Adaptive Neural Network Control of Serial Variable Stiffness Actuators

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This paper focuses on modeling and control of a class of serial variable stiffness actuators (SVSAs) based on level mechanisms for robotic applications. A multi-input multi-output complex nonlinear dynamic model is derived to fully describe SVSAs and the relative degree of the model is determined accordingly. Due to nonlinearity, high coupling, and parametric uncertainty of SVSAs, a neural network-based adaptive control strategy based on feedback linearization is proposed to handle system uncertainties. The feasibility of the proposed approach for position and stiffness tracking of SVSAs is verified by simulation results.

1. Introduction

Variable stiffness actuators (VSAs), which can increase safety in physical human-robot interaction and meet the dynamic requirements of robots in unknown environments, have been developed for a new generation of robots [1–11]. VSAs present advantages in terms of mechanical stiffness adjusting, energy storing, and force sensing capabilities [3]. According to the design configurations, there are two typical mechanical arrangements for VSAs: one is the antagonistic configuration with a pair of actuators coupled via nonlinear springs [10], and the other is the serial configuration with two independent motors, where a principal motor drives the joint position through a compliant transmission, and a secondary motor adjusts the stiffness [11].

Antagonistic-type VSAs need two driving units to change the stiffness and position synchronously, which lead to high energy consumption and complex control design. Serial-type VSAs (SVSAs) do not have this requirement and have gained more attention. Recently, new types of SVSAs have been developed based on level arm mechanisms, where the mechanical stiffness is regulated by moving one of the level points: a pivot point, a spring located point, or a force point [12–14]. Thus, the SVSAs are not significantly influenced by the coupling between the load and the stiffness transmission mechanism. It also brings higher energy efficiency in unloaded conditions [15]. Based on the variable ratio lever principle by changing the pivot position, a novel compact rotational SVSA with an Archimedean spiral relocation mechanism (ASRM) was developed to obtain compact mechanical structure, large adjustable stiffness range, and better force transmission ability in [15]. In this innovative design, the ASRM is applied to transfer the rotation of the Archimedean spiral cam into the linear motion of the pivot, and an elastic force transmission mechanism with a spring shaft vertical to the output link is proposed to achieve large deflection angle and high energy storage ability.

VSAs are multi-input multi-output (MIMO), highly coupled, and complex nonlinear systems which include both structured and unstructured uncertainties [16]. In VSAs, the stiffness variation brings physical modifications, requiring the control system to quickly transit among different operating conditions. In addition, the coupling between stiffness and positioning mechanisms and the increased system dimensionality complicate the entire control system [16]. As a result, it is challenging to design control strategies for VSAs...
in robotic applications. Different control strategies have been proposed to for VSAs, where the simplest one is the PD control in [16]. However, the performance of the PD control highly depends on PD gains due to the high nonlinearity of VSAs. In [17, 18], a classical nonlinear control technique termed feedback linearization was explored for VSAs. Nevertheless, feedback linearization is very sensitive to modeling accuracy so that significant efforts in system modeling and parameter identification are required to achieve a desired performance. Based on the feedback linearization, some advanced control methods, such as gain-scheduling control [19], active damping control [20], backstepping control [21], nonlinear model predictive control [22], and output feedback control [23], have been integrated to improve stiffness and position tracking performances of VSAs. Although these control methods are proved to be effective for VSAs, they may exhibit undesired performances especially if the VSA dynamics are highly nonlinear and experience large parameter variations due to the changes of the actuator stiffness and load conditions [24].

Neural networks (NNs) have been widely applied to control complex nonlinear systems, especially complex robotic systems in recent years [25–37]. Compared with the traditional adaptive control, NN adaptive control (NNAC) has two prominent attractive features [38]. Firstly, due to the universal approximation property of NNs, the difficulty of system modeling in many practical control problems can be largely alleviated. Secondly, due to the practical persistently exciting (PE) condition of NNs, parameter convergence is easier to be obtained during control resulting in an exact online modeling and superior exponential tracking. Although NNAC is promising for complex nonlinear systems like VSAs, it is seldom applied to VSAs except [39], where antagonistic VSAs are considered in [39].

This paper focuses on modeling and control of a class of VSAs based on level mechanisms. Firstly, a MIMO complex nonlinear dynamic model is derived to fully describe VSAs and the relative degree of the model is determined accordingly; secondly, based on the feedback linearization, a direct NNAC strategy is proposed to govern the SVSA model; finally, high-fidelity simulations are provided to verify the effectiveness of the proposed approach. Compared with other control strategies for VSAs, the proposed NNAC can deal with high coupling and parametric uncertainties of VSAs. In addition, the control on both position and stiffness tracking remain effective under load changing conditions.

2. Serial Variable Stiffness Actuator Modeling

As illustrated in Figure 1, a compact rotational SVSA consists of a variable stiffness mechanism (VSM), a principal motor, and a secondary motor, where the principal motor drives the output link motion through the spring transmission, and a secondary motor adjusts the stiffness of the actuator by changing the position of the pivot with an ASRM [15].

By considering gravity and external loads, the dynamic model of VSAs is represented as follows:

\[ M\ddot{q} + D\dot{q} + \tau_e(\theta_2, \varphi) + \tau_g(q) = \tau_{ext} \]
\[ B_1\ddot{\theta}_1 + D_1\dot{\theta}_1 - \tau_e(\theta_2, \varphi) = u_1 \]
\[ B_2\ddot{\theta}_2 + D_2\dot{\theta}_2 + \tau_e(\theta_2, \varphi) = u_2, \]

where \( q \) is a position of the output link, \( \theta_i \) with \( i = 1, 2 \) is a position associated with each motor, \( \varphi = q - \theta_i \) is a deflection angle of the elastic transmission, \( M \) is an inertia of the output link, \( B_i \) is a reflected inertia of each motor, \( D \) is a reflected damping of the spring link, \( D_i \) is a reflected damping of each motor, \( \tau_{ext} \) is an external torque, \( \tau_e \) is a coupling reaction torque, and \( \tau_g(\theta) \) is a gravity torque.

The elastic torque across the transmission is given by

\[ \tau_e = K_s R^2 \mu^2 \frac{\sin \varphi \cos \varphi}{(1 - \mu \cos \varphi)^2}, \]

where \( K_s \) is a spring stiffness, \( R \) is a radius of the output link, and \( \mu \) is a lever length ratio. The coupled flexibility torque, representing the transmission deformation reacting on the secondary motor, is given by

\[ \tau_r = K_s R^2 \frac{\mu^2 \sin 2\beta \sin^2 \varphi}{2(R - a \cos \varphi)(a^2 + R^2 - 2aR \cos \varphi)}, \]

where \( \beta = \arctan(-\theta_2/\gamma) \) is a tangent angle of the Archimedean spiral gear, \( \gamma \) is a reduction gear ratio of the secondary motor, and \( a = \mu R = R\theta_2/2\pi \) is a distance from the pivot point to the joint center. The stiffness of this SVSA is formulated by

\[ \sigma(\theta_2, \varphi) = K_s R^2 \frac{\cos 2\varphi - \mu \cos \varphi}{(1 - \mu \cos \varphi)^2}. \]

The level length ratio \( \mu \) can be represented by the position of the secondary motor as follows:

\[ \mu = \frac{\theta_2}{2\pi \gamma} + \mu_0, \]

where \( \mu_0 \) is an initial level length ratio.

3. Neural Network Control Design

3.1. Problem Formulation. The dynamic model of VSAs given by (1)–(5) can be represented in the standard form

\[ \dot{x} = f(x) + g(x) u \]
\[ y = h(x), \]
where \( x = [q, \dot{q}, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T \) is a vector of system states, \( u = [u_1, u_2]^T \) is a control input, \( y = [q, \sigma]^T \) is a system output, and

\[
f(x) = \begin{bmatrix}
    \dot{q} \\
    M^{-1}(-D\dot{q} - \tau_e + \tau_{ext} - \tau_g) \\
    \dot{\theta}_1 \\
    B_1^{-1}(-D_1\dot{\theta}_1 + \tau_e) \\
    \dot{\theta}_2 \\
    B_2^{-1}(-D_2\dot{\theta}_2 - \tau_e)
\end{bmatrix},
\]

\[
g(x) = \begin{bmatrix}
    0 & 0 \\
    0 & 0 \\
    0 & 0 \\
    B_1^{-1} & 0 \\
    0 & 0 \\
    0 & B_2^{-1}
\end{bmatrix},
\]

\[
h(x) = \begin{bmatrix}
    q \\
    \sigma
\end{bmatrix}.
\]
By simple calculation, the input relative degree of system (6) is (4, 2) so that \( G(x) \) can be rearranged as follows:

\[
G(x) = \begin{bmatrix}
L_{s1}L_{s1}^3h_{11} & L_{s1}L_{s1}^3h_{12} \\
L_{s2}L_{s2}^3h_{21} & L_{s2}L_{s2}^3h_{22}
\end{bmatrix}.
\] (9)

Therefore, there exists a diffeomorphism

\[
\begin{bmatrix}
\dot{q} \\
\dot{\sigma}
\end{bmatrix} = \begin{bmatrix}
h_1(x) \\
L_{s1}h_1(x)
\end{bmatrix} \quad \begin{bmatrix}
\dot{q} \\
\dot{\sigma}
\end{bmatrix} = \begin{bmatrix}
L_{s1}^3h_{11}(x) \\
L_{s1}^3h_{12}(x)
\end{bmatrix} 
\] (10)

that transforms system (6) into

\[
\begin{bmatrix}
q^{(4)} \\
\sigma^{(2)}
\end{bmatrix} = F(x) + G(x) u
\] (11)

with \( F: R^6 \rightarrow R^2 \) being a vector field.

Let \( e_q := [e_{q1}, e_{q2}, e_{q3}, e_{q4}]^T \) and \( e_\sigma := [e_\sigma, e_\sigma]^T \). Define

\[
e_{q1} = \frac{d}{dt} + \lambda_1 \quad e_q = \begin{bmatrix}
\lambda_1^3, 3\lambda_1^2, 3\lambda_1, 1
\end{bmatrix} e_q
\]

\[
e_{q2} = \frac{d}{dt} + \lambda_2 \quad e_\sigma = [\lambda_2, 1] e_\sigma
\] (12)

with \( \lambda_1, \lambda_2 \in R^+ \). Then, one gets

\[
e = F(x) + G(x) u + v
\] (13)

with \( e := [e_q, e_\sigma]^T \), \( v := [v_1, v_2]^T \), \( v_1 := [0, \lambda_1^3, 3\lambda_1^2, 3\lambda_1, 1] e_q - q_q^{(4)} \) and \( v_2 := [0, \lambda_2] e_\sigma - \dot{\sigma}_d \).

A feedback linearization-based ideal control law is given by

\[
u = G^{-1}(x)(-K e - F(x) - v)
\] (14)

with \( K \in R^{2x2} \) being a positive-definite diagonal matrix of control gains.

3.3. Adaptive Neural Network Control. As \( F(x) \) in control law (14) is not exactly known, a radial basis function- (RBF-) NN represented as follows:

\[
\hat{f}(x, \widehat{W}) = \Phi^T(x) \widehat{W}
\] (15)

is applied to approximate \( F(x) \), where \( \widehat{W} = [\widehat{W}_1, \widehat{W}_2] \in R^{N \times 2} \) with \( \widehat{W}_i = [\hat{w}_{i1}, \hat{w}_{i2}, \ldots, \hat{w}_{iN}]^T (i = 1, 2) \) is a matrix of NN weights, \( \Phi(x) = \{\phi_1(x), \phi_2(x), \ldots, \phi_N(x)\}^T \in R^N \) is a vector of regression functions, \( N \) is the number of neural nodes, and \( \phi_j : R^6 \rightarrow R \) are commonly chosen to be Gaussian RBFs:

\[
\phi_j(x) = \exp \left( -\frac{\|x - c_j\|^2}{2r_j^2} \right)
\] (16)

with \( j = 1, 2, \ldots, N \) and \( c_j = [c_{j1}, c_{j2}]^T \), where \( c_{ij} \in R \) \((i = 1, 2)\) and \( r_j \in R^+ \) are centers and widths of the Gaussian functions, respectively.

Let \( \Omega_w = \{ |\hat{W}| \leq c_w \} \) and \( \Omega_x = \{ x | |x| \leq c_x \} \). Define an optimal NN approximation error

\[
e(x) = f(x) - \hat{f}(x, W^*) \]

with a constant matrix of optimal weights

\[
W^* = \arg \min_{\hat{W} \in \Omega_w} \left\{ \sup_{x \in \Omega_x} |f(x) - \hat{f}(x, \hat{W})| \right\}.
\] (18)

According to the universal approximation property of RBF-NNs, given any small constant \( e^* \in R^+ \), there is a sufficiently large \( N \) so that \( |e(x)| \leq e^* \), \( \forall x \in \Omega_x \) [41].

Assumption 4. There exists a matrix \( G_0 = \text{diag}(g_{10}, g_{20}) \) with \( g_{10}, g_{20} \in R^+ \) such that \( G(x) \geq G_0 \) holds.

Now, the actual control law is designed as follows:

\[
u = G_0^{-1}(-K e - \Phi^T(x) \hat{W} - v),
\] (19)

and the update law of \( \hat{W} \) is given by

\[
\hat{W} = \Gamma \mathcal{P}(\Phi(x) e),
\] (20)

where \( \Gamma \in R^{2 \times 2} \) is a positive-definite matrix of learning rates, and \( \mathcal{P} \) is a projection operator given by

\[
\mathcal{P}(*) = \begin{cases} *
\end{cases}
\] (21)

\[
\text{if } \|\hat{W}\|_p < c_w \text{ or } \|\hat{W}\|_p = c_w \& \|\hat{W}^T\|_p \leq 0
\]

\[
\text{otherwise.}
\]

Note that the choice of the gain matrix \( G_0 \) is based on a rough estimation of the inertia matrices \( M, B_1, \) and \( B_2, \) where the exact values of \( M, B_1, \) and \( B_2 \) are not required in practice. It follows from the standard NNAC result [41] that system (11) driven by control law (19) with (20) achieves practical asymptotic stability in the sense that \( e_q \) and \( e_\sigma \) converge to small neighborhoods of zero determined by \( K \) and \( e^* \).

4. Numerical Results

A general specification of SVSAs in Table 1 [15] is used for simulation. The construction of the proposed NNAC follows the following steps: (1) to construct regression functions \( \phi_j(x) \) in (16), select three Gaussian functions to cover each universe of \( \Omega_x \) such that \( j = 1 \) to 729; (2) set the filtered error parameters \( \lambda_1 = \lambda_2 = 20 \) in (12); (3) set the lower bound \( G_0 = \text{diag}(50000, 1000) \) and the control gain \( K = \text{diag}(20, 20) \) for the control law in (19); (4) set the learning rate matrix \( \Gamma = \text{diag}(300, 100) \) for the adaptive law in (20).

A tracking task with the desired position \( q_d(t) = 120 \sin(\pi t) \) and the desired stiffness \( c_q(t) = 50 \sin(1.25\pi + \pi t) \) is used for simulation. The desired position is bounded by \( q_d(t) \leq 120 \sin(\pi t) \). The desired position is bounded by \( q_d(t) \leq 120 \sin(\pi t) \).
Table 1: SVSA parameter specifications.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output link inertia</td>
<td>$M$ (kg·m²)</td>
<td>0.0063</td>
</tr>
<tr>
<td>Motor M1 + gearbox + intermediate inertia</td>
<td>$B_1$ (kg·m²)</td>
<td>0.0134</td>
</tr>
<tr>
<td>Motor M2 + gearbox + ASRM inertia</td>
<td>$B_2$ (kg·m²)</td>
<td>0.0110</td>
</tr>
<tr>
<td>Output link damping</td>
<td>$D$ (N·ms/rad)</td>
<td>$1.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>Motor M1 damping</td>
<td>$D_1$ (N·ms/rad)</td>
<td>$3.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>Motor M2 damping</td>
<td>$D_2$ (N·ms/rad)</td>
<td>$2.4 \times 10^{-3}$</td>
</tr>
<tr>
<td>Inherent spring stiffness</td>
<td>$K_s$ (N/m)</td>
<td>3764.5</td>
</tr>
<tr>
<td>Radius (mm)</td>
<td>$R$ (m)</td>
<td>0.05</td>
</tr>
<tr>
<td>Range of motion</td>
<td>(deg.)</td>
<td>0–360</td>
</tr>
<tr>
<td>Range of deflection angle</td>
<td>(deg.)</td>
<td>0–30</td>
</tr>
<tr>
<td>Range of stiffness</td>
<td>(N·m/rad)</td>
<td>1.7–150.5</td>
</tr>
</tbody>
</table>

Figure 2: Control trajectories by the PD control without load.

$\pi t + 55$ (Nm/rad) (frequency 0.5 Hz) is chosen to show the tracking performance of the proposed controller for SVSAs, where the desired stiffness $\sigma_d$ ranges between 5 and 105 Nm/rad. To make comparisons, a PD controller is selected as a baseline controller, where PD gains are optimized to minimize errors under a reasonable control $u$.

Tracking results by the PD control and the proposed NNAC are demonstrated in Figures 2 and 3, respectively, and comparisons of system errors are given in Figure 5, where no load is applied on the actuator in this case. It is shown that the proposed controller achieves a much higher position tracking accuracy than the PD control under a comparable control input $u$ despite the change of the stiffness. The position tracking accuracies by the PD control and the proposed NNAC are about 0.4547° and 7.3490°, respectively. The stiffness tracking result also shows better performance of the proposed NNAC than the PD control, where the stiffness tracking accuracy reduces from 1.1200 Nm/rad to 0.2857 Nm/rad. In addition, Figure 4 shows that the norms of the NN weights $\hat{W}_1$ and $\hat{W}_2$ converge to some constants, which implies that an exact estimation of the system uncertainty is achieved according to the NN learning theory [38].

To investigate the adaptability of the proposed NNAC, a 2 kg load is added to the link at $t = 15$ s. System errors...
and control inputs within 30 s are depicted in Figure 6. It is shown that, at the time 15 s, both the position error $e_q$ and the stiffness error $e_\sigma$ increase dramatically but then reduce gradually to small values. After an adaptation about 15 s, the high position tracking accuracy is still maintained by the proposed NNAC, where the maximum position error $e_q$ (0.8744°) is even smaller than that before adding the load (0.9265°). The adaptability of the proposed NNAC is apparent as position error $e_q$ continues decreasing to small range despite the unknown load added. Note that adding loads does not affect the stiffness tracking accuracy.

5. Conclusions

This paper has successfully applied a NNAC method to control SVSAs. Simulation results verify the ability of the
**Figure 5**: Comparisons of system errors without load.

**Figure 6**: Control trajectories by the proposed NNAC with load.
proposed approach to cope with system variability, by showing remarkable control performances for both position and
stiffness tracking under load changes. In future work, the practicality of the proposed NNAC would be enhanced by
the composite learning [42] and the observer design [43], and experimental validity of the proposed controller will be
carried out on a physical SVSA system.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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