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Research Article

Centralized and Decentralized Data-Sampling Principles for Outer-Synchronization of Fractional-Order Neural Networks

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This paper aims to investigate the outer-synchronization of fractional-order neural networks. Using centralized and decentralized data-sampling principles and the theory of fractional differential equations, sufficient criteria about outer-synchronization of the controlled fractional-order neural networks are derived for structure-dependent centralized data-sampling, state-dependent centralized data-sampling, and state-dependent decentralized data-sampling, respectively. A numerical example is also given to illustrate the superiority of theoretical results.

1. Introduction

Fractional operator has become visible in application domains [1–15]. As the demanding performance expectations with uncertainty, fractional operator offers more degrees of freedom to designers to meet some predefined performance indexes. After gradually recognizing the importance of fractional operator, it is found that the description of fractional-order model is more accurate and totally different from that of the corresponding integer-order model. As a direct application, the characteristic of fractional-order model can be used to identify possible behavior of electrical signals from neurons. In physical implementation of neurodynamic systems, arbitrary order analog fractance circuit is most appropriate, which reveals profoundly the relationships among neural circuit elements [9–11]. In that way, real neurodynamic systems should be addressed by fractional-order models. Fractional-order neurodynamic systems can better describe how action potentials in neurons are launched and spread. In addition, fractional-order neurodynamic systems possess infinite memory, and yet, integer-order neurodynamic systems are not of such feature [3–8, 12–15]. Therefore, fractional-order neurodynamic systems have the potential to accomplish what integer-order ones can not do. More feasible analysis methods and easy-to-use techniques to be deal with fractional-order neurodynamic systems are worth looking into.

As a coherent behavior within nonlinear systems, synchronization of nonlinear systems has attracted phenomenal worldwide attention. Many studies have shown that synchronization mechanism is a universal phenomenon and has a wide range of applications in engineering systems. Generally, two schemes for synchronization are frequently used: inner-synchronization and outer-synchronization. For inner-synchronization, all nodes within a network will achieve a coherent behavior. However, for outer-synchronization, all individuals in two networks will achieve identical behaviors. In many application fields, outer-synchronization may seem practical [16–23]. For example, in heuristic computational intelligence, it is known that outer-synchronization is rooted in brain-inspired computing from evolutionary strategies to cognitive tasks. Nevertheless, results focusing on outer-synchronization of complex control systems have seldom been reported [19]. Control strategy for outer-synchronization deserves more investigation.

Sampled-data control through only using the local information has recently generated significant research interest [24–38]. Unlike continuous-time control, which requires the continuous communication data, sampled-data control is more appropriate under networked environment. For control systems, once we can give effective sampling policies and schedule, then the sampled-data control will reduce communication data and save energy dramatically. Thus, how to develop high-efficiency, heuristic information-based sampled-data control with the ultimate aim of maximizing the data collected is worth studying [38]. However, relevant studies of the data-sampling strategy for control systems are still in early stage.

Motivated by the above discussions, in this paper, we introduce the centralized and decentralized data-sampling principles to achieve outer-synchronization between coupled fractional-order neural networks. The efficient allocation of the limited energy resources of centralized and decentralized data-sampling principles that maximizes the information value of the data collected is clearly a step forward. Meanwhile, to more efficiently design the sampling method, we merge the structure and state clusters through centralized and decentralized data-sampling principles and then select the best sampling time. On the basis of some analytical tools of fractional differential equations, a series of criteria on outer-synchronization are derived. It should be noted that such criteria capture the information on sampling pattern and may have much wider application range.

The rest of the paper is organized as follows. In Section 2, we present the preliminaries and problem formulation. In Section 3, we state main results in detail. In Section 4, simulation example is illustrated. Finally, Section 5 concludes the paper.

2. Preliminaries and Problem Formulation

First, some preliminaries of fractional operator are given.

Fractional integral $I^q(\cdot)$ for $\mathscr{H}(t)$ with order $q > 0$ is described as

$$I^q \mathscr{H}(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} \mathscr{H}(s)\, ds, \quad t \geq t_0, \tag{1}$$

where $\Gamma(\cdot)$ is Gamma function and t_0 is the initial time.

Caputo fractional derivative ${}^{C}D_{t_0}^{q}(\cdot)$ for $\mathscr{H}(t) \in \mathscr{C}^{m+1}([t_0,+\infty),\mathfrak{R})$ with order $q > 0$ is described as

$${}^{C}D_{t_0}^{q} \mathscr{H}(t) = \frac{1}{\Gamma(m-q)} \int_{t_0}^{t} \frac{\mathscr{H}^{(m)}(s)}{(t-s)^{q-m+1}} ds, \quad t \geq t_0, \tag{2}$$

where $\Gamma(\cdot)$ is Gamma function, $m-1 < q < m$, m is a positive integer, and t_0 is the initial time.

One-parameter Mittag-Leffler function $E_q(\cdot)$ is described as

$$E_q(s) = \sum_{k=0}^{+\infty} \frac{s^k}{\Gamma(kq+1)}, \tag{3}$$

where $\Gamma(\cdot)$ is Gamma function, $q > 0$, and s is a complex number.

Consider a class of fractional-order neural networks

$$\begin{aligned} {}^{C}D_{t_0}^{q} x_i(t) = {} & -a_i(t)\, x_i(t) + \sum_{j=1}^{n} b_{ij}(t)\, f_j\left(x_j(t)\right) \\ & + u_i(t), \quad i = 1,2,\ldots,n, \end{aligned} \tag{4}$$

where $0 < q < 1$, $a_i(t) > 0$, $b_{ij}(t)$ and $u_i(t)$ are piecewise continuous and bounded, and feedback function $f_j(\cdot)$ satisfies

$$\begin{gathered} 0 \leq \frac{f_j(\mathscr{U}_1) - f_j(\mathscr{U}_2)}{\mathscr{U}_1 - \mathscr{U}_2} \leq F_j, \\ \forall \mathscr{U}_1, \mathscr{U}_2 \in \mathfrak{R},\ \mathscr{U}_1 \neq \mathscr{U}_2, \end{gathered} \tag{5}$$

in which $F_j > 0$, $j = 1,2,\ldots,n$.

For the centralized data-sampling principle, (4) is rewritten as

$$\begin{aligned} {}^{C}D_{t_0}^{q} x_i(t) = {} & -a_i(t)\, x_i(t_k) + \sum_{j=1}^{n} b_{ij}(t)\, f_j\left(x_j(t_k)\right) \\ & + u_i(t), \quad i = 1,2,\ldots,n, \end{aligned} \tag{6}$$

where t_k is simple notion of $t_{k(t)}$ with $k(t) = \max\{\mathscr{K} : t_{\mathscr{K}} \leq t\}$ and $0 = t_0 < t_1 < \cdots < t_k < \cdots$ is uniform for all the system states. Every neuron intersperses its state to its out-neighbors and receives the state information from its in-neighbors at the same time point t_k.

For the decentralized data-sampling principle, (4) is rewritten as

$$\begin{aligned} {}^{C}D_{t_0}^{q} x_i(t) = {} & -a_i(t)\, x_i(t_k^i) + \sum_{j=1}^{n} b_{ij}(t)\, f_j\left(x_j(t_k^j)\right) \\ & + u_i(t), \quad i = 1,2,\ldots,n, \end{aligned} \tag{7}$$

where t_k^i is simple notion of $t_{k(t)}^i$ with $k(t) = \max\{\mathscr{K} : t_{\mathscr{K}}^i \leq t\}$ and $0 = t_0^i < t_1^i < \cdots < t_k^i < \cdots$ is distributed for $i \in \{1,2,\ldots,n\}$. Each neuron i pushes its state information to its out-neighbors at time t_k^i when it updates its state. It receives the information of in-neighbor state at time t_k^j when the neighbor neuron j updates its state.

Now, we state definition and problem formulation.

Definition 1 (see [19]). For any two trajectories $w(t) = (w_1(t), w_2(t), \ldots, w_n(t))^T$ and $\overline{w}(t) = (\overline{w}_1(t), \overline{w}_2(t), \ldots, \overline{w}_n(t))^T$ of (4) starting from different initial values $w(0)$ and $\overline{w}(0)$, if there exists some control scheme such that

$$\lim_{t \to +\infty} \| w(t) - \overline{w}(t) \| = 0, \tag{8}$$

then we call system (4) can achieve outer-synchronization, where $\| \cdot \|$ denotes norm.

Let $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ and $v(t) = (v_1(t), v_2(t), \ldots, v_n(t))^T$ be two trajectories of (6) starting from different initial values $u(0)$ and $v(0)$. Defining $z_i(t) = u_i(t) - v_i(t)$, $i = 1,2,\ldots,n$, it follows that

$$\begin{aligned} {}^{C}D_{t_0}^{q} z_i(t) = {} & -a_i(t)\, z_i(t_k) + \sum_{j=1}^{n} b_{ij}(t)\, h_j(t_k), \\ & i = 1,2,\ldots,n, \end{aligned} \tag{9}$$

where $h_j(t) = f_j(u_j(t)) - f_j(v_j(t))$, $j = 1, 2, \ldots, n$, for all $t \in [t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$.

When we adopt the centralized data-sampling principle via structure to achieve outer-synchronization of (6), according to Definition 1, we need to design control strategy based on system structure of (9) such that

$$\lim_{t \to +\infty} \|z(t)\| = 0, \quad (10)$$

where $\|\cdot\|$ denotes norm, $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$.

When we adopt the centralized data-sampling principle via state to achieve outer-synchronization of (6), in this case, consider state measurement error

$$e_i(t) = z_i(t_k) - z_i(t), \quad i = 1, 2, \ldots, n, \quad (11)$$

where $t \in [t_k, t_{k+1})$, $k = 0, 1, 2, \ldots$. According to Definition 1, we need to design control strategy based on state measurement error (11) such that

$$\lim_{t \to +\infty} \|z(t)\| = 0, \quad (12)$$

where $\|\cdot\|$ denotes norm, $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$.

Let $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ and $v(t) = (v_1(t), v_2(t), \ldots, v_n(t))^T$ be two trajectories of (7) starting from different initial values $u(0)$ and $v(0)$. Defining $z_i(t) = u_i(t) - v_i(t)$, $i = 1, 2, \ldots, n$, it follows that

$$\begin{aligned} {}^{C}D_{t_0}^{q} z_i(t) &= -a_i(t) z_i(t_k^i) + \sum_{j=1}^{n} b_{ij}(t) h_j(t_k^j), \\ &\qquad i = 1, 2, \ldots, n, \end{aligned} \quad (13)$$

where $h_j(t) = f_j(u_j(t)) - f_j(v_j(t))$, $j = 1, 2, \ldots, n$, for all $t \in [t_k^j, t_{k+1}^j)$, $k = 0, 1, 2, \ldots$.

When we adopt the decentralized data-sampling principle via state to achieve outer-synchronization of (7), in this case, consider state measurement error

$$e_i(t) = z_i(t_k^i) - z_i(t), \quad i = 1, 2, \ldots, n, \quad (14)$$

where $t \in [t_k^i, t_{k+1}^i)$, $k = 0, 1, 2, \ldots$. According to Definition 1, we need to design control strategy based on state measurement error (14) such that

$$\lim_{t \to +\infty} \|z(t)\| = 0, \quad (15)$$

where $\|\cdot\|$ denotes norm, $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$.

Next, we present relevant lemmas.

Lemma 2 (see [1]). *Let $0 < q < 1$. If $\mathscr{H}(t) \in \mathscr{C}^1[t_0, +\infty]$, then*

$$ {}^{C}D_{t_0}^{q} |\mathscr{H}(t)| \le \operatorname{sgn}(\mathscr{H}(t)) \, {}^{C}D_{t_0}^{q} \mathscr{H}(t), \quad t \ge t_0, \quad (16)$$

where

$$ {}^{C}D_{t_0}^{q} |\mathscr{H}(t)| = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{(d/ds)|\mathscr{H}(s)|}{(t-s)^q} ds. \quad (17)$$

Lemma 3 (see [39]). *Given $q > 0$, let $\mathscr{A}(t)$ be nonnegative and locally integrable on $[a, b)$; let $\mathscr{G}(t)$ be continuous, bounded, nonnegative, and nondecreasing on $[a, b)$. Assuming $\mathscr{Y}(t)$ to be nonnegative and locally integrable on $[a, b)$ with*

$$\begin{aligned} \mathscr{Y}(t) &\le \mathscr{A}(t) + \mathscr{G}(t) \int_0^t (t-s)^{q-1} \mathscr{Y}(s) ds, \\ &\qquad t \in [a, b), \end{aligned} \quad (18)$$

then

$$\begin{aligned} &\mathscr{Y}(t) \\ &\quad \le \mathscr{A}(t) \\ &\qquad + \int_0^t \sum_{k=1}^{+\infty} \left[\frac{[\mathscr{G}(t)\Gamma(q)]^k}{\Gamma(kq)} (t-s)^{kq-1} \mathscr{A}(s) \right] ds, \\ &\qquad t \in [a, b). \end{aligned} \quad (19)$$

Moreover, if $\mathscr{A}(t)$ is nondecreasing on $[a, b)$, then

$$\mathscr{Y}(t) \le \mathscr{A}(t) E_q(\mathscr{G}(t)\Gamma(q) t^q), \quad t \in [a, b), \quad (20)$$

where $\Gamma(\cdot)$ is Gamma function and $E_q(\cdot)$ is one-parameter Mittag-Leffler function.

In the following, we end this section with some notations that are needed later.

Let $\beta_i > 0$ $(i = 1, 2, \ldots, n)$ be positive constants, throughout this paper; denote

$$\begin{aligned} \theta_i(t) &= a_i(t) - F_i b_{ii}^{+}(t), \\ \lambda_j(\beta, t) &= \theta_j(t) - F_j \sum_{j=1, j \ne i}^{n} \frac{\beta_i}{\beta_j} |b_{ij}(t)|, \\ \rho(t) &= \max_{1 \le i \le n} \{a_i(t) - F_i b_{ii}^{-}(t)\}, \end{aligned} \quad (21)$$

$$M = \max_{1 \le j \le n} \sup_{t \ge t_0} \left\{ a_j(t) + \sum_{i=1}^{n} \frac{\beta_i}{\beta_j} |b_{ij}(t)| F_j \right\}, \quad (22)$$

where $b_{ii}^{+}(t) = \max\{0, b_{ii}(t)\}$, $b_{ii}^{-}(t) = \min\{0, b_{ii}(t)\}$. For vector $\mathscr{V} = (\mathscr{V}_1, \mathscr{V}_2, \ldots, \mathscr{V}_n)^T$, vector norm $\|\mathscr{V}\| = \sum_{i=1}^{n} \beta_i |\mathscr{V}_i|$. In addition, by the boundedness of $a_i(t)$ and $b_{ij}(t)$, there exist positive constants B and C such that

$$\begin{aligned} \sup_{t \ge t_0} \rho(t) &\le B, \\ \max_{1 \le j \le n} \sup_{t \ge t_0} \lambda_j(\beta, t) &\le C. \end{aligned} \quad (23)$$

3. Main Results

For problem formulation in preceding section, in this section, we propose the corresponding control schemes for centralized data-sampling principle and decentralized data-sampling principle, respectively.

To facilitate the narrative, we first address the control designs, then review, and analyze the theoretical results.

3.1. Centralized Data-Sampling Principle

Theorem 4. *Let $0 < \varepsilon < 1$ and $\iota > 0$ be positive constants with $B\varepsilon \leq \iota$ and $C\varepsilon \leq \iota(2-\varepsilon)$. Assume that there exist positive constants $\beta_i > 0$ $(i = 1, 2, \ldots, n)$ such that $\lambda_j(\beta, t) \geq \iota$ for all $j = 1, 2, \ldots, n$ and $t \in [t_0, +\infty)$. Set t_{k+1} as a time point such that*

$$\begin{aligned} t_{k+1} &= \sup_{\tau \geq t_k} \left\{ \tau : \min_{1 \leq j \leq n} \left(\frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} \lambda_j(\beta, s)\, ds \right) \right. \\ &\left. \leq \varepsilon,\ \forall t \in (t_k, \tau] \right\} \end{aligned} \tag{24}$$

for $k = 0, 1, 2, \ldots$. Then system (6) reaches outer-synchronization.

Proof. From $\lambda_j(\beta, t) \geq \iota$ for all $j = 1, 2, \ldots, n$ and $t \in [t_0, +\infty)$, together with (23), it follows that

$$\begin{aligned} &\frac{\iota}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1}\, ds \\ &\leq \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} \lambda_j(\beta, s)\, ds \\ &\leq \frac{C}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1}\, ds; \end{aligned} \tag{25}$$

then

$$\begin{aligned} \frac{\iota (t-t_k)^q}{\Gamma(q+1)} &\leq \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} \lambda_j(\beta, s)\, ds \\ &\leq \frac{C (t-t_k)^q}{\Gamma(q+1)}, \end{aligned} \tag{26}$$

for all $j = 1, 2, \ldots, n$ and any $t \in [t_k, t_{k+1})$. According to (9), $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$ will not update until

$$\min_{1 \leq j \leq n} \left(\frac{1}{\Gamma(q)} \int_{t_k}^{t} (t-s)^{q-1} \lambda_j(\beta, s)\, ds \right) = \varepsilon \tag{27}$$

at time point $t = t_{k+1}$. Thus, we get $\iota(t_{k+1} - t_k)^q / \Gamma(q+1) \leq \varepsilon \leq C(t_{k+1} - t_k)^q / \Gamma(q+1)$, which implies

$$\frac{\varepsilon}{C} \leq \frac{(t_{k+1} - t_k)^q}{\Gamma(q+1)} \leq \frac{\varepsilon}{\iota}, \tag{28}$$

for $k = 0, 1, 2, \ldots$. Then

$$t_k \geq t_0 + k \left(\frac{\Gamma(q+1)\, \varepsilon}{C} \right)^{1/q}; \tag{29}$$

therefore, the Zeno behavior can be excluded. Combining with (24) and (28),

$$\begin{aligned} &\frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \lambda_j(\beta, s)\, ds \leq \frac{C\varepsilon}{\iota} \leq 2 - \varepsilon, \\ &\frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \lambda_j(\beta, s)\, ds \\ &\quad \geq \min_{1 \leq j \leq n} \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \lambda_j(\beta, s)\, ds = \varepsilon, \end{aligned} \tag{30}$$

so

$$\varepsilon \leq \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \lambda_j(\beta, s)\, ds \leq 2 - \varepsilon, \tag{31}$$

for $k = 0, 1, 2, \ldots$. By the definition of vector norm in this paper, from (9), now let us consider $z_i(t)$ $(i = 1, 2, \ldots, n)$ at time $t = t_{k+1}$,

$$\begin{aligned} \|z(t_{k+1})\| &= \sum_{i=1}^{n} \beta_i |z_i(t_{k+1})| = \sum_{i=1}^{n} \beta_i \left| z_i(t_k) + \frac{1}{\Gamma(q)} \right. \\ &\cdot \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \\ &\cdot \left. \left[-a_i(s) z_i(t_k) + \sum_{j=1}^{n} b_{ij}(s) h_j(t_k) \right] ds \right| = \sum_{i=1}^{n} \beta_i \\ &\cdot \left| z_i(t_k) + \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \left[-a_i(s) z_i(t_k) \right. \right. \\ &+ b_{ii}(s) m_i(t_k) z_i(t_k) \\ &+ \left. \left. \sum_{j=1, j \neq i}^{n} b_{ij}(s) m_j(t_k) z_j(t_k) \right] ds \right| = \sum_{i=1}^{n} \beta_i \\ &\cdot \left| z_i(t_k) \left\{ 1 - \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \right. \right. \\ &\cdot \left. [a_i(s) - b_{ii}(s) m_i(t_k)]\, ds \right\} + \frac{1}{\Gamma(q)} \\ &\cdot \sum_{j=1, j \neq i}^{n} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \\ &\cdot \left. \left[b_{ij}(s) m_j(t_k) z_j(t_k) \right] ds \right|, \end{aligned} \tag{32}$$

where

$$m_i(t) = \begin{cases} \dfrac{h_i(t)}{z_i(t)}, & z_i(t) \neq 0, \\ 0, & z_i(t) = 0. \end{cases} \tag{33}$$

According to (5), obviously, $0 \leq m_i(t) \leq F_i$ for all $i = 1, 2, \ldots, n$, $t > t_0$, and

$$b_{ii}^{-}(s) F_i \leq b_{ii}(s) m_i(t_k) \leq b_{ii}^{+}(s) F_i. \quad (34)$$

Notice that $B\varepsilon \leq \iota$; then for any $t \in [t_k, t_{k+1}]$,

$$\begin{aligned} &\frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} [a_i(s) - b_{ii}(s) m_i(t_k)] \, ds \\ &\leq \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} [a_i(s) - b_{ii}^{-}(s) F_i] \, ds \\ &\leq \frac{B}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \, ds = \frac{B(t_{k+1} - t_k)^q}{\Gamma(q+1)} \\ &\leq \frac{B\varepsilon}{\iota} \leq 1; \end{aligned} \quad (35)$$

thus

$$\begin{aligned} 1 &- \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} [a_i(s) - b_{ii}(s) m_i(t_k)] \, ds \\ &\geq 0. \end{aligned} \quad (36)$$

By (32) and (36),

$$\begin{aligned} \|z(t_{k+1})\| &= \sum_{i=1}^{n} \beta_i |z_i(t_{k+1})| \leq \sum_{i=1}^{n} \beta_i z_i(t_k) \left\{ 1 - \frac{1}{\Gamma(q)} \right. \\ &\cdot \left. \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} [a_i(s) - b_{ii}(s) m_i(t_k)] \, ds \right\} \\ &+ \frac{1}{\Gamma(q)} \sum_{i=1}^{n} \beta_i \\ &\cdot \sum_{j=1, j\neq i}^{n} \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} b_{ij}(s) m_j(t_k) z_j(t_k) \, ds \\ &\leq \sum_{j=1}^{n} \beta_j |z_j(t_k)| \left\{ 1 - \frac{1}{\Gamma(q)} \right. \\ &\cdot \left. \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} [a_j(s) - b_{jj}^{+}(s) F_j] \, ds \right\} + \frac{1}{\Gamma(q)} \\ &\cdot \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \beta_j |z_j(t_k)| \\ &\cdot \int_{t_k}^{t_{k+1}} \left[(t_{k+1} - s)^{q-1} \frac{\beta_i}{\beta_j} |b_{ij}(s)| F_j \right] ds \\ &\leq \sum_{j=1}^{n} \beta_j |z_j(t_k)| \left\{ 1 - \frac{1}{\Gamma(q)} \right. \\ &\cdot \left. \int_{t_k}^{t_{k+1}} (t_{k+1} - s)^{q-1} \lambda_j(\beta, s) \, ds \right\} \leq (1 - \varepsilon) \\ &\cdot \sum_{j=1}^{n} \beta_j |z_j(t_k)| = (1 - \varepsilon) \|z(t_k)\|, \end{aligned} \quad (37)$$

which leads to

$$\|z(t_k)\| = o\left(t_k^{-q-1}\right), \quad t_k \longrightarrow +\infty, \quad (38)$$

hence

$$\lim_{t_k \to +\infty} \|z(t_k)\| = 0. \quad (39)$$

Recalling system (9), we have

$$\begin{aligned} \lim_{t\to+\infty} \|z(t)\| &= \lim_{t\to+\infty} \sum_{i=1}^{n} \beta_i |z_i(t) - z_i(t_k) + z_i(t_k)| \\ &= \lim_{t\to+\infty} \|z(t_k)\| + \lim_{t\to+\infty} \sum_{i=1}^{n} \beta_i \left| \frac{1}{\Gamma(q)} \int_{t_k}^{t_{k+1}} (t - s)^{q-1} \right. \\ &\cdot \left. \left[-a_i(s) z_i(t_k) + \sum_{j=1}^{n} b_{ij}(s) h_j(t_k) \right] ds \right| \\ &\leq \lim_{k\to+\infty} \|z(t_k)\| + \lim_{t\to+\infty} M \sum_{j=1}^{n} \beta_j |z_j(t_k)| \frac{1}{\Gamma(q)} \int_{t_k}^{t} (t \\ &- s)^{q-1} \, ds \leq \lim_{k\to+\infty} \|z(t_k)\| + M \\ &\cdot \lim_{t\to+\infty} \frac{(t_{k+1} - t_k)^q}{\Gamma(q+1)} \|z(t_k)\| = 0; \end{aligned} \quad (40)$$

where M is defined in (22). It can be concluded that outer-synchronization of system (6) is proved. □

Remark 5. From inequality (28), we can see

$$\left[\frac{\Gamma(q+1)\varepsilon}{C} \right]^{1/q} \leq t_{k+1} - t_k \leq \left[\frac{\Gamma(q+1)\varepsilon}{\iota} \right]^{1/q} \quad (41)$$

for all $k = 0, 1, 2, \ldots$, which excludes the Zeno behavior for rule (24).

Theorem 6. *Let $\varphi(t)$ be a positive and continuous function on $[t_0, +\infty)$. Set t_{k+1} as a time point such that*

$$t_{k+1} = \sup_{\tau \geq t_k} \{ \tau : \|e(t)\| \leq \varphi(t), \ \forall t \in (t_k, \tau] \} \quad (42)$$

for all $k = 0, 1, 2, \ldots$, where $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$ is defined in (11). If there exist positive constants $\beta_i > 0$ ($i = 1, 2, \ldots, n$) such that $\min_{1\leq j\leq n} \lambda_j(\beta, t) \geq N$ for some $N > 0$ and all $t \geq t_0$, $\sup_{t\geq t_0} (1/\Gamma(q)) \int_{t_0}^{t} (t - s)^{q-1} \varphi(s) ds < +\infty$, then system (6) reaches outer-synchronization.

Proof. According to Lemma 2, from (9) and (42),

$$\begin{aligned}
{}^{C}D_{t_k}^{q}\|z(t)\| &= \sum_{i=1}^{n}\beta_i {}^{C}D_{t_k}^{q}|z_i(t)| \le \sum_{i=1}^{n}\operatorname{sgn}(z_i(t)) \\
&\cdot \beta_i {}^{C}D_{t_k}^{q} z_i(t) = \sum_{i=1}^{n}\operatorname{sgn}(z_i(t))\beta_i\Bigg\{-a_i(t)z_i(t_k) \\
&+ \sum_{j=1}^{n} b_{ij}(t)h_j(t_k)\Bigg\} = \sum_{i=1}^{n}\operatorname{sgn}(z_i(t))\beta_i\Bigg\{-a_i(t) \\
&\cdot [z_i(t_k) - z_i(t) + z_i(t)] + \sum_{j=1}^{n} b_{ij}(t) \\
&\cdot [m_j(t_k)z_j(t_k) - m_j(t)z_j(t) + m_j(t)z_j(t)]\Bigg\} \\
&= \sum_{i=1}^{n}\operatorname{sgn}(z_i(t))\beta_i\Bigg\{-a_i(t)e_i(t) + \sum_{j=1}^{n} b_{ij}(t)m_j(t_k) \\
&\cdot e_j(t) - a_i(t)z_i(t) + b_{ii}(t)m_i(t_k)z_i(t) \\
&+ \sum_{j=1,j\ne i}^{n} b_{ij}(t)m_j(t_k)z_j(t)\Bigg\} \le \sum_{j=1}^{n}\Bigg\{a_j(t) \\
&+ \sum_{i=1}^{n}\frac{\beta_i}{\beta_j}|b_{ij}(t)|F_j\Bigg\}\beta_j|e_j(t)| \\
&- \sum_{j=1}^{n}\Bigg\{a_j(t) - b_{jj}^{+}(t)F_j - \sum_{i=1,i\ne j}\frac{\beta_i}{\beta_j}|b_{ij}(t)|F_j\Bigg\} \\
&\cdot \beta_j|z_j(t)| \le -N\|z(t)\| + M\|e(t)\| \le -N\|z(t)\| \\
&+ M\varphi(t),
\end{aligned} \tag{43}$$

where M is defined in (22), and

$$m_i(t) = \begin{cases} \dfrac{h_i(t)}{z_i(t)}, & z_i(t) \ne 0, \\ 0, & z_i(t) = 0. \end{cases} \tag{44}$$

On the other hand, by (43),

$$\begin{aligned}
&\|z(t)\| \\
&\le \|z(t_0)\| \\
&+ \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}[-N\|z(s)\| + M\varphi(s)]\,ds \\
&= \|z(t_0)\| - \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}N\|z(s)\|\,ds \\
&+ \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}M\varphi(s)\,ds \\
&\le \|z(t_0)\| - \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}N\|z(s)\|\,ds + M\delta,
\end{aligned} \tag{45}$$

for $s \in [t_k, t)$, $t \in [t_k, t_{k+1})$, where $(1/\Gamma(q))\int_{t_0}^{t}(t-s)^{q-1}\varphi(s)ds \le \delta < +\infty$.

Using Lemma 3, from (45), it follows

$$\lim_{t\to+\infty}\|z(t)\| \le \lim_{t\to+\infty}\Theta E_q\left(-N(t-t_0)^q\right) = 0, \quad t \ge t_0, \tag{46}$$

where $\Theta = \|z(t_0)\| + M\delta$, which implies that $\|z(t)\|$ converges to 0 by the sampling time sequence $\{t_k\}_{k=0}^{+\infty}$. Therefore, system (6) reaches out-synchronization. □

3.2. Decentralized Data-Sampling Principle

Theorem 7. *Let $\phi(t) = (\phi_1(t), \phi_2(t), \dots, \phi_n(t))^T$ be positive and continuous on $[t_0, +\infty)$. Set t_{k+1} as a time point such that*

$$t_{k+1} = \sup_{\tau\ge t_k}\left\{\tau : |e_i(t)| \le \phi_i(t),\ \forall t \in \left(t_k^i, \tau\right]\right\} \tag{47}$$

for $i = 1, 2, \dots, n$ and all $k = 0, 1, 2, \dots$, where $e(t) = (e_1(t), e_2(t), \dots, e_n(t))^T$ is defined in (14). If there exist positive constants $\beta_i > 0$ $(i = 1, 2, \dots, n)$ such that $\min_{1\le j\le n}\lambda_j(\beta, t) \ge K$ for some $K > 0$ and all $t \ge t_0$, and $\sup_{t\ge t_0}(1/\Gamma(q))\int_{t_0}^{t}(t-s)^{q-1}\|\phi(s)\|ds < +\infty$, then system (7) reaches outer-synchronization.

Proof. According to Lemma 2, from (13) and (47),

$$\begin{aligned}
{}^{C}D_{t_k}^{q}\|z(t)\| &= \sum_{i=1}^{n}\beta_i {}^{C}D_{t_k}^{q}|z_i(t)| \le \sum_{i=1}^{n}\operatorname{sgn}(z_i(t)) \\
&\cdot \beta_i {}^{C}D_{t_k}^{q} z_i(t) = \sum_{i=1}^{n}\operatorname{sgn}(z_i(t))\beta_i\Bigg\{-a_i(t)z_i\left(t_k^i\right) \\
&+ \sum_{j=1}^{n} b_{ij}(t)h_j\left(t_k^j\right)\Bigg\} = \sum_{i=1}^{n}\operatorname{sgn}(z_i(t))\beta_i\Bigg\{-a_i(t) \\
&\cdot \left[z_i\left(t_k^i\right) - z_i(t) + z_i(t)\right] + \sum_{j=1}^{n} b_{ij}(t)
\end{aligned}$$

$$
\begin{aligned}
&\cdot \left[m_j\left(t_k^j\right) z_j\left(t_k^j\right) - m_j\left(t_k^j\right) z_j(t) \right.\\
&\left.\left. + m_j\left(t_k^j\right) z_j(t)\right]\right\} = \sum_{i=1}^{n} \operatorname{sgn}\left(z_i(t)\right) \beta_i \left\{-a_i(t)\right.\\
&\cdot e_i(t) - a_i(t) z_i(t) + b_{ii}(t) m_i\left(t_k^i\right) z_i(t)\\
&+ \sum_{j=1}^{n} b_{ij}(t) m_j\left(t_k^j\right) e_j(t)\\
&\left.+ \sum_{j=1, j\neq i}^{n} b_{ij}(t) m_j\left(t_k^j\right) z_j(t)\right\} \leq \sum_{j=1}^{n} \left\{a_j(t)\right.\\
&\left.+ \sum_{i=1}^{n} \frac{\beta_i}{\beta_j}\left|b_{ij}(t)\right| F_j\right\} \beta_j \left|e_j(t)\right| - \sum_{j=1}^{n}\left\{a_j(t)\right.\\
&\left.- b_{jj}^{+}(t) F_j - \sum_{i=1, i\neq j} \frac{\beta_i}{\beta_j}\left|b_{ij}(t)\right| F_j\right\} \beta_j \left|z_j(t)\right|\\
&\leq -K\|z(t)\| + M\|e(t)\| \leq -K\|z(t)\| + M\|\phi(t)\|,
\end{aligned} \tag{48}
$$

where M is defined in (22), and

$$
m_j\left(t_k^j\right) = \begin{cases} \dfrac{h_j\left(t_k^j\right)}{z_j\left(t_k^j\right)}, & z_j\left(t_k^j\right) \neq 0, \\ 0, & z_j\left(t_k^j\right) = 0. \end{cases} \tag{49}
$$

On the other hand, by (48),

$$
\begin{aligned}
&\|z(t)\|\\
&\quad \leq \left\|z(t_0)\right\|\\
&\quad\quad + \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} \left[-K\|z(s)\| + M\|\phi(s)\|\right] ds\\
&\quad = \left\|z(t_0)\right\| - \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} K \|z(s)\| \, ds\\
&\quad\quad + \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} M \|\phi(s)\| \, ds\\
&\quad \leq \left\|z(t_0)\right\| - \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-s)^{q-1} K \|z(s)\| \, ds + M\rho,
\end{aligned} \tag{50}
$$

for $s \in [t_k^i, t)$, $t \in [t_k^i, t_{k+1}^i)$, where $(1/\Gamma(q)) \int_{t_0}^{t} (t-s)^{q-1} \|\phi(s)\| ds \leq \rho < +\infty$.

Using Lemma 3, from (50), it follows

$$
\lim_{t\to+\infty} \|z(t)\| \leq \lim_{t\to+\infty} \Pi E_q\left(-K\left(t-t_0\right)^q\right) = 0, \quad t \geq t_0, \tag{51}
$$

where $\Pi = \|z(t_0)\| + M\rho$, which implies that $\|z(t)\|$ converges to 0 by the sampling time sequence $\{t_k^i\}_{k=0}^{+\infty}$, $i = 1, 2, \ldots, n$. Therefore, system (7) reaches out-synchronization. □

Remark 8. As Theorem 5 in [19], under the data-sampling rule in Theorem 6 or Theorem 7, the interevent interval of each system state is strictly positive and possesses a common positive lower bound. Furthermore, the Zeno behavior is excluded.

Remark 9. For the sampled-data control, how to choose the proper scheme with the ultimate aim of maximizing the data collected to control the system is challenging. For example, as revealed in [9, 10], it is extremely difficult to design the sampling time point inherited from the sampled-data control strategy. However, according to Theorems 4–7, this situation can be effectively solved if the centralized and decentralized data-sampling principles are cleverly utilized.

Remark 10. For three control schemes in Theorems 4–7, these are just the type and level of points, not the merits of good points of difference. Theorem 4 is entirely focused around the centralized data-sampling principle via structure. Theorem 6 is concerned with the centralized data-sampling principle via state. Theorem 7 is to place emphasis on the decentralized data-sampling principle via state.

Remark 11. Note that the sampled-data control in Theorems 4–7 exerts only at the sampling time point, that is, every system state employs only its neighbors' information at t_k or t_k^i. Thus, compared with the continuous-time control strategy, the control schemes in Theorems 4–7 can effectively save the bandwidth and reduce the communication cost. Moreover, the results obtained here are the first ones on centralized and decentralized data-sampling principles for outer-synchronization of fractional-order neural networks.

Remark 12. The key features of outer-synchronization in Theorems 4–7 are follows. (1) Each outer-synchronization scheme is closely related to the sampling time point. Once the sampling time point is given, the states of the controlled fractional-order neural networks will achieve outer-synchronization. (2) Centralized data-sampling principle via structure makes full use of the characteristic of system itself, while centralized or decentralized data-sampling principle via state skillfully combines the feature of state measurement error.

Remark 13. The analytical methods for outer-synchronization in Theorems 4–7 are quite different from conventional complete synchronization, projective synchronization, phase synchronization, distributed synchronization, pinning synchronization, and cluster synchronization.

4. A Numerical Example

In this section, a numerical example is utilized to show the effectiveness of the results obtained.

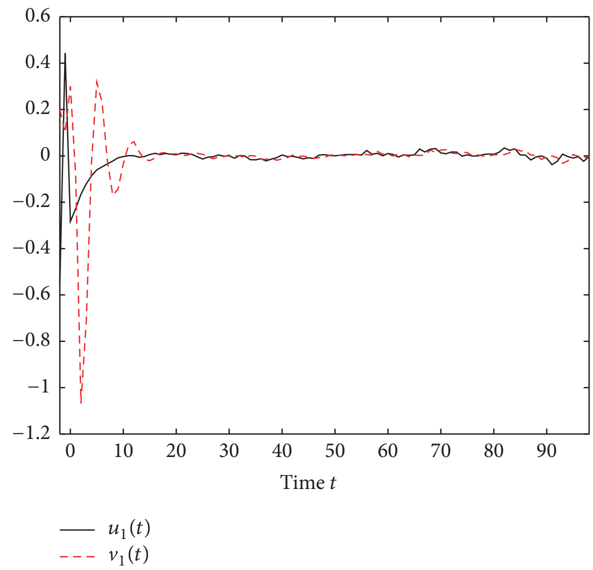

FIGURE 1: Dynamics of $u_1(t)$ and $v_1(t)$ in the triggering mechanism as Theorem 4.

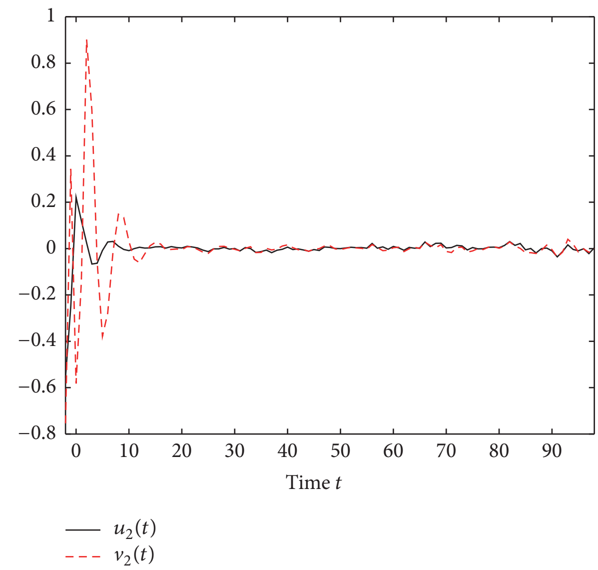

FIGURE 2: Dynamics of $u_2(t)$ and $v_2(t)$ in the triggering mechanism as Theorem 4.

Consider a class of fractional-order neural networks as follows:

$$\begin{aligned} {}^{C}D^{q}_{t_0}x_1(t) &= -a_1(t)x_1(t) + b_{11}(t)f_1(x_1(t)) \\ &\quad + b_{12}(t)f_2(x_2(t)) + u_1(t), \\ {}^{C}D^{q}_{t_0}x_2(t) &= -a_2(t)x_2(t) + b_{21}(t)f_1(x_1(t)) \\ &\quad + b_{22}(t)f_2(x_2(t)) + u_2(t), \end{aligned} \quad (52)$$

where $q = 1/2$, $t_0 = 0$, $A = \begin{pmatrix} a_1(t) & 0 \\ 0 & a_2(t) \end{pmatrix} = \begin{pmatrix} 1.5 & 0 \\ 0 & 1.5 \end{pmatrix}$, $B = \begin{pmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{pmatrix} = \begin{pmatrix} 0.3 & -0.7 \\ -1.2 & -0.1 \end{pmatrix}$, $u = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}$, $f_1(\mathbb{X}) = f_2(\mathbb{X}) = 1/(1+e^{-\mathbb{X}})$.

By direct calculation, we can obtain

$$\begin{aligned} \lambda_1(\beta, t) &= a_1(t) - F_1 b^{+}_{11}(t) - \frac{F_1\beta_2}{\beta_1}\left|b_{21}(t)\right|, \\ \lambda_2(\beta, t) &= a_2(t) - F_2 b^{+}_{22}(t) - \frac{F_2\beta_1}{\beta_2}\left|b_{12}(t)\right|. \end{aligned} \quad (53)$$

To choose $\varepsilon = 0.2$, $\iota = 0.8$, $\beta_1 = \beta_2 = 1$, then it follows that $B = 1.6$, $C = 1.05$. Hence the following inequalities hold:

$$\begin{aligned} B\varepsilon - \iota &\le 0, \\ C\varepsilon - \iota(2-\varepsilon) &\le 0. \end{aligned} \quad (54)$$

According to Theorem 4, system (52) reaches outer-synchronization. Figures 1 and 2 depict the dynamics of $u_1(t)$ and $v_1(t)$, $u_2(t)$ and $v_2(t)$ in the triggering time points as Theorem 4, respectively. Figure 3 describes the release time points and release intervals.

To select $\varphi(t) = 1/(t+1)^{1/2}$, together with

$$\sup_{t\ge 0} \frac{1}{\Gamma(1/2)} \int_0^t (t-s)^{q-1} \frac{1}{(s+1)^{1/2}} ds < +\infty, \quad (55)$$

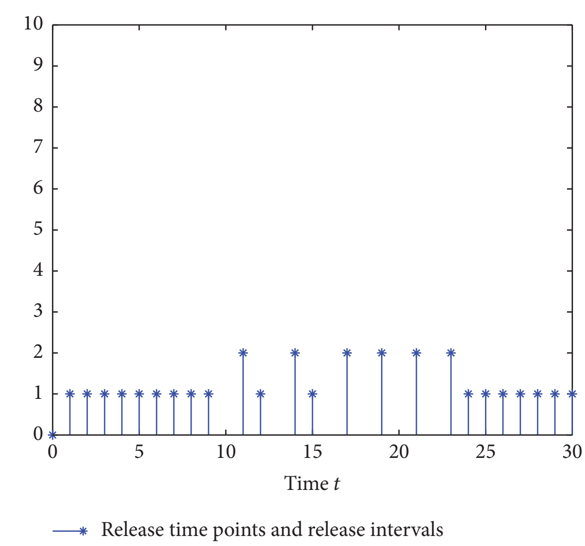

FIGURE 3: The release time points and release intervals in the triggering mechanism as Theorem 4.

according to Theorem 6, system (52) reaches outer-synchronization. Figures 4 and 5 depict the dynamics of $u_1(t)$ and $v_1(t)$, $u_2(t)$ and $v_2(t)$ in the triggering time points as Theorem 6, respectively. Figure 6 describes the release time points and release intervals.

To select $\phi_1(t) = 1/(t+1)^{1/2}$, $\phi_2(t) = 1/(t+2)^{1/2}$, together with

$$\begin{aligned} &\sup_{t\ge 0} \frac{1}{\Gamma(1/2)} \int_0^t (t-s)^{q-1} \frac{1}{(s+1)^{1/2}} ds < +\infty, \\ &\sup_{t\ge 0} \frac{1}{\Gamma(1/2)} \int_0^t (t-s)^{q-1} \frac{1}{(s+2)^{1/2}} ds < +\infty, \end{aligned} \quad (56)$$

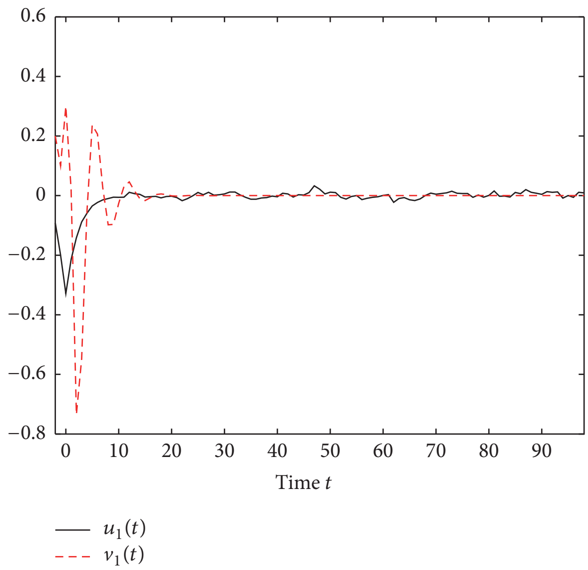

Figure 4: Dynamics of $u_1(t)$ and $v_1(t)$ in the triggering mechanism as Theorem 6.

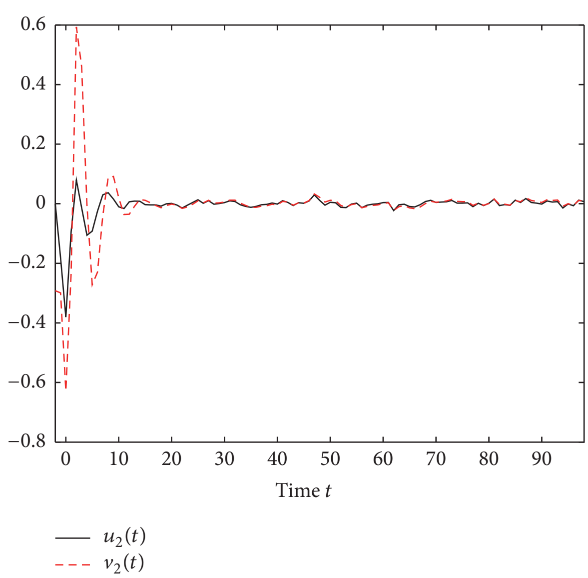

Figure 5: Dynamics of $u_2(t)$ and $v_2(t)$ in the triggering mechanism as Theorem 6.

according to Theorem 7, system (52) reaches outer-synchronization. Figures 7 and 8 depict the dynamics of $u_1(t)$ and $v_1(t)$, $u_2(t)$ and $v_2(t)$ in the triggering time points as Theorem 7, respectively. Figure 9 describes the release time points and release intervals.

Remark 14. In existing publications, there has been no theoretic criterion to achieve outer-synchronization of (52). In addition, using centralized or decentralized data-sampling principle to analyze and control fractional-order systems is also rare.

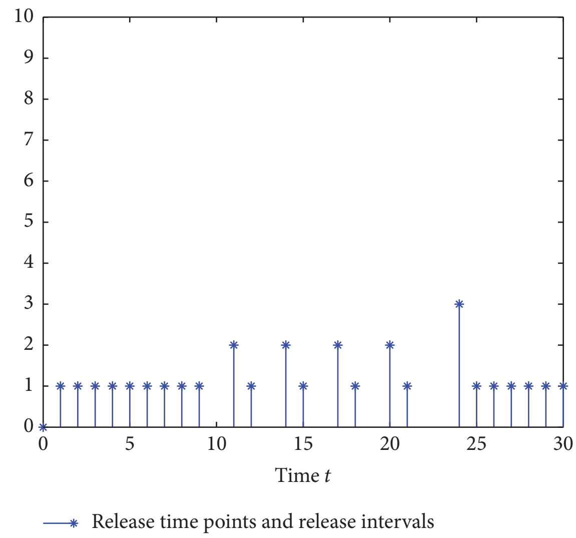

Figure 6: The release time points and release intervals in the triggering mechanism as Theorem 6.

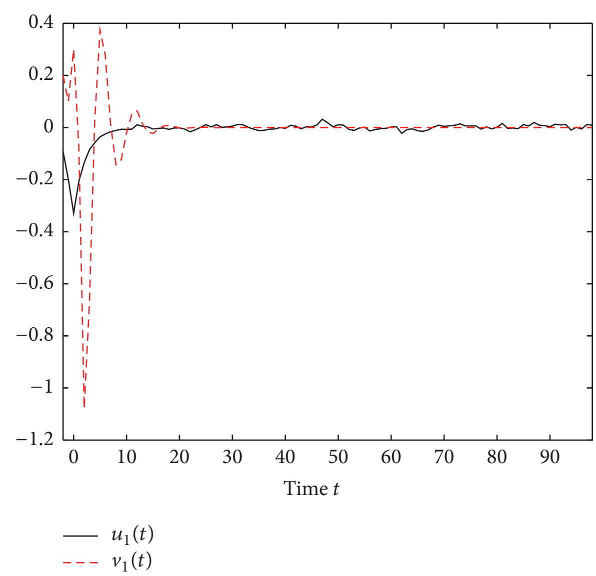

Figure 7: Dynamics of $u_1(t)$ and $v_1(t)$ in the triggering mechanism as Theorem 7.

Remark 15. According to simulation analysis in Figures 1–9, there is no essential difference regarding outer-synchronization performance in three control schemes as Theorems 4–7. By comparative analysis of Figures 3, 6, and 9, the release intervals via control scheme as Theorem 4 are relatively minor, and the triggering time points via control scheme as Theorem 7 are spread more thinly.

5. Concluding Remarks

In this paper, we show that outer-synchronization of fractional-order neural networks can be achieved by applying appropriate centralized and decentralized data-sampling

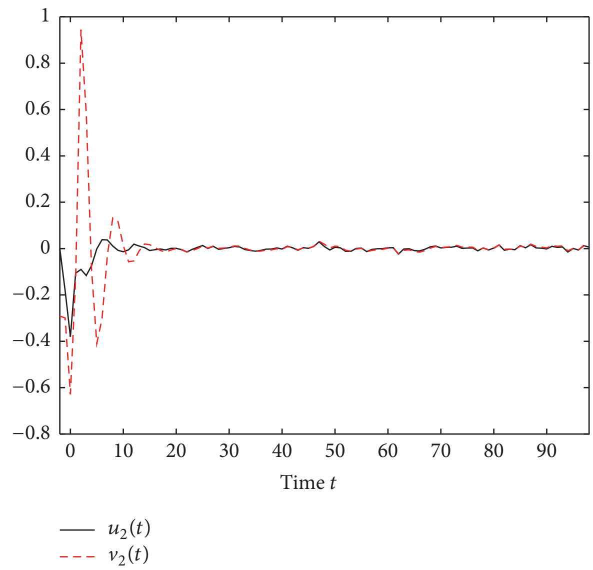

FIGURE 8: Dynamics of $u_2(t)$ and $v_2(t)$ in the triggering mechanism as Theorem 7.

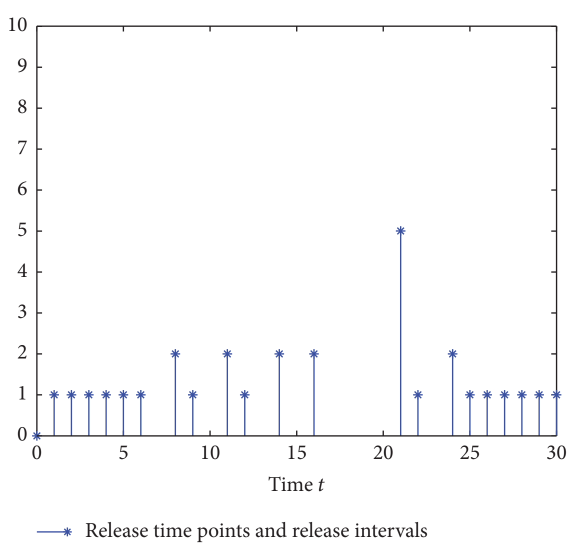

FIGURE 9: The release time points and release intervals in the triggering mechanism as Theorem 7.

principles. Such theoretical results improve and supplement some existing related results. The results obtained here are sufficient conditions for outer-synchronization of fractional-order neural networks and may remain room for improvement. Further extensions would be welcome: (1) outer-synchronization of fractional-order neural networks considering both conservativeness and complexity; (2) analyzing the outer-synchronization of fractional-order neural networks subject to time-delay; (3) analyzing the outer-synchronization of fractional-order neural networks subject to stochastic disturbance.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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