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Research Article

Finite-Time Stability of Large-Scale Systems with Interval Time-Varying Delay in Interconnection

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We investigate finite-time stability of a class of nonlinear large-scale systems with interval time-varying delays in interconnection. Time-delay functions are continuous but not necessarily differentiable. Based on Lyapunov stability theory and new integral bounding technique, finite-time stability of large-scale systems with interval time-varying delays in interconnection is derived. The finite-time stability criteria are delays-dependent and are given in terms of linear matrix inequalities which can be solved by various available algorithms. Numerical examples are given to illustrate effectiveness of the proposed method.

1. Introduction

It is well known that several real-world processes often depend on time-delay; namely, the present state depends on the past states. Consequently, time-delay systems have been investigated extensively in the past decades. Time-delay is often the main source of instability and poor performance of the systems (see [1–5]). Therefore, it is important to study time-delay systems, especially those with time-varying delays.

There are two types of stability of time-varying delays systems, namely, delay-dependent and delay-independent ones. The delay-dependent conditions are usually less conservative than delay-independent ones, especially when the time-delays are relatively small. One of the main purposes of delay-dependent stability criteria is to estimate the maximum possible allowable delay bounds.

For stability analysis of time-delay systems, a common approach used is by choosing an appropriate Lyapunov-Krasovskii functional and then properly estimating upper bounds of its derivative along trajectories of solutions of the system (see [1, 6]). In order to estimate such upper bounds of derivative of Lyapunov-Krasovskii functional, various mathematical tools have been used such as the Jensen inequality [3, 4, 7], lower bound lemma for reciprocal convexity [8–11], delay partitioning method [11], and free-weighting matrix variables method [6, 12, 13]. Recently, a new integral inequality which improves Jensen inequality, namely, Wirtinger-based integral inequality, has been proposed in [14]. In [15], a novel integral inequality has been proposed which has been shown to be less conservative than Jensen inequality. In [16, 17], some new bounding techniques have been developed and enhanced stability criteria for time-delay systems have been derived using these bounding techniques which are shown to be less conservative than some existing results which used Wirtinger-based or Jensen inequalities. In [18], by using Wirtinger-based integral inequality combined with the reciprocally convex approach, some less conservative stability criteria for time-delay systems are derived.

Large-scale interconnected systems have been the subject of considerable research (see [19–25]), which can be characterized by a large number of variables representing the systems, a strong interaction between subsystem variables, and a complex interaction between subsystems. The large-scale interconnected system can be found in many practical control problems such as transportation systems, electrical power systems, communication systems, and economic systems. The operation of large-scale interconnected systems requires the capability of monitoring and stabilizing in the face of uncertainties, disturbances, failures, and attacks through the

utilization of internal system states. However, even with the assumption that all the state variables are available for feedback control, the task of effective controlling of a large-scale interconnected system using a global (centralized) state feedback controller is still not easy as there is a necessary requirement for information transfer between the subsystems [22].

Generally, studies of dynamical systems are involved with stability analysis which is defined over an infinite-time interval. Nevertheless, in many real-world applications, the main aim is concerned with the behavior of the system over a fixed finite-time interval, for instance, the problem of sending a rocket from the neighborhood of point A to the neighborhood of point B over a fixed time interval [26]. In this example, the concept of finite-time stability (FTS) is proposed. The problem of FTS has been revisited using linear matrix inequality (LMI) technique which allows us to find feasible condition guaranteeing FTS (see [26–29]). Some interesting results on stability and stabilization in the context of time-delay systems have been obtained by using the Lyapunov-Krasovskii functional technique (see [7, 11, 30]). Most existing work assumed that the time-delays are either constant or differentiable. In [20, 24], delay-dependent sufficient conditions for stability of large-scale system with time-varying delay were studied. However, the approach used there cannot be applied to systems with interval nondifferentiable time-varying delays. Moreover, to the best of our knowledge, there are few results for FTS of large-scale interconnected systems, especially systems with interval nondifferential time-varying delay.

Motivated by the above discussions, we shall derive new FTS criteria for large-scale systems with interval time-varying delays in interconnection. The main contributions of our studies are as follows. (i) The time-delay functions are only required to be continuous but not necessarily differentiable. (ii) By employing an improved integral inequality in [18], we derive new and less conservative FTS for large-scale interconnected systems with time-varying delays in terms of LMIs.

The rest of this paper is organized as follows. Section 2 presents notations, definitions, and auxiliary lemmas required for the proof of the main results. In Section 3, the FTS criteria for large-scale interconnected system with time-varying delays are obtained. Illustrative numerical examples are presented in Section 4. Section 5 gives the conclusion of the paper.

2. Problem Formulation and Preliminaries

Under the practical constraint of decentralized information coupled with the facts that the measurement of all the states and the real-time knowledge of the interval time-varying delays are not available, we consider the following nonlinear large-scale interconnected system:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + \sum_{j\neq i,\, j=1}^{N} A_{ij} x_j\left(t - h_{ij}(t)\right) \\ &\quad + f_i\left(t, x_i(t), \left\{x_j\left(t - h_{ij}(t)\right)\right\}_{j=1, j\neq i}^{N}\right), \\ &\qquad t \geq 0 \\ x_i(s) &= \phi_i(s), \quad s \in [-h_2, 0], \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$, $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector, $\phi_i \in C^1([-h_2, 0], \mathbb{R}^{n_i})$ are initial conditions with the norm defined by

$$\|\phi_i\| = \sup_{s\in[-h_2,0]} \left\{\|\phi_i(s)\|, \|\dot{\phi}_i(s)\|\right\}, \tag{2}$$

and $h_{ij}(t)$ are time-delay functions which are continuous and satisfy

$$0 \leq h_1 \leq h_{ij}(t) \leq h_2, \tag{3}$$

for all $t \geq 0$ and all $i, j = 1, 2, \ldots, N$. The state matrices A_i and A_{ij} are of appropriate dimensions. The nonlinear perturbations $f_i(\cdot)$ are continuous, satisfying the following conditions.

There exist $a_i, a_{ij} > 0$ such that

$$\begin{aligned} &\left\|f_i\left(t, x_i(t), \left\{x_j\left(t - h_{ij}(t)\right)\right\}_{j=1,\, j\neq i}^{N}\right)\right\| \\ &\quad \leq a_i \|x_i(t)\| + \sum_{j\neq i,\, j=1}^{N} a_{ij} \left\|x_j\left(t - h_{ij}(t)\right)\right\|. \end{aligned} \tag{4}$$

For $\phi = [\phi_1, \phi_2, \ldots, \phi_N]^T \in C^1([-h_2, 0], \mathbb{R}^L)$, where $\phi_i \in C^1([-h_2, 0], \mathbb{R}^{n_i})$ and $L = \sum_{i=1}^{N} n_i$, we define the norm of ϕ as follows:

$$\|\phi\|_{C^1} = \sqrt{\sum_{i=1}^{N} \|\phi_i\|_{C^1}}. \tag{5}$$

The segment of the trajectory of $x(t)$ denoted by x_t is defined by $x_t = \{x(t+s) : s \in [-h_2, 0]\}$ with the norm defined by $\|x_t\| = \sup_{s\in[-h_2,0]} \|x(t+s)\|$.

Definition 1 (see [29]). For given positive numbers c_1, c_2, T and a symmetric positive definite matrix M, the nonlinear large-scale system (1) is finite-time stable (FTS) with respect to (c_1, c_2, T, M) if the following condition holds:

$$\begin{aligned} &\sup_{-h_2\leq s\leq 0} \left\{\phi(s)^T M \phi(s), \dot{\phi}(s)^T M \dot{\phi}(s)\right\} < c_1 \Longrightarrow \\ &\qquad x^T(t) M x(t) < c_2, \\ &\qquad\qquad \forall t \in [0, T]. \end{aligned} \tag{6}$$

Proposition 2 (see [22] Schur complement lemma). *Given matrices X, Y, Z, where $Y = Y^T > 0$ and $X = X^T$, $X + Z^T Y^{-1} Z < 0$ if and only if $\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0$.*

Proposition 3 (see [22] Jensen-type integral inequality). *For any constant matrix $Z = Z^T > 0$ and scalar $h > 0$ such that the following integrals are well defined,*

$$-\int_{t-h}^{t} x(s)^T Z x(s)\, ds \le -\frac{1}{h}\left(\int_{t-h}^{t} x(s)\, ds\right)^T Z \left(\int_{t-h}^{t} x(s)\, ds\right). \tag{7}$$

Proposition 4 (see [22] Cauchy matrix inequality). *For any $x, y \in \mathbb{R}^n$ and positive definite matrix $M \in \mathbb{R}^{n\times n}$, we have*

$$2x^T y \le y^T M y + x^T M^{-1} x. \tag{8}$$

Lemma 5 (see [18]). *For a given matrix $R > 0$, $h_m \le h(t) \le h_M$ and any appropriate dimension matrix χ which satisfies $\begin{bmatrix} \overline{R} & \chi \\ \chi^T & \overline{R} \end{bmatrix} \ge 0$. The following inequality holds for all continuously differentiable function $x(t)$:*

$$-(h_M - h_m)\int_{t-h_M}^{t-h_m} \dot{x}(s)^T R \dot{x}(s)\, ds \le -\alpha^T(t)\begin{bmatrix} \overline{R} & \chi \\ \chi^T & \overline{R} \end{bmatrix}\alpha(t), \tag{9}$$

where

$$\begin{aligned}
\alpha(t) &= \left[\alpha_1^T(t), \alpha_2^T(t), \alpha_3^T(t), \alpha_4^T(t)\right]^T, \\
\alpha_1(t) &= x(t-h_m) - x(t-h(t)), \\
\alpha_2(t) &= x(t-h_m) + x(t-h(t)) - \left(\frac{2}{h(t)-h_m}\right)\int_{t-h(t)}^{t-h_m} x(s)\, ds, \\
\alpha_3(t) &= x(t-h(t)) - x(t-h_M), \\
\alpha_4(t) &= x(t-h(t)) + x(t-h_M) - \left(\frac{2}{h_M-h(t)}\right)\int_{t-h_M}^{t-h(t)} x(s)\, ds, \\
\overline{R} &= \begin{bmatrix} R & 0 \\ * & 3R \end{bmatrix}.
\end{aligned} \tag{10}$$

3. Finite-Time Stability of Nonlinear Large-Scale Systems

In this section, we derive FTS criteria for large-scale interconnected system with time-varying delays. For the sake of simplicity, the following notations will be used:

For $i, j, k = 1, \ldots, N$, $j \ne i$ and $k \ne j$, we let

$$\Theta_{i,i}^i = P_i A_i + A_i^T P_i + 2Q_i - \left(e^{\beta h_1} + e^{\beta h_2}\right) R_i - \beta P + a_i I;$$

$$\Theta_{i,N+1}^i = e^{\beta h_1} R_i;$$

$$\Theta_{i,N+2}^i = e^{\beta h_2} R_i;$$

$$\Theta_{i,N+3}^i = A_i^T P_i;$$

$$\Theta_{j,j}^i = \left(2 + 2a_{ji}\right) I + e^{\beta h_2}\left(-8U_i + T_{1i} + T_{1i}^T + \frac{T_{2i}}{2} + \frac{T_{2i}^T}{2} + T_{3i} + T_{3i}^T - \frac{T_{4i}}{2} - \frac{T_{4i}^T}{2}\right);$$

$$\Theta_{j,N+1}^i = e^{\beta h_2}\left(4U_i - T_{1i} - T_{2i} - T_{3i} - T_{4i}\right);$$

$$\Theta_{j,N+2}^i = e^{\beta h_2}\left(-4U_i - T_{1i} + \frac{T_{2i}}{2} + \frac{T_{2i}^T}{2} + T_{3i} - T_{4i} - \frac{T_{4i}^T}{2}\right);$$

$$\Theta_{j,N+4}^i = e^{\beta h_2}\left(3U_i + T_{3i}^T + \frac{T_{4i}^T}{2}\right);$$

$$\Theta_{j,N+5}^i = e^{\beta h_2}\left(-3U_i + T_{2i} + \frac{T_{4i}^T}{2}\right);$$

$$\Theta_{N+1,N+1}^i = -e^{\beta h_1} Q_i - e^{\beta h_1} R_i - 4e^{\beta h_1} U_i,$$

$$\Theta_{N+1,N+2}^i = e^{\beta h_2}\left(T_{1i} - T_{2i} + T_{3i} - T_{4i}\right);$$

$$\Theta_{N+1,N+4}^i = 3e^{\beta h_2} U_i;$$

$$\Theta_{N+1,N+5}^i = e^{\beta h_2}\left(T_{2i} + T_{4i}\right);$$

$$\Theta_{N+2,N+2}^i = -2e^{\beta h_2} Q_i + e^{\beta h_2}\left(-R_i - 4U_i - T_{2i} - T_{2i}^T\right);$$

$$\Theta_{N+2,N+4}^i = e^{\beta h_2}\left(T_{3i}^T + \frac{T_{4i}}{2}\right);$$

$$\Theta_{N+2,N+5}^i = e^{\beta h_2}\left(-3U_i + \frac{T_{4i}}{2}\right),$$

$$\Theta_{N+3,N+3}^i = \left(h_1^2 + h_2^2\right) R_i - 2P_i + \left(h_2^2 - h_1^2\right) U_i;$$

$$\Theta_{N+4,N+4}^i = -3e^{\beta h_2} U_i;$$

$$\Theta_{N+4,N+5}^i = -e^{\beta h_2} T_{4i},$$

$$\Theta_{N+5,N+5}^i = -3e^{\beta h_2} U_i;$$

$$\Theta_{N+5+j,N+5+j}^i = -I;$$

$$\Theta_{i,N+5+j}^i = P_i A_{ij};$$

$$\Theta_{N+5+i,N+5+i}^i = -I;$$

$$\Theta_{i,N+5+i}^i = \sqrt{\xi_i} P_i;$$

$$\Theta_{2N+5+j,2N+5+j}^i = -I;$$

$$\Theta_{N+5,2N+5+j}^i = P_i A_{ij};$$

$$\Theta_{2N+5+i,2N+5+i}^i = -I;$$

$$\Theta^i_{N+5,\,2N+5+i} = \sqrt{\xi_i}P_i;$$

$$\zeta_i^T(t) = \left[\vartheta_i^1, \ldots, \vartheta_i^N, x_i(t-h_1)^T, x_i(t-h_2)^T, \dot{x}(t)^T, \omega_1(t)^T, \omega_2(t)^T\right];$$

$$\vartheta_i^j = x_i\left(t-h_{ji}(t)\right)^T, \quad i \neq j, \ \vartheta_i^i = x_i^T(t);$$

$$\omega_1(t) = \left(\frac{2}{h(t)-h_1}\right)\int_{t-h(t)}^{t-h_1} x(s)\,ds;$$

$$\omega_2(t) = \left(\frac{2}{h_2-h(t)}\right)\int_{t-h_2}^{t-h(t)} x(s)\,ds;$$

$$\Omega^i = \begin{bmatrix} \Omega^i_{11} & \Omega^i_{12} & \cdots & \Omega^i_{1(N+5)} \\ * & \Omega^i_{22} & \cdots & \Omega^i_{2(N+5)} \\ \cdot & \cdot & \cdots & \cdot \\ * & * & \cdots & \Omega^i_{(N+5)(N+5)} \end{bmatrix};$$

$$\Omega^i_{i,i} = P_iA_i + A_i^TP_i + 2Q_i - \left(e^{\beta h_1} + e^{\beta h_2}\right)R_i - \beta P + a_iI + \sum_{j\neq i,\,j=1}^{N} P_iA_iA_i^TP_i + \xi_iP_i^2;$$

$$\Omega^i_{i,\,N+1} = e^{\beta h_1}R_i;$$

$$\Omega^i_{i,\,N+2} = e^{\beta h_2}R_i;$$

$$\Omega^i_{i,\,N+3} = A_i^TP_i;$$

$$\Omega^i_{j,j} = \left(2+2a_{ji}\right)I + e^{\beta h_2}\left(-8U_i + T_{1i} + T_{1i}^T + \frac{T_{2i}}{2} + \frac{T_{2i}^T}{2} + T_{3i} + T_{3i}^T - \frac{T_{4i}}{2} - \frac{T_{4i}^T}{2}\right);$$

$$\Omega^i_{j,\,N+1} = e^{\beta h_2}\left(4U_i - T_{1i} - T_{2i} - T_{3i} - T_{4i}\right);$$

$$\Omega^i_{j,\,N+2} = e^{\beta h_2}\left(-4U_i - T_{1i} + \frac{T_{2i}}{2} + \frac{T_{2i}^T}{2} + T_{3i} - T_{4i} - \frac{T_{4i}^T}{2}\right);$$

$$\Omega^i_{j,\,N+4} = e^{\beta h_2}\left(3U_i + T_{3i}^T + \frac{T_{4i}^T}{2}\right);$$

$$\Omega^i_{j,\,N+5} = e^{\beta h_2}\left(-3U_i + T_{2i} + \frac{T_{4i}^T}{2}\right);$$

$$\Omega^i_{N+1,\,N+1} = -e^{\beta h_1}Q_i - e^{\beta h_1}R_i - 4e^{\beta h_1}U_i;$$

$$\Omega^i_{N+1,\,N+2} = e^{\beta h_2}\left(T_{1i} - T_{2i} + T_{3i} - T_{4i}\right);$$

$$\Omega^i_{N+1,\,N+4} = 3e^{\beta h_2}U_i;$$

$$\Omega^i_{N+1,\,N+5} = e^{\beta h_2}\left(T_{2i} + T_{4i}\right);$$

$$\Omega^i_{N+2,\,N+2} = -2e^{\beta h_2}Q_i + e^{\beta h_2}\left(-R_i - 4U_i - T_{2i} - T_{2i}^T\right);$$

$$\Omega^i_{N+2,\,N+4} = e^{\beta h_2}\left(T_{3i}^T + \frac{T_{4i}}{2}\right);$$

$$\Omega^i_{N+2,\,N+5} = e^{\beta h_2}\left(-3U_i + \frac{T_{4i}}{2}\right);$$

$$\Omega^i_{N+3,\,N+3} = \left(h_1^2 + h_2^2\right)R_i - 2P_i + \left(h_2^2 - h_1^2\right)U_i + \sum_{j\neq i,\,j=1}^{N} P_iA_iA_i^TP_i + \xi_iP_i^2;$$

$$\Omega^i_{N+4,\,N+4} = -3e^{\beta h_2}U_i;$$

$$\Omega^i_{N+4,\,N+5} = -e^{\beta h_2}T_{4i};$$

$$\Omega^i_{N+5,\,N+5} = -3e^{\beta h_2}U_i;$$

$$f_i(\cdot) = f_i\left(t, x_i(t), \left\{x_j\left(t-h_{ij}(t)\right)\right\}_{j=1,\,j\neq i}^{N}\right);$$

$$\xi_i = a_i + \sum_{j\neq i,\,j=1}^{N} a_{ij};$$

$$\overline{P}_i = M^{-1/2}P_iM^{-1/2};$$

$$\alpha_1 = \min_{i=1,\ldots,N}\lambda_{\min}\left(\overline{P}_i\right);$$

$$\alpha_2 = \max_{i=1,\ldots,N}\left\{\lambda_{\max}(P_i) + \beta^{-1}\lambda_{\max}(Q_i) + \left(h_1^3 + h_2^3\right)\lambda_{\max}(R_i) + \left(h_2 - h_1\right)^3\lambda_{\max}(U_i)\right\};$$

$$\overline{U_i} = \begin{bmatrix} U_i & 0 \\ * & 3U_i \end{bmatrix};$$

$$\chi_i = \begin{bmatrix} T_{1i} & T_{2i} \\ T_{3i} & T_{4i} \end{bmatrix}. \tag{11}$$

The other matrices $\Theta^i_{j,k}$ and $\Omega^i_{j,k}$ which do not appear above would be defined to be zero matrices with appropriate dimensions.

Theorem 6. *For given positive numbers c_1, c_2, T and a symmetric positive definite matrix M, if there exist symmetric positive definite matrices P_i, Q_i, R_i, U_i and matrices $T_{1i}, \ldots, T_{4i}$ of appropriate dimensions such that the conditions*

$$\Theta^i = \begin{bmatrix} \Theta^i_{11} & \Theta^i_{12} & \cdots & \Theta^i_{1(3N+5)} \\ * & \Theta^i_{22} & \cdots & \Theta^i_{2(3N+5)} \\ \cdot & \cdot & \cdots & \cdot \\ * & * & \cdots & \Theta^i_{(3N+5)(3N+5)} \end{bmatrix} < 0, \tag{12}$$

$$i = 1, 2, \ldots, N$$

$$\frac{\alpha_2 c_1}{\alpha_1} \le c_2 e^{-\beta T}, \tag{13}$$

hold, then system (1) is FTS with respect to (c_1, c_2, T, M).

Proof. We choose the following Lyapunov-Krasovskii functional:

$$V(t, x_t) = \sum_{i=1}^{N} \sum_{j=1}^{6} V_{ij}(t, x_t), \tag{14}$$

where

$$\begin{aligned}
V_{i1}(t, x_t) &= x_i^T(t) P_i x_i(t), \\
V_{i2}(t, x_t) &= \int_{t-h_1}^{t} e^{\beta(t-s)} x^T(s) Q_i x(s)\, ds, \\
V_{i3}(t, x_t) &= \int_{t-h_2}^{t} e^{\beta(t-s)} x^T(s) Q_i x(s)\, ds, \\
V_{i4}(t, x_t) &= h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{\beta(t-\tau)} \dot{x}_i^T(\tau) R_i \dot{x}_i(\tau)\, d\tau\, ds, \\
V_{i5}(t, x_t) &= h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{\beta(t-\tau)} \dot{x}_i^T(\tau) R_i \dot{x}_i(\tau)\, d\tau\, ds, \\
V_{i6}(t, x_t) &\\
&= (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+s}^{t} e^{\beta(t-\tau)} \dot{x}_i^T(\tau) U_i \dot{x}_i(\tau)\, d\tau\, ds.
\end{aligned} \tag{15}$$

We first show that

$$\alpha_1 x^T(t) M x(t) \le V(t, x_t), \quad \forall t : 0 \le t \le T. \tag{16}$$

From

$$\begin{aligned}
V_{i1}(t, x_t) &= x_i^T(t) P_i x_i(t) \\
&= x_i^T(t) M^{1/2} M^{-1/2} P_i M^{-1/2} M^{1/2} x_i(t) \\
&= x_i^T(t) M^{1/2} \overline{P}_i M^{1/2} x_i(t) \\
&\ge \lambda_{\min}(\overline{P}_i) x_i^T(t) M x_i(t),
\end{aligned} \tag{17}$$

we have

$$V(t, x_t) \ge \alpha_1 x^T(t) M x(t), \tag{18}$$

where $\alpha_1 = \min_{i=1,\dots,N} \lambda_{\min}(\overline{P}_i)$. Thus, (16) holds. Similarly, we obtain the following estimation:

$$\begin{aligned}
V(0, x_0) &\le \alpha_2 \sup_{-h_2 \le s \le 0} \left\{ \phi(s)^T M \phi(s), \dot{\phi}(s)^T M \dot{\phi}(s) \right\} \\
&\le \alpha_2 c_1.
\end{aligned} \tag{19}$$

Taking the derivative of $V(t, x_t)$ with respect to t along the trajectory of solution of the system (1), we obtain

$$\begin{aligned}
\dot{V}_{i1}(\cdot) &= 2x_i^T(t) P_i \dot{x}(t) = 2x_i^T(t) \\
&\quad \cdot P_i \left[A_i x_i(t) + \sum_{j \ne i, j=1}^{N} A_{ij} x_j(t - h_{ij}(t)) + f_i(\cdot) \right], \\
\dot{V}_{i2}(\cdot) &= x_i^T(t) Q_i x_i(t) - e^{\beta h_1} x_i^T(t - h_1) Q_i x_i(t - h_1) \\
&\quad + \beta V_{i2}(\cdot), \\
\dot{V}_{i3}(\cdot) &= x_i^T(t) Q_i x_i(t) - e^{\beta h_2} x_i^T(t - h_1) Q_i x_i(t - h_2) \\
&\quad + \beta V_{i3}(\cdot), \\
\dot{V}_{i4}(\cdot) &\le h_1^2 \dot{x}_i^T(t) R_i \dot{x}_i(t) \\
&\quad - h_1 e^{\beta h_1} \int_{t-h_1}^{t} \dot{x}_i^T(s) R_i \dot{x}_i(s)\, ds + \beta V_{i4}(\cdot), \\
\dot{V}_{i5}(\cdot) &\le h_2^2 \dot{x}_i^T(t) R_i \dot{x}_i(t) \\
&\quad - h_2 e^{\beta h_2} \int_{t-h_2}^{t} \dot{x}_i^T(s) R_i \dot{x}_i(s)\, ds + \beta V_{i5}(\cdot), \\
\dot{V}_{i6}(\cdot) &\le (h_2 - h_1)^2 \dot{x}_i^T(t) U_i \dot{x}_i(t) - (h_2 - h_1) \\
&\quad \cdot e^{\beta h_2} \int_{t-h_2}^{t-h_1} \dot{x}_i^T(s) U_i \dot{x}_i(s)\, ds + \beta V_{i6}(\cdot).
\end{aligned} \tag{20}$$

From Proposition 4, we have

$$\begin{aligned}
&2x_i^T(t) P_i \sum_{j \ne i, j=1}^{N} A_{ij} x_j(t - h_{ij}(t)) \\
&\quad \le \sum_{j \ne i, j=1}^{N} x_i(t)^T P_i A_i A_i^T P_i x_i(t) \\
&\qquad + \sum_{j \ne i, j=1}^{N} x_i(t - h_{ij}(t))^T x_i(t - h_{ij}(t)).
\end{aligned} \tag{21}$$

It follows from (4) and Proposition 4 that

$$\begin{aligned}
&2x_i^T(t) P_i f_i(\cdot) \le 2 \left\| x_i^T P_i \right\| \\
&\quad \cdot \left\{ a_i \left\| x_i(t) \right\| + \sum_{j \ne i, j=1}^{N} a_{ij} \left\| x_j(t - h_{ij}(t))_i \right\| \right\}, \\
&\quad \le \xi_i \left\| x_i(t)^T P_i \right\|^2 + a_i \left\| x_i(t) \right\|^2 \\
&\quad + \sum_{j \ne i, j=1}^{N} a_{ij} \left\| x_j(t - h_{ij}(t)) \right\|^2.
\end{aligned} \tag{22}$$

From (22), we have

$$\begin{aligned}\dot{V}_{i1}(\cdot) \le{}& x_i^T(t)\left[P_iA_i + A_i^TP_i\right]x_i(t)\\&+\sum_{j\neq i,\,j=1}^{N} x_i(t)^T P_iA_iA_i^TP_ix_i(t)\\&+\sum_{j\neq i,\,j=1}^{N} x_i\left(t-h_{ij}(t)\right)^T x_i\left(t-h_{ij}(t)\right)\\&+\xi_i\left\|x_i(t)^TP_i\right\|^2 + a_i\left\|x_i(t)\right\|^2\\&+\sum_{j\neq i,\,j=1}^{N} a_{ij}\left\|x_j\left(t-h_{ij}(t)\right)\right\|^2.\end{aligned} \tag{23}$$

From Proposition 3 and Newton-Leibniz formula, we obtain

$$\begin{aligned}&-h_ie^{-\beta h_k}\int_{t-h_k}^{t}\dot{x}_i^T(s)R_i\dot{x}_i(s)\,ds\\&\le-\left[\int_{t-h_k}^{t}\dot{x}_i(s)\,ds\right]^T R_i\left[\int_{t-h_k}^{t}\dot{x}_i(s)\,ds\right]\\&=-\left[x_i(t)-x_i(t-h_k)\right]^T R\left[x_i(t)-x_i(t-h_k)\right],\end{aligned} \tag{24}$$

for $k = 1, 2$. It follows that

$$\begin{aligned}\dot{V}_{i4}(\cdot) \le{}& h_1^2\dot{x}_i^T(t)R_i\dot{x}_i(t)\\&-h_1e^{\beta h_1}\left[x_i(t)-x_i(t-h_1)\right]^T\\&\cdot R\left[x_i(t)-x_i(t-h_1)\right]+\beta V_{i4}(\cdot),\\\dot{V}_{i5}(\cdot) \le{}& h_2^2\dot{x}_i^T(t)R_i\dot{x}_i(t)\\&-h_2e^{\beta h_2}\left[x_i(t)-x_i(t-h_2)\right]^T\\&\cdot R\left[x_i(t)-x_i(t-h_2)\right]+\beta V_{i5}(\cdot).\end{aligned} \tag{25}$$

From Lemma 5, we have

$$\begin{aligned}&-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}_i^T(s)U_i\dot{x}_i(s)\,ds\\&\le-\alpha^T(t)\begin{bmatrix}\overline{U_i} & \chi_i\\ \chi_i^T & \overline{U_i}\end{bmatrix}\alpha(t).\end{aligned} \tag{26}$$

Thus, from (26), we obtain

$$\begin{aligned}\dot{V}_{i6}(\cdot) \le{}& (h_2-h_1)^2\dot{x}_i^T(t)U_i\dot{x}_i(t)+\beta V_{i6}(\cdot)\\&-e^{\beta h_2}\alpha^T(t)\begin{bmatrix}\overline{U_i} & \chi_i\\ \chi_i^T & \overline{U_i}\end{bmatrix}\alpha(t).\end{aligned} \tag{27}$$

Hence, we obtain the estimation of $\dot{V}(t, x_t)$ as

$$\begin{aligned}\dot{V}(t,x_t) \le{}& x_i^T(t)\left[P_iA_i + A_i^TP_i\right]x_i(t)\\&+\sum_{j\neq i,\,j=1}^{N} x_i(t)^T P_iA_iA_i^TP_ix_i(t)\\&+\sum_{j\neq i,\,j=1}^{N} x_i\left(t-h_{ij}(t)\right)^T x_i\left(t-h_{ij}(t)\right)\\&+\xi_i\left\|x_i(t)^TP_i\right\|^2 + a_i\left\|x_i(t)\right\|^2\\&+\sum_{j\neq i,\,j=1}^{N} a_{ij}\left\|x_j\left(t-h_{ij}(t)\right)\right\|^2 + x_i^T(t)\\&\cdot Q_ix_i(t)-e^{\beta h_1}x_i^T(t-h_1)Q_ix_i(t-h_1)\\&+x_i^T(t)Q_ix_i(t)-e^{\beta h_2}x_i^T(t-h_1)\\&\cdot Q_ix_i(t-h_2)+h_1^2\dot{x}_i^T(t)R_i\dot{x}_i(t)\\&-h_1e^{\beta h_1}\left[x_i(t)-x_i(t-h_1)\right]^T\\&\cdot R\left[x_i(t)-x_i(t-h_1)\right]+h_2^2\dot{x}_i^T(t)R_i\dot{x}_i(t)\\&-h_2e^{\beta h_2}\left[x_i(t)-x_i(t-h_2)\right]^T\\&\cdot R\left[x_i(t)-x_i(t-h_2)\right]+(h_2-h_1)^2\\&\cdot\dot{x}_i^T(t)U_i\dot{x}_i(t)-e^{\beta h_2}\alpha^T(t)\\&\cdot\begin{bmatrix}\overline{U_i} & \chi_i\\ \chi_i^T & \overline{U_i}\end{bmatrix}\alpha(t).\end{aligned} \tag{28}$$

From (1) and Proposition 4, we get

$$\begin{aligned}0 ={}& -2\dot{x}(t)^TP_i\left[\dot{x}(t)-A_ix_i(t)\right.\\&\left.-\sum_{j\neq i,\,j=1}^{N}A_{ij}x_j\left(t-h_{ij}(t)\right)-f_i(\cdot)\right]\le-2\dot{x}(t)^T\\&\cdot P_i\left[\dot{x}(t)-A_ix_i(t)\right]\\&+\sum_{j\neq i,\,j=1}^{N}\dot{x}_i(t)^TP_iA_iA_i^TP_i\dot{x}_i(t)\\&+\sum_{j\neq i,\,j=1}^{N} x_i\left(t-h_{ij}(t)\right)^T x_i\left(t-h_{ij}(t)\right)\\&+\xi_i\left\|\dot{x}_i(t)^TP_i\right\|^2 + a_i\left\|x_i(t)\right\|^2\\&+\sum_{j\neq i,\,j=1}^{N} a_{ij}\left\|x_j\left(t-h_{ij}(t)\right)\right\|^2.\end{aligned} \tag{29}$$

Adding inequality (29) into the left-hand side of (28) and from the fact that

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j\neq i,\, j=1}^{N} x_j\left(t-h_{ij}(t)\right)^T x_j\left(t-h_{ij}(t)\right)\\
&\quad=\sum_{j=1}^{N}\sum_{i\neq j,\, i=1}^{N} x_i\left(t-h_{ji}(t)\right)^T x_i\left(t-h_{ji}(t)\right)\\
&\quad=\sum_{i=1}^{N}\left[\sum_{j\neq i,\, j=1}^{N} x_i\left(t-h_{ji}(t)\right)^T x_i\left(t-h_{ji}(t)\right)\right],\qquad(30)\\
&\sum_{i=1}^{N}\sum_{j\neq i,\, j=1}^{N} a_{ij}\left\|x_j\left(t-h_{ij}(t)\right)\right\|^2\\
&\quad=\sum_{i=1}^{N}\sum_{j\neq i,\, j=1}^{N} a_{ji}\left\|x_i\left(t-h_{ji}(t)\right)\right\|^2,
\end{aligned}$$

we have

$$\dot{V}(t,x_t)-\beta V(t,x_t)\leq\sum_{i=1}^{N}\zeta_i^T(t)\,\Omega^i\zeta_i(t).\qquad(31)$$

From assumption (12), it follows from Proposition 2 that $\Omega^i<0$, $\forall i=1,\ldots,N$.

Therefore, we have

$$\dot{V}(t,x_t)-\beta V(t,x_t)<0,\quad\forall t:0\leq t\leq T.\qquad(32)$$

Multiply both sides of (32) by $e^{-\beta t}$; we obtain

$$e^{-\beta t}\dot{V}(t,x_t)-\beta e^{-\beta t}\beta V(t,x_t)<0,\quad\forall t:0\leq t\leq T.\qquad(33)$$

Integrating both sides of (33) from 0 to t, we have

$$e^{-\beta t}V(t,x_t)<V(0,x_0),\quad\forall t:0\leq t\leq T.\qquad(34)$$

Hence, from (16) and (19), it follows that

$$\alpha_1 e^{-\beta t}x(t)^T Mx(t)<e^{-\beta t}V(t,x_t)\leq\alpha_2 c_1.\qquad(35)$$

From (13), we have

$$x^T Mx(t)<\frac{\alpha_2 c_1}{\alpha_1}e^{\beta t}\leq c_2,\quad\forall t:0\leq t\leq T.\qquad(36)$$

Therefore, system (1) is FTS with respect to (c_1,c_2,T,M). □

Remark 7. The obtained FTS criteria (12) are delay-dependent and are formulated in terms of solutions of LMIs which can be easily solved by various available algorithms such as MATLAB LMI Toolbox [31].

Remark 8. In Theorem 6, the chosen Lyapunov-Krasovskii functional depends only on the lower and upper bounds of time-delay functions to remove restrictions on the differentiability of time-delay functions. In several existing results on stability of large-scale systems, for example, in [20, 24], the restrictions on the differentiability of time-delay functions are required. As a result, the methods proposed in [20, 24] are not applicable to system (1).

Remark 9. In [6, 12, 13], the free-weighting matrix variables have been introduced to obtain stability conditions for time-delay systems. On the other hand, we have not introduced free-weighting matrix variables in order to derive stability criteria which results in less matrix variables regarding computational burdens.

Remark 10. In Theorem 6, we have used Proposition 3 (Jensen inequality) to estimate some integrals and to derive FTS criterion for large-scale interconnected systems with time-varying delays. Nonetheless, Proposition 3 is rather restrictive and further improvement can be made if new refined Jensen-based inequalities reported in the recent literature, for example, in [16, 17], are used.

Remark 11. In (1), if $f_i(\cdot)\equiv 0$, then we have the following linear large-scale system with interval time-varying delays in interconnection:

$$\begin{aligned}
&\dot{x}_i(t)=A_i x_i(t)+\sum_{j\neq i,\, j=1}^{N}A_{ij}x_j\left(t-h_{ij}(t)\right),\quad t\geq 0,\\
&x_i(s)=\phi_i(s),\quad s\in[-h_2,0].
\end{aligned}\qquad(37)$$

Then, from Theorem 6, we obtain FTS criteria for linear large-scale system (37) as in the following corollary.

Corollary 12. *For given $\beta,T>0$, $c_2>c_1>0$, and let M be a symmetric positive definite matrix. If there exist symmetric positive definite matrices P_i,Q_i,R_i,U_i and matrices $T_{1i},\ldots,T_{4i}$ of appropriate dimensions such that the conditions*

$$\Gamma^i=\begin{bmatrix}\Gamma_{11}^i & \Gamma_{12}^i & \cdots & \Gamma_{1(3N+5)}^i\\ * & \Gamma_{22}^i & \cdots & \Gamma_{2(3N+5)}^i\\ \cdot & \cdot & \cdots & \cdot\\ * & * & \cdots & \Gamma_{(3N+5)(3N+5)}^i\end{bmatrix}<0,\qquad(38)$$

$$i=1,2,\ldots,N$$

$$\frac{\alpha_2 c_1}{\alpha_1}\leq c_2 e^{-\beta T}$$

hold, where, for $i,j,k=1,\ldots,N$, $j\neq i$ and $k\neq j$, we have

$$\begin{aligned}
&\Gamma_{i,i}^i=P_iA_i+A_i^TP_i+2Q_i-\left(e^{\beta h_1}+e^{\beta h_2}\right)R_i-\beta P;\\
&\Gamma_{i,N+1}^i=e^{\beta h_1}R_i;\\
&\Gamma_{i,N+2}^i=e^{\beta h_2}R_i;\\
&\Gamma_{i,N+3}^i=A_i^TP_i;\\
&\Gamma_{j,j}^i=2I+e^{\beta h_2}\left(-8U_i+T_{1i}+T_{1i}^T+\frac{T_{2i}}{2}+\frac{T_{2i}^T}{2}+T_{3i}\right.\\
&\qquad\left.+T_{3i}^T-\frac{T_{4i}}{2}-\frac{T_{4i}^T}{2}\right);\\
&\Gamma_{j,N+1}^i=e^{\beta h_2}\left(4U_i-T_{1i}-T_{2i}-T_{3i}-T_{4i}\right);
\end{aligned}$$

$$\begin{aligned}
\Gamma^i_{j,N+2} &= e^{\beta h_2}\left(-4U_i - T_{1i} + \frac{T_{2i}}{2} + \frac{T^T_{2i}}{2} + T_{3i} - T_{4i} - \frac{T^T_{4i}}{2}\right);\\
\Gamma^i_{j,N+4} &= e^{\beta h_2}\left(3U_i + T^T_{3i} + \frac{T^T_{4i}}{2}\right);\\
\Gamma^i_{j,N+5} &= e^{\beta h_2}\left(-3U_iT_{2i} + \frac{T^T_{4i}}{2}\right);\\
\Gamma^i_{N+1,N+1} &= -e^{\beta h_1}Q_i - e^{\beta h_1}R_i - 4e^{\beta h_1}U_i;\\
\Gamma^i_{N+1,N+2} &= e^{\beta h_2}\left(T_{1i} - T_{2i} + T_{3i} - T_{4i}\right);\\
\Gamma^i_{N+1,N+4} &= 3e^{\beta h_2}U_i;\\
\Gamma^i_{N+1,N+5} &= e^{\beta h_2}\left(T_{2i} + T_{4i}\right);\\
\Gamma^i_{N+2,N+2} &= -2e^{\beta h_2}Q_i + e^{\beta h_2}\left(-R_i - 4U_i - T_{2i} - T^T_{2i}\right);\\
\Gamma^i_{N+2,N+4} &= e^{\beta h_2}\left(T^T_{3i} + \frac{T_{4i}}{2}\right);\\
\Gamma^i_{N+2,N+5} &= e^{\beta h_2}\left(-3U_i + \frac{T_{4i}}{2}\right);\\
\Gamma^i_{N+3,N+3} &= \left(h_1^2 + h_2^2\right)R_i - 2P_i + \left(h_2^2 - h_1^2\right)U_i;\\
\Gamma^i_{N+4,N+4} &= -3e^{\beta h_2}U_i;\\
\Gamma^i_{N+4,N+5} &= -e^{\beta h_2}T_{4i};\\
\Gamma^i_{N+5,N+5} &= -3e^{\beta h_2}U_i;\\
\Gamma^i_{N+5+j,N+5+j} &= -I;\\
\Gamma^i_{i,N+5+j} &= P_iA_{ij};\\
\Gamma^i_{2N+5+j,2N+5+j} &= -I;\\
\Gamma^i_{N+5,2N+5+j} &= P_iA_{ij};
\end{aligned} \tag{39}$$

and other matrices $\Gamma^i_{j,k}$ which do not appear above would be defined to be zero matrices with appropriate dimensions, then system (37) is FTS with respect to (c_1, c_2, T, M).

4. Numerical Examples

In this section, we provide numerical examples to illustrate the effectiveness of our theoretical results.

Example 1. Consider the following nonlinear large-scale systems (1) composed of two machine subsystems which are studied in [23]:

$$\begin{aligned}
\dot{x}_1(t) &= A_1x_1(t) + A_{12}x_2(t - h_{12}(t)) + f_1(t, x_1(t), x_2(t - h_{12}(t))),\\
x_1(s) &= \phi_1(s), \quad s \in [-h_2, 0],\\
\dot{x}_2(t) &= A_2x_2(t) + A_{21}x_2(t - h_{21}(t)) + f_2(t, x_2(t), x_1(t - h_{21}(t))),\\
x_2(s) &= \phi_2(s), \quad s \in [-h_2, 0],
\end{aligned} \tag{40}$$

where the absolute rotor angle and angular velocity of the machine in each subsystem are denoted by $x_1 = [x_{11} \;\; x_{12}]^T$ and $x_2 = [x_{21} \;\; x_{22}]^T$, respectively. The ith subsystem state matrices A_i, the nonlinear perturbations $f_i(\cdot)$, the modulus of the transfer admittance A_{ij}, and the time-varying delays $h_{ij}(t)$ between the two machines in the subsystems are given by

$$\begin{aligned}
A_1 &= \begin{bmatrix} -1.2 & 0.1 \\ 0.2 & -1.3 \end{bmatrix},\\
A_{12} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\\
A_2 &= \begin{bmatrix} -1.1 & 0.2 \\ 0.1 & -1 \end{bmatrix},\\
A_{21} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\\
h_{12}(t) &= 0.1 + 0.4\left|\sin(t)\right|,\\
h_{21}(t) &= 0.1 + 0.4\left|\sin^2(t)\right|,\\
f_1(\cdot) &= 0.1\begin{bmatrix} \sqrt{x_{11}(t)^2 + x_{21}(t - h_{12})^2} \\ \sqrt{x_{12}(t)^2 + x_{22}(t - h_{12})^2} \end{bmatrix},\\
f_2(\cdot) &= 0.1\begin{bmatrix} \sqrt{x_{21}(t)^2 + x_{11}(t - h_{21})^2} \\ \sqrt{x_{22}(t)^2 + x_{12}(t - h_{21})^2} \end{bmatrix}.
\end{aligned} \tag{41}$$

By using LMI Control Toolbox, LMIs (12) and (13) are feasible with solutions given by

$$\begin{aligned}
h_1 &= 0.1,\\
h_2 &= 0.5,\\
\beta &= 0.01,\\
c_1 &= 0.5,\\
c_2 &= 4,\\
T &= 10,\\
M &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\end{aligned}$$

$$P_1 = \begin{bmatrix} 0.6660 & 0.0726 \\ 0.0726 & 0.6042 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.2148 & 0.0140 \\ 0.0140 & 0.2001 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.1426 & 0.0388 \\ 0.0388 & 0.1737 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.9293 & 0.0998 \\ 0.0998 & 0.8446 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 1.4417 & 0.0525 \\ 0.0525 & 1.3487 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 1.9777 & 0.2090 \\ 0.2090 & 1.7997 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.6081 & 0.0421 \\ 0.0421 & 0.5719 \end{bmatrix},$$

$$U_2 = \begin{bmatrix} 2.5098 & 0.2700 \\ 0.2700 & 2.2802 \end{bmatrix},$$

$$T_{11} = \begin{bmatrix} 1.1797 & 0.0822 \\ 0.0892 & 1.0967 \end{bmatrix},$$

$$T_{12} = \begin{bmatrix} 2.8450 & 0.2867 \\ 0.2868 & 2.6001 \end{bmatrix},$$

$$T_{21} = \begin{bmatrix} 1.1520 & 0.0629 \\ 0.0660 & 1.0956 \end{bmatrix},$$

$$T_{22} = \begin{bmatrix} 2.9639 & 0.2834 \\ 0.2835 & 2.7224 \end{bmatrix},$$

$$T_{31} = \begin{bmatrix} -1.5532 & -0.0755 \\ -0.0792 & -1.4895 \end{bmatrix},$$

$$T_{32} = \begin{bmatrix} -3.5433 & -0.3274 \\ -0.3273 & -3.2645 \end{bmatrix},$$

$$T_{41} = \begin{bmatrix} 0.2621 & 0.0168 \\ 0.0163 & 0.2459 \end{bmatrix},$$

$$T_{42} = \begin{bmatrix} 0.1844 & 0.0138 \\ 0.0138 & 0.1729 \end{bmatrix}. \tag{42}$$

Thus, from Theorem 6, large-scale interconnected system is FTS with respect to (c_1, c_2, T, M). The trajectories of solutions of the two subsystems are given in Figure 1. In Figure 2, it is shown that if the initial condition satisfies

$$\sup_{-h_2 \le s \le 0} \left\{ \phi(s)^T M \phi(s), \dot{\phi}(s)^T M \dot{\phi}(s) \right\} < c_1, \tag{43}$$

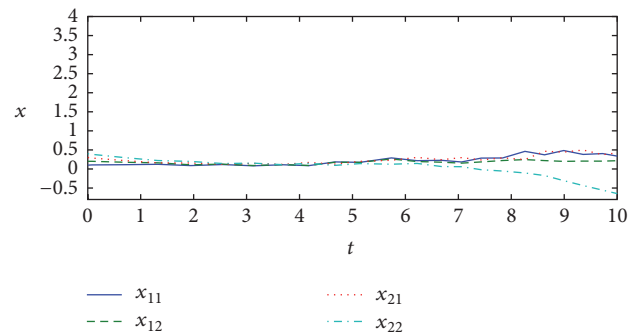

Figure 1: The trajectory of solutions of large-scale systems (40).

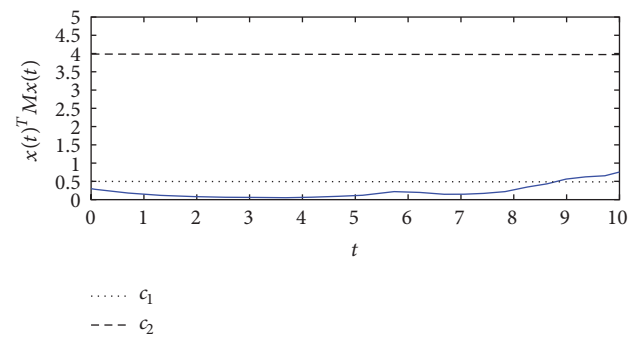

Figure 2: Bounds of $x(t)^T Mx(t)$ where $x(t)$ is the state of large-scale systems (40).

then $x^T(t)Mx(t) < c_2, \forall t \in [0, 10]$, where $x_1(0) = [0.1 \quad 0.2]^T$, $x_2(0) = [0.3 \quad 0.4]^T$, $\phi_1(s) = [0.1e^s \quad 0.2e^s]^T$, and $\phi_2(s) = [0.3e^s \quad 0.4e^s]^T$, $s \in [-h_2, 0)$.

Example 2. Consider the following bidirectional associative memory (BAM) neural networks which are studied in [32]:

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + Cf(y(t)) + Ef(y(t - h(t))) \\ &\quad + u(t), \\ z_1(t) &= f(y(t)) + f(y(t - h(t))) + u(t), \\ \dot{y}(t) &= -By(t) + Dg(x(t)) + Fg(x(t - \tau(t))) \\ &\quad + v(t) \\ z_2(t) &= g(x(t)) + g(x(t - \tau(t))) + v(t), \\ &\qquad t \ge t_0, \end{aligned} \tag{44}$$

where $A = \left[\begin{smallmatrix} 1.8 & 0 \\ 0 & 2.2 \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]$, $E = \left[\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.1 \end{smallmatrix}\right]$, $B = \left[\begin{smallmatrix} 2.5 & 0 \\ 0 & 2.2 \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right]$, $F = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $L = \left[\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.1 \end{smallmatrix}\right]$, $H = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $f_i(s) = s$, $g_i(s) = 0.1s$, $(i = 1, 2)$, and $h(t)$ and $\tau(t)$ are time-varying delays which are continuous but not necessarily differentiable. In Table 1, we give a comparison for maximum allowable upper bounds of time-varying delay, $h_1 \le h(t) \le h_2$ and $h_1 \le \tau(t) \le h_2$, which are obtained from Theorem 3.1 in [32] and our main result, Theorem 6.

Table 1: Maximum allowable upper bounds h_2 of the time-varying delay for different values of the lower bounds h_1 in example of [32].

h_1	Theorem 3.1 in [32]	Theorem 6
0.1	9.4999	9.5167
0.5	10.2013	12.0028
0.75	10.2068	11.7519
1	10.2110	11.4846

5. Conclusion

In this paper, new delay-dependent FTS criteria of nonlinear large-scale interconnected systems with time-varying delay have been derived in terms of solutions of LMIs which could be solved by various available algorithms. By choosing an appropriate Lyapunov-Krasovskii functional and then by using an improved integral inequality, it has been shown by two numerical examples that the obtained FTS criteria are effective and less conservative than some existing results.

Notations

$\mathbb{R}^n$:	The n-dimensional Euclidean space
$\mathbb{R}^{n\times n}$:	The set of all $n \times n$ real matrices
$\mathbb{N}$:	The set of all positive integers
diag $\{\cdot\}$:	The block diagonal matrix
A^T:	The transposition of matrix A
$A \geq 0\,(A > 0)$:	A is a semipositive definite matrix (positive)
$A \geq B\,(A > B)$:	$A - B \geq 0\,(A - B > 0)$
$\lambda_{\max}(A)$:	$\max\{\mathrm{Re}(\lambda) : \lambda$ is eigenvalue of $A\}$
$\lambda_{\min}(A)$:	$\min\{\mathrm{Re}(\lambda) : \lambda$ is eigenvalue of $A\}$
$\lVert\cdot\rVert$:	The usual Euclidean norm (or the induced matrix norm).

Competing Interests

The authors declare that they have no competing interests.

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