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Research Article

The Fractional Kalman Filter-Based Asynchronous Multirate Sensor Information Fusion

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A fractional Kalman filter-based multirate sensor fusion algorithm is presented to fuse the asynchronous measurements of the multirate sensors. Based on the characteristics of multirate and delay measurement, the state is reestimated at the time when the delayed measurement occurs by using weighted fractional Kalman filter, and then the state estimation is updated at the current time when the delayed measurement arrives following the similar pattern of Kalman filter. The simulation examples are given to illustrate the effectiveness of the proposed fusion method.

1. Introduction

Multisensor information fusion has been a key issue in sensor research since the 1970s and it has been applied in many fields, such as navigation, tracking, control, and wireless sensor networks. Due to the limitations of sensors, the single sensor cannot accurately estimate the system states; various asynchronous sensors with multiple sampling rates are used, such as visual sensors, position sensors, and inertial sensors. Moreover, with the improvements of the complexity of multisensor systems and design accuracy, the designs of integer-order estimators cannot also meet the existing requirements. Considering the high-accuracy estimation, fractional-order and multisensor fusion are studied by more and more scholars.

Because systems can be accurately described by fractional calculus operator, fractional order has been widely applied in many fields, such as electromagnetism, thermal, electrochemical, robot, control system, and image processing [1]. Sierociuk and Dzieliski [2] proposed fractional-order Kalman filter to estimate the states and parameters of discrete fractional-order state models. In [3], the fractional-order Kalman filter was introduced to fuse the MEMS (microelectromechanical systems) sensor data, which was successfully applied in the estimation of motion problems. In [4], the extended fractional Kalman filter is utilized for state estimation strategy for fractional-order systems with noises and multiple time delayed measurements. In [5], the control, estimation, and stability analysis for fractional-order system were investigated. In [6–8], the stability theories of fractional-order systems were studied to provide theoretical basis for fractional-order systems state estimation methods. The discrete-time differential system modeling was improved in [9, 10]; fractional-order system modeling lays the foundation for the discrete filter design. Although fractional-order filters are used in some fields, the fractional-order-based asynchronous multirate sensor fusion is not considered.

The state estimate plays an important role in practical application, such as tracking control [11, 12]; multiple sensors are utilized to achieve the higher estimate performance. Since multisensor fusion can get more comprehensive and refined information than any single sensor alone, the asynchronous multirate sensor information fusion becomes an important problem in the actual system. In [13], the asynchronous multirate information fusion was modeled, and Kalman filter-based information fusion algorithm was proposed. In [14], the Kalman filter-based asynchronous optimal estimation was presented for a class of 2D gauss

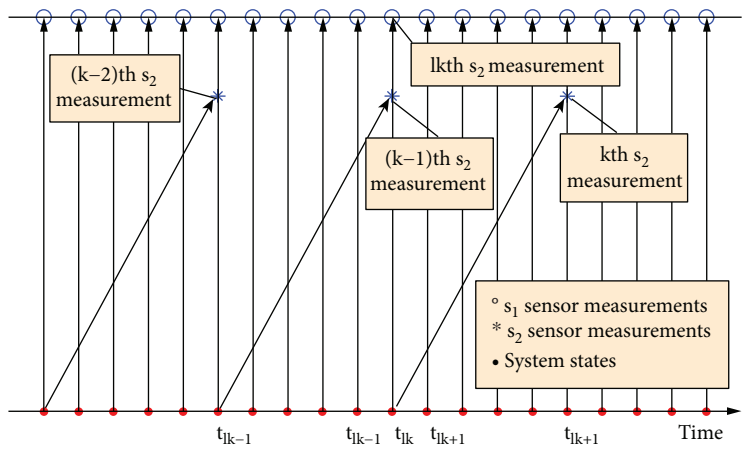

Figure 1: The asynchronous measurement schematic diagram between different sampling rates.

Markov process. Xia et al. [15] designed multiple-lag out-of-sequence measurement filtering algorithms for fusing the network delayed data. The main drawback of the above methods is only a consideration for the integer Kalman filters; thus, the system states cannot be approximated accurately.

In this paper, we present a novel fractional fusion algorithm to the asynchronous multirate sensor systems. The fractional multirate sensor system is addressed, and the fractional Kalman filter is used for asynchronous fusion algorithm, such that the fusion results achieve high-precision and economic storage space.

2. Problem Formulations

2.1. Discrete Linear System Model. Consider the state equation and measurement equation of integer systems as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k), \\ y_i(k) &= C_i x(k) + v(k), \end{aligned} \tag{1}$$

where C_i denotes the measurement matrix; $i = 1$ and $i = 2$ represent fast and slow sampling rate measurements, respectively; $v(k)$ is the measurement noise; $w(k)$ is the process noise.

Due to slow rate and delay, system states cannot be measured accurately by slow sensor. Thus, fast sensor is introduced to improve control performance of robotic systems. The relationship between slow sensor and fast sensor is depicted in Figure 1, where l is a positive integer, and s_1 and s_2 are defined as the different rates of sensors, respectively, and satisfy the following equation:

$$s_1 = ls_2. \tag{2}$$

Because systems can be accurately described by fractional calculus, the fractional-order Grunwald-Letnikov difference is introduced to transfer the original systems to fractional systems.

According to the definition of the fractional-order Grunwald-Letnikov difference,

$$\Delta^n x_k = \frac{1}{h^n} \sum_{j=0}^{k} (-1)^j \binom{n}{j} x_{k-j}, \tag{3}$$

where $n \in R$ is the order of fractional difference; R is the set of real numbers; h is the sampling interval, later assumed to be 1, and k is the number of samples for which the derivative is calculated. The factor can be obtained from

$$\binom{n}{j} = \begin{cases} 1 & j = 0, \\ \dfrac{n(n-1)\cdots(n-j+1)}{j!} & j > 0. \end{cases} \tag{4}$$

Using the definition in (3), the traditional discrete linear stochastic state-space system can be rewritten as follows:

$$\Delta^{\Upsilon} x(k+1) = A_d x(k) + Bu(k) + w(k), \tag{5}$$

$$x(k+1) = \Delta^{\Upsilon} x(k+1) - \sum_{j=1}^{k+1} (-1)^j \Upsilon_j x(k+1-j), \tag{6}$$

where

$$\Upsilon_j = \begin{bmatrix} \binom{n_1}{j} & 0 & \cdots & 0 \\ 0 & \binom{n_2}{j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \binom{n_N}{j} \end{bmatrix}, \tag{7}$$

$$\Delta^{\Upsilon} x(k+1) = \begin{bmatrix} \Delta^{n_1} x_1(k+1) \\ \vdots \\ \Delta^{n_N} x_N(k+1) \end{bmatrix}, \tag{8}$$

with $A_d = A - I$ and $n_1, \ldots, n_N$ represent the orders of systems.

According to (5)–(8), the system model is summarized as follows:

$$x(k+1) = A_d x(k) + Bu(k) + w(k) - \sum_{j=1}^{k+1} (-1)^j \Upsilon_j x(k+1-j), \tag{9}$$

$$y_i(k) = C_i x(k) + v(k). \tag{10}$$

Assumption 1. The process noise $w(k)$ and measurement noise $v(k)$ are Gaussian noises, which are independent from each other and satisfy

$$\begin{cases} E\{w(k)\} = 0, \\ E\{v(k)\} = 0, \\ E\{v(j) w^{\mathrm{T}}(k)\} = 0, \\ E\{w(j) w^{\mathrm{T}}(k)\} = R(k)\delta_{jk}, \\ E\{v(j) v^{\mathrm{T}}(k)\} = Q(k)\delta_{jk}, \end{cases} \tag{11}$$

where R and Q are variances of $w(k)$ and $v(k)$, respectively.

Assumption 2. The initial state $x(0)$ is irrelevant with process noise $w(k)$ and measurement noise $v(k)$; moreover, it satisfies

$$E\{x(0)\} = \mu_0, E\left\{[x(0) - \mu_0][x(0) - \mu_0]^{\mathrm{T}}\right\} = P_0. \tag{12}$$

2.2. Fractional Kalman Filter

Lemma 1 (see [2]). *Considering the fractional discrete state (9) and measurement (10), the fractional recursive Kalman filter is devised as follows:*

$$\hat{x}(k+1 \mid k) = A_d \hat{x}(k \mid k) + Bu(k) - \sum_{j=1}^{k+1} (-1)^j \Upsilon_j \hat{x}(k+1-j), \tag{13}$$

$$P(k+1 \mid k) = (A_d + \Upsilon_1) P(k \mid k) (A_d + \Upsilon_1)^{\mathrm{T}} + Q(k) + \sum_{j=1}^{k+1} \Upsilon_j P(k+1-j \mid k+1-j) \Upsilon_j^{\mathrm{T}}, \tag{14}$$

$$\hat{x}(k+1 \mid k+1) = \hat{x}(k+1 \mid k) + K(k+1) \cdot [y(k+1) - C_i \hat{x}(k+1 \mid k)], \tag{15}$$

$$P(k+1 \mid k+1) = [I - K(k+1)] P(k+1 \mid k), \tag{16}$$

$$K(k+1) = P(k+1 \mid k) C_i^{\mathrm{T}} \left[C_i P(k+1 \mid k) C_i^{\mathrm{T}} + R(k+1)\right]^{-1}, \tag{17}$$

$$\begin{aligned} \hat{x}(0 \mid 0) &= \mu_0, \\ P(0 \mid 0) &= P_0, \end{aligned} \tag{18}$$

where

$$\begin{aligned} P(k+1 \mid k) &= E\left\{[\hat{x}(k+1 \mid k) - x(k+1)] \cdot [\hat{x}(k+1 \mid k) - x(k+1)]^{\mathrm{T}}\right\} \\ &= (A_d + \Upsilon_1) E\left\{[\hat{x}(k \mid k) - x(k)] \cdot [\hat{x}(k \mid k) - x(k)]^{\mathrm{T}}\right\} (A_d + \Upsilon_1)^{\mathrm{T}} \\ &\quad + E\left[w_k w_k^{\mathrm{T}}\right] + \sum_{j=1}^{k+1} \Upsilon_j E\left\{[\hat{x}(k-j \mid k-j) - x(k-j)] \times [\hat{x}(k-j \mid k-j) - x(k-j)]^{\mathrm{T}}\right\} \Upsilon_j^{\mathrm{T}} \\ &= (A_d + \Upsilon_1) P(k \mid k) (A_d + \Upsilon_1)^{\mathrm{T}} + Q(k) \\ &\quad + \sum_{j=1}^{k+1} \Upsilon_j P(k+1-j \mid k+1-j) \Upsilon_j^{\mathrm{T}}. \end{aligned} \tag{19}$$

From above, the prediction of the covariance error matrix depends on the values of the covariance matrices in previous time samples. This is the main difference in comparison with an integer-order Kalman filter.

In the following section, the fractional Kalman filter-based asynchronous multirate sensor fusion algorithm is elaborated.

3. Asynchronous Multirate Sensor Information Fusion

According to the relationship between sensors shown in Figure 1, it is known that the slow rate delay measurement is the state measurement at the time $t_\kappa = t_{lk-l}$. As shown in Figure 2, the key task of the fusion method is to utilize the delay measurement with slow sampling rate to calculate $\hat{x}(lk)$.

The realization of the algorithm is mainly divided into the following two steps:

Step 1 As shown in Figure 3, the slow rate delay measurement and fast rate measurement at the time t_{lk} are applied to reestimate the state at the time t_κ by a weighted fusion method

Step 2 The fusion estimate $x_F(\kappa \mid lk)$ is utilized to update the current state estimate

Considering the smoothing and the weighted optimal fusion, the weighted fusion method in [16] is introduced to fuse multirate sensor measurements for reestimating the state at the time t_κ. Moreover, the optimal estimation at the time t_κ is used to update the state at the time t_{lk}.

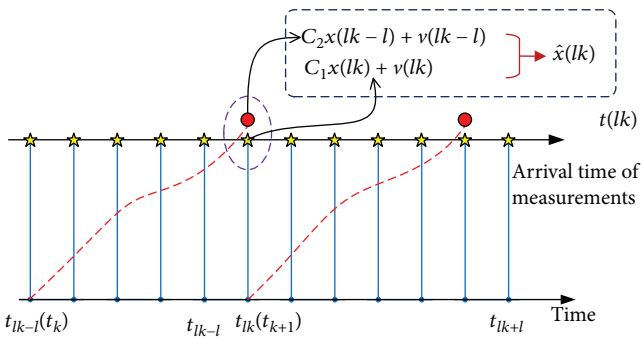

Figure 2: The core purpose of the proposed method.

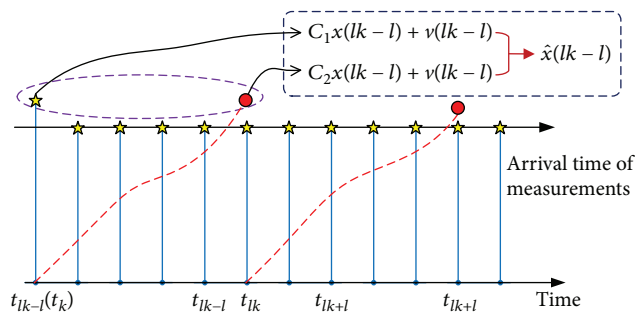

Figure 3: Calculate the $x_F(\kappa \mid lk)$ by using weighted fusion method.

Lemma 2 (see [16]). *Considering the state at the time t_κ, multisensor measurements are available simultaneously, and then, system fusion estimation is formulated as follows:*

$$\hat{x}_F(\kappa) = a_1\hat{x}_1(\kappa) + a_2\hat{x}_2(\kappa), \tag{20}$$

$$P_F = a_1^2P_1 + a_2^2P_2 + 2a_1a_2, \tag{21}$$

where

$$\begin{aligned} P_{m(=1,2,F)} &= E\left[(\hat{x}_m - x)(\hat{x}_m - x)^{\mathrm{T}}\right], \\ P_{12} &= E\left[(\hat{x}_1 - x)(\hat{x}_2 - x)^{\mathrm{T}}\right], \\ a_1 &= \frac{\mathrm{tr}(P_2) - \mathrm{tr}(P_{12})}{\mathrm{tr}(P_1) + \mathrm{tr}(P_2) - 2\mathrm{tr}(P_{12})}, \\ a_2 &= \frac{\mathrm{tr}(P_1) - \mathrm{tr}(P_{12})}{\mathrm{tr}(P_1) + \mathrm{tr}(P_2) - 2\mathrm{tr}(P_{12})}, \end{aligned} \tag{22}$$

where $\mathrm{tr}(\cdot)$ denotes trace of matrix. $\mathrm{tr}(P_F)$ satisfies the following equation:

$$\begin{aligned} \mathrm{tr}(P_F) &\le \mathrm{tr}(P_1), \\ \mathrm{tr}(P_F) &\le \mathrm{tr}(P_2). \end{aligned} \tag{23}$$

Remark 1. For convenience, the fusion weights can also be chosen as follows:

$$a_1 = \frac{\mathrm{tr}(P_2)}{\mathrm{tr}(P_1) + \mathrm{tr}(P_2)}, \tag{24}$$

$$a_2 = \frac{\mathrm{tr}(P_1)}{\mathrm{tr}(P_1) + \mathrm{tr}(P_2)}. \tag{25}$$

In order to solve the fusion estimation (20), the frictional Kalman filters are introduced to obtain $\hat{x}_1(\kappa)$ and $\hat{x}_2(\kappa)$.

Firstly, considering the slow sensor, the state estimation $\hat{x}_2(k)$ is solved at the time t_κ, where the corresponding state and measurement equations are denoted as follows:

$$\begin{aligned} x(lk) = {} & A_d^l x(lk-l) + Bu(lk) + \sum_{j=1}^{l} A^j w(lk) \\ & - \sum_{j=1}^{lk} (-1)^j \Upsilon_j x(lk-j), \end{aligned} \tag{26}$$

$$y_2(lk) = C_2x(lk) + v(lk). \tag{27}$$

According to (26) and (27), $\hat{x}_2(k)$ can be solved by (13)–(18) as follows:

$$\begin{aligned} \hat{x}_2(\kappa \mid \kappa-1) = {} & A_d^l\hat{x}_2(\kappa-1) + Bu(\kappa) \\ & - \sum_{j=1}^{\kappa} (-1)^j \Upsilon_j \hat{x}_2(\kappa-j), \end{aligned} \tag{28}$$

$$\begin{aligned} P_2(\kappa \mid \kappa-1) = {} & \left(A_d^l + \Upsilon_1\right)P_2(\kappa-1|\kappa-1)\left(A_d^l + \Upsilon_1\right)^{\mathrm{T}} \\ & + \left(\sum_{j=0}^{l-1} A_d^{j-1}\right)Q(\kappa-1)\left(\sum_{j=0}^{l-1} A_d^{j-1}\right)^{\mathrm{T}} \\ & + \sum_{j=1}^{\kappa} \Upsilon_j P_2(\kappa-j \mid \kappa-j)\Upsilon_j^{\mathrm{T}}, \end{aligned} \tag{29}$$

$$\begin{aligned} \hat{x}_2(\kappa \mid \kappa) = {} & \hat{x}_2(\kappa \mid \kappa-1) + K_2(\kappa) \\ & \cdot \left[y_2(\kappa) - C_2\hat{x}_2(\kappa \mid \kappa-1)\right], \end{aligned} \tag{30}$$

$$P_2(\kappa \mid \kappa) = [I - K_2(\kappa)]P_2(\kappa \mid \kappa-1), \tag{31}$$

$$\begin{aligned} K_2(\kappa) = {} & P_2(\kappa \mid \kappa-1)C_2^{\mathrm{T}} \\ & \cdot \left[C_2P_2(\kappa \mid \kappa-1)C_2^{\mathrm{T}} + R(\kappa)\right]^{-1}, \end{aligned} \tag{32}$$

$$\begin{aligned} \hat{x}_2(0 \mid 0) &= \mu_0, \\ P_2(0 \mid 0) &= P_0. \end{aligned} \tag{33}$$

In the following, based on a reverse filter algorithm, the state at the time t_κ is estimated by fast rate measurement at the time t_{lk}. The fractional Kalman filter (13)–(18) is adopted to obtain the estimation $\hat{x}_1(lk \mid lk)$ at the time t_{lk}.

$$\begin{aligned} \hat{x}_1(lk \mid lk-1) = {} & A_d\hat{x}_1(lk-1) + Bu(lk-1) \\ & - \sum_{j=1}^{lk} (-1)^j \Upsilon_j \hat{x}_1(lk-j), \end{aligned} \tag{34}$$

$$P(lk \mid lk-1) = (A_d + \Upsilon_1)P_1(lk-1 \mid lk-1) \cdot (A_d + \Upsilon_1)^{\mathrm{T}} + Q(lk-1) + \sum_{j=1}^{lk} \Upsilon_j P(lk-j \mid lk-j)\Upsilon_j^{\mathrm{T}}, \tag{35}$$

$$\hat{x}_1(lk \mid lk) = \hat{x}_1(lk \mid lk-1) + K_1(lk) \cdot [y_1(lk) - C_1\hat{x}_1(lk \mid lk-1)], \tag{36}$$

$$P_1(lk \mid lk) = [I - K_1(lk)]P_1(lk \mid lk-1), \tag{37}$$

$$K_1(lk) = P_1(lk \mid lk-1)C_1^{\mathrm{T}} \cdot \left[C_1P_1(lk \mid lk-1)C_1^{\mathrm{T}} + R(lk)\right]^{-1}, \tag{38}$$

$$\hat{x}_1(0 \mid 0) = \mu_0, \quad P_1(0 \mid 0) = P_0. \tag{39}$$

All the state transition matrixes of actual systems are exponential matrixes, which are reversible, defined as A_d^{-1}. Reverse state transition equation from time t_{lk} to t_κ is shown as follows:

$$x_1(\kappa) = A_d^{-l}\left[x_1(lk) - \sum_{j=1}^{l-1} A_d^{j-1} w_1(lk-j)\right], \tag{40}$$

and the solution of $\hat{x}_1(\kappa \mid \kappa)$ is revealed as follows:

$$\hat{x}_1(\kappa \mid \kappa) = A_d^{-l}\left[\hat{x}_1(lk \mid lk) - \sum_{j=1}^{l-1} A_d^{j-1} \hat{w}_1(lk-j)\right], \tag{41}$$

which can be transformed to

$$\hat{x}_1(\kappa \mid \kappa) = A_d^{-l}[\hat{x}_1(lk \mid lk-1) + K_1(lk)y_1(lk) - C_1\hat{x}_1(lk \mid lk-1)]. \tag{42}$$

Substituting (30) and (42) into (20), we can yield the optimal fusion estimation at the time t_κ.

$$\hat{x}_F(\kappa) = a_1A_d^{-l}[\hat{x}_1(lk \mid lk-1) + K_1(lk)y_1(lk) - C_1\hat{x}_1(lk \mid lk-1)] + a_2[\hat{x}_2(\kappa \mid \kappa-1) + K_2(\kappa)y_2(\kappa) - C_2\hat{x}_2(\kappa \mid \kappa-1)]. \tag{43}$$

Defining $\tilde{x}(k) = x(k) - \hat{x}(k)$, one can find

$$\hat{x}_F(\kappa) - x(\kappa) = a_1A_d^{-l}[\hat{x}_1(lk \mid lk-1) + K_1(lk)y_1(lk) - C_1\hat{x}_1(lk \mid lk-1)] + a_2[\hat{x}_2(\kappa \mid \kappa-1) + K_2(\kappa)(y_2(\kappa) - C_2\hat{x}_2(\kappa \mid \kappa-1))] - (a_1 + a_2)x(\kappa), \tag{44}$$

$$\tilde{x}_1(lk \mid lk-1) = A_d\tilde{x}(\kappa) + K_1(lk)[y_1(lk) - C_1\hat{x}_1(lk \mid lk-1)] + a_1^{-1}A_d^lK_2(\kappa)[y_2(\kappa) - C_2\hat{x}_2(\kappa \mid \kappa-1)]. \tag{45}$$

It is known from (45) that there is relationship between the current prediction error (or estimation error) and delay measurement. Therefore, the slow rate delay measurement can be applied to reestimate the current state.

Theorem 1. *Consider asynchronous multirate sensor system (9) and (10). The minimum variance unbiased estimator can be described as (46)–(48) when the delayed measurement arrives.*

$$\hat{x}^*(lk \mid lk) = \hat{x}_1(lk \mid lk) + W(lk, \kappa)\tilde{y}_F(\kappa), \tag{46}$$

$$\tilde{y}_F(\kappa) = y_2(\kappa) - C_2(\kappa)\hat{x}_F(\kappa \mid lk), \tag{47}$$

$$W(lk, \kappa) = \frac{1}{2}\left[2P_{1xx}(lk \mid lk) + P_{1xx}(lk, \kappa \mid lk) + P_{1xw}(lk, lk)C^{\mathrm{T}}\right]\left[P_{1xx}(lk, \kappa \mid lk) + P_{1xx}(lk, \kappa \mid lk) + P_{1xx}(lk \mid lk) + a_2^2P_2(\kappa) + CQ_1(\kappa, lk) + CP_{1xw}(lk, lk)C^{\mathrm{T}} - R(\kappa)\right]^{-1}, \tag{48}$$

where

$$\begin{aligned} \underline{C} &= a_1C_1(\kappa)A_1(\kappa, lk), \\ P_{1xx}(lk \mid lk) &= \mathrm{cov}\ \{x_1(lk)\}, \\ P_{1xw}(lk, \kappa \mid lk) &= \mathrm{cov}\ \{x(lk), w(lk, \kappa)\}, \\ P_2(\kappa) &= \mathrm{cov}\ \{x_2(\kappa)\}, \\ Q_1(lk, \kappa) &= \mathrm{cov}\ \{w_2(lk, \kappa)\}. \end{aligned} \tag{49}$$

Proof 1. In order to prove the unbiasedness of proposed fuse algorithm, the update of coefficient $W(lk, \kappa)$ of unbiased estimator is derived as follows:

$P_2(\kappa)$ is calculated in (31), and P_{1xx} can be given by (37); $P_{1xw}(lk, \kappa \mid lk)$ and $Q_1(lk, \kappa)$ are given as follows:

$$\begin{aligned} P_{1xw}(lk, \kappa \mid lk) &= Q_1(lk, \kappa) - K_1(lk)C_1(\kappa)Q_1(lk, \kappa), \\ Q_1(lk, \kappa) &= \sum_{j=1}^{l-1} A_d^{j-l-1}Q_1(lk-j)\left(A_d^{j-l-1}\right)^{\mathrm{T}}. \end{aligned} \tag{50}$$

Combining (31), (46), and (47), one has

$$\tilde{x}^*(lk \mid lk) = \tilde{x}_1(lk \mid lk) - W(lk, \kappa)\tilde{y}_F(\kappa), \tag{51}$$

where $\tilde{x}^*(lk \mid lk) = x(lk \mid lk) - \hat{x}(lk \mid lk)$.

For simplicity, $W(lk, \kappa)$ is rewritten as W and we yield

$$\tilde{x}^*(lk \mid lk) = \tilde{x}_1(lk \mid lk) - W[y_2(\kappa) - C_2(\kappa)\hat{x}_F(\kappa \mid lk)]. \tag{52}$$

Substituting the coefficient given by (24) and (25) yields

$$\tilde{x}^*(lk \mid lk) = \tilde{x}_1(lk \mid lk) - Wy_2(\kappa) + WC_2(\kappa)[a_2\hat{x}_2(\kappa \mid lk) + a_1\hat{x}_1(\kappa \mid lk)], \tag{53}$$

which can be approximatively regard as follows:

$$\tilde{x}^*(lk \mid lk) \approx \tilde{x}_1(lk \mid lk) - W[a_2 y_2(\kappa) - C_2(\kappa) a_2 \hat{x}_2(\kappa \mid lk)] + W[a_1 y_1(\kappa) - C_1(\kappa) a_1 \hat{x}_1(\kappa \mid lk)]. \tag{54}$$

Moreover, one can obtain that

$$\tilde{x}^*(lk \mid lk) = \tilde{x}_1(lk \mid lk) - W a_2[\tilde{x}_2(\kappa) + \upsilon(\kappa)] - W \underbrace{a_1 C_1(\kappa) A_1(\kappa, lk)}_{\underline{C}} [\tilde{x}_1(lk) - \tilde{\omega}_1(lk, \kappa)]. \tag{55}$$

Define $\underline{C}$ as $a_1 C_1(\kappa) A_1(\kappa, lk)$; thus, (55) can be represented as follows:

$$\tilde{x}^*(lk \mid lk) = (I - W\underline{C})\tilde{x}_1(lk \mid lk) - W a_2 \tilde{x}_2(\kappa) - W\upsilon(\kappa) + W\underline{C}\tilde{\omega}_1(\kappa, lk). \tag{56}$$

According to (56), $P^*(lk \mid lk) = E\{\tilde{x}^*(lk \mid lk)\tilde{x}^*(lk \mid lk)^{\mathrm{T}}\}$ can be deduced as follows:

$$\begin{aligned} P^*(lk \mid lk) = {} & [I - W\underline{C}]P_{1xx}(lk \mid lk)[I - W\underline{C}]^{\mathrm{T}} \\ & - [I - W\underline{C}]P_{1xx}(lk, \kappa \mid lk)W^{\mathrm{T}} \\ & - [I - W\underline{C}]P_{1xw}(lk, lk)[W\underline{C}]^{\mathrm{T}} \\ & + a_2^2 W P_2(\kappa) W^{\mathrm{T}} + W R(\kappa) W^{\mathrm{T}} \\ & + W\underline{C} Q_1(\kappa, lk)[W\underline{C}]^{\mathrm{T}}. \end{aligned} \tag{57}$$

Then, the trace of $P^*(lk \mid lk)$ is introduced to solve W in the sense of linear minimum variance, and one can obtain

$$\begin{aligned} \frac{\partial\{tr[P^*(lk \mid lk)]\}}{\partial W} = {} & 2WP_{1xx}(lk \mid lk) + 2a_2^2 W P_2(\kappa) \\ & - 2WR(\kappa) + 2W\underline{C}WQ_1(\kappa, lk)\underline{C}^{\mathrm{T}} \\ & + 2WP_{1xx}(lk, \kappa \mid lk) \\ & + 2W\underline{C}P_{1xw}(lk, lk)\underline{C}^{\mathrm{T}} \\ & - P_{1xx}(lk, \kappa \mid lk) - P_{1xw}(lk, lk)\underline{C}^{\mathrm{T}}. \end{aligned} \tag{58}$$

Let $\partial\{tr[P^*(lk \mid lk)]\}/\partial W = 0$, then it can be found that $2P_{Nxx}(lk \mid lk) + P_{Nxx}(lk, \kappa \mid lk) + P_{Nxw}(lk, lk)\underline{C}^{\mathrm{T}} = 2W[P_{Nxx}(lk \mid lk) + a_V^2 P_V(\kappa) + \underline{C}Q_N(\kappa, lk) + P_{Nxx}(lk, \kappa \mid lk) + \underline{C}P_{Nxx}(lk \mid lk)\underline{C}^{\mathrm{T}} - R(\kappa)]$. Moreover, W can be shown as follows:

$$\begin{aligned} W = {} & \frac{1}{2}\big[2P_{1xx}(lk \mid lk) + P_{1xx}(lk, \kappa \mid lk) \\ & + P_{1xw}(lk, lk)\underline{C}^{\mathrm{T}}\big]\big[P_{1xx}(lk, \kappa \mid lk) \\ & + P_{1xx}(lk \mid lk) + a_2^2 P_2(\kappa) + \underline{C}Q_1(\kappa, lk) \\ & + \underline{C}P_{1xw}(lk, lk)\underline{C}^{\mathrm{T}} - R(\kappa)\big]^{-1}. \end{aligned} \tag{59}$$

With the equation of W, $P^*(lk \mid lk)$ satisfies the following inequality:

$$0 \le P^*(lk \mid lk) \le P_i(lk \mid lk), \tag{60}$$

where $i = 1, 2$.

4. The Stability Analysis of Fractional Kalman Filter

The stability of fractional Kalman filter is analyzed based on Lyapunov stability theory in the section.

Theorem 2. *Considering asynchronous multirate sensor system (9) and (10), the fractional Kalman filter is given by (13)–(18). Then, the error of estimation is exponentially bounded in sense of mean square.*

Proof 2. The Lyapunov function is chosen as follows:

$$V(k) = \tilde{x}^{\mathrm{T}}(k)P^{-1}(k)\tilde{x}(k), \tag{61}$$

where $\tilde{x}(k) = x(k) - \hat{x}(k)$ is error of estimation, and $P(k)$ denotes covariance matrix.

Due to the principle of stability, when k increases, if $\Delta V(k+1) = V(k+1) - V(k)$ remains negative besides $\tilde{x}(k) = 0$, then $\tilde{x}(k)$ converges to 0.

$$\begin{aligned} \Delta V(k+1) &= V(k+1) - V(k) \\ &= \tilde{x}^{\mathrm{T}}(k+1)P^{-1}(k+1)\tilde{x}(k+1) \\ &\quad - \tilde{x}^{\mathrm{T}}(k)P^{-1}(k)\tilde{x}(k). \end{aligned} \tag{62}$$

It is obtained from the fractional Kalman filter equation that

$$\tilde{x}(k+1) = \underbrace{A_d[I - K(k+1)C]}_{\underline{A}}\tilde{x}(k) - \sum_{j=1}^{k+1}(-1)^j \Upsilon_j \tilde{x}(k+1-j), \tag{63}$$

where C is the measurement matrix; $A_d[I - K(k+1)C]$ is written as $\underline{A}$.

Moreover, $\Delta V(k+1)$ satisfies

$$\Delta V(k+1) < \tilde{x}^{\mathrm{T}}(k)\underbrace{\left[\underline{A}^{\mathrm{T}}P^{-1}(k+1)\underline{A} - P^{-1}(k)\right]}_{\mathbb{P}}\tilde{x}(k). \tag{64}$$

$\mathbb{P} < 0$ is a sufficient condition for $\Delta V(k+1) < 0$. One has

$$\underline{A}^{\mathrm{T}}P^{-1}(k+1)\underline{A} - P^{-1}(k) < 0. \tag{65}$$

It is deduced from (65) that

$$P^{-1}(k+1) - \underline{A}^{-\mathrm{T}}P^{-1}(k)\underline{A}^{-1} < 0. \tag{66}$$

Moreover, multiplying $P(k+1)$ on both sides of (66) obtains

$$I - P(k+1)\underline{A}^{-\mathrm{T}}P^{-1}(k)\underline{A}^{-1} < 0. \tag{67}$$

According to (14) and (20), one can get

$$\begin{aligned} P(k+1) = {} & \underline{A}P(k)\underline{A}^{\mathrm{T}} + A_d K(k)R(k)K^{\mathrm{T}}(k)A_d^{\mathrm{T}} \\ & + Q(k) + \sum_{j=1}^{k+1} \Upsilon_j P(k+1-j \mid k+1-j)\Upsilon_j^{\mathrm{T}}. \end{aligned} \tag{68}$$

Substituting (68) to (67), we can derive

$$\begin{aligned} & -\Bigg[A_d K(k)R(k)K^{\mathrm{T}}T(k)A_d^{\mathrm{T}} + Q(k) \\ & \quad + \sum_{j=1}^{k+1} \Upsilon_j P(k+1-j \mid k+1-j)\Upsilon_j^{\mathrm{T}}\Bigg] \\ & \quad \times \underline{A}^{-\mathrm{T}}P^{-1}(k)\underline{A}^{-1} < 0. \end{aligned} \tag{69}$$

Due to $\underline{A}^{-\mathrm{T}}P^{-1}(k)\underline{A}^{-1} > 0$, (69) satisfies

$$\begin{aligned} & \left[A_d K(k)R(k)K^{\mathrm{T}}(k)A_d^{\mathrm{T}} + Q(k)\right] \\ & \quad + \sum_{j=1}^{k+1} \Upsilon_j P(k+1-j \mid k+1-j)\Upsilon_j^{\mathrm{T}} > 0. \end{aligned} \tag{70}$$

Since $P(k+1)$ is positive definite, it is found that

$$\begin{aligned} P(k+1) = {} & A_d P(k)A_d^{\mathrm{T}} - A_d K(k)CP(k)A_d^{\mathrm{T}} + Q(k) \\ & + \sum_{j=1}^{k+1} \Upsilon_j P(k+1-j \mid k+1-j)\Upsilon_j^{\mathrm{T}} > 0. \end{aligned} \tag{71}$$

According to (70), to make sure the inequality (69) is satisfied, one obtains that

$$A_d P(k)A_d^{\mathrm{T}} - A_d K(k)CP(k)A_d^{\mathrm{T}} < A_d K(k)R(k)K^{T}(k)A_d^{\mathrm{T}}, \tag{72}$$

where $P(\cdot)$ represents the positive definite error variance matrix, and $\Upsilon_j P(k+1-j \mid k+1-j)\Upsilon_j^{\mathrm{T}}$ is positive definite.

Due to the positive definite of $R(k)$, it can be derived from (72) that

$$\begin{aligned} & A_d\left[K(k)R(k)K^{\mathrm{T}}(k) + K(k)CP(k) - P(k)\right]A_d^{\mathrm{T}} > 0, \\ & \underbrace{K(k)R(k)K^{\mathrm{T}}(k)}_{>0} + \underbrace{[K(k)C - I]}_{>0}P(k) > 0. \end{aligned} \tag{73}$$

Thus, $K(k)C - I > 0$ can ensure the stability of fractional Kalman filter.

$$K(k)C - I > 0. \tag{74}$$

According to the fractional Kalman filter gain matrix $K(k)$, it can be obtained that

$$\begin{aligned} K(k)C - I &= P(k)C^{\mathrm{T}}\left[CP(k)C^{\mathrm{T}} + R(k)\right]^{-1}C - I \\ &> \underbrace{P(k)C^{\mathrm{T}}\left[CP(k)C^{\mathrm{T}}\right]^{-1}C - I}_{=0}, \end{aligned} \tag{75}$$

where $R(k)$ is positive definite to guarantee the stability of fractional Kalman filter.

5. Simulation Results

To demonstrate the applicability of proposed method, the fractional Kalman filter-based fusion algorithm with various values is simulated. The system parameters and the system initial values are shown as follows:

$$\begin{aligned} & A_d = \begin{bmatrix} 0 & 0.1 \\ -0.035 & -0.01 \end{bmatrix}, \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}, \\ & B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, C_1 = [0.3 \quad 0.4], C_2 = [0.6 \quad 0.9], \\ & w(k) \sim N\left(0, \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}\right), v(k) \sim N(0 \quad 0.1), \\ & l = 4, P_0 = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{76}$$

The control law is designed as $u = u_r - [-0.015 \quad 0.03]x$, and u_r is a square signal with the period of 10s.

Based on the conclusions in [2], the order of system equation should be defined previously and the control accuracy is infected by the value range of j in the sum term $\sum_{j=1}^{k+1}(-1)^j \Upsilon_j x(k+1-j)$. Thus, the upper bound of j must be defined previously in practice. In the simulation, as the width of a circular buffer of past state vectors for fractional-order difference, memory length L is chosen as 200 or 50.

The fractional Kalman filter-based asynchronous multirate sensor information fusion results are described from Figures 4–8. The reference input and output signal in the system as well as the multisensor measurement are shown in Figure 4. Moreover, the system state fusion estimation, measurement estimation results of sensor 1 and sensor 2 are shown in Figures 5 and 6, where sensors 1 and 2 represent fast sensor and slow sensor, respectively. The estimation errors of x_1 and x_2 are described by Figures 5 and 6 in detail.

The simulation results of $n_N = 50$ are depicted from Figures 9–13. Figure 9 provides input signal, output signal, and the measurement of output. The state estimation based on fusion method and a single sensor are provided by Figures 10 and 11, respectively. Finally, Figures 12 and 13 describe the error curves with different algorithms. From the simulation results, one can conclude that the proposed algorithm gives better control performance.

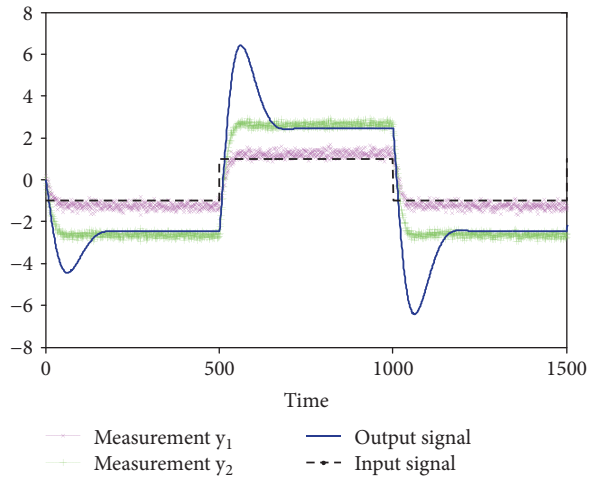

FIGURE 4: Input signal, output signal, and multisensor measurements (L=200).

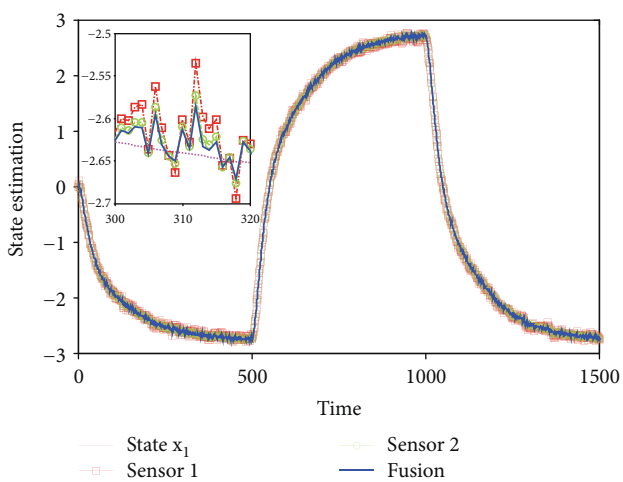

FIGURE 5: The estimation of x_1 (sensor 1, sensor 2, and fusion) (L=200).

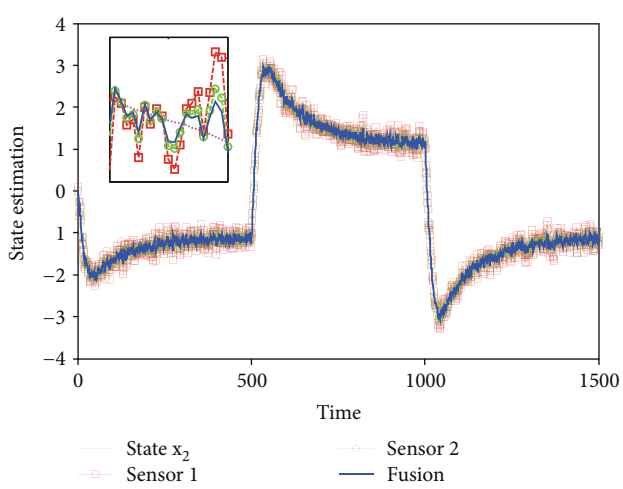

FIGURE 6: The estimation of x_2 (sensor 1, sensor 2, and fusion) (L=200).

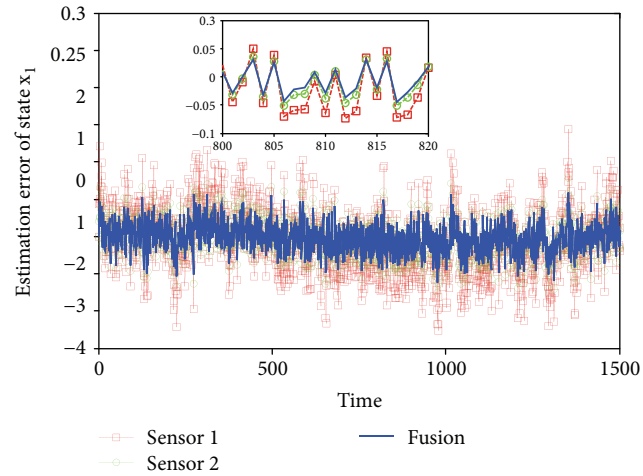

FIGURE 7: The estimate error of x_1 (sensor 1, sensor 2, and fusion) (L=200).

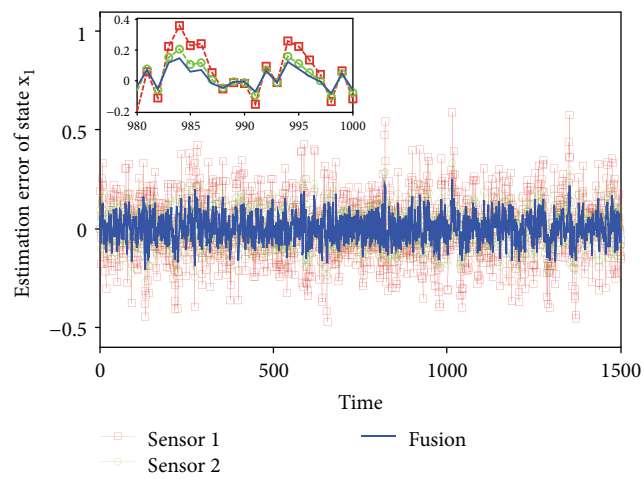

FIGURE 8: The error of x_2 estimation (sensor 1, sensor 2, and fusion) (L=200).

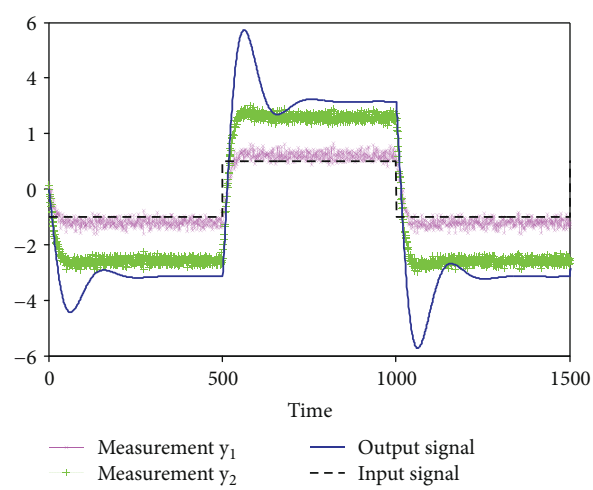

FIGURE 9: Input signal, output signal, and multisensor measurement (L=50).

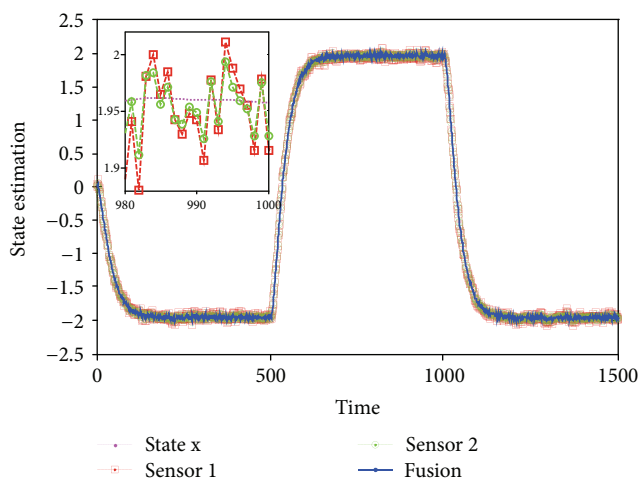

Figure 10: The estimation of x_1 (sensor 1, sensor 2, and fusion) (L=50).

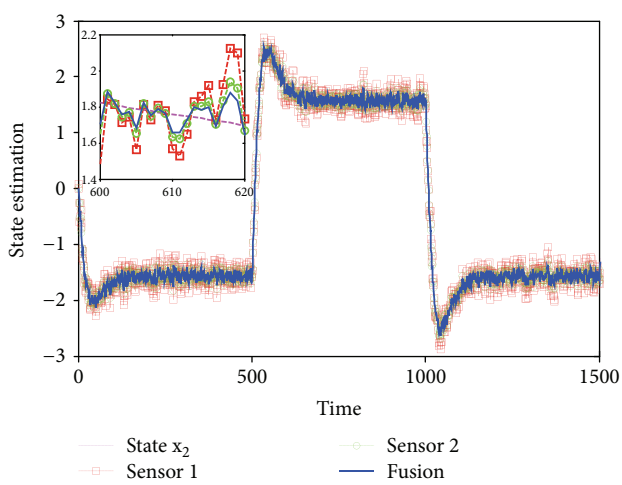

Figure 11: The estimation of x_2 (sensor 1, sensor 2, and fusion) (L=50).

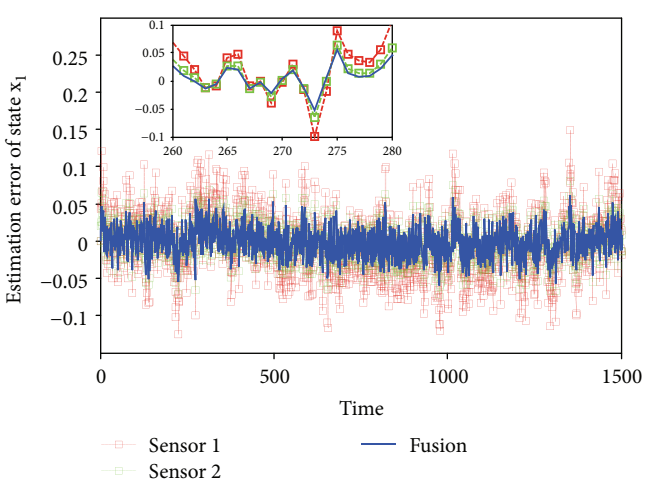

Figure 12: The estimate error of x_1 (sensor 1, sensor 2, and fusion) (L=50).

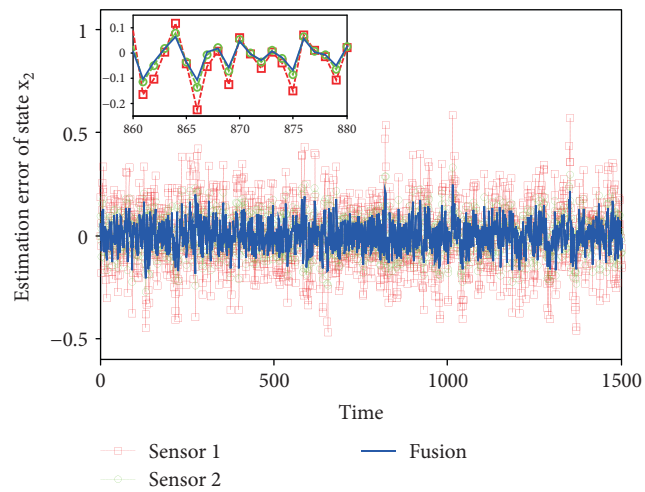

Figure 13: The estimate error of x_2 estimation (sensor 1, sensor 2, and fusion) (L=50).

In simulation results, the effectiveness of the proposed fusion method is verified, and the system output value and fusion accuracy are compared. It can be obtained that high-precision and economic storage space requirement is satisfied as $L = 50$.

6. Conclusion

The fractional Kalman filter-based fusion algorithm is presented to solve the problem of asynchronous multirate sensor information fusion. According to the memory performance of fractional order, which can accurately describe the essential characteristics of the system, the fractional filter is introduced to improve the estimation accuracy. Based on the relationship between slow rate delay measurement and the current state estimation, the minimum variance unbiased estimator is designed to update the current estimation. The simulation results prove the superiority of this method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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