Research Article
Multiscale Chebyshev Neural Network Identification and Adaptive Control for Backlash-Like Hysteresis System

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Received 18 May 2018; Accepted 14 August 2018; Published 3 October 2018

Academic Editor: Zhile Yang

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An adaptive control based on a new Multiscale Chebyshev Neural Network (MSCNN) identification is proposed for the backlash-like hysteresis nonlinearity system in this paper. Firstly, a MSCNN is introduced to approximate the backlash-like nonlinearity of the system, and then, the Lyapunov theorem assures the identification approach is effective. Afterward, to simplify the control design, tracking error is transformed into a scalar error with Laplace transformation. Therefore, an adaptive control strategy based on the transformed scalar error is proposed, and all the signals of the closed-loop system are uniformly ultimately bounded (UUB). Finally, simulation results have demonstrated the performance of the proposed control scheme.

1. Introduction

As one of the typical nonlinearity, hysteresis exists in many intelligent materials, servo motor systems, mechanical systems, and others [1–3], and it has attracted much attention for many years. Many hysteresis models have been investigated including mathematical model and physical model. For these models, Preisach model [1, 4], Prandtl-Ishlinskii (P-I) model [5, 6], Bouc-Wen model [7, 8], and backlash-like model [2, 3] are important in hysteresis research. Backlash-like model has been firstly investigated for hysteretic systems with a differential equation in [2]. A robust adaptive control strategy was designed without constructing inverse hysteresis nonlinearity in [2]. Zhou et al. [3] have utilized backstepping technique designing a robust adaptive backstepping control to deal with the unknown backlash-like hysteresis nonlinearity with bounded external disturbances. H. J. Lee and J. J. Lee [9] applied the results for Shape memory alloy (SMA) actuators, and a control strategy was adopted for the time delay control of the backlash-like hysteresis system.

The investigation of backlash-like hysteresis model has attracted attention for these years. Compared with the backstepping technique, sliding mode control has less calculated amount. Zhang et al. [10] have designed a sliding mode control for the backlash-like hysteresis system based on Ackermann’s formula. Different from the above literatures, Dong et al. [11] investigated nonsmooth predictive control backlash-like hysteresis mechanical transmission systems. The backlash-like model was described by a nonsmooth function, and a model predictive control was presented with a nonsmooth receding horizon strategy. Liu et al. [12] have investigated multiple-input multiple-output (MIMO) systems with backlash-like hysteresis model and proposed an adaptive prescribed performance control to precision control the system. More and more researchers pay great attentions to the prescribed performance control [13–20] in recent years. But prescribed performance control for hysteresis system investigation needs more attention from researchers.

Chebyshev neural network (CNN) is one of the functional-link neural network (FLNN) which is only one layer and omit the hidden layer compared with other multineural networks. Therefore, CNN has lower computation for its simple structure. The CNN has been widely utilized in permanent magnet synchronous motor (PMSM) system [21, 22], spacecraft system, and others [23, 24]. Zou et al. [23] proposed a terminal sliding mode control (TSMC) and
CNN for spacecraft. The attitude and velocity error have been transformed into a double integrator for the spacecraft system, and then, a switch function was adopted to generate a switching between the adaptive NN and the robust controller. An adaptive TSMC-based CNN was proposed for tracking the attitude of the spacecraft. Different from [23], Lin [21] investigated a hybrid recurrent CNN for the PMSM of continuously variable transmission (CVT) system. The gradient descent method of the adaptive law in the HRCNN was introduced, and the stability was assured by Lyapunov function. Sun et al. [22] adopted a modified dynamic surface control with high-order sliding mode (HOSM) differentiator approximating the unknown velocity of the multimotor servomechanism. The unknown friction and disturbance were estimated by CNN. Above literatures all investigated system control with CNN but not applied to system identification. Patra and Kot [25] firstly proposed CNN to identify the dynamic nonlinear system where it needed less computation. As a whole, system identification with CNN has few research for the researchers. Meanwhile, adaptive control [26–29] is an extensive control strategy that has been investigated for many years. In this paper, an adaptive CNN controller will be investigated for the backlash-like hysteresis nonlinear system.

Multiscale identification was adopted in [30, 31] for singularly perturbed nonlinear system with multitime-scale recurrent high-order neural networks (MSRHONN). A new optimal bounded ellipsoid algorithm was applied for MSRHONN which had faster convergence due to the adaptively adjusted learning rate. Different from Zheng et al. [29], in this paper, we will propose a CNN to identify the backlash-like hysteresis system and design an adaptive control strategy for this system. Firstly, the CNN identification will be designed and guaranteed by Lyapunov function with update law. Then, the tracking error will be transformed into scalar form through Laplace transportation to simplify the controller design. Finally, an adaptive neural network will be proposed for the backlash-like hysteresis nonlinearity system and assured by Lyapunov theorem.

2. Problem Formulation

The backlash-like hysteresis system of this paper is described as follows:

\[
\begin{aligned}
\dot{x} &= f_1(x) + f_2(x)v, \\
\dot{v} &= f_3(v, u),
\end{aligned}
\]

where \(f_1, f_2\) are unknown nonlinear smooth functions, and \(x, v\) have different time scale; \(v\) represents hysteresis nonlinearity; it is formulated by nonlinear smooth function \(f_3\); in this paper, the function \(f_3\) is defined by backlash-like model, i.e., \(f_3\) is described by the following equation:

\[
\dot{v} = f_3(v, u) = \alpha |u| (\beta u - v) + \gamma \dot{u},
\]

where \(\alpha\) is positive coefficient, and \(u\) means the input signal of the nonlinear system; \(\beta, \gamma\) are constants of the hysteresis nonlinearity, and \(\beta > 0\) satisfying \(\beta > \gamma\).

Figure 1: The curve of the backlash-like hysteresis model.

Table 1: The backlash-like model parameters.

<table>
<thead>
<tr>
<th>Backlash-like parameters</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(K_1)</th>
<th>(K_2)</th>
<th>(K_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1.6</td>
<td>0.22</td>
<td>2.1</td>
<td>6.5</td>
<td>4.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

According to the literature [2], (2) is solved as follows:

\[
v = \beta u + \Xi(u),
\]

where

\[
\Xi(u) = (v_0 - \beta u_0)e^{-\alpha((v-u_0) \text{ sign}(\dot{u}))}
+ e^{-\alpha \text{ sign}(\dot{u})} \int_{u_0}^{u} (v - \beta) e^\alpha \text{ sign}(\dot{u}) d\xi.
\]

The backlash-like hysteresis curves are illustrated in Figure 1 where the model parameters are listed in Table 1. The input signals of the backlash-like hysteresis model are selected as \(u(t) = K_i \sin (2.5t), i = 1, 2, 3\), respectively; \(K_i\) is chosen in Table 1.

3. Multiscale Neural Network Identification Design

In order to identify the hysteresis system (1), a multiscale CNN is introduced and then is applied for the backlash-like hysteresis nonlinearity to approximate the system.

3.1. Chebyshev Neural Network. The CNN is one kind of FLN based on Chebyshev polynomials actually which is only single-layer and omits hidden layer compared with other neural network such as Multilayer Neural Network (MNN). CNN structure includes two sections: Chebyshev expansion block and learning block. A structure of CNN is shown in...
Figure 2 in this paper. Then, the CNN is defined as

$$F(x) = W^* \Phi(x) + \varepsilon,$$

where $W^* \in \mathbb{R}^{m \times n}$ means the weight matrix of the CNN, and $\varepsilon$ denotes the bounded approximation error of the CNN.

$$\Phi(x) = [1, C_1(x_1), \ldots, C_n(x_1), C_1(x_2), \ldots, C_n(x_2), \ldots, C_1(x_m), \ldots, C_n(x_m)]^T,$$

and $[x_1, x_2, \ldots, x_m]$ is the $m$-dimensional input vector; $n$ denotes the order of the Chebyshev polynomial; the $k+1$th order Chebyshev polynomial $C_{k+1}(x)$ is selected as the well-known two-term recursive formula:

$$C_{k+1}(x) = 2x C_k(x) - C_{k-1}(x),$$

where $C_1(x) = x$, $C_0(x) = 1$.

3.2. Multiscale Identification. Considering the hysteresis system (1) with backlash-like hysteresis model (2), the new CNNs are utilized to estimate the system as follows:

$$\dot{x} = A\tilde{x} + \tilde{W}_1 \Phi_1(x) + \tilde{W}_2 \Phi(2)x,$$

where $\tilde{x}$ is the estimation of the hysteresis system states; $A$ is the designed diagonal stable matrix; $\tilde{W}_1 \in \mathbb{R}^{m \times n}$, $\tilde{W}_2 \in \mathbb{R}^{n \times n}$ are the weights of the CNN.

The backlash-like hysteresis nonlinearity system can be approximated by the optimal CNN as

$$\dot{x} = Ax + W_1^* \Phi_1(x) + W_2^* \Phi_2(x) + \varepsilon,$$

where $\varepsilon$ represents the modeling error.

Define the error $e$ as

$$e = x - \tilde{x}.$$
Next, the identification of the backlash-like hysteresis system will be guaranteed by Lyapunov theorem; a Lyapunov function is selected as

$$V_0 = \frac{1}{2} e^T Pe + \frac{1}{2\delta_1} tr\{\hat{W}_1^T \hat{W}_1\} + \frac{1}{2\delta_3} tr\{\hat{W}_2^T \hat{W}_2\}. \quad (16)$$

Considering Assumption 1 and Assumption 2, (12), (14), and the CNN update law (15), the derivative of $V_0$ is deduced as

$$\dot{V}_0 = \frac{1}{2} (e^T Pe + e^T \dot{p}e) - \frac{1}{2\delta_1} tr\{\hat{W}_1^T \hat{W}_1\} - \frac{1}{2\delta_3} tr\{\hat{W}_2^T \hat{W}_2\} \leq -\frac{1}{2} e^T Qe - \epsilon + \frac{\delta_2}{2} tr\{\hat{W}_1^T \hat{W}_1\} + \frac{\delta_4}{2} tr\{\hat{W}_2^T \hat{W}_2\}$$

$$\leq -\frac{1}{2} e^T Qe + \epsilon_M + \frac{\delta_2 W_{1M}}{2} + \frac{\delta_4 W_{2M}}{2}, \quad (17)$$

where the fact that

$$\frac{\delta_2}{2} tr\{\hat{W}_1^T \hat{W}_1\} \leq \frac{\delta_2 W_{1M}}{2},$$

$$\frac{\delta_4}{2} tr\{\hat{W}_2^T \hat{W}_2\} \leq \frac{\delta_4 W_{2M}}{2} \quad (18)$$

are utilized, and $\epsilon_M = (\delta_2 W_{1M}/2) + (\delta_4 W_{2M}/2)$; thus, $\dot{V}_0$ satisfies the following inequality:

$$\dot{V}_0 \leq -\rho_1 V_0 + \rho_2, \quad (19)$$

where $\rho_1 > 0$, $\rho_2 > 0$.

Therefore, the error $e$ can be solved as

$$\|e\| \leq \sqrt{2 \left(\frac{\rho_2}{\rho_1} + V_0(0)\right)} e^{-\rho_1 t}. \quad (20)$$

Considering $t \to \infty$, we have

$$\|e\|_{t \to \infty} \leq \lim_{t \to \infty} \sqrt{2 \left(\frac{\rho_2}{\rho_1} + V_0(0)\right)} e^{-\rho_1 t} = \sqrt{2 \frac{\rho_2}{\rho_1}}. \quad (21)$$

Then, the proposed CNNs can identify the backlash-like hysteresis nonlinearity system (1), and all the signals of the closed-loop are UUB. Next, an adaptive controller will be designed to control the system, and the Lyapunov theorem will guarantee the UUB of the closed-loop system.

**Remark 1.** CNN is a FLNN consists of two parts: the numerical transformation and learning. It has been proven that CNN has stronger approximation capabilities and has less calculate quantities since it is a single-layer neural network based on Chebyshev polynomials. In this paper, we designed two CNNs to estimate the unknown system parameters where the two CNNs have different scales. The multiscale CNN can be separated into the different unknown nonlinear dynamics where it will reduce the system order and simplify the structure of the controller.

### 4. Adaptive Controller Design

#### 4.1. Error Transformation

The control objective of this paper is to design an adaptive controller for the backlash-like hysteresis nonlinearity system such that the tracking follows the reference model as

$$\dot{x}_r = A_r x_r + b_r u_r, \quad (22)$$

where $A_r$, $B_r$ are known controllable state coefficient matrices, and $u_r$ represents the reference input signal.

In order to simplify the controller design, the following assumption and lemma are introduced.

**Assumption 3.** There exist positive constant vector $\delta_x^*$ and known nonzero constant $\delta_r^*$ such that the following equations are satisfied:

$$\begin{align*}
A_r &= A + B \delta_x^*, \\
b_r &= B \delta_r^*.
\end{align*} \quad (23)$$

Then, the following lemma exists:

**Lemma 1** [32]. Let

$$\begin{align*}
\dot{x} &= A_m x + b_m, \\
\lambda(s) &= (s + r)H(s),
\end{align*} \quad (24)$$

where $A_m$ is a diagonal stable matrix; $\lambda(s)$ denotes characteristic polynomial of $A_m$; $r$ represents a designed positive constant; $H(s)$ is Hurwitz, and $A_m$, $b$ are controllable. Then, $\exists \eta$ such that

$$\eta^T (s I - A_m)^{-1} b = \frac{1}{s + r}. \quad (25)$$

The tracking error is defined as

$$e_t = x - x_r. \quad (26)$$

In view of (3), (9), (22), and (23), the derivative of $e_t$ can be expressed as

$$\dot{e}_t = \dot{x} - \dot{x}_r$$

$$= A_r e_t + b_r (u_t - \delta_r^* x) + W_1^* \Phi_1(x) + W_2^* \Phi_2(x)(\delta_r u + \Xi(u)) + \epsilon. \quad (27)$$

The Laplace transformation is utilized for the derivative of the tracking error $e_t$ in (27) as
\[ e_i(s) = (sI - A_f)^{-1} b_r \left( u_r(s) - \frac{\theta_r^*}{\theta_r} x(s) \right) + W_1^* \Phi_1(x(s)) + W_2^* \Phi_2(x(s))(\beta u + \Xi(u(s))) + \epsilon(s). \] (28)

Defining \( e_i = \eta^t e_i \), considering (25), both sides of (28) multiply \( \eta^t \), and then utilize inverse Laplace transformation, one has
\[
\dot{e}_i = -re_i + (\theta_r^* u_r - \theta_r^* x) + \frac{W_1^* \Phi_1(x) + W_2^* \Phi_2(x)(\beta u + \Xi(u)) + \epsilon}{b_r}.
\] (29)

4.2. Adaptive Controller Design. In order to design an adaptive controller for the backlash-like hysteresis nonlinearity system, the following variables are firstly defined as
\[
\begin{align*}
\tilde{\theta}_x &= \theta_x^* - \tilde{\theta}_x, \\
\tilde{\theta}_r &= \theta_r^* - \tilde{\theta}_r,
\end{align*}
\] (30)

where \( \tilde{\theta}_x, \tilde{\theta}_r \) are the estimation of \( \theta_x^*, \theta_r^* \), respectively. The controller is designed as
\[
u = \frac{b_r}{W_1^* \Phi_1(x) \beta} \left( \tilde{\theta}_x - \tilde{\theta}_x u_r \right) - \frac{\tilde{W}_1 \Phi_1(x) - \Xi(u)}{W_2^* \Phi_2(x) \beta}.
\] (31)

and the adaptive law is selected as
\[
\dot{\tilde{\theta}}_r = \delta_5 u_r,
\] (32)

where \( \delta_5 > 0 \) is designed constants.

Then, the following theorem holds.

**Theorem 1.** The backlash-like hysteresis nonlinearity system is described as (1), which is approximated by CNN in (8); the reference model is selected as (22); the controller is designed as (31), and the adaptive law is selected as (32). Then, the tracking error and other signals in closed-loop are UUB.

**Proof 1.** Select a Lyapunov function as
\[
V = \frac{1}{2} e_r^2 + \frac{1}{2\delta_5} \tilde{\theta}_r^2.
\] (33)

Considering (29) and (30), the derivative of \( V \) can be deduced as
\[
\dot{V} = \epsilon \dot{e}_i + \frac{1}{\delta_5} \left( \tilde{\theta}_r \dot{\tilde{\theta}}_r \right)
\]
\[
= \epsilon \left( -re_i + (\theta_r^* u_r - \theta_r^* x) + \frac{W_1^* \Phi_1(x) + W_2^* \Phi_2(x)(\beta u + \Xi(u)) + \epsilon}{b_r} \right)
\]
\[
+ \frac{1}{\delta_5} \left( \tilde{\theta}_r \dot{\tilde{\theta}}_r \right).
\] (34)

Substituting (31) and (32) into (34), one has
\[
V = -re_i^2 + \epsilon \left( \theta_r^* u_r - \theta_r^* x + \frac{W_1^* \Phi_1(x) - \tilde{\theta}_x u_r + \tilde{\theta}_x}{b_r} \right)
\]
\[
- \frac{\tilde{W}_1 \Phi_1(x) + \Xi(u) + \epsilon}{b_r} + \frac{1}{\delta_5} \left( \dot{\tilde{\theta}} - \dot{\tilde{\theta}} \right)
\)
\[
\leq -re_i^2 + e_i \left( \frac{\tilde{W}_1 \Phi_1(x) - \tilde{\theta}_x u_r + \tilde{\theta}_x}{b_r} \right) + \frac{W_1^* \Phi_1(x) - \tilde{\theta}_x u_r + \tilde{\theta}_x + \Xi(u)}{b_r} + \epsilon.
\] (35)

Therefore, we have
\[
\dot{V} \leq -\psi_1 V + \psi_2,
\] (36)

Then, the error \( e_i \) is deduced as
\[
|e_i| \leq \sqrt{\frac{\psi_1}{\psi_2} + V(0)} e^{-\psi_1 t}.
\] (37)

According to Lyapunov theorem, the error \( e_i \) is bounded and converged to a compact set, and others signals are all bounded.

**Remark 2.** The proposed approach can be tracked in the reference model through the transformed tracking scalar error under Assumptions 1, 2, and Assumption 3. For CNN, Assumption 1 and Assumption 2 require that the approximation error \( \epsilon \) and the weight matrix \( W \) are bounded. It suits the real condition since the weight cannot select infinity. Assumption 3 and Lemma 1 can transform the vector error into a scalar error to simplify the controller design but degrade the control precision in fact.

5. Simulations

To verify the effectiveness of the CNN identification and adaptive control, the following system is considered:
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= g_1(x) + g_2(x) v, \\
\dot{v} &= 1.4 |u| (4.2 u - v) + 0.38 u,
\end{align*}
\] (38)

where \( g_1(x) = 0.5 \sin (x_1) x_2^2 + 1.8 \cos (x_1) \), \( g_2(x) = 0.2 \cos (x_1) + 1 \), \( \alpha = 1.4 \), \( \beta = 4.2 \), \( \gamma = 0.38 \), and \( u = 2 \sin (2t) \).

The reference model is selected as
\[
\dot{x}_r = A_r x_r + b_r u_r,
\] (39)
where

$$A_r = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix},$$

and $b_3 = 3$.

The parameters of the CNN and adaptive law are chosen; $W_1(0) = 0$, $W_2(0) = 0$, $\epsilon_m = 0.05$, $\delta_1 = 100$, $\delta_2 = 0.002$, $\delta_3 = 150$, $\delta_4 = 0.001$, $\delta_5 = 0.15$, and $r = 3$. Figures 3–5 illustrate the proposed adaptive control results in this paper. Figure 3 shows the adaptive control results versus the reference model; Figure 4 shows the tracking errors of the proposed approaches, and Figure 5 illustrates the control input.

From Figure 3, it is clearly shown that the adaptive control results are well tracking the reference model for $u_r = (\pi/6) \sin(t)$. The tracking errors are illustrated in Figure 4 and that demonstrates CNN identified the backlash-like hysteresis system, and the proposed approaches are effective. From these figures, we can find that oscillation is reduced at the beginning and this is due to the fact for the adaptive control.

We chose $u_r = (\pi/5) \sin(t)$; the tracking results and errors are shown in Figures 6–8. With large amplitude of the sinusoid wave reference input, the tracking results are shown in Figure 6; the tracking errors are illustrated in Figure 7, and the control input is shown in Figure 8.
Compared with the results for \( u_r = (\pi/6) \sin (t) \), it is shown that the tracking precision is declined with large sinusoid wave amplitude. The same results are clearly shown in Figure 7, and the oscillation time is longer than the results of \( u_r = (\pi/6) \sin (t) \). It is declared that more signal amplitude has more adjustment time and less control precision. This phenomenon is due to the fact and it verified the effectiveness of the proposed approaches.

In order to verify the effectiveness of the proposed adaptive CNN controller, we compare the proposed adaptive CNN controller, adaptive fuzzy controller [33–35], and PID controller in this paper. The system and parameters are chosen same as before, and the reference input is \( u_r = (\pi/6) \sin (t) \). For PID controller, we chose \( K_p = 50, K_i = 0.83, K_d = 0.6 \), and the adaptive fuzzy controller similar reference [35] is used in this paper. The results are illustrated in Figures 9 and 10. It is clearly shown that the proposed adaptive CNN controller and adaptive fuzzy controller are better than the PID controller. PID controller has longer dynamic procedure and more violent fluctuation. Adaptive CNN controller and adaptive fuzzy have similar performance in \( x_1 \), but adaptive CNN controller has more accuracy in dynamic procedure than adaptive fuzzy in \( x_2 \). It is illustrated that the proposed adaptive CNN controller has better performance for the hysteresis nonlinear system with backlash-like model.

6. Conclusion

This paper proposed a CNN multiscale identification and an adaptive control strategy to control the hysteresis nonlinearity where the hysteresis model was described by backlash-like model. The proposed multiscale CNN was firstly introduced approximating the backlash-like hysteresis.
system; a Lyapunov function guaranteed the effectiveness of the proposed identification method. Afterward, the tracking error was transformed into scalar error through Laplace and inverse Laplace transformation for simplifying the control design. Then, a new adaptive controller was designed with CNN based on the transformed error, and the Lyapunov theorem assured that all the signals included the tracking error in the closed-loop were UUB. Finally, simulations are designed that have verified the effectiveness of the proposed CNN identification and adaptive control.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

This work is supported by the Shandong Provincial Natural Science Foundation of China (ZR2017MF048), the National Natural Science Foundation of China (61433003), Shandong Science and Technology program of higher education (J17KA214), Scientific Research Foundation of Shandong University of Science and Technology for Recruited Talents (2016RCJ035), Tai’an Science and Technology development program (2017GX0017).

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