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Research Article

Extinction and Persistence in Mean of a Novel Delay Impulsive Stochastic Infected Predator-Prey System with Jumps

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In this paper, we explore an impulsive stochastic infected predator-prey system with Lévy jumps and delays. The main aim of this paper is to investigate the effects of time delays and impulse stochastic interference on dynamics of the predator-prey model. First, we prove some properties of the subsystem of the system. Second, in view of comparison theorem and limit superior theory, we obtain the sufficient conditions for the extinction of this system. Furthermore, persistence in mean of the system is also investigated by using the theory of impulsive stochastic differential equations (ISDE) and delay differential equations (DDE). Finally, we carry out some simulations to verify our main results and explain the biological implications.

1. Introduction

With the development of the economy, environmental pollution is caused by various industries and other activities of human, which has been one of the most important social problems in the world today. Many species have gone extinct due to the toxicant in the environment. Therefore, controlling the environmental pollution has been the important topics around the world. There are many researchers which have investigated the pollution models in recent years [1–3]. In addition, a lot of animal populations suffer from infectious disease, so some scholars investigated the predator-prey systems with diseases [4–8]. For example, a deterministic predator-prey model with infected predator in an impulsive polluted environment is described by the following equation:

$$\begin{aligned}
&\mathrm{d}X(t) = X(t)\left[r - \delta_1 c_1(t) - a_{11}X(t) - a_{12}S(t)\right]\mathrm{d}t,\\
&\mathrm{d}S(t) = S(t)\\
&\quad\cdot\left[-\mu_1 - \delta_2 c_2(t) + a_{21}X(t) - \beta I(t) - a_{22}S(t)\right]\mathrm{d}t,\\
&\mathrm{d}I(t) = I(t)\left[-\mu_2 - \delta_3 c_3(t) + \beta S(t) - a_{33}I(t)\right]\mathrm{d}t,\\
&\dot{c}_1(t) = k_1 c_e(t) - g_1 c_1(t) - m_1 c_1(t),\\
&\dot{c}_2(t) = k_2 c_e(t) - g_2 c_2(t) - m_2 c_2(t),\\
&\dot{c}_3(t) = k_3 c_e(t) - g_3 c_3(t) - m_3 c_3(t),\\
&\dot{c}_e(t) = -h c_e(t),\\
&\qquad\qquad\qquad\qquad\qquad\qquad t \neq nT,\\
&\Delta X(t) = 0,\\
&\Delta S(t) = 0,\\
&\Delta I(t) = 0,\\
&\Delta c_i(t) = 0,\\
&\Delta c_e(t) = u,\\
&\qquad\qquad\qquad\qquad\qquad\qquad i = 1,2,3,\ t = nT,\ n \in \mathbb{Z}^+,
\end{aligned} \tag{1}$$

where $\Delta X(t) = X(t^+) - X(t)$, $\Delta S(t) = S(t^+) - S(t)$, $\Delta I(t) = I(t^+) - I(t)$, $\Delta c_i(t) = c_i(t^+) - c_i(t)$, $\Delta c_e(t) = c_e(t^+) - c_e(t)$, $X(t)$ is the density of the prey at time t, and $S(t)$ and $I(t)$ represent the density of susceptible predator and infected predator at time t, respectively. $c_i(t)$ $(i = 1, 2, 3)$ and $c_e(t)$ stand for the concentrations of the toxicant in the organism and the environment at time t, respectively. δ_i $(i = 1, 2, 3)$ are dose-response parameter to the toxicant. r stands for the intrinsic growth rate of the prey. μ_i $(i = 1, 2)$ denote the death rates of $S(t)$ and $I(t)$, respectively. β is the infection rate. a_{ii} $(i = 1, 2, 3)$ are density-dependent coefficients, and a_{12} and a_{21} represent predation rate and ingestion rate, respectively. k_i $(i = 1, 2, 3)$ are environmental toxicant uptake rates, m_i $(i = 1, 2, 3)$ denote organismal net depuration rates, g_i $(i = 1, 2, 3)$ represent organismal net ingestion rates, h is the loss rate of toxicant from environment, and u denotes the amount of pulsed input concentration of the toxicant at every time. The above parameters are all positive constants. Next, we propose a new mathematical model by taking more factors into account based on model (1).

In the natural world, time delay often occurs in almost every situation. Thus it is significant to take time delay into consideration [9–14]. As we know, deterministic model is not enough to describe the species activities. Sometimes, the species activities may be disturbed by environmental noises. May [15] revealed that the birth rates, death rates, carrying capacities, competition coefficients, and other parameters involved in the system should exhibit random fluctuation to a greater or lesser extent. Hence some parameters should be stochastic [16–26]. First, we assume that the intrinsic growth rate and the death rates of species are disturbed by white noise, then r and μ_i can be replaced by

$$\begin{aligned} r &\longrightarrow r + \sigma_1 \dot{B}_1(t), \\ -\mu_1 &\longrightarrow -\mu_1 + \sigma_2 \dot{B}_2(t), \\ -\mu_2 &\longrightarrow -\mu_2 + \sigma_3 \dot{B}_3(t), \end{aligned} \tag{2}$$

where $B_i(t)$ $(i = 1, 2, 3)$ are standard Brownian motions and σ_i are the intensities of $B_i(t)$. $B_i(t)$ are mutually independent defined on a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t\geq 0}, \mathscr{P})$.

Furthermore, populations may suffer from sudden environmental fluctuations, such as floods and earthquakes, which cannot be described by Brownian motions. To explain these phenomena, introducing a jump process into the underlying population dynamics is one of the important methods. Thus, there are many scholars introduce Lévy jumps into the population system [27–31]. Taking all above influences into consideration, we focus on the infected stochastic predator-prey system with Lévy jumps and delays in a polluted environment

$$\begin{aligned}
\mathrm{d}X(t) &= X(t^-)\left[r - \delta_1 c_1(t) - a_{11}X(t^-) - a_{12}e^{-d_1\tau_1}S(t^- - \tau_1)\right]\mathrm{d}t + \sigma_1 X(t)\,\mathrm{d}B_1(t) \\
&\quad + \int_{\mathbb{Y}} X(t^-)\gamma_1(u)\widetilde{N}(\mathrm{d}t, \mathrm{d}u), \\
\mathrm{d}S(t) &= S(t^-)\left[-\mu_1 - \delta_2 c_2(t) + a_{21}e^{-d_2\tau_2}X(t^- - \tau_2) - \beta e^{-d_3\tau_3}I(t^- - \tau_3) - a_{22}S(t^-)\right]\mathrm{d}t \\
&\quad + \sigma_2 S(t)\,\mathrm{d}B_2(t) + \int_{\mathbb{Y}} S(t^-)\gamma_2(u)\widetilde{N}(\mathrm{d}t, \mathrm{d}u), \\
\mathrm{d}I(t) &= I(t^-)\left[-\mu_2 - \delta_3 c_3(t) + \beta e^{-d_4\tau_4}S(t^- - \tau_4) - a_{33}I(t^-)\right]\mathrm{d}t + \sigma_3 I(t)\,\mathrm{d}B_3(t) \\
&\quad + \int_{\mathbb{Y}} I(t^-)\gamma_3(u)\widetilde{N}(\mathrm{d}t, \mathrm{d}u), \\
\dot{c}_1(t) &= k_1 c_e(t) - g_1 c_1(t) - m_1 c_1(t), \\
\dot{c}_2(t) &= k_2 c_e(t) - g_2 c_2(t) - m_2 c_2(t), \\
\dot{c}_3(t) &= k_3 c_e(t) - g_3 c_3(t) - m_3 c_3(t), \\
\dot{c}_e(t) &= -h c_e(t), \qquad t \neq nT, \\
\Delta X(t) &= 0, \\
\Delta S(t) &= 0, \\
\Delta I(t) &= 0, \\
\Delta c_i(t) &= 0, \\
\Delta c_e(t) &= u, \qquad i = 1, 2, 3,\ t = nT,\ n \in \mathbb{Z}^+,
\end{aligned} \tag{3}$$

where $X(t^-)$, $S(t^-)$, and $I(t^-)$ stand for the left limits of $X(t)$, $S(t)$, and $I(t)$, respectively. $N(\mathrm{d}t, \mathrm{d}u)$ denotes a Poisson counting measure with characteristic measure ν which defines a measurable bounded subset $\mathbb{Y}$ of $(0, \infty)$ with $\nu(\mathbb{Y}) < \infty$ and $\widetilde{N}(\mathrm{d}t, \mathrm{d}u) = N(\mathrm{d}t, \mathrm{d}u) - \nu(\mathrm{d}u)\mathrm{d}t$, and γ_i is bounded and continuous with respect to ν and is $\mathfrak{B}(\mathbb{Y}) \times \mathscr{F}_t$-measurable, and $1 + \gamma_i > 0$ $(i = 1, 2, 3)$ (see [27–30]). Moreover, $B_i(t)$ $(i = 1, 2, 3)$ are independent of N, d_i are death rates of species, and $\tau_i \geq 0$ $(i = 1, 2, 3, 4)$ represent the time delay. Other parameters are defined as system (1).

The rest of this paper is arranged as follows. Section 2 introduces some lemmas which will be used in our main results. In Section 3, we show the main results. We examine the extinction of system (3) in Section 3.1; in Section 3.2 we also prove the permanence in mean of this system. Finally, we present some simulations and conclusions in Section 4.

2. Preliminary Results

Throughout the paper, we assume that $X(t)$, $S(t)$, $I(t)$, and $c_i(t)$ are continuous at $t = nT$ and $c_e(t)$ is left continuous at $t = nT$ and $c_e(nT^+) = \lim_{t\to nT^+} c_e(t)$. Moreover, let $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t\geq 0}, \mathscr{P})$ be a complete probability space with a

filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the common conditions (i.e., it is increasing and right continuous while $\mathscr{F}_0$ contains all $\mathscr{P}$-null sets).

For the sake of convenience, we introduce some notions and some lemmas which will be used for the main results. We define

$$\begin{aligned}
\tau &= \max\{\tau_1,\tau_2,\tau_3,\tau_4\},\\
b_1 &= r-\frac{1}{2}\sigma_1^2\\
&\quad-\int_{\mathbb{Y}}\left(\gamma_1(u)-\ln\left(1+\gamma_1(u)\right)\right)\nu(\mathrm{d}u),\\
b_2 &= \mu_1+\frac{1}{2}\sigma_2^2\\
&\quad+\int_{\mathbb{Y}}\left(\gamma_2(u)-\ln\left(1+\gamma_2(u)\right)\right)\nu(\mathrm{d}u),\\
b_3 &= \mu_2+\frac{1}{2}\sigma_3^2\\
&\quad+\int_{\mathbb{Y}}\left(\gamma_3(u)-\ln\left(1+\gamma_3(u)\right)\right)\nu(\mathrm{d}u),\\
\Delta_1 &= \left(b_1-\delta_1\bar{c}_1\right)a_{21}e^{-d_2\tau_2}-\left(b_2+\delta_2\bar{c}_2\right)a_{11},\\
\Delta' &= a_{11}a_{22}+a_{12}a_{21}e^{-d_1\tau_1-d_2\tau_2},\\
\Delta_2 &= \left(b_1-\delta_1\bar{c}_1\right)\left(a_{22}a_{33}+\beta^2e^{-d_3\tau_3-d_4\tau_4}\right)\\
&\quad+\left(b_2+\delta_2\bar{c}_2\right)a_{33}a_{12}e^{-d_1\tau_1}\\
&\quad-\left(b_3+\delta_3\bar{c}_3\right)a_{12}\beta e^{-d_1\tau_1-d_3\tau_3},\\
\Delta_3 &= a_{33}\Delta_1+\left(b_3+\delta_3\bar{c}_3\right)a_{11}\beta e^{-d_3\tau_3},\\
\Delta_4 &= \beta e^{-d_4\tau_4}\Delta_1-\left(b_3+\delta_3\bar{c}_3\right)\Delta',\\
\Delta &= a_{33}\Delta'+a_{11}\beta^2e^{-(d_3\tau_3+d_4\tau_4)},\\
\mathscr{R} &= \frac{\beta e^{-d_4\tau_4}\Delta_1}{\left(b_3+\delta_3\bar{c}_3\right)a_{11}a_{22}},\\
\langle f(t)\rangle &= \frac{1}{t}\int_0^t f(t)\,\mathrm{d}t,\\
f^* &= \limsup_{t\to+\infty} f(t),\\
f_* &= \liminf_{t\to+\infty} f(t),
\end{aligned}\tag{4}$$

where $f(t)$ is a bounded continuous function on $[0,+\infty)$.

Then we show some basic properties of the subsystem of system (3)

$$\begin{aligned}
\dot{c}_1(t) &= k_1c_e(t)-g_1c_1(t)-m_1c_1(t),\\
\dot{c}_2(t) &= k_2c_e(t)-g_2c_2(t)-m_2c_2(t),\\
\dot{c}_3(t) &= k_3c_e(t)-g_3c_3(t)-m_3c_3(t),\\
\dot{c}_e(t) &= -hc_e(t),\\
&\qquad\qquad t\neq nT,\ n\in Z^+,\\
\Delta c_i(t) &= 0,\\
\Delta c_e(t) &= u,\\
&\qquad\qquad i=1,2,3,\ t=nT,\ n\in Z^+.
\end{aligned}\tag{5}$$

Lemma 1 (see [3]). *System (5) has a unique positive T-periodic solution $(\tilde{c}_1(t),\tilde{c}_2(t),\tilde{c}_3(t),\tilde{c}_e(t))^T$ which is globally asymptotical stable. Furthermore, if $c_i(0)>\tilde{c}_i(0)$ and $c_e(0)>\tilde{c}_e(0)$, then $c_i(t)>\tilde{c}_i(t)$ and $c_e(t)>\tilde{c}_e(t)$ for all $t\geq 0$, where*

$$\begin{aligned}
\tilde{c}_i(t) &= \tilde{c}_i(0)\,e^{-(g_i+m_i)(t-nT)}\\
&\quad+\frac{k_iu\left(e^{-(g_i+m_i)(t-nT)}-e^{-h(t-nT)}\right)}{\left(h-g_i-m_i\right)\left(1-e^{-hT}\right)},\\
\tilde{c}_e(t) &= \frac{ue^{-h(t-nT)}}{1-e^{-hT}},\\
\tilde{c}_i(0) &= \frac{k_iu\left(e^{-(g_i+m_i)T}-e^{-hT}\right)}{\left(h-g_i-m_i\right)\left(1-e^{-(g_i+m_i)T}\right)\left(1-e^{-hT}\right)},\\
\tilde{c}_e(0) &= \frac{u}{1-e^{-hT}},
\end{aligned}\tag{6}$$

for $t\in(nT,(n+1)T]$, $i=1,2,3$, and $n\in Z^+$.

It can be obtained from a simple calculation that

$$\int_{nT}^{(n+1)T}\tilde{c}_i(t)\,\mathrm{d}t=\frac{k_iu}{h\left(g_i+m_i\right)}.\tag{7}$$

Since $\tilde{c}_i(t)$ is a periodic function, then we get

$$\begin{aligned}
\lim_{t\to\infty}\langle c_i(t)\rangle^* &\leq \lim_{n\to\infty}\frac{1}{nT}\int_0^{(n+1)T}\tilde{c}_i(t)\,\mathrm{d}t\\
&= \lim_{n\to\infty}\frac{n+1}{nT}\int_{nT}^{(n+1)T}\tilde{c}_i(t)\,\mathrm{d}t\\
&= \frac{k_iu}{h\left(g_i+m_i\right)T},\\
\lim_{t\to\infty}\langle c_i(t)\rangle_* &\geq \lim_{n\to\infty}\frac{1}{(n+1)T}\int_0^{nT}\tilde{c}_i(t)\,\mathrm{d}t\\
&= \lim_{n\to\infty}\frac{n}{(n+1)T}\int_{(n-1)T}^{nT}\tilde{c}_i(t)\,\mathrm{d}t\\
&= \frac{k_iu}{h\left(g_i+m_i\right)T}.
\end{aligned}\tag{8}$$

Thus we have

$$\lim_{t\to+\infty}\langle c_i(t)\rangle=\frac{k_iu}{h\left(g_i+m_i\right)T}\coloneqq\bar{c}_i.\tag{9}$$

From Lemma 1, we know that the long time dynamical behaviors of system (3) can be replaced by the dynamical behaviors of the following limiting system:

$$\begin{aligned}
\mathrm{d}X(t) &= X(t^-)\left[r - \delta_1\tilde{c}_1(t) - a_{11}X(t^-)\right.\\
&\quad \left. - a_{12}e^{-d_1\tau_1}S(t^- - \tau_1)\right]\mathrm{d}t + \sigma_1 X(t)\,\mathrm{d}B_1(t)\\
&\quad + \int_{\mathbb{Y}} X(t^-)\gamma_1(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u),\\
\mathrm{d}S(t) &= S(t^-)\left[-\mu_1 - \delta_2\tilde{c}_2(t) + a_{21}e^{-d_2\tau_2}X(t^- - \tau_2)\right.\\
&\quad \left. - \beta e^{-d_3\tau_3}I(t^- - \tau_3) - a_{22}S(t^-)\right]\mathrm{d}t\\
&\quad + \sigma_2 S(t)\,\mathrm{d}B_2(t)\\
&\quad + \int_{\mathbb{Y}} S(t^-)\gamma_2(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u),\\
\mathrm{d}I(t) &= I(t^-)\left[-\mu_2 - \delta_3\tilde{c}_3(t) + \beta e^{-d_4\tau_4}S(t^- - \tau_4)\right.\\
&\quad \left. - a_{33}I(t^-)\right]\mathrm{d}t + \sigma_3 I(t)\,\mathrm{d}B_3(t)\\
&\quad + \int_{\mathbb{Y}} I(t^-)\gamma_3(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u).
\end{aligned} \tag{10}$$

Now we give an assumption which will be used in the following proof.

Assumption 2. There exist constants C_i such that

$$\int_{\mathbb{Y}} \left[\gamma_i - \ln(1+\gamma_i)\right]\nu(\mathrm{d}u) \le C_i, \quad (i = 1, 2, 3). \tag{11}$$

Lemma 3. *For any given initial value* $((\phi_1(t), \phi_2(t), \phi_3(t)) : -\tau \le t \le 0,\ \tau = \max\{\tau_1, \tau_2, \tau_3, \tau_4\}) \in C([-\tau, 0]; R_+^3)$, *there is a unique solution* $(X(t), S(t), I(t))$ *of (10) on* $t \ge \tau$ *and the solution will remain in* $\mathbb{R}_+^2$ *with probability 1.*

Proof. This proof is the same as Theorem 3.1 in [11] by defining

$$V(X, S, I) = V_1(X, S, I) + V_2(X, S, I), \tag{12}$$

where

$$\begin{aligned}
V_1(X, S, I) &= X(t) - 1 - \ln X(t) + S(t) - 1 - \ln S(t)\\
&\quad + I(t) - 1 - \ln I(t),\\
V_2(X, S, I) &= a_{12}e^{-d_1\tau_1}\int_t^{t+\tau_1} S(s - \tau_1)\,\mathrm{d}s\\
&\quad + \frac{1}{2}\int_t^{t+\tau_2} X^2(s - \tau_2)\,\mathrm{d}s\\
&\quad + \beta e^{-d_3\tau_3}\int_t^{t+\tau_3} I(s - \tau_3)\,\mathrm{d}s\\
&\quad + \frac{1}{2}\int_t^{t+\tau_4} S^2(s - \tau_4)\,\mathrm{d}s.
\end{aligned} \tag{13}$$

Thus, we omit it here. □

The stochastic comparison theorem and limit superior and limit inferior theory are given as follows.

Lemma 4 (see [27]). *Suppose that* $Y(t) \in C(\Omega \times [0, \infty), \mathbb{R}_+)$.

(i) If there exist three positive constants T, λ, *and* λ_0 *such that*

$$\begin{aligned}
\ln Y(t) &\le \lambda t - \lambda_0\int_0^t Y(s)\,\mathrm{d}s + \alpha B(t)\\
&\quad + \sum_{i=1}^n \eta_i \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u))\widetilde{N}(\mathrm{d}s, \mathrm{d}u) \quad a.s.
\end{aligned} \tag{14}$$

for all $t \ge T$, *where* α *and* η_i *are constants, then*

$$\begin{aligned}
&\limsup_{t\to\infty}\langle Y(t)\rangle = \langle Y(t)\rangle^* \le \frac{\lambda}{\lambda_0} \quad a.s., \text{ if } \lambda \ge 0,\\
&\lim_{t\to\infty} Y(t) = 0 \quad a.s., \text{ if } \lambda < 0.
\end{aligned} \tag{15}$$

(ii) If there exist three positive constants T, λ, *and* λ_0 *such that*

$$\begin{aligned}
\ln Y(t) &\ge \lambda t - \lambda_0\int_0^t Y(s)\,\mathrm{d}s + \alpha B(t)\\
&\quad + \sum_{i=1}^n \eta_i \int_0^t \int_{\mathbb{Y}} \ln(1 + \gamma_i(u))\widetilde{N}(\mathrm{d}s, \mathrm{d}u) \quad a.s.
\end{aligned} \tag{16}$$

for all $t \ge T$, *then* $\liminf_{t\to\infty}\langle Y(t)\rangle = \langle Y(t)\rangle_* \ge \lambda/\lambda_0$ *a.s.*

First, we explore the following auxiliary system:

$$\begin{aligned}
\mathrm{d}X_1(t) &= X(t^-)\left[r - \delta_1\tilde{c}_1(t) - a_{11}X_1(t^-)\right]\mathrm{d}t\\
&\quad + \sigma_1 X_1(t)\,\mathrm{d}B_1(t)\\
&\quad + \int_{\mathbb{Y}} X_1(t^-)\gamma_1(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u),\\
\mathrm{d}S_1(t) &= S_1(t^-)\left[-\mu_1 - \delta_2\tilde{c}_2(t)\right.\\
&\quad \left. + a_{21}e^{-d_2\tau_2}X_1(t^- - \tau_2) - a_{22}S_1(t^-)\right]\mathrm{d}t\\
&\quad + \sigma_2 S_1(t)\,\mathrm{d}B_2(t)\\
&\quad + \int_{\mathbb{Y}} S_1(t^-)\gamma_2(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u),\\
\mathrm{d}I_1(t) &= I_1(t^-)\left[-\mu_2 - \delta_3\tilde{c}_3(t) + \beta e^{-d_4\tau_4}S_1(t^- - \tau_4)\right.\\
&\quad \left. - a_{33}I_1(t^-)\right]\mathrm{d}t + \sigma_3 I_1(t)\,\mathrm{d}B_3(t)\\
&\quad + \int_{\mathbb{Y}} I_1(t^-)\gamma_3(u)\widetilde{N}(\mathrm{d}t,\mathrm{d}u).
\end{aligned} \tag{17}$$

Lemma 5. *For system (17), let* $(X_1(t), S_1(t), I_1(t))$ *be the solution of this system with initial value* $((\phi_1(t), \phi_2(t), \phi_3(t)) : -\tau \le t \le 0) \in C([-\tau, 0]; R_+^3)$.

(i) If $b_1 < \delta_1\bar{c}_1$, then

$$\begin{aligned}\lim_{t\to\infty} X_1(t) &= 0,\\ \lim_{t\to\infty} S_1(t) &= 0,\\ \lim_{t\to\infty} I_1(t) &= 0,\end{aligned} \tag{18}$$

a.s.

(ii) If $b_1 > \delta_1\bar{c}_1$, when $\Delta_1 < 0$, then

$$\begin{aligned}\lim_{t\to\infty} \langle X_1(t)\rangle &= \frac{b_1-\delta_1\bar{c}_1}{a_{11}},\\ \lim_{t\to\infty} S_1(t) &= 0,\\ \lim_{t\to\infty} I_1(t) &= 0,\\ \lim_{t\to\infty} \frac{\ln X_1(t)}{t} &= 0,\end{aligned} \tag{19}$$

a.s.

(iii) If $b_1 > \delta_1\bar{c}_1$, $\Delta_1 > 0$, when $\mathscr{R} < 1$, then

$$\begin{aligned}\lim_{t\to\infty} \langle X_1(t)\rangle &= \frac{b_1-\delta_1\bar{c}_1}{a_{11}},\\ \lim_{t\to\infty} \langle S_1(t)\rangle &= \frac{\Delta_1}{a_{11}a_{22}},\\ \lim_{t\to\infty} I_1(t) &= 0,\\ \lim_{t\to\infty} \frac{\ln S_1(t)}{t} &= 0,\end{aligned} \tag{20}$$

a.s.

(iv) If $b_1 > \delta_1\bar{c}_1$, when $\mathscr{R} > 1$, then

$$\begin{aligned}\lim_{t\to\infty} \langle X_1(t)\rangle &= \frac{b_1-\delta_1\bar{c}_1}{a_{11}},\\ \lim_{t\to\infty} \langle S_1(t)\rangle &= \frac{\Delta_1}{a_{11}a_{22}},\end{aligned}$$

a.s,

$$\begin{aligned}\lim_{t\to\infty} \langle I_1(t)\rangle &= \frac{\beta e^{-d_4\tau_4}\Delta_1 - (b_3+\delta_3\bar{c}_3)a_{11}a_{22}}{a_{11}a_{22}a_{33}},\\ \lim_{t\to\infty} \frac{\ln I_1(t)}{t} &= 0,\end{aligned} \tag{21}$$

a.s.

Proof. Applying Itô's formula to system (17) leads to

$$\begin{aligned}\mathrm{d}\ln X_1 &= [b_1 - \delta_1\tilde{c}_1(t) - a_{11}X_1(t)]\,\mathrm{d}t + \sigma_1\mathrm{d}B_1(t)\\ &\quad + \int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u),\\ \mathrm{d}\ln S_1 &= \left[-b_2 - \delta_2\tilde{c}_2(t) + a_{21}e^{-d_2\tau_2}X_1(t-\tau_2)\right.\\ &\quad \left. - a_{22}S_1(t)\right]\mathrm{d}t + \sigma_2\mathrm{d}B_2(t)\\ &\quad + \int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u),\\ \mathrm{d}\ln I_1 &= \left[-b_3 - \delta_3\tilde{c}_3(t) + \beta e^{-d_4\tau_4}S_1(t-\tau_4)\right.\\ &\quad \left. - a_{33}I_1(t)\right]\mathrm{d}t + \sigma_3\mathrm{d}B_3(t)\\ &\quad + \int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u).\end{aligned} \tag{22}$$

Integrating both sides of (22), we have

$$\begin{aligned}\ln X_1(t) - \ln X_1(0) &= b_1 t - \delta_1\int_0^t \tilde{c}_1(s)\,\mathrm{d}s\\ &\quad - a_{11}\int_0^t X_1(s)\,\mathrm{d}s + \sigma_1 B_1(t)\\ &\quad + \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u),\\ \ln S_1(t) - \ln S_1(0) &= -b_2 t - \delta_2\int_0^t \tilde{c}_2(s)\,\mathrm{d}s\\ &\quad + a_{21}e^{-d_2\tau_2}\int_0^t X_1(t-\tau_2) - a_{22}\int_0^t S_1(s)\,\mathrm{d}s\\ &\quad + \sigma_2 B_2(t) + \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\\ &= -b_2 t - \delta_2\int_0^t \tilde{c}_2(s)\,\mathrm{d}s + a_{21}e^{-d_2\tau_2}\int_0^t X_1(s)\,\mathrm{d}s\\ &\quad - a_{22}\int_0^t S_1(s)\,\mathrm{d}s\\ &\quad - a_{21}e^{-d_2\tau_2}\left(\int_t^{t+\tau_2} X_1(s-\tau_2)\,\mathrm{d}s\right.\\ &\quad \left. - \int_0^{\tau_2} X_1(s-\tau_2)\,\mathrm{d}s\right) + \sigma_2 B_2(t)\\ &\quad + \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u),\\ \ln I_1(t) - \ln I_1(0) &= -b_3 t - \delta_3\int_0^t \tilde{c}_3(s)\,\mathrm{d}s\\ &\quad + \beta e^{-d_4\tau_4}\int_0^t S_1(s-\tau_4)\,\mathrm{d}s - a_{33}\int_0^t I_1(s)\,\mathrm{d}s\\ &\quad + \sigma_3 B_3(t) + \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\\ &= -b_3 t - \delta_3\int_0^t \tilde{c}_3(s)\,\mathrm{d}s + \beta e^{-d_4\tau_4}\int_0^t S_1(s)\,\mathrm{d}s\\ &\quad - a_{33}\int_0^t I_1(s)\,\mathrm{d}s - \beta e^{-d_4\tau_4}\left(\int_t^{t+\tau_4} S_1(s-\tau_4)\,\mathrm{d}s\right.\\ &\quad \left. - \int_0^{\tau_4} S_1(s-\tau_4)\,\mathrm{d}s\right) + \sigma_3 B_3(t)\\ &\quad + \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u).\end{aligned} \tag{23}$$

Then we can obtain

$$\frac{\ln X_1(t)-\ln X_1(0)}{t}=b_1-\delta_1\langle\widetilde{c}_1(t)\rangle-a_{11}\langle X_1(t)\rangle+\frac{\sigma_1B_1(t)}{t}+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_1(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u), \tag{24}$$

$$\frac{\ln S_1(t)-\ln S_1(0)}{t}+a_{21}e^{-d_2\tau_2}\cdot\frac{\left(\int_t^{t+\tau_2}X_1(s-\tau_2)\,\mathrm{d}s-\int_0^{\tau_2}X_1(s-\tau_2)\,\mathrm{d}s\right)}{t}=-b_2-\delta_2\langle\widetilde{c}_2(t)\rangle+a_{21}e^{-d_2\tau_2}\langle X_1(t)\rangle-a_{22}\langle S_1(t)\rangle+\frac{\sigma_2B_2(t)}{t}+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_2(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u), \tag{25}$$

$$\frac{\ln I_1(t)-\ln I_1(0)}{t}+\beta e^{-d_4\tau_4}\cdot\frac{\left(\int_t^{t+\tau_4}S_1(s-\tau_4)\,\mathrm{d}s-\int_0^{\tau_4}S_1(s-\tau_4)\,\mathrm{d}s\right)}{t}=-b_3-\delta_3\langle\widetilde{c}_3(t)\rangle+\beta e^{-d_4\tau_4}\langle S_1(t)\rangle-a_{33}\langle I_1(t)\rangle+\frac{\sigma_3B_3(t)}{t}+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_3(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u). \tag{26}$$

Case (i). From Lemma 4 and (24), if $b_1<\delta_1\overline{c}_1$, then we have

$$\lim_{t\to\infty}X_1(t)=0\quad\text{a.s.} \tag{27}$$

Obviously, we find

$$\lim_{t\to\infty}S_1(t)=0,\qquad\lim_{t\to\infty}I_1(t)=0,\qquad\text{a.s.} \tag{28}$$

Case (ii). By Lemma 4, it is derived from (24) and conditions that

$$\langle X_1(t)\rangle^*\le\frac{b_1-\delta_1\langle\widetilde{c}_1(t)\rangle_*}{a_{11}},\qquad\langle X_1(t)\rangle_*\ge\frac{b_1-\delta_1\langle\widetilde{c}_1(t)\rangle^*}{a_{11}}. \tag{29}$$

Since

$$\lim_{t\to\infty}\langle\widetilde{c}_i(t)\rangle_*=\lim_{t\to\infty}\langle\widetilde{c}_i(t)\rangle^*=\overline{c}_i,\quad i=1,2,3, \tag{30}$$

then we obtain

$$\lim_{t\to\infty}\langle X_1(t)\rangle=\frac{b_1-\delta_1\overline{c}_1}{a_{11}}. \tag{31}$$

Using Assumption 2 and the strong law of large numbers for local martingales, one has

$$\lim_{t\to+\infty}\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_i(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u)=0,\qquad\lim_{t\to+\infty}\frac{\sigma_iB_i(t)}{t}=0,\qquad\text{a.s},\ i=1,2,3. \tag{32}$$

Then, substituting (31) into (24) yields

$$\lim_{t\to\infty}\frac{\ln X_1(t)}{t}=0,\quad\text{a.s.} \tag{33}$$

Since

$$\lim_{t\to\infty}\frac{\int_t^{t+\tau_2}X_1(s-\tau_2)\,\mathrm{d}s}{t}=\lim_{t\to\infty}\frac{1}{t}\int_{t-\tau_2}^tX_1(s)\,\mathrm{d}s=0\qquad\text{a.s.}, \tag{34}$$

$$\lim_{t\to\infty}\frac{\int_0^{\tau_2}X_1(s-\tau_2)\,\mathrm{d}s}{t}=\lim_{t\to\infty}\frac{\int_{-\tau_2}^0X_1(s)\,\mathrm{d}s}{t}=0\qquad\text{a.s.}, \tag{35}$$

then we have that, for any $0<\varepsilon_1<\Delta_1$, there exists $T_1>0$ such that

$$-\varepsilon_1<a_{21}e^{-d_2\tau_2}\cdot\frac{\left(\int_t^{t+\tau_2}X_1(s-\tau_2)\,\mathrm{d}s-\int_0^{\tau_2}X_1(s-\tau_2)\,\mathrm{d}s\right)}{t}<\varepsilon_1,\qquad t\ge T_1. \tag{36}$$

Combining (25) with (36) yields

$$\frac{\ln S_1(t)-\ln S_1(0)}{t}\le-b_2-\delta_2\langle\widetilde{c}_2(t)\rangle_*+a_{21}e^{-d_2\tau_2}\langle X_1(t)\rangle^*+\varepsilon_1-a_{22}\langle S_1(t)\rangle+\frac{\sigma_2B_2(t)}{t}+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_2(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u). \tag{37}$$

From Lemma 4 and conditions, we have

$$\lim_{t\to\infty}S_1(t)=0\quad\text{a.s.} \tag{38}$$

Obviously, we have

$$\lim_{t\to\infty}I_1(t)=0\quad\text{a.s.} \tag{39}$$

Case (iii). By (25), we obtain

$$\begin{aligned}\frac{\ln S_1(t)-\ln S_1(0)}{t} &\geq -b_2-\delta_2\langle\tilde{c}_2(t)\rangle^*+a_{21}e^{-d_2\tau_2}\langle X_1(t)\rangle_*-\varepsilon_1\\ &\quad -a_{22}\langle S_1(t)\rangle+\frac{\sigma_2B_2(t)}{t}\\ &\quad +\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_2(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u).\end{aligned}\tag{40}$$

It can infer from (37), (40), and Lemma 4 that

$$\begin{aligned}\langle S_1(t)\rangle^* &\leq \frac{\Delta_1+\varepsilon_1}{a_{11}a_{22}},\\ \langle S_1(t)\rangle_* &\geq \frac{\Delta_1-\varepsilon_1}{a_{11}a_{22}}.\end{aligned}\tag{41}$$

Since ε_1 is an arbitrary number, then we get

$$\lim_{t\to\infty}\langle S_1(t)\rangle=\frac{\Delta_1}{a_{11}a_{22}}\quad \text{a.s.}\tag{42}$$

Combining this equality with (25) leads to

$$\lim_{t\to\infty}\frac{\ln S_1(t)}{t}=0,\quad \text{a.s.}\tag{43}$$

Similar to (34) and (35), we have that, for any $0<\varepsilon_2<\beta e^{-d_4\tau_4}\Delta_1-(b_3+\delta_3\bar{c}_3)a_{11}a_{22}$, there exists $T_2>0$ such that

$$\begin{aligned}\frac{\ln I_1(t)-\ln I_1(0)}{t} &\leq -b_3-\delta_3\langle\tilde{c}_3(t)\rangle_*+\beta e^{-d_4\tau_4}\langle S_1(t)\rangle^*+\varepsilon_2\\ &\quad -a_{33}\langle I_1(t)\rangle+\frac{\sigma_3B_3(t)}{t}\\ &\quad +\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_3(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u).\end{aligned}\tag{44}$$

Note that $\mathscr{R}<1$, so we have

$$\lim_{t\to\infty}I_1(t)=0\quad \text{a.s.}\tag{45}$$

Case (iv). By (26), we get

$$\begin{aligned}\frac{\ln I_1(t)-\ln I_1(0)}{t} &\geq -b_3-\delta_3\langle\tilde{c}_3(t)\rangle^*+\beta e^{-d_4\tau_4}\langle S_1(t)\rangle_*-\varepsilon_2\\ &\quad -a_{33}\langle I_1(t)\rangle+\frac{\sigma_3B_3(t)}{t}\\ &\quad +\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln(1+\gamma_3(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u).\end{aligned}\tag{46}$$

When $\mathscr{R}>1$, from (44) and (46), using Lemma 4 results in

$$\begin{aligned}&\langle I_1(t)\rangle^*\\ &\quad\leq\frac{\beta e^{-d_4\tau_4}\Delta_1-(b_3+\delta_3\langle\tilde{c}_3(t)\rangle_*)a_{11}a_{22}+\varepsilon_2}{a_{11}a_{22}a_{33}},\\ &\langle I_1(t)\rangle_*\\ &\quad\geq\frac{\beta e^{-d_4\tau_4}\Delta_1-(b_3+\delta_3\langle\tilde{c}_3(t)\rangle^*)a_{11}a_{22}-\varepsilon_2}{a_{11}a_{22}a_{33}}.\end{aligned}\tag{47}$$

Since ε_2 is an arbitrary number, then we can obtain

$$\lim_{t\to\infty}\langle I_1(t)\rangle=\frac{\beta e^{-d_4\tau_4}\Delta_1-(b_3+\delta_3\bar{c}_3)a_{11}a_{22}}{a_{11}a_{22}a_{33}}\quad \text{a.s.}\tag{48}$$

Combining this equality with (26) leads to

$$\lim_{t\to\infty}\frac{\ln I_1(t)}{t}=0,\quad \text{a.s.}\tag{49}$$

This completes the proof of Lemma 1. □

3. Main Results

3.1. Extinction. Now we are going to show our main results. By Lemma 5, we can get the extinction of system (10).

Theorem 6. *For system (10), let $(X(t),S(t),I(t))$ be the solution of this system with initial value $((\phi_1(t),\phi_2(t),\phi_3(t)):-\tau\leq t\leq 0)\in C([-\tau,0];R_+^3)$.*

(i) If $b_1<\delta_1\bar{c}_1$, then

$$\begin{aligned}\lim_{t\to\infty}X(t)&=0,\\ \lim_{t\to\infty}S(t)&=0,\\ \lim_{t\to\infty}I(t)&=0,\end{aligned}\quad \textit{a.s.}\tag{50}$$

(ii) If $b_1>\delta_1\bar{c}_1$, when $\Delta_1<0$, then

$$\begin{aligned}\lim_{t\to\infty}\langle X(t)\rangle&=\frac{b_1-\delta_1\bar{c}_1}{a_{11}},\\ \lim_{t\to\infty}S(t)&=0,\\ \lim_{t\to\infty}I(t)&=0,\end{aligned}\quad \textit{a.s.}\tag{51}$$

(iii) If $b_1>\delta_1\bar{c}_1$, $\Delta_1>0$, when $\mathscr{R}<1$, then

$$\begin{aligned}\lim_{t\to\infty}\langle X(t)\rangle&=\frac{(b_1-\delta_1\bar{c}_1)a_{22}+(b_2+\delta_2\bar{c}_2)a_{12}e^{-d_1\tau_1}}{\Delta'}\quad \textit{a.s},\\ \lim_{t\to\infty}\langle S(t)\rangle&=\frac{\Delta_1}{\Delta'}\quad \textit{a.s},\\ \lim_{t\to\infty}I(t)&=0\quad \textit{a.s.}\end{aligned}\tag{52}$$

Proof. By stochastic comparison theorem, we have

$$\begin{aligned} X(t) &\le X_1(t), \\ S(t) &\le S_1(t), \\ I(t) &\le I_1(t). \end{aligned} \tag{53}$$

By (34), we derive

$$\lim_{t\to\infty} \frac{\int_t^{t+\tau_2} X(s-\tau_2)\,\mathrm{d}s}{t} = 0. \tag{54}$$

In the same way, we can verify that

$$\begin{aligned} \lim_{t\to\infty} \frac{\int_t^{t+\tau_2} S(s-\tau_2)\,\mathrm{d}s}{t} &= 0, \\ \lim_{t\to\infty} \frac{\int_t^{t+\tau_2} I(s-\tau_2)\,\mathrm{d}s}{t} &= 0. \end{aligned} \tag{55}$$

It follows from (33), (43), and (49) that

$$\left[\frac{\ln X(t)}{t}\right]^* \le \lim_{t\to\infty} \frac{\ln X_1(t)}{t} = 0, \tag{56}$$

$$\left[\frac{\ln S(t)}{t}\right]^* \le \lim_{t\to\infty} \frac{\ln S_1(t)}{t} = 0, \tag{57}$$

$$\left[\frac{\ln I(t)}{t}\right]^* \le \lim_{t\to\infty} \frac{\ln I_1(t)}{t} = 0. \tag{58}$$

Applying Itô's formula to system (10) leads to

$$\begin{aligned} \mathrm{d}\ln X &= \left[b_1 - \delta_1\tilde{c}_1(t) - a_{11}X(t) - a_{12}e^{-d_1\tau_1}S(t-\tau_1)\right]\mathrm{d}t + \sigma_1\mathrm{d}B_1(t) + \int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}\ln S &= \left[-b_2 - \delta_2\tilde{c}_2(t) + a_{21}e^{-d_2\tau_2}X(t-\tau_2) - \beta e^{-d_3\tau_3}I(t-\tau_3) - a_{22}S(t)\right]\mathrm{d}t + \sigma_2\mathrm{d}B_2(t) + \int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}\ln I &= \left[-b_3 - \delta_3\tilde{c}_3(t) + \beta e^{-d_4\tau_4}S(t-\tau_4) - a_{33}I(t)\right]\mathrm{d}t + \sigma_3\mathrm{d}B_3(t) + \int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u). \end{aligned} \tag{59}$$

Then we can obtain

$$\begin{aligned} \frac{\ln X(t) - \ln X(0)}{t} &= b_1 - \delta_1\langle\tilde{c}_1(t)\rangle - a_{12}e^{-d_1\tau_1}\langle S(t)\rangle - a_{11}\langle X(t)\rangle - a_{12}e^{-d_1\tau_1} \cdot \frac{\left(\int_0^{\tau_1} X(s-\tau_1)\,\mathrm{d}s - \int_t^{t+\tau_1} X(s-\tau_1)\,\mathrm{d}s\right)}{t} \\ &\quad + \frac{\sigma_1 B_1(t)}{t} + \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u), \end{aligned} \tag{60}$$

$$\begin{aligned} \frac{\ln S(t) - \ln S(0)}{t} &= -b_2 - \delta_2\langle\tilde{c}_2(t)\rangle + a_{21}e^{-d_2\tau_2}\langle X(t)\rangle - \beta e^{-d_3\tau_3}\langle I(t)\rangle - a_{22}\langle S(t)\rangle - a_{21}e^{-d_2\tau_2} \cdot \frac{\left(\int_t^{t+\tau_2} X(s-\tau_2)\,\mathrm{d}s - \int_0^{\tau_2} X(s-\tau_2)\,\mathrm{d}s\right)}{t} \\ &\quad + \beta e^{-d_3\tau_3}\frac{\left(\int_t^{t+\tau_2} I(s-\tau_2)\,\mathrm{d}s - \int_0^{\tau_2} I(s-\tau_2)\,\mathrm{d}s\right)}{t} + \frac{\sigma_2 B_2(t)}{t} + \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u), \end{aligned} \tag{61}$$

$$\begin{aligned} \frac{\ln I(t) - \ln I(0)}{t} &= -b_3 - \delta_3\langle\tilde{c}_3(t)\rangle + \beta e^{-d_4\tau_4}\langle S(t)\rangle - a_{33}\langle I(t)\rangle - \beta e^{-d_4\tau_4} \cdot \frac{\left(\int_t^{t+\tau_4} S(s-\tau_4)\,\mathrm{d}s - \int_0^{\tau_4} S(s-\tau_4)\,\mathrm{d}s\right)}{t} \\ &\quad + \frac{\sigma_3 B_3(t)}{t} + \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u). \end{aligned} \tag{62}$$

Computing (60) $\times\, a_{21}e^{-d_2\tau_2}$ + (61) $\times\, a_{11}$ leads to

$$\begin{aligned} a_{21}e^{-d_2\tau_2}\frac{\ln X(t) - \ln X(0)}{t} + a_{11}\frac{\ln S(t) - \ln S(0)}{t} &= \left(b_1 - \delta_1\langle\tilde{c}_1(t)\rangle\right)a_{21}e^{-d_2\tau_2} - \left(b_2 + \delta_2\langle\tilde{c}_2(t)\rangle\right)a_{11} - \Delta'\langle S(t)\rangle \\ &\quad - a_{11}\beta e^{-d_3\tau_3}\langle I(t)\rangle + \Gamma_1, \end{aligned} \tag{63}$$

where Γ_1 is given in Appendix.

Case (i). By Lemma 5, we know that $\lim_{t\to\infty} X_1(t) = 0$, if $b_1 < \delta_1\bar{c}_1$. From (53), we have

$$\lim_{t\to\infty} X(t) = 0 \quad \text{a.s.} \tag{64}$$

Obviously, we get

$$\begin{aligned} \lim_{t\to\infty} S(t) &= 0, \\ \lim_{t\to\infty} I(t) &= 0, \end{aligned} \tag{65}$$

a.s.

Case (ii). By Lemma 5, we know that $\lim_{t\to\infty} S_1(t) = 0$, if $b_1 > \delta_1\bar{c}_1$ and $\Delta_1 < 0$. From (53), we have

$$\lim_{t\to\infty} S(t) = 0 \quad \text{a.s.} \tag{66}$$

Then we obtain

$$\lim_{t\to\infty} I(t) = 0 \quad \text{a.s.} \tag{67}$$

By using (60) and Lemma 4, we can prove

$$\lim_{t\to\infty} \langle X(t)\rangle = \frac{b_1 - \delta_1\bar{c}_1}{a_{11}} \quad \text{a.s.} \tag{68}$$

Case (iii). Similarly, by Lemma 5, we get that $\lim_{t\to\infty} I_1(t) = 0$, if $b_1 > \delta_1\bar{c}_1$, $\Delta_1 > 0$, and $\mathscr{R} < 1$. By (53), we obtain

$$\lim_{t\to\infty} I(t) = 0 \quad \text{a.s.} \tag{69}$$

Combining Lemma 4 with (56) and (63), we get

$$\langle S(t)\rangle_* \geq \frac{\Delta_1}{\Delta'}. \tag{70}$$

From (60), we have

$$\begin{aligned}
&\frac{\ln X(t) - \ln X(0)}{t} \\
&\quad \leq b_1 - \delta_1 \langle \tilde{c}_1(t)\rangle_* - a_{12}e^{-d_1\tau_1}\langle S(t)\rangle_* - a_{11}\langle X(t)\rangle \\
&\qquad + \frac{\sigma_1 B_1(t)}{t} + \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u).
\end{aligned} \tag{71}$$

Then we infer from Lemma 4 that

$$\begin{aligned}
&\langle X(t)\rangle^* \\
&\leq \frac{(b_1 - \delta_1\langle \tilde{c}_1(t)\rangle_*)a_{22} + (b_2 + \delta_2\langle \tilde{c}_2(t)\rangle^*)a_{12}e^{-d_1\tau_1}}{\Delta'}.
\end{aligned} \tag{72}$$

By (61), we get

$$\begin{aligned}
&\frac{\ln S(t) - \ln S(0)}{t} \\
&\quad \leq -b_2 - \delta_2\langle \tilde{c}_2(t)\rangle_* + a_{21}e^{-d_2\tau_2}\langle X(t)\rangle^* \\
&\qquad - a_{22}\langle S(t)\rangle + \frac{\sigma_2 B_2(t)}{t} \\
&\qquad + \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u).
\end{aligned} \tag{73}$$

From Lemma 4, we obtain

$$\langle S(t)\rangle^* \leq \frac{\Delta_1}{\Delta'}. \tag{74}$$

By (60), we have

$$\begin{aligned}
&\frac{\ln X(t) - \ln X(0)}{t} \\
&\quad \geq b_1 - \delta_1\langle \tilde{c}_1(t)\rangle^* - a_{12}e^{-d_1\tau_1}\langle S(t)\rangle^* \\
&\qquad - a_{11}\langle X(t)\rangle + \frac{\sigma_1 B_1(t)}{t} \\
&\qquad + \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u).
\end{aligned} \tag{75}$$

In view of Lemma 4, we can see that

$$\begin{aligned}
&\langle X(t)\rangle_* \\
&\geq \frac{(b_1 - \delta_1\langle \tilde{c}_1(t)\rangle^*)a_{22} + (b_2 + \delta_2\langle \tilde{c}_2(t)\rangle_*)a_{12}e^{-d_1\tau_1}}{\Delta'}.
\end{aligned} \tag{76}$$

Consequently,

$$\begin{aligned}
&\lim_{t\to\infty}\langle X(t)\rangle = \frac{(b_1 - \delta_1\bar{c}_1)a_{22} + (b_2 + \delta_2\bar{c}_2)a_{12}e^{-d_1\tau_1}}{\Delta'} \quad \text{a.s,} \\
&\lim_{t\to\infty}\langle S(t)\rangle = \frac{\Delta_1}{\Delta'} \quad \text{a.s.}
\end{aligned} \tag{77}$$

This completes the proof of Theorem 6. □

3.2. Permanence in Mean. In this section, we prove the permanence in mean of system (10).

Theorem 7. *Let $(X(t), S(t), I(t))$ be the solution of system (10) with initial value $((\phi_1(t), \phi_2(t), \phi_3(t)) : -\tau \leq t \leq 0) \in C([-\tau, 0]; R_+^3)$. If $\Delta_2 > 0$ and $\Delta_3 > 0$, when $\widehat{\mathscr{R}} = (a_{11}a_{22}/\Delta')\mathscr{R} > 1$, then*

$$\begin{aligned}
\lim_{t\to\infty}\langle X(t)\rangle &= \frac{\Delta_2}{\Delta}, \\
\lim_{t\to\infty}\langle S(t)\rangle &= \frac{\Delta_3}{\Delta}, \\
\lim_{t\to\infty}\langle I(t)\rangle &= \frac{\Delta_4}{\Delta},
\end{aligned} \tag{78}$$

a.s.

Proof. Computing (62) × Δ' + (63) × $\beta e^{-d_4\tau_4}$, we obtain

$$\begin{aligned}
&\Delta'\frac{\ln I(t) - \ln I(0)}{t} \\
&\qquad + \beta a_{21}e^{-d_2\tau_2 - d_4\tau_4}\frac{\ln X(t) - \ln X(0)}{t} \\
&\qquad + a_{11}\beta e^{-d_4\tau_4}\frac{\ln S(t) - \ln S(0)}{t} \\
&\quad = -(b_3 + \delta_3\langle \tilde{c}_3(t)\rangle)\Delta' \\
&\qquad + (b_1 - \delta_1\langle \tilde{c}_1(t)\rangle)a_{21}\beta e^{-d_2\tau_2 - d_4\tau_4} \\
&\qquad - (b_2 + \delta_2\langle \tilde{c}_2(t)\rangle)a_{11}\beta e^{-d_4\tau_4} \\
&\qquad - (a_{33}\Delta' + a_{11}\beta^2 e^{-d_3\tau_3 - d_4\tau_4})\langle I(t)\rangle + \Gamma_2,
\end{aligned} \tag{79}$$

where Γ_2 is given in Appendix.

When $\widehat{\mathscr{R}} = (a_{11}a_{22}/\Delta')\mathscr{R} > 1$, by Lemma 4, we have

$$\langle I(t)\rangle_* \geq \frac{\Delta_4}{\Delta}. \tag{80}$$

So

$$\lim_{t\to\infty}\frac{\ln I(t)}{t}=0 \quad \text{a.s.} \tag{81}$$

Computing (63) $\times\, a_{33} -$ (62) $\times\, a_{11}\beta e^{-d_3\tau_3}$, we have

$$\begin{aligned}
&a_{33}a_{21}e^{-d_2\tau_2}\frac{\ln X(t)-\ln X(0)}{t}\\
&\quad+a_{11}a_{33}\frac{\ln S(t)-\ln S(0)}{t}\\
&\quad-a_{11}\beta e^{-d_3\tau_3}\frac{\ln I(t)-\ln I(0)}{t}\\
&=\left(b_3+\delta_3\left\langle\widetilde{c}_3(t)\right\rangle\right)a_{11}\beta e^{-d_3\tau_3}\\
&\quad+\left(b_1-\delta_1\left\langle\widetilde{c}_1(t)\right\rangle\right)a_{33}a_{21}e^{-d_2\tau_2}\\
&\quad-\left(b_2+\delta_2\left\langle\widetilde{c}_2(t)\right\rangle\right)a_{11}a_{33}\\
&\quad-\left(a_{33}\Delta'+a_{11}\beta^2e^{-d_3\tau_3-d_4\tau_4}\right)\langle S(t)\rangle+\Gamma_3,
\end{aligned} \tag{82}$$

where Γ_3 is given in Appendix.

When $\Delta_3>0$, by Lemma 4, we have

$$\langle S(t)\rangle_*\geq\frac{\Delta_3}{\Delta}. \tag{83}$$

From (71), when $\Delta_2>0$, we obtain

$$\langle X(t)\rangle^*\leq\frac{\Delta_2}{\Delta}. \tag{84}$$

By (61), we get

$$\begin{aligned}
&\frac{\ln S(t)-\ln S(0)}{t}\\
&\quad\leq-b_2-\delta_2\left\langle\widetilde{c}_2(t)\right\rangle_*+a_{21}e^{-d_2\tau_2}\langle X(t)\rangle^*\\
&\quad-\beta e^{-d_3\tau_3}\langle I(t)\rangle_*-a_{22}\langle S(t)\rangle+\frac{\sigma_2B_2(t)}{t}\\
&\quad+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln\left(1+\gamma_2(u)\right)\widetilde{N}(\mathrm{d}s,\mathrm{d}u),
\end{aligned} \tag{85}$$

then from Lemma 4 we prove that

$$\langle S(t)\rangle^*\leq\frac{\Delta_3}{\Delta}. \tag{86}$$

By (60), we have

$$\begin{aligned}
&\frac{\ln X(t)-\ln X(0)}{t}\\
&\quad\geq b_1-\delta_1\left\langle\widetilde{c}_1(t)\right\rangle^*-a_{12}e^{-d_1\tau_1}\langle S(t)\rangle^*\\
&\quad-a_{11}\langle X(t)\rangle+\frac{\sigma_1B_1(t)}{t}\\
&\quad+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln\left(1+\gamma_1(u)\right)\widetilde{N}(\mathrm{d}s,\mathrm{d}u),
\end{aligned} \tag{87}$$

and then from Lemma 4 we get

$$\langle X(t)\rangle_*\geq\frac{\Delta_2}{\Delta}. \tag{88}$$

It can be inferred from (62) that

$$\begin{aligned}
&\frac{\ln I(t)-\ln I(0)}{t}\\
&\quad\leq-b_3-\delta_3\left\langle\widetilde{c}_3(t)\right\rangle_*+\beta e^{-d_4\tau_4}\langle S(t)\rangle^*-a_{33}\langle I(t)\rangle\\
&\quad-\beta e^{-d_4\tau_4}\frac{\left(\int_t^{t+\tau_4}S(s-\tau_4)\,\mathrm{d}s-\int_0^{\tau_4}S(s-\tau_4)\,\mathrm{d}s\right)}{t}\\
&\quad+\frac{\sigma_3B_3(t)}{t}+\frac{1}{t}\int_0^t\int_{\mathbb{Y}}\ln\left(1+\gamma_3(u)\right)\widetilde{N}(\mathrm{d}s,\mathrm{d}u),
\end{aligned} \tag{89}$$

and then from Lemma 4 we have

$$\langle I(t)\rangle^*\leq\frac{\Delta_4}{\Delta}. \tag{90}$$

Consequently,

$$\begin{aligned}
\lim_{t\to\infty}\langle X(t)\rangle&=\frac{\Delta_2}{\Delta},\\
\lim_{t\to\infty}\langle S(t)\rangle&=\frac{\Delta_3}{\Delta},\\
\lim_{t\to\infty}\langle I(t)\rangle&=\frac{\Delta_4}{\Delta},
\end{aligned} \tag{91}$$

a.s.

This completes the proof of Theorem 7. □

4. Conclusion and Simulations

This paper explores the dynamics of a stochastic predator-prey model with time delays in the polluted environment. We show some properties of the subsystem of the predator-prey system. Then by using comparison theorem and limit superior theory, we obtain the sufficient conditions for the extinction of this system. From Theorem 6, we know that if environmental noises are large enough, the species will be extinct (see Figure 1) and if the amount of pulsed input concentration of the toxicant is large enough or the pulse period is small enough, the species will also be extinct (see Figures 2 and 3). In addition, Theorem 7 indicates that the permanence of populations has high correlation with the intensity of environmental noises (see Figure 2(a)). The main results show that time delays can lead to extinction of this system. Therefore, we realize that the environmental noises, the toxicant input, and delays are harmful to the permanence of the populations. Now we show some simulations to verify our main results.

Choose some parameters as follows $r=1.6$, $\mu_1=0.1$, $\mu_2=0.1$, $\delta_1=0.3$, $\delta_2=0.1$, $\delta_3=0.1$, $a_{11}=0.3$, $a_{12}=0.3$, $a_{21}=0.3$, $a_{22}=0.3$, $a_{33}=0.1$, $d_1=0.1$, $d_2=0.1$, $d_3=0.1$,

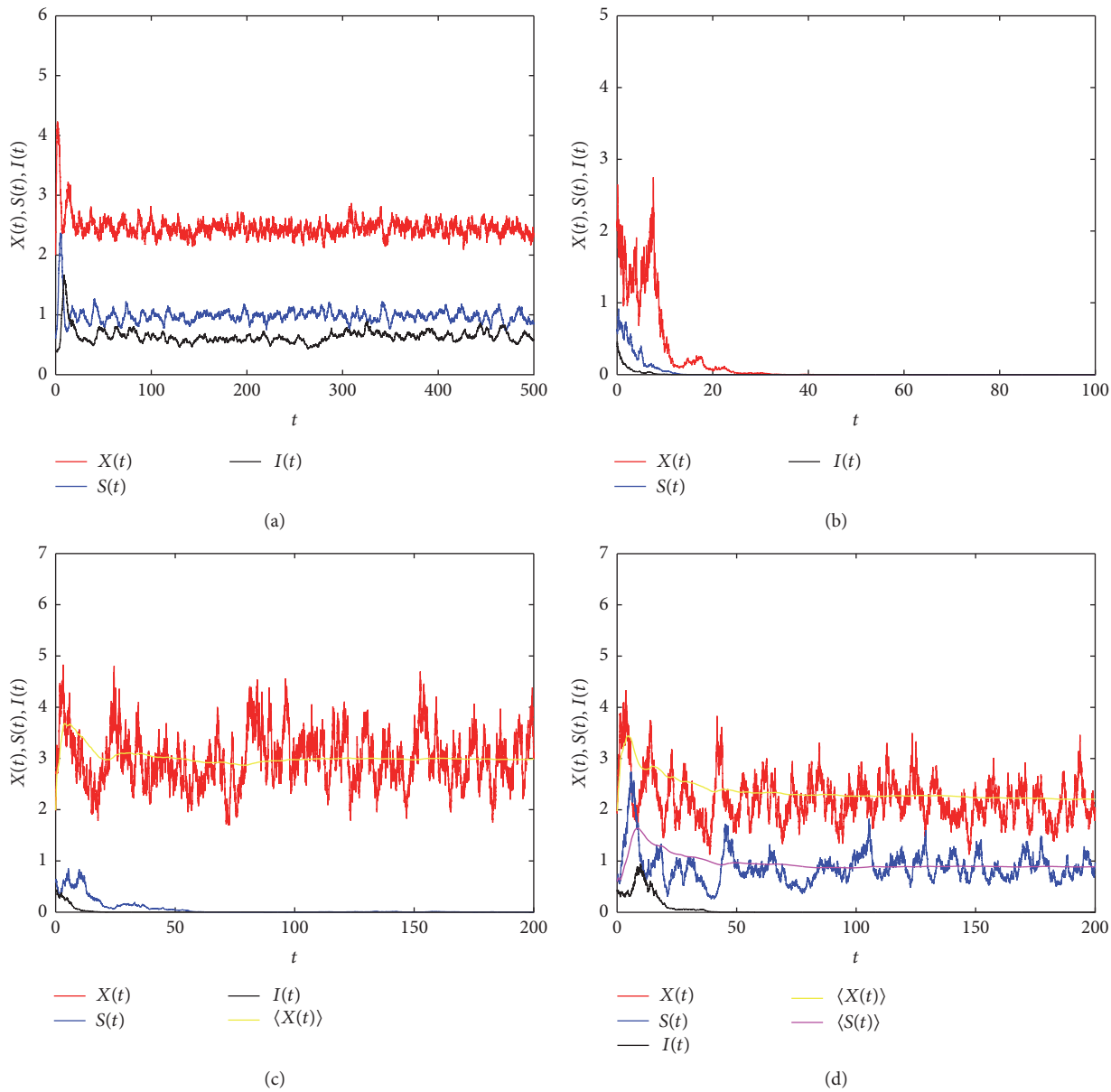

FIGURE 1: In (a), $\gamma_1 = 0$, $\gamma_2 = 0$, $\gamma_3 = 0$, and the species are permanent; in (b), $\gamma_1 = 2.2$, $\gamma_2 = 1.5$, $\gamma_3 = 1.2$, and then the species are extinct; in (c), $\gamma_1 = 0.5$, $\gamma_2 = 1.5$, $\gamma_3 = 1.2$, $X(t)$ is permanent, $S(t)$ and $I(t)$ are extinct, and $\lim_{t\to\infty}\langle X(t)\rangle = 2.953$; in (d), $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, $\gamma_3 = 1.2$, $X(t)$ and $S(t)$ are permanent, $I(t)$ is extinct, $\lim_{t\to\infty}\langle X(t)\rangle = 2.144$, and $\lim_{t\to\infty}\langle S(t)\rangle = 0.892$.

$d_4 = 0.1$, $\tau_1 = 1$, $\tau_2 = 1$, $\tau_3 = 1$, $\tau_4 = 1$, $h = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.2$, $\sigma_3 = 0.1$, $k_1 = 0.3$, $g_1 = 0.1$, $m_1 = 0.1$, $k_2 = 0.3$, $g_2 = 0.2$, $m_2 = 0.2$, $k_3 = 0.3$, $g_3 = 0.2$, and $m_3 = 0.2$. Based on the above parameters, we give simulations to explain the biological implications.

In Figure 1, we keep $u = 0.4$ and $T = 1$. In Figure 1(a), choose $\gamma_1 = 0$, $\gamma_2 = 0$, and $\gamma_3 = 0$, there is no Lévy noises, and we can see that the species are permanent. In Figure 1(b), if $\gamma_1 = 2.2$, $\gamma_2 = 1.5$, and $\gamma_3 = 1.2$, the intensities of Lévy noises are large and then we get $b_1 = 0.543 < \delta_1\bar{c}_1 = 0.600$, which means that the productiveness of the prey is less than its death loss rate; from Theorem 6, the species go extinction. In Figure 1(c), if $\gamma_1 = 0.5$, $\gamma_2 = 1.5$, and $\gamma_3 = 1.2$, then $b_1 = 0.886 > \delta_1\bar{c}_1 = 0.600$ and $\Delta_1 = -0.009 < 0$, from Theorem 6 we have that $X(t)$ is permanent and $S(t)$ and $I(t)$ are extinct; furthermore, we have $\lim_{t\to\infty}\langle X(t)\rangle = 2.953$. In Figure 1(d), if $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, and $\gamma_3 = 1.2$, then $b_1 = 0.886 > \delta_1\bar{c}_1 = 0.600$, $\Delta_1 = 0.146 > 0$, and $\mathscr{R} = 0.946 < 1$; from Theorem 6 we obtain that $X(t)$ and $S(t)$ are permanent and $I(t)$ is extinct and we get $\lim_{t\to\infty}\langle X(t)\rangle = 2.144$ and $\lim_{t\to\infty}\langle S(t)\rangle = 0.892$.

In Figure 2, we keep $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, $\gamma_3 = 0.1$, and $T = 1$. In Figure 2(a), we choose $u = 0.4$, and then $\Delta_2 = 0.114 > 0$, $\Delta_3 = 0.037 > 0$, and $\mathscr{R} = 2.809 > (a_{11}a_{22}/\Delta')\mathscr{R} = 1.559 > 1$. From Theorem 7, we have that $X(t)$, $S(t)$, and $I(t)$ are permanent and we obtain that $\lim_{t\to\infty}\langle X(t)\rangle = 2.072$, $\lim_{t\to\infty}\langle S(t)\rangle = 0.680$ and $\lim_{t\to\infty}\langle I(t)\rangle = 0.345$. In

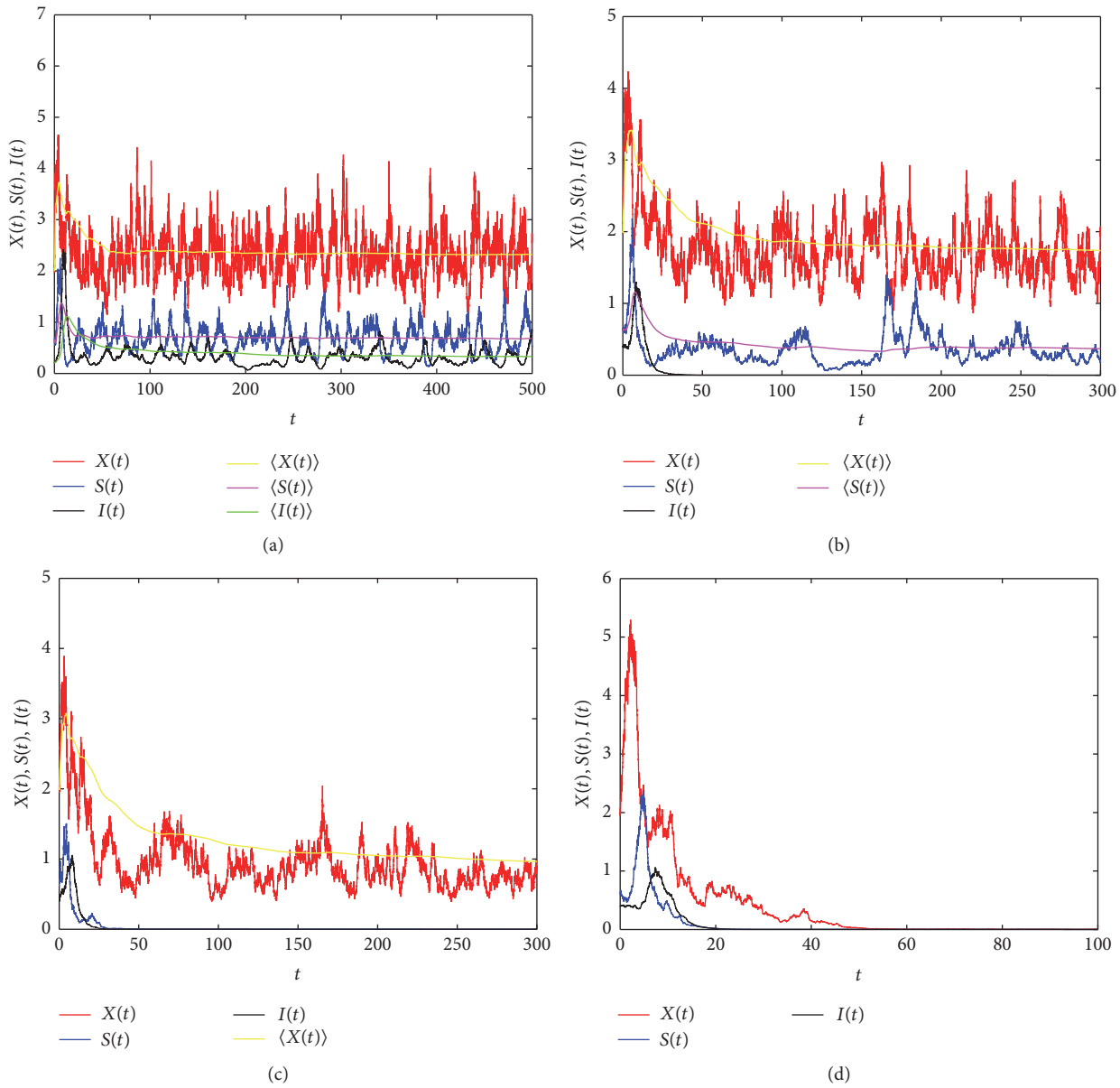

FIGURE 2: Keep $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, $\gamma_3 = 0.1$, and $T = 1$. In (a), $u = 0.4$, $X(t)$, $S(t)$, and $I(t)$ are permanent, $\lim_{t\to\infty}\langle X(t)\rangle = 2.072$, $\lim_{t\to\infty}\langle S(t)\rangle = 0.680$, and $\lim_{t\to\infty}\langle I(t)\rangle = 0.345$; in (b), $u = 0.6$, $X(t)$ and $S(t)$ are permanent, $I(t)$ is extinct, $\lim_{t\to\infty}\langle X(t)\rangle = 1.719$, and $\lim_{t\to\infty}\langle S(t)\rangle = 0.313$; in (c), $u = 0.8$, $X(t)$ is permanent, $S(t)$ and $I(t)$ are extinct, and $\lim_{t\to\infty}\langle X(t)\rangle = 0.952$; in (d), $u = 1$ and then the species are extinct.

Figure 2(b), if $u = 0.6$, then $\Delta_1 = 0.050 > 0$ and $\mathscr{R} = 0.768 < 1$, and from Theorem 6, we get that $X(t)$ and $S(t)$ are permanent and $I(t)$ is extinct and $\lim_{t\to\infty}\langle X(t)\rangle = 1.719$ and $\lim_{t\to\infty}\langle S(t)\rangle = 0.313$. In Figure 2(c), if $u = 0.8$, then $\Delta_1 = -0.047 < 0$, $X(t)$ is permanent, $S(t)$ and $I(t)$ are extinct by Theorem 6, and $\lim_{t\to\infty}\langle X(t)\rangle = 0.952$. In Figure 2(d), we choose $u = 1$, and then $b_1 = 1.4855 < \delta_1\bar{c}_1 = 1.5$; from Theorem 6 the species are extinct.

In Figure 3, we keep $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, $\gamma_3 = 0.1$, and $u = 0.4$. In Figure 3(a), we choose $T = 1$ and then $\Delta_2 = 0.114 > 0$, $\Delta_3 = 0.037 > 0$, and $\mathscr{R} = 2.809 > (a_{11}a_{22}/\Delta')\mathscr{R} = 1.559 > 1$. From Theorem 7, we have that $X(t)$, $S(t)$, and $I(t)$ are permanent and we obtain that $\lim_{t\to\infty}\langle X(t)\rangle = 2.072$, $\lim_{t\to\infty}\langle S(t)\rangle = 0.680$, and $\lim_{t\to\infty}\langle I(t)\rangle = 0.345$. In Figure 3(b), if $T = 0.7$, then $\Delta_1 = 0.063 > 0$ and $\mathscr{R} = 0.710 < 1$, and from Theorem 6, we get that $X(t)$ and $S(t)$ are permanent and $I(t)$ is extinct and $\lim_{t\to\infty}\langle X(t)\rangle = 1.743$ and $\lim_{t\to\infty}\langle S(t)\rangle = 0.386$. In Figure 3(c), if $T = 0.5$, then $\Delta_1 = -0.047 < 0$, $X(t)$ is permanent, $S(t)$ and $I(t)$ are extinct by Theorem 6, and $\lim_{t\to\infty}\langle X(t)\rangle = 0.952$. In Figure 3(d), we choose $T = 0.4$ and then $b_1 = 1.4855 < \delta_1\bar{c}_1 = 1.5$; from Theorem 6 the species are extinct.

From Figures 1–3, we can obtain the following conclusions:

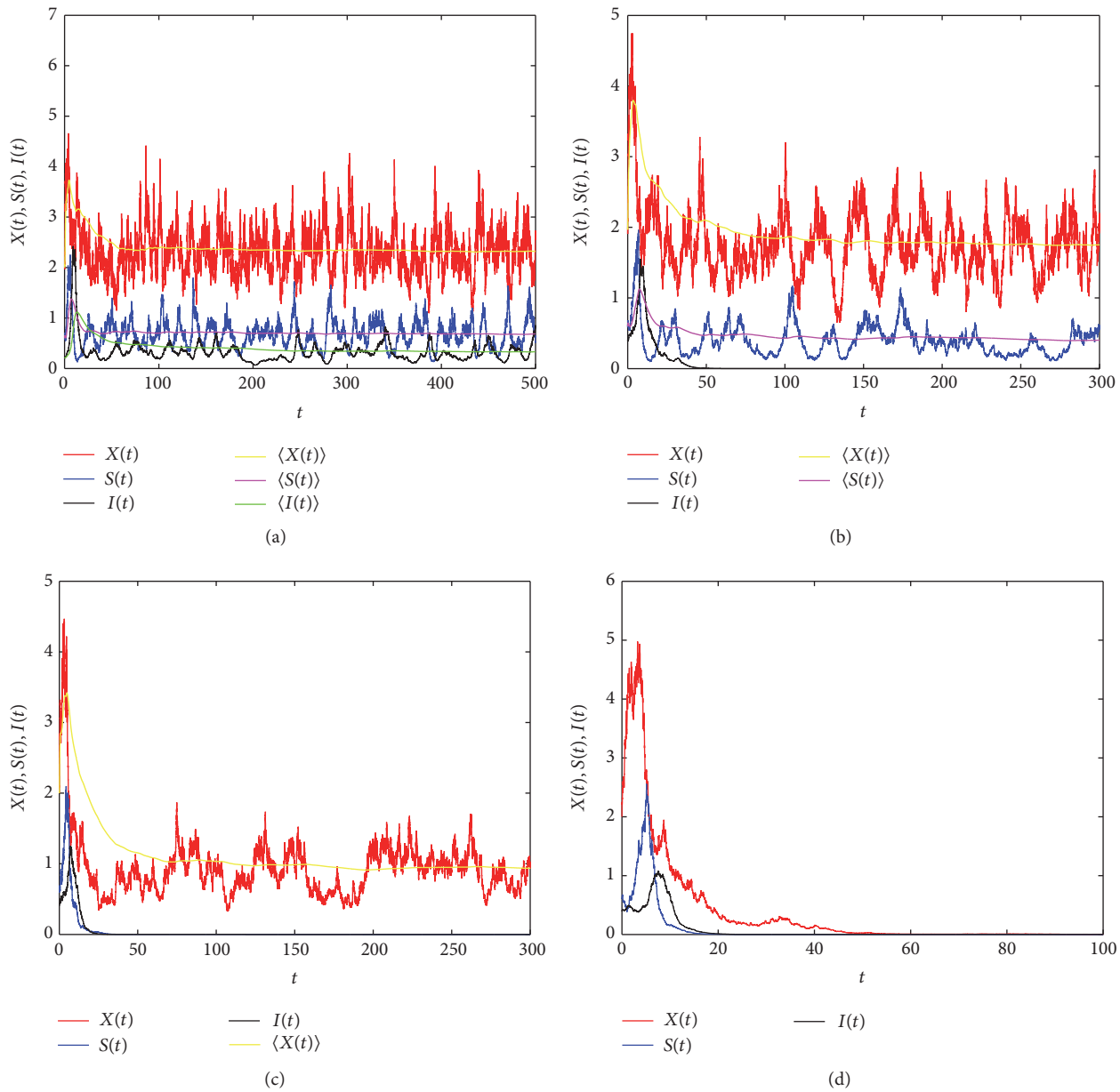

Figure 3: Keep $\gamma_1 = 0.5$, $\gamma_2 = 0.5$, $\gamma_3 = 0.1$, and $u = 0.4$. In (a), $T = 1$, $X(t)$, $S(t)$ and $I(t)$ are permanent, $\lim_{t\to\infty}\langle X(t)\rangle = 2.072$, $\lim_{t\to\infty}\langle S(t)\rangle = 0.680$, and $\lim_{t\to\infty}\langle I(t)\rangle = 0.345$; in (b), $T = 0.7$, $X(t)$ and $S(t)$ are permanent, $I(t)$ is extinct, $\lim_{t\to\infty}\langle X(t)\rangle = 1.743$, and $\lim_{t\to\infty}\langle S(t)\rangle = 0.386$; in (c), $T = 0.5$, $X(t)$ is permanent, $S(t)$ and $I(t)$ are extinct, and $\lim_{t\to\infty}\langle X(t)\rangle = 0.952$; in (d), $u = 0.4$ and then the species are extinct.

(1) Large stochastic disturbance can cause the populations to go to extinction; that is, the persistent population of a deterministic system can become extinct due to the white noise stochastic disturbance.

(2) Large impulsive input concentration of the toxicant or small impulsive period of the exogenous input of toxicant can cause the populations to go to extinction.

Therefore, the above numerical simulations illustrate the performance of the theoretical results, and the biological results show that the white noise stochastic disturbance and impulsive toxicant input are disadvantage for the permanence of system.

Appendix

For the sake of convenience, we define

$$\Gamma_1 = -a_{12}a_{21}e^{-d_1\tau_1-d_2\tau_2} \cdot \frac{\left(\int_0^{\tau_1} X(s-\tau_1)\,ds - \int_t^{t+\tau_1} X(s-\tau_1)\,ds\right)}{t}$$

$$
\begin{aligned}
&- a_{11}a_{21}e^{-d_2\tau_2} \\
&\cdot \frac{\left(\int_t^{t+\tau_2} X(s-\tau_2)\,\mathrm{d}s - \int_0^{\tau_2} X(s-\tau_2)\,\mathrm{d}s\right)}{t} \\
&+ a_{11}\beta e^{-d_3\tau_3} \\
&\cdot \frac{\left(\int_t^{t+\tau_3} I(s-\tau_3)\,\mathrm{d}s - \int_0^{\tau_3} I(s-\tau_3)\,\mathrm{d}s\right)}{t} \\
&+ a_{21}e^{-d_2\tau_2}\left(\frac{\sigma_1 B_1(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right) \\
&+ a_{11}\left(\frac{\sigma_2 B_2(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right),
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_2 = &-\beta a_{12}a_{21}e^{-d_1\tau_1-d_2\tau_2-d_4\tau_4} \\
&\cdot \frac{\left(\int_0^{\tau_1} X(s-\tau_1)\,\mathrm{d}s - \int_t^{t+\tau_1} X(s-\tau_1)\,\mathrm{d}s\right)}{t} \\
&- a_{11}a_{21}\beta e^{-d_2\tau_2-d_4\tau_4} \\
&\cdot \frac{\left(\int_t^{t+\tau_2} X(s-\tau_2)\,\mathrm{d}s - \int_0^{\tau_2} X(s-\tau_2)\,\mathrm{d}s\right)}{t} \\
&+ a_{11}\beta^2 e^{-d_3\tau_3-d_4\tau_4} \\
&\cdot \frac{\left(\int_t^{t+\tau_3} I(s-\tau_3)\,\mathrm{d}s - \int_0^{\tau_3} I(s-\tau_3)\,\mathrm{d}s\right)}{t} \\
&- \Delta'\beta e^{-d_4\tau_4} \\
&\cdot \frac{\left(\int_t^{t+\tau_4} S(s-\tau_4)\,\mathrm{d}s - \int_0^{\tau_4} S(s-\tau_4)\,\mathrm{d}s\right)}{t} \\
&+ a_{21}\beta e^{-d_2\tau_2-d_4\tau_4}\left(\frac{\sigma_1 B_1(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right) \\
&+ a_{11}\beta e^{-d_4\tau_4}\left(\frac{\sigma_2 B_2(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right) \\
&+ \Delta'\left(\frac{\sigma_3 B_3(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right),
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_3 = &-a_{33}a_{12}a_{21}e^{-d_1\tau_1-d_2\tau_2-d_4\tau_4} \\
&\cdot \frac{\left(\int_0^{\tau_1} X(s-\tau_1)\,\mathrm{d}s - \int_t^{t+\tau_1} X(s-\tau_1)\,\mathrm{d}s\right)}{t} \\
&- a_{11}a_{33}a_{21}e^{-d_2\tau_2} \\
&\cdot \frac{\left(\int_t^{t+\tau_2} X(s-\tau_2)\,\mathrm{d}s - \int_0^{\tau_2} X(s-\tau_2)\,\mathrm{d}s\right)}{t} \\
&+ a_{11}a_{33}\beta e^{-d_3\tau_3} \\
&\cdot \frac{\left(\int_t^{t+\tau_3} I(s-\tau_3)\,\mathrm{d}s - \int_0^{\tau_3} I(s-\tau_3)\,\mathrm{d}s\right)}{t} \\
&+ a_{11}\beta^2 e^{-d_3\tau_3-d_4\tau_4} \\
&\cdot \frac{\left(\int_t^{t+\tau_4} S(s-\tau_4)\,\mathrm{d}s - \int_0^{\tau_4} S(s-\tau_4)\,\mathrm{d}s\right)}{t} \\
&+ a_{33}a_{21}\beta e^{-d_2\tau_2}\left(\frac{\sigma_1 B_1(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_1(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right) \\
&+ a_{11}a_{33}\left(\frac{\sigma_2 B_2(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_2(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right) \\
&- a_{11}\beta e^{-d_3\tau_3}\left(\frac{\sigma_3 B_3(t)}{t}\right. \\
&\left.+ \frac{1}{t}\int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_3(u))\,\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right).
\end{aligned}
\tag{A.1}
$$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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