integrator implemented in Kintex 7 (device = 7kl60t; package = fbg484 S).

FIGURE 20: (a) Power utilized and (b) power utilization versus fractional order of FOMNE system.

FIGURE 21: 2D state portraits of the FOMENS system implemented using Xilinx system generator (XYZ plane; YZW plane).

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