Research Article

Adaptive Neural Tracking Control for Discrete-Time Switched Nonlinear Systems with Dead Zone Inputs

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In this paper, the adaptive neural controllers of subsystems are proposed for a class of discrete-time switched nonlinear systems with dead zone inputs under arbitrary switching signals. Due to the complicated framework of the discrete-time switched nonlinear systems and the existence of the dead zone, it brings about difficulties for controlling such a class of systems. In addition, the radial basis function neural networks are employed to approximate the unknown terms of each subsystem. Switched update laws are designed while the parameter estimation is invariable until its corresponding subsystem is active. Then, the closed-loop system is stable and all the signals are bounded. Finally, to illustrate the effectiveness of the proposed method, an example is employed.

1. Introduction

In the past few decades, intelligent control of uncertain nonlinear systems has attracted much more attention. Based on the universal approximation properties of neural networks (NNs) and fuzzy logic systems (FLSs), they are always used to approximate the unknown system functions or the control inputs. So far, there are a lot of results in intelligent control [1–6]. For example, the NN-based adaptive control methods were proposed for uncertain nonlinear systems in [7, 8] by using backstepping technique. The above results are all about nonlinear systems in continuous-time form. On the contrary, many researchers devoted much effort to study the adaptive control problem of discrete-time systems by using intelligent methods on the basis of these works [9–11].

However, the previous results do not consider the switching phenomenon of the system. Actually, the switching phenomenon often exists in practical systems. We call this class of systems switched systems. Switched systems, which are used to model a wide variety of physical systems, consist of a family of continuous-time or discrete-time subsystems and a switching rule to govern the switching between the subsystems. Generally, the stability and stabilisation problems are the main concerns in the study of switched systems [12–16]. Subsequently, adaptive control and intelligent control

of switched systems have been studied more and more [17–22]. But to the best of our knowledge, there are no results on discrete-time switched systems, and let alone results on discrete-time switched nonlinear systems.

In addition, the dead zone input, as one of the most important input nonlinearity, widely exists in lots of practical systems. The existence of the dead zone input may damage the system performance or even destroy the system stability. Thus, the related robust control attracted high attention [8, 23–25]. Since the dead zone parameters are poorly known in most practical systems, adaptive control techniques are naturally used to deal with this problem. An adaptive tracking control strategy was proposed for a nonlinear system with nonsymmetric dead zone input having unknown but bounded parameters in [26].

Motivated by the above discussion, we study the adaptive neural control problem of a class of discrete-time switched nonlinear systems with dead zone inputs under arbitrary switching signals. We use the radial basis function neural networks to approximate the unknown terms of each subsystem. Dead zone inputs of subsystems and switched update laws are designed such that the closed-loop system is stable and all the signals are bounded. Finally, an example is employed to illustrate the effectiveness of the proposed method.

The main contributions of this paper compared with the existing results on switched and nonswitched nonlinear systems are in two aspects:

- (1) Compared with the existing results [8, 23–25], in which the considered systems are all nonswitched systems or continuous-time switched systems, in this paper, we study a discrete-time switched uncertain nonlinear system.
- (2) The dead zone input is considered in the discrete-time switched uncertain nonlinear system. As far as we know, there are no results on discrete-time switched systems with dead zone inputs. This is mainly due to the complicated framework of the discrete-time switched nonlinear systems, the existence of the dead zone, and the interaction between the system structure and switching.

2. System Description

In this paper, we consider the following discrete-time switched uncertain nonlinear system:

$$\begin{aligned} \xi_1(k+1) &= \xi_2(k), \\ &\vdots \\ \xi_{n-1}(k+1) &= \xi_n(k), \\ \xi_n(k+1) &= f_\sigma(\xi(k)) + g_\sigma(\xi(k)) u_\sigma(k), \\ y(k) &= \xi_1(k), \end{aligned} \quad (1)$$

where $\xi(k) = [\xi_1(k), \xi_2(k), \dots, \xi_n(k)]^T \in \mathfrak{R}^n$ and $y \in \mathfrak{R}$ are the system state and output, respectively. Moreover, the state of switched system (1) is assumed not to jump at the switching instants, which is a standard assumption in the switched system [27]. σ is the switching signal taking values in $P = \{1, 2, \dots, p\}$ with p being the subsystems number of switched system (1). $f_i(x(k))$ and $g_i(x(k))$, $i \in P$ are unknown smooth functions. $u_i(k)$ is the dead zone output of the i th subsystem, which is described by

$$u_i(k) = D(v_i(k)) = \begin{cases} h_{ir}(v_i(k) - w_{ir}), & \text{if } v_i(k) \geq w_{ir}, \\ 0, & \text{if } -w_{il} < v_i(k) < w_{ir}, \\ h_{il}(v_i(k) + w_{il}), & \text{if } v_i(k) \leq -w_{il}. \end{cases} \quad (2)$$

In (2), $v_i(k)$ is the dead zone input of the i th subsystem. h_{ir} and h_{il} stand for the right and left slopes of the dead zone characteristic, respectively. w_{ir} and w_{il} represent the breakpoints of the input nonlinearity.

According to [28], the dead zone can be further expressed as

$$u_i(k) = D(v_i(k)) = h_i(k) v_i(k) + w_i(k), \quad i \in P, \quad (3)$$

where

$$h_i(k) = \begin{cases} h_{ir}, & \text{if } v_i(k) > 0, \\ h_{il}, & \text{if } v_i(k) \leq 0, \end{cases}$$

$$w_i(k) = \begin{cases} -h_{ir}w_{ir}, & \text{if } v_i(k) \geq w_{ir}, \\ -h_i(k) v_i(k), & \text{if } -w_{il} < v_i(k) < w_{ir}, \\ h_{il}w_{il}, & \text{if } v_i(k) \leq -w_{il}. \end{cases} \quad (4)$$

Assumption 1. The parameters of dead zone are all positive, and they are unknown but bounded, that is, $\underline{h}_i = \min\{h_{ir}, h_{il}\}$, $\bar{h}_i = \max\{h_{ir}, h_{il}\}$, and $|w_i(k)| \leq \bar{w}_i$.

From Assumption 1, we can easily know that

$$\underline{h}_i \leq h_i(k) \leq \bar{h}_i, \quad |w_i(k)| \leq \bar{w}_i = \max\{h_{ir}w_{ir}, h_{il}w_{il}\}. \quad (5)$$

Assumption 2 (see [29–31]). $g_i(\xi(k))$, $i \in P$, are strictly positive or negative and the signs of $g_i(\xi(k))$ are known. Without loss of generality, we suppose that $g_i(\xi(k))$ are positive with $\underline{g}_i$ and $\bar{g}_i$ being the lower bound and upper bound, respectively. It implies that $\bar{g}_i \geq \underline{g}_i > 0$.

The control objective of this paper is to design dead zone inputs $v_i(k)$ of subsystems, such that

- (1) the system state $\xi(k)$ can track a given signal $\xi^d(k) = [\xi_1^d(k), \xi_2^d(k), \dots, \xi_n^d(k)]^T$, or equivalently speaking, the output can track a desired trajectory $y^d(k)$,
- (2) all the signals in the closed-loop system are bounded under arbitrary switching signals.

Assumption 3 (see [32]). The desired trajectory of the system state $\xi^d(k) = [\xi_1^d(k), \xi_2^d(k), \dots, \xi_n^d(k)]^T \in \mathfrak{R}^n$ satisfies the condition $\xi_j^d(k+1) = \xi_{j+1}^d(k)$, $j = 1, \dots, n-1$. Moreover $y^d(k)$ is a known smooth bounded function.

According to Assumption 2 and system (1), we can obtain that $\xi_j^d(k) = y^d(k+j-1)$, $j = 1, \dots, n$.

3. Neural Networks

Similar to FLSs, NNs also have the approximation properties. In this paper, we consider the case that there exist unknown functions in the systems. Here, we use RBFNNs to approximate the unknown functions of subsystems.

For any unknown continuous function $F(X)$ defined on $\Omega \subset \mathfrak{R}^n$, there exists NN $\theta^T \varphi(X)$, such that

$$F(X) = \theta^T \varphi(X) + \varepsilon(X), \quad (6)$$

where $X \in \Omega \subset \mathfrak{R}^n$ is the input variable of the NN, $\theta = [\theta_1, \dots, \theta_\ell]^T$ is the weight vector with ℓ being the node number of NNs, $\varepsilon(X)$ is the approximation error, and $\varphi(X) =$

$[\phi_1(X), \dots, \phi_\ell(X)]^T$ is the basis function. In RBFNNs, $\phi_i(X)$, $i = 1, \dots, \ell$, are chosen as Gaussian functions

$$\phi_i(X) = \exp\left(-\frac{(X - c_i)^T(X - c_i)}{\mu_i^2}\right), \quad (7)$$

where $c_i = [c_{i1}, \dots, c_{in}]^T$ and μ_i , $i = 1, \dots, \ell$, are the centers and widths of the Gaussian functions, respectively.

According to (7), we have $0 < \phi_i(X) \leq 1$. Thus, we can further obtain that $\phi_i^2(X) \leq 1$, $i = 1, \dots, \ell$. This leads to the following inequality:

$$\varphi^T(X) \varphi(X) \leq \ell. \quad (8)$$

In this paper, since there are unknown functions in (11), we cannot obtain v_i^0 , $i = 1, \dots, n$, directly. Thus, the neural networks are used to approximate the unknown function v_i^0 , $i = 1, \dots, n$. In fact, the neural networks in this paper are used for identification.

4. Dead Zone Input Design

Define the tracking error at instant k as

$$e_j(k) = \xi_j(k) - \xi_j^d(k+1) = \xi_j(k) - y^d(k+j-1). \quad (9)$$

Then, we can get the n th state error $e_n(k) = \xi_n(k) - y^d(k+n-1)$. Furthermore, we have

$$\begin{aligned} e_n(k+1) &= f_i(\xi(k)) + g_i(\xi(k))u_i(k) - y^d(k+n) \\ &= f_i(k) + g_i(k)h_i(k)v_i(k) + g_i(k)w_i(k) \\ &\quad - y^d(k+n), \end{aligned} \quad (10)$$

where $f_i(k)$ and $g_i(k)$ are the abbreviations of $f_i(\xi(k))$ and $g_i(\xi(k))$, respectively.

Define functions

$$v_i^0(k) = -\frac{1}{g_i(k)m_i(k)}(f_i(k) - y^d(k+n)), \quad i \in P. \quad (11)$$

Because of the existence of the unknown functions in (11), $v_i^0(k)$ cannot be directly obtained. Here, we use RBFNNs to approximate them

$$v_i^0(k) = \theta_i^T \varphi(S(k)) + \varepsilon_i(S(k)), \quad i \in P, \quad (12)$$

where θ_i are the weight matrices which are assumed to be bounded, that is, $\|\theta_i\| \leq \bar{\theta}_i$, and $\varphi(S(k))$ and $\varepsilon_i(S(k))$ are the basis function vector and the approximation errors, respectively. Furthermore, $\|\varphi(S(k))\|^2 \leq \ell$ and $\|\varepsilon_i(S(k))\| \leq \bar{\varepsilon}_i$. $S(k) = [\xi^T(k), y^{dT}(k), y^{dT}(k+n)]^T$ is the input vector of RBFNNs.

By adding and subtracting $g_i(k)h_i(k)v_i^0(k)$ on the right side of equation (10), we can conclude that

$$\begin{aligned} e_n(k+1) &= g_i(k)h_i(k) \left[v_i(k) - \theta_i^T \varphi(k) - \varepsilon_i(k) + \frac{w_i(k)}{h_i(k)} \right], \end{aligned} \quad (13)$$

where $\varphi(k)$ and $\varepsilon_i(k)$ are the shorthand for $\varphi(S(k))$ and $\varepsilon_i^*(S(k))$, respectively. Denote that $w_i(k)/h_i(k) \leq \bar{w}_i/\bar{h}_i \triangleq \eta_i$ and η_i , $i \in P$, are unknown constants. Let $\hat{\eta}_i$ be the estimation of η_i and $\text{lwt } \tilde{\eta}_i(k) = \hat{\eta}_i(k) - \eta_i$ be the estimation error.

According to (13), we design the controllers as

$$v_i(k) = \tilde{\theta}_i^T(k) \varphi(k) + \hat{\eta}_i(k), \quad (14)$$

where $\tilde{\theta}_i(k)$ are the estimation of $\theta_i(k)$. Let the estimation error satisfy $\tilde{\theta}_i(k) = \hat{\theta}_i(k) - \theta_i$.

The switched update laws are designed as

$$\begin{aligned} \hat{\theta}_i(k+1) &= \begin{cases} \hat{\theta}_i(k) - \Gamma_i [\delta_i \hat{\theta}_i(k) + \varphi(k) e_n(k+1)], & \sigma = i, \\ 0, & \sigma \neq i, \end{cases} \\ \hat{\eta}_i(k+1) &= \begin{cases} \hat{\eta}_i(k) - \mu_i [\tau_i \hat{\eta}_i(k) + e_n(k+1)], & \sigma = i, \\ 0, & \sigma \neq i, \end{cases} \end{aligned} \quad (15)$$

where Γ_i and μ_i are positive matrices and positive design parameters, respectively.

Based on (14), (13) becomes

$$\begin{aligned} e_n(k+1) &= g_i(k)h_i(k) \\ &\cdot \left[\tilde{\theta}_i^T \varphi(k) - \varepsilon_i(k) + \hat{\eta}_i(k) + \frac{w_i(k)}{h_i(k)} \right] = g_i(k)h_i(k) \\ &\cdot \left[\tilde{\theta}_i^T \varphi(k) + \tilde{\eta}_i(k) - \varepsilon_i(k) + \eta_i + \frac{w_i(k)}{h_i(k)} \right]. \end{aligned} \quad (16)$$

5. Stability Analysis

In this section, the stability of switched system (1) and the boundedness of all the signals in the closed-loop system are proved under arbitrary switching signals.

Theorem 4. Consider the discrete-time switched system (1), the dead zone (3), the switched update laws given by (15), and the input of the i th dead zone selected as in (14) under Assumptions 1–3. If the design parameters satisfy the conditions

$$\begin{aligned} \gamma_\sigma &< \frac{1 - \mu_\sigma \bar{g} \bar{h}}{3 + l \bar{g} \bar{h} + l + \mu_\sigma}, \\ \delta_\sigma &< \frac{1}{\gamma_\sigma + \gamma_\sigma \bar{g} \bar{h}}, \\ \mu_\sigma &< \frac{\gamma_\sigma}{\gamma_\sigma + \bar{g} \bar{h} \tau_\sigma}, \end{aligned} \quad (17)$$

then the boundedness of all the signals in the closed-loop switched system can be ensured and the state $\xi(k)$ can track the reference signal $\xi^d(k)$ and the system output can follow the desired trajectory $y^d(k)$.

Proof. Choose the following common Lyapunov function for system (1) as

$$V(k) = \frac{1}{\bar{g}h} e_n^2(k) + \sum_{i=1}^p \text{tr} \{ \tilde{\theta}_i^T(k) \Gamma_i^{-1} \tilde{\theta}_i(k) \} + \sum_{i=1}^p \frac{1}{\mu_i} \tilde{\eta}_i^2(k), \quad i \in P, \quad (18)$$

where $\bar{g} = \max_{i \in P} \bar{g}_i$ and $\bar{h} = \max_{i \in P} \bar{h}_i$.

The first difference of $V(k)$ is

$$\begin{aligned} \Delta V(k) &= \frac{1}{\bar{g}h} [e_n^2(k+1) - e_n^2(k)] \\ &+ \sum_{i=1}^p \text{tr} \{ \tilde{\theta}_i^T(k+1) \Gamma_i^{-1} \tilde{\theta}_i(k+1) \} \\ &- \sum_{i=1}^p \text{tr} \{ \tilde{\theta}_i^T(k) \Gamma_i^{-1} \tilde{\theta}_i(k) \} + \sum_{i=1}^p \frac{1}{\mu_i} \tilde{\eta}_i^2(k+1) \\ &- \sum_{i=1}^p \frac{1}{\mu_i} \tilde{\eta}_i^2(k). \end{aligned} \quad (19)$$

Substituting (15) into (19), one has

$$\begin{aligned} \Delta V(k) &= \frac{1}{\bar{g}h} [e_n^2(k+1) - e_n^2(k)] - 2 \text{tr} \{ \tilde{\theta}_\sigma^T(k) \\ &\cdot [\delta_\sigma \hat{\theta}_\sigma(k) + \varphi(k) e_n(k+1)] \} \\ &+ \text{tr} \{ [\delta_\sigma \hat{\theta}_\sigma(k) + \varphi(k) e_n(k+1)]^T \\ &\cdot \Gamma_\sigma [\delta_\sigma \hat{\theta}_\sigma(k) + \varphi(k) e_n(k+1)] \} - 2 \tilde{\eta}_\sigma(k) \\ &\cdot [\tau_\sigma \hat{\eta}_\sigma(k) + e_n(k+1)] + \mu_\sigma [\tau_\sigma \hat{\eta}_\sigma(k) + e_n(k+1)]^2 \\ &= \frac{1}{\bar{g}h} [e_n^2(k+1) - e_n^2(k)] - 2 \delta_\sigma \tilde{\theta}_\sigma^T(k) \hat{\theta}_\sigma(k) \\ &- 2 \tilde{\theta}_\sigma^T(k) \varphi(k) e_n(k+1) + \text{tr} \{ \delta_\sigma^2 \tilde{\theta}_\sigma^T(k) \Gamma_\sigma \hat{\theta}_\sigma(k) \} \\ &+ \text{tr} \{ e_n(k+1) \varphi^T(k) \Gamma_\sigma \varphi(k) e_n(k+1) \} + 2 \\ &\cdot \text{tr} \{ \delta_\sigma \tilde{\theta}_\sigma^T(k) \Gamma_\sigma \varphi(k) e_n(k+1) \} - 2 \tau_\sigma \tilde{\eta}_\sigma(k) \hat{\eta}_\sigma(k) \\ &+ \mu_\sigma \tau_\sigma \hat{\eta}_\sigma^2(k) - 2 \tilde{\eta}_\sigma(k) e_n(k+1) + \mu_\sigma e_n^2(k+1) \\ &+ 2 \mu_\sigma \tau_\sigma \hat{\eta}_\sigma(k) e_n(k+1), \end{aligned} \quad (20)$$

with γ_σ being the maximum eigenvalue of Γ_σ .

According to (16), we can conclude that

$$\begin{aligned} \tilde{\theta}_i^T(k) \varphi(k) + \tilde{\eta}_i(k) &= \frac{e_n(k+1)}{g_i(k) h_i(k)} + \varepsilon_i(k) - \eta_i \\ &- \frac{w_i(k)}{h_i(k)}. \end{aligned} \quad (21)$$

Then, $\Delta V(k)$ becomes

$$\begin{aligned} \Delta V(k) &\leq \frac{1}{\bar{g}h} [e_n^2(k+1) - e_n^2(k)] \\ &- \frac{2}{\bar{g}_\sigma(k) \bar{h}_\sigma(k)} e_n^2(k+1) \\ &- 2 \varepsilon_\sigma(k) e_n(k+1) + 2 \eta_\sigma e_n(k+1) \\ &+ \frac{w_\sigma(k)}{h_\sigma(k)} e_n(k+1) - 2 \delta_\sigma \tilde{\theta}_\sigma^T(k) \hat{\theta}_\sigma(k) \\ &+ \mu_\sigma \tau_\sigma \hat{\eta}_\sigma^2(k) + \gamma_\sigma \delta_\sigma^2 \|\hat{\theta}_\sigma(k)\|^2 \\ &+ \gamma_\sigma l e_n^2(k+1) \\ &+ 2 \gamma_\sigma \delta_\sigma \tilde{\theta}_\sigma^T(k) \varphi(k) e_n(k+1) \\ &- 2 \tau_\sigma \tilde{\eta}_\sigma(k) \hat{\eta}_\sigma(k) + \mu_\sigma e_n^2(k+1) \\ &+ 2 \mu_\sigma \tau_\sigma \hat{\eta}_\sigma(k) e_n(k+1). \end{aligned} \quad (22)$$

Based on Assumption 2, it holds that

$$-\frac{2}{g_i(k) h_i(k)} e_n^2(k+1) \leq -\frac{2}{\bar{g}h} e_n^2(k+1). \quad (23)$$

By using Young's inequality, we have the following inequalities:

$$\begin{aligned} -2 \varepsilon_\sigma(k) e_n(k+1) &\leq \frac{\bar{g}h}{\gamma_\sigma} \varepsilon_\sigma^2 + \frac{\gamma_\sigma}{\bar{g}h} e_n^2(k+1), \\ 2 \eta_\sigma e_n(k+1) &\leq \frac{\bar{g}h}{\gamma_\sigma} \eta_\sigma^2 + \frac{\gamma_\sigma}{\bar{g}h} e_n^2(k+1), \\ 2 \frac{w_i(k)}{h_i(k)} e_n(k+1) &\leq \frac{\bar{g}h}{\gamma_\sigma} \eta_i^2 + \frac{\gamma_\sigma}{\bar{g}h} e_n^2(k+1), \\ 2 \gamma_\sigma \delta_\sigma \tilde{\theta}_\sigma^T(k) \varphi(k) e_n(k+1) \\ &\leq \bar{g}h \gamma_\sigma \delta_\sigma^2 \|\hat{\theta}_\sigma(k)\|^2 + \frac{\gamma_\sigma l}{\bar{g}h} e_n^2(k+1), \\ 2 \mu_\sigma \tau_\sigma \hat{\eta}_\sigma(k) e_n(k+1) \\ &\leq \frac{\bar{g}h \mu_\sigma \tau_\sigma^2}{\gamma_\sigma} \hat{\eta}_\sigma^2(k) + \frac{\gamma_\sigma \mu_\sigma}{\bar{g}h} e_n^2(k+1), \end{aligned} \quad (24)$$

and we consider the following equations:

$$\begin{aligned} 2 \tilde{\theta}_\sigma^T(k) \hat{\theta}_\sigma(k) &= \|\tilde{\theta}_\sigma(k)\|^2 + \|\hat{\theta}_\sigma(k)\|^2 - \|\theta_\sigma\|^2, \\ 2 \tilde{\eta}_\sigma(k) \hat{\eta}_\sigma(k) &= \tilde{\eta}_\sigma^2(k) + \hat{\eta}_\sigma^2(k) - \eta_\sigma^2. \end{aligned} \quad (25)$$

Then, we have

$$\begin{aligned}
\Delta V(k) &\leq \frac{1}{\bar{g}h} [e_n^2(k+1) - e_n^2(k)] - \frac{2}{\bar{g}h} e_n^2(k+1) \\
&\quad + \frac{\bar{g}h}{\gamma_\sigma} \bar{\varepsilon}_\sigma^2 + \frac{\gamma_\sigma}{\bar{g}h} e_n^2(k+1) + \frac{2\bar{g}h}{\gamma_\sigma} \eta_\sigma^2 \\
&\quad + \frac{2\gamma_\sigma}{\bar{g}h} e_n^2(k+1) - \delta_\sigma \|\tilde{\theta}_\sigma(k)\|^2 \\
&\quad - \delta_\sigma \|\hat{\theta}_\sigma(k)\|^2 + \delta_\sigma \|\theta_\sigma\|^2 + \mu_\sigma \tau_\sigma \hat{\eta}_\sigma^2(k) \\
&\quad + \gamma_\sigma \delta_\sigma^2 \|\hat{\theta}_\sigma(k)\|^2 + \gamma_\sigma l e_n^2(k+1) \\
&\quad + \bar{g}h \gamma_\sigma \delta_\sigma^2 \|\hat{\theta}_\sigma(k)\|^2 + \frac{\gamma_\sigma l}{\bar{g}h} e_n^2(k+1) \\
&\quad - \tau_\sigma \hat{\eta}_\sigma^2(k) - \tau_\sigma \hat{\eta}_\sigma^2(k) + \tau_\sigma \eta_\sigma^2 \\
&\quad + \mu_\sigma e_n^2(k+1) \frac{\bar{g}h \mu_\sigma \tau_\sigma^2}{\gamma_\sigma} \hat{\eta}_\sigma^2(k) \\
&\quad + \frac{\gamma_\sigma \mu_\sigma}{\bar{g}h} e_n^2(k+1) \\
&\leq -\frac{1}{\bar{g}h} M_1 e_n^2(k+1) - \delta_\sigma M_2 \|\hat{\theta}_\sigma(k)\|^2 \\
&\quad - \tau_\sigma M_3 \hat{\eta}_\sigma^2(k) - \frac{1}{\bar{g}h} e_n^2(k) - \delta_\sigma \|\tilde{\theta}_\sigma(k)\|^2 \\
&\quad - \tau_\sigma \hat{\eta}_\sigma^2(k) + \beta,
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
M_1 &= 1 - 3\gamma_\sigma - \gamma_\sigma l \bar{g}h - \gamma_\sigma l - \mu_\sigma \bar{g}h - \gamma_\sigma \mu_\sigma, \\
M_2 &= 1 - \gamma_\sigma \delta_\sigma - \gamma_\sigma \delta_\sigma \bar{g}h, \\
M_3 &= 1 - \mu_\sigma - \frac{\bar{g}h \mu_\sigma \tau_\sigma}{\gamma_\sigma}, \\
\beta &= \frac{\bar{g}h}{\gamma_\sigma} \bar{\varepsilon}_\sigma^2 + \frac{2\bar{g}h}{\gamma_\sigma} \eta_\sigma^2 + \delta_\sigma \|\theta_\sigma\|^2 + \tau_\sigma \eta_\sigma^2.
\end{aligned} \tag{27}$$

If the design parameters satisfy condition (17), then we can deduce that

$$\Delta V_i(k) \leq -\frac{1}{\bar{g}h} e_n^2(k) - \delta_\sigma \|\tilde{\theta}_\sigma(k)\|^2 - \tau_\sigma \hat{\eta}_\sigma^2(k) + \beta. \tag{28}$$

If $|e_n(k)| > \sqrt{\bar{g}h\beta}$ or $\|\tilde{\theta}_\sigma(k)\| > \sqrt{\beta/\delta_\sigma}$ or $|\hat{\eta}_\sigma(k)| > \sqrt{\beta/\tau_\sigma}$ holds, we have $\Delta V_i(k) < 0$. According to the standard Lyapunov extension theorem, the tracking error $e_n(k)$ and the estimation errors $\tilde{\theta}_\sigma(k)$ and $\tilde{\eta}_\sigma(k)$ are bounded. Since $e_j(k) = \xi_j(k) - \xi_j^d(k+1) = e_n(k-n+j)$, $j = 1, \dots, n$, we can easily know that $e_j(k)$, $j = 1, \dots, n-1$, are also bounded. Furthermore, because θ_σ and η_σ are bounded, the

boundedness of the estimated parameters $\hat{\theta}_\sigma(k)$ and $\hat{\eta}_\sigma(k)$ is guaranteed by $\hat{\theta}_\sigma(k) = \tilde{\theta}_\sigma(k) + \theta_\sigma$ and $\hat{\eta}_\sigma(k) = \tilde{\eta}_\sigma(k) + \eta_\sigma$. Thus, from (14), we know that the dead zone inputs v_σ are bounded. Based on the definition of dead zone, one can conclude that the control inputs u_σ are also bounded. Therefore, all the signals in the closed-loop system are bounded under arbitrary switching signals. $\square$

Remark 5. As we all know that a switched system might be unstable even if all the subsystems are stable, arbitrary switching method can guarantee the stability of the switched system in this case. A sufficient condition for analysing the stability of switched systems under arbitrary switching signals is that there exists a common Lyapunov function. In many cases, it is easy to find a Lyapunov function for each subsystem. However, a common Lyapunov is hard to find for switched systems. In this paper, we have successfully found a common Lyapunov function shown as in (18).

Remark 6. There are many parameters in the design procedure. h_{ir} , h_{il} , w_{ir} , and w_{il} , $i = 1, 2$, are parameters of dead zone. They are all positive based on Assumption 1. In addition, Γ_i , δ_i , μ_i , and τ_i , $i = 1, 2$, are introduced to improve the flexibility of the update laws. In this paper, they are chosen to be positive. But how to choose a set of optimal ones to ensure the satisfactory performance is still an open problem; they need to be flexibly selected. In the following, a constructive suggestion on how to choose these parameters is given. From the above stability analysis, we can see that the bounds of $e_n(k)$, $\tilde{\theta}_\sigma(k)$, and $\tilde{\eta}_\sigma(k)$ depend on the values of $\bar{g}h$, β , $\delta_\sigma(k)$, and $\tau_\sigma(k)$. Actually, we can select smaller $\bar{g}h$ and larger $\delta_\sigma(k)$ and $\tau_\sigma(k)$ to obtain smaller bounds of $e_n(k)$, $\tilde{\theta}_\sigma(k)$, and $\tilde{\eta}_\sigma(k)$ to further get good performance. But, it is important to note that these parameters must satisfy conditions (17). In the simulation, a trial-and-error method is used.

6. Simulation

In this section, we apply the proposed approach to the following discrete-time switched nonlinear system:

$$\begin{aligned}
\xi_1(k+1) &= \xi_2(k), \\
\xi_2(k+1) &= f_\sigma^1(\xi(k)) + g_\sigma^1(\xi(k)) u_\sigma(k), \\
\xi_3(k+1) &= \xi_4(k), \\
\xi_4(k+1) &= f_\sigma^2(\xi(k)) + g_\sigma^2(\xi(k)) u_\sigma(k), \\
y_1(k) &= \xi_1(k), \\
y_2(k) &= \xi_3(k),
\end{aligned} \tag{29}$$

where $f_1^1(\bar{\xi}(k)) = \xi_1(k) + \xi_2^2(k)$, $f_1^2(\bar{\xi}(k)) = 1/2(\xi_1(k) + \xi_2(k))$, $g_1^1(\bar{\xi}(k)) = \xi_1(k)\xi_2(k)$, $g_1^2(\bar{\xi}(k)) = \xi_1(k)\xi_2(k) + \xi_1(k)\xi_2^2(k)$, $f_2^1(\bar{\xi}(k)) = \xi_3(k) + \xi_4^2(k)$, $f_2^2(\bar{\xi}(k)) = \xi_1(k) + \xi_3(k)$, $g_2^1(\bar{\xi}(k)) = \xi_1(k)\xi_3^2(k) + \xi_3(k)\xi_4^2(k)$, and $g_2^2(\bar{\xi}(k)) = \xi_1(k)\xi_3(k) + \xi_3(k)\xi_4(k)$. The switching signal takes values in $P = \{1, 2\}$.

The dead zone is described as

$$u_i(k) = D(v_i(k))$$

$$= \begin{cases} h_{ir}(v_i(k) - w_{ir}), & \text{if } v_i(k) \geq w_{ir}, \\ 0, & \text{if } -w_{il} < v_i(k) < w_{ir}, \\ h_{il}(v_i(k) + w_{il}), & \text{if } v_i(k) \leq -w_{il}. \end{cases} \quad (30)$$

The parameters of the dead zone inputs are selected as $h_{1r} = 0.3$, $h_{2r} = 0.4$, $h_{1l} = 0.2$, $h_{2l} = 0.3$, $w_{1r} = 0.5$, $w_{2r} = 0.7$, $w_{1l} = 0.6$, and $w_{2l} = 0.8$.

Define the tracking errors $e_1(k) = y_1(k) - y_{d1}(k) = \xi_1(k) - y_{d1}(k)$ and $e_2(k) = y_2(k) - y_{d2}(k) = \xi_2(k) - y_{d2}(k)$. The desired signals $y_{d1}(k)$ and $y_{d2}(k)$ are selected as $y_{d1}(k) = 0.28[\sin(k\pi/10) + \cos(k\pi/10)]$ and $y_{d2}(k) = 0.22[\sin(k\pi/10) + \cos(k\pi/10)]$, respectively. The initial values for the system states are $\xi_1(0) = 0.01$, $\xi_2(0) = 0.01$, $\xi_1(0) = 0.01$, and $\xi_2(0) = 0.01$.

The dead zone input is designed for each subsystem

$$v_i(k) = \tilde{\theta}_i^T(k) \varphi(k) + \hat{\eta}_i(k), \quad (31)$$

where $\varphi(k)$ is the Gaussian function.

The switched update laws are designed for $i = 1, 2$, as

$$\begin{aligned} & \hat{\theta}_i(k+1) \\ &= \begin{cases} \hat{\theta}_i(k) - \Gamma_i [\delta_i \hat{\theta}_i(k) + \varphi(k) e_n(k+1)], & \sigma = i, \\ 0, & \sigma \neq i, \end{cases} \\ & \hat{\eta}_i(k+1) \\ &= \begin{cases} \hat{\eta}_i(k) - \mu_i [\tau_i \hat{\eta}_i(k) + e_n(k+1)], & \sigma = i, \\ 0, & \sigma \neq i, \end{cases} \end{aligned} \quad (32)$$

where $\Gamma_1 = 0.5I$, $\Gamma_2 = 0.2I$, $\delta_1 = 2$, $\delta_2 = 2.5$, $\mu_1 = 0.5$, $\mu_2 = 0.7$, $\tau_1 = 2.3$, and $\tau_2 = 1.8$.

Furthermore, we give the simulation results in Figures 1–7. Figure 1 gives the curves of y_1 and y_{d1} . Figure 2 shows the trajectories of y_2 and y_{d2} . Figures 3 and 4 display the tracking errors e_1 and e_2 , respectively. From Figures 1–4, we can observe that a good tracking performance is obtained. Moreover, the dead zone inputs are shown in Figures 5 and 6, which explain the boundedness of the dead zone inputs. Figure 7 is the switching signal.

7. Conclusion

We design a controller of each subsystem for a class of discrete-time switched nonlinear systems with dead zone inputs under arbitrary switching signals in this paper. The unknown functions are approximated by RBFNNs. Switched update laws are designed while the parameter estimation is invariable until its corresponding subsystem is active. Then, the closed-loop system is stable and all the signals are bounded. Due to the complicated framework of the discrete-time switched nonlinear systems and the existence of the dead zone, it brings about great difficulties for controlling such a class of systems.

FIGURE 1: The trajectories of y_1 and y_{d1} .

FIGURE 2: The trajectories of y_2 and y_{d2} .

FIGURE 3: The trajectory of tracking error e_1 .

FIGURE 4: The trajectory of tracking error e_2 .

FIGURE 5: Control input u_1 .

FIGURE 6: Control input u_2 .

FIGURE 7: Switching signal σ .

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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