Research Article

Formation-Containment Control of Second-Order Multiagent Systems via Intermittent Communication

Mao-Dong Xia , Cheng-Lin Liu , and Fei Liu

Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Institute of Automation, Jiangnan University, Wuxi 214122, China

Correspondence should be addressed to Cheng-Lin Liu; liucl1981@sohu.com

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This paper investigates the formation-containment control of second-order multiagent systems with intermittent communication. Distributed coordination control algorithms are proposed under aperiodic intermittent communication, where each agent only communicates with its neighboring agents on some disconnected time intervals. By means of constructing Lyapunov functions, sufficient convergence conditions are obtained for the leaders reaching a prescribed formation asymptotically and the followers converging into the convex hull formed by leaders asymptotically, respectively. Besides, sufficient convergence conditions are also provided for second-order multiagent systems converging to the desired formation-containment under time-varying communication delay and intermittent communication. Finally, the validity of theoretical results is illustrated by numerical simulations.

1. Introduction

In the past decades, coordination control of multiagent systems has drawn considerable attention due to its wide engineering applications, such as sensor networks [1], railway traffic control [2], formation control of robots [3], and so on. Furthermore, consensus seeking, formation control, and containment control have become the hot issues of coordination control of multiagent systems and have been extensively studied in various research fields, e.g., biology, physics, control theory, etc.

Formation control concerned in this paper requires the agents to reach a prescribed formation by cooperating with each other locally, and the traditional formation control strategies include behavior [4], leader-follower [5, 6], virtual structure approach [7, 8], and consensus-based algorithms [9–12]. Up to now, the formation control problem has been widely studied for homogeneous and heterogeneous multiagent systems [13] with time-invariant or time-varying formation [14] and fixed or switching topologies [15].

As a special leader-following coordination control of multiagent systems with multiple leaders, containment control requires the following agents to converge into a convex

hull generated by the leaders asymptotically. Wang et al. [16] investigated the containment control of second-order multiagent systems with time-varying communication delays and proved that the containment control was achieved asymptotically as long as the interconnection topology was jointly connected. Mu et al. [17] studied the containment control problem of general linear multiagent systems with fixed topology and switching topologies and obtained some sufficient conditions by constructing Lyapunov functions. Hamed et al. [18] proposed the containment control algorithms for linear heterogeneous multiagent systems, and the convergence conditions were obtained for the followers converging into the convex hull formed by leaders based on output regulation techniques. Kan et al. [19] studied containment control of multiagent systems over directed random graphs to deal with random communication failure and got conditions based on a stochastic version of LaSalle's Invariance Principle. In addition, Kan et al. [20] proposed a balanced containment control algorithm for driving the followers to the desired region but also equally space the agents in the desired region when driving the agents towards the target. Moreover, containment control problems were studied for second-order nonlinear multiagent systems, and a

distributed algorithm was designed and analyzed on the basis of high-frequency feedback robust control [21].

In some practical applications, the leaders not only provide a global reference state, but also are required to accomplish some assigned tasks cooperatively. However, there are no cooperation requirements on the leaders in [16–19, 21]. Specially, formation-containment control means that the leaders' states achieve an expected formation while the followers' states converge into the convex hull formed by leaders. Ferrari et al. [22] investigated a formation-containment control for first-order multiagent systems, and convergence conditions were obtained by using the partial difference equation on graph. Zheng et al. [23] studied the formation-containment control problem of second-order multiagent systems with only sampled position data, and got sufficient formation-containment conditions based on algebraic graph theory and matrix theory. In addition, Xia et al. [24] proposed the formation-containment control algorithms for second-order multiagent systems under time-varying delays, and two delay-dependent convergence conditions were obtained for the leaders and followers, respectively, based on Lyapunov-Krasovskii functional. Moreover, Dong et al. [25] analyzed the formation-containment problem of a high-order linear multiagent system with identical time-invariant delay and the convergence conditions were gained in the form of linear matrix inequalities. Dong et al. applied algorithms into a UAV platform, simulation and experimental results were shown the effectiveness of theoretical research in [26].

In the above-mentioned works, evidently, the neighboring agents communicate with each other continuously. However, some practical situations require that the information can not be transmitted all the time because of instability of communication devices and limitation of technology. In addition, it is useful to save energy and reduce information exchange times, so some intermittent communication algorithms are applied. Therefore, the coordination control of multiagent systems with intermittent communication has received great interest recently. Huang et al. [27] investigated the consensus problem of second-order systems with partly and completely intermittent communications, and the delay-dependent consensus conditions were obtained by using graph theory and Lyapunov functions. Hu et al. [28] studied the leader-following consensus seeking via the intermittent control, and some sufficient consensus conditions were obtained based on the Lyapunov stability theory. Cheng et al. [29] investigated a decentralized formation control under limited and intermittent communication, and the convergence conditions were obtained by using a navigation function framework. Containment control algorithm under the intermittent sampled data was designed for second-order multiagent systems [30], and the necessary and sufficient conditions were dependent on the gain parameters, the sampling period and the communication width.

Motivated by the above survey, we will consider the formation-containment problem of second-order multiagent systems with intermittent communication in this paper. The multiagent system consists of the leaders and followers, where each leader updates the states based on its neighboring leaders and the followers receive information from

neighboring agents and leaders. The distributed coordination control algorithms are designed for the leaders and followers, respectively, based on the intermittent information. By transforming the formation-containment problem into an asymptotic stability problem and constructing proper Lyapunov functions, the sufficient convergence conditions are established for the leaders and followers achieving the desired formation-containment asymptotically.

Compared with the algorithms proposed in this paper, the existing works [27, 31, 32] are seen as a special case. The contributions of this paper can be summarized into three aspects. Firstly, we study the formation-containment problem with intermittent communication, which is different from [25, 26], where the formation-containment problems were under the common assumption that information was transmitted among agents all the time. Secondly, the leaders and followers in this paper reach the desired formation-containment asymptotically within aperiodic intervals. However, intermittent communications were periodical in [27, 33, 34]. Thirdly, the leaders and followers suffer from the time-varying delays in this paper, but the reference [32], which investigated the consensus with intermittent communication, ignored the influence of communication delay.

The rest of this paper is organized as follows. In Section 2, some preliminaries are given, and the multiagent system with intermittent communication is formulated. In Section 3, sufficient conditions are obtained for the leaders reaching a prescribed formation and the followers converging into the convex hull formed by leaders, respectively. The effectiveness of our proposed control strategies is illustrated by numerical simulations in Section 4. The conclusion is presented in Section 5.

Notation: R^m and $R^{m \times m}$ represent $m \times 1$ column vectors and $m \times m$ matrices, respectively. I_n and 1_n denote $n \times n$ identity matrix and $n \times 1$ column vectors with all entries equivalent to 1. $\lambda(L)$ denotes the eigenvalues of matrix L , and $\sigma(L)$ indicates the largest singular value of matrix L . $\otimes$ stands for Kronecker product, and Q^T represents the transpose matrix of Q .

2. Preliminaries

In this section, the formation-containment control of second-order multiagent systems is formulated, and some useful lemmas and coordination control algorithms with intermittent communication are presented.

2.1. Formation-Containment Control of Second-Order Multiagent Systems. In this paper, the second-order multiagent systems consist of M followers indexed by $1, 2, \dots, M$ and $N - M$ leaders indexed by $M + 1, M + 2, \dots, N$. The agents' dynamics are given by

$$\begin{aligned} \dot{p}_i(t) &= q_i(t), \\ \dot{q}_i(t) &= u_i(t), \quad i = 1, 2, \dots, N, \end{aligned} \tag{1}$$

where $p_i(t) \in R^n$, $q_i(t) \in R^n$ and $u_i(t) \in R^n$ are the position, velocity and input of agent i , respectively.

Definition 1 (see [23–26]). The multiagent system (1) is said to achieve the desired formation-containment asymptotically, if

$$\lim_{t \rightarrow \infty} ((p_i(t) - g_i) - (p_j(t) - g_j)) = 0, \quad (2)$$

$$i, j \in \{M+1, M+2, \dots, N\},$$

and

$$\lim_{t \rightarrow \infty} \left(p_k(t) - \sum_{j=M+1}^N \beta_{k,j} p_j(t) \right) = 0, \quad (3)$$

$$k = 1, 2, \dots, M,$$

where $g_i \in \mathbb{R}^n$ denotes the desired formation reference vector, and $\beta_{k,j} > 0$ satisfies $\sum_{j=M+1}^N \beta_{k,j} = 1$.

The interconnection topology of N agents (1) is usually denoted by a digraph $G(V, E, A)$ with a set of nodes $V = \{1, \dots, N\}$, a set of edges $E \subseteq V \times V$, and a weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ with $a_{ij} \geq 0$. $e_{ij} = (i, j) \in E$ is a direct edge from node i to node j . $a_{ij} > 0$ if and only if $e_{ij} \in E$, and we define $a_{ii} = 0, i = 1, \dots, N$. A set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. The Laplacian matrix L is defined as $L = \text{diag}\{\sum_{j=1}^N a_{ij}, i \in V\} - A$. A digraph has a *directed spanning tree* if there exists at least one node having a directed path to all the other nodes.

In the digraph G of agents (1), G_F and G_L denote the interaction topologies of the followers and leaders separately.

Then, the Laplacian matrix L is decomposed as $L = \begin{bmatrix} L_1 & L_2 \\ 0 & L_3 \end{bmatrix}$, where $L_1 \in \mathbb{R}^{M \times M}$ and $L_3 \in \mathbb{R}^{(N-M) \times (N-M)}$ can be regarded as the Laplacian matrices of G_F and G_L , respectively, and $L_2 \in \mathbb{R}^{M \times (N-M)}$.

In this paper, we will consider the following topology of agents (1).

Assumption 2. The leaders' interaction topology G_L composed of the leaders has a directed spanning tree, and each follower has at least one leader that has a directed path to it.

2.2. Intermittent Coordination Control Algorithms. To solve the formation-containment control problem of agents (1), most of the existing algorithms are implemented on the basis of a common assumption that each agent communicates continuously with neighbors. In practical situations, however, the information may not be transmitted all the time due to instability of communication devices and limitation of technology. Hence, the following algorithms for the leaders and followers with intermittent communication are proposed, respectively.

The main idea of intermittent communication is shown in Figure 1. The communication periods are divided into work time and rest time, where the inputs are nonzero on $[t_k, t_k + \theta_k)$ and zero on $[t_k + \theta_k, t_{k+1})$. The values of θ_k are not constants, which is different from [27, 33, 34].

For the followers, the coordination control algorithms are designed as

$$u_i(t) = \begin{cases} k_{11} \sum_{j \in N_i} a_{ij} (p_i(t) - p_j(t)) + k_{12} \sum_{j \in N_i} a_{ij} (q_i(t) - q_j(t)), & t \in [t_k, t_k + \theta_k), \\ 0, & t \in [t_k + \theta_k, t_{k+1}), \end{cases} \quad i = 1, 2, \dots, M, \quad (4)$$

where $k_{11} < 0$ and $k_{12} < 0$, N_i means the neighbors of agent i , $a_{ij} > 0, j \in N_i$ and $0 < \theta_k \leq t_{k+1} - t_k, k = 0, 1, \dots, n$.

The formation control algorithms for the leaders are constructed as

$$u_i(t) = \begin{cases} k_{21} \sum_{j \in N_i} a_{ij} (p_i(t) - g_i) - (p_j(t) - g_j) + k_{22} \sum_{j \in N_i} a_{ij} (q_i(t) - q_j(t)), & t \in [t_k, t_k + \theta_k), \\ 0, & t \in [t_k + \theta_k, t_{k+1}), \end{cases} \quad (5)$$

$$i = M+1, M+2, \dots, N,$$

where $k_{21} < 0$ and $k_{22} < 0$, N_i means the neighbors of agent i , $a_{ij} > 0, j \in N_i$, and $0 < \theta_k \leq t_{k+1} - t_k, k = 0, 1, \dots, n$.

The closed-loop forms of agents (1) under algorithms (4) and (5) are

$$\dot{p}_i(t) = q_i(t),$$

$$\dot{q}_i(t) = k_{11} \sum_{j \in N_i} a_{ij} (p_i(t) - p_j(t))$$

$$\dot{q}_i(t) = 0, \quad t \in [t_k + \theta_k, t_{k+1}),$$

$$t \in [t_k, t_k + \theta_k),$$

$$i = 1, 2, \dots, M, \quad (6)$$

$$+ k_{12} \sum_{j \in N_i} a_{ij} (q_i(t) - q_j(t)),$$

FIGURE 1: Intermittent communication.

and

$$\begin{aligned}
 \dot{p}_i(t) &= q_i(t), \\
 \dot{q}_i(t) &= k_{21} \sum_{j \in N_i} a_{ij} ((p_i(t) - g_i) - (p_j(t) - g_j)) \\
 &\quad + k_{22} \sum_{j \in N_i} a_{ij} (q_i(t) - q_j(t)), \\
 t &\in [t_k, t_k + \theta_k), \\
 \dot{q}_i(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}), \\
 i &= M+1, \dots, N.
 \end{aligned} \tag{7}$$

2.3. Useful Lemmas. The following lemmas will play important roles in deriving the main results.

Lemma 3 (see [35]). *If the Assumption 2 holds, L_1 is nonsingular matrix, each entry of $-L_1^{-1}L_2$ is nonnegative, and all row sums of $-L_1^{-1}L_2$ equal to one.*

Lemma 4 (see [36]). *Suppose that $M \in \mathbb{R}^{n \times n}$ is a symmetric positive-definite matrix and $N \in \mathbb{R}^{n \times n}$ is symmetric. Then for any vector $x \in \mathbb{R}^n$, the following inequality holds:*

$$\begin{aligned}
 \lambda_{\min}(M^{-1}N) x^T M x &\leq x^T N x \\
 &\leq \lambda_{\max}(M^{-1}N) x^T M x.
 \end{aligned} \tag{8}$$

Lemma 5 (see [37]). *Let $g : [t_0 - \tau, \infty) \rightarrow [0, \infty)$ be a continuous function such that*

$$\dot{g}(t) \leq -ag(t) + b \sup_{t-\tau < \theta < t} g(\theta) \tag{9}$$

holds for $t > t_0$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$. If $a > b > 0$, then,

$$g(t) \leq \sup_{t-\tau < \theta < t} g(\theta) e^{-\rho(t-t_0)}, \tag{10}$$

where $\rho > 0$ is the smallest real root of the equation $a - \rho - be^{\rho\tau} = 0$.

Lemma 6 (see [38]). *For any vectors $x, y \in \mathbb{R}^n$ and a positive-definite matrix $Q \in \mathbb{R}^{n \times n}$,*

$$x^T y + y^T x \leq x^T Q x + y^T Q^{-1} y \tag{11}$$

holds.

3. Main Results

In this section, the formation-containment problem is analyzed for second-order multiagent systems with aperiodic intermittent communication.

3.1. Intermittent Communication without Communication Delay. In this subsection, we will prove that systems (6) and (7) reach the desired formation-containment asymptotically within aperiodic intermittent intervals.

Express the multiagent systems (6) and (7) in a multivariable form as

$$\begin{aligned}
 \dot{p}_F(t) &= q_F(t), \\
 \dot{q}_F(t) &= k_{11}(L_1 \otimes I_n) p_F(t) + k_{11}(L_2 \otimes I_n) p_L(t) \\
 &\quad + k_{12}(L_1 \otimes I_n) q_F(t) \\
 &\quad + k_{12}(L_2 \otimes I_n) q_L(t), \quad t \in [t_k, t_k + \theta_k), \\
 \dot{q}_F(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}),
 \end{aligned} \tag{12}$$

and

$$\begin{aligned}
 \dot{p}_L(t) &= q_L(t), \\
 \dot{q}_L(t) &= k_{21}(L_3 \otimes I_n) p_L(t) - k_{21}(L_3 \otimes I_n) g_L \\
 &\quad + k_{22}(L_3 \otimes I_n) q_L(t), \quad t \in [t_k, t_k + \theta_k), \\
 \dot{q}_L(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}),
 \end{aligned} \tag{13}$$

where $p_F = [p_1^T, p_2^T, \dots, p_M^T]^T$, $p_L = [p_{M+1}^T, p_{M+2}^T, \dots, p_N^T]^T$, $q_F = [q_1^T, q_2^T, \dots, q_M^T]^T$, $q_L = [q_{M+1}^T, q_{M+2}^T, \dots, q_N^T]^T$, and $g_L = [g_{M+1}^T, g_{M+2}^T, \dots, g_N^T]^T$.

Define $\bar{p}_F(t) = p_F(t) - ((-L_1^{-1}L_2) \otimes I_n) p_L(t)$ and $\bar{q}_F(t) = q_F(t) - ((-L_1^{-1}L_2) \otimes I_n) q_L(t)$, and we reformulate the multiagent system (12) as

$$\begin{aligned}
 \dot{\bar{p}}_F(t) &= \bar{q}_F(t), \\
 \dot{\bar{q}}_F(t) &= k_{11}(L_1 \otimes I_n) \bar{p}_F(t) + k_{12}(L_1 \otimes I_n) \bar{q}_F(t) \\
 &\quad - ((-L_1^{-1}L_2) \otimes I_n) \dot{q}_L(t), \\
 t &\in [t_k, t_k + \theta_k), \\
 \dot{\bar{q}}_F(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}),
 \end{aligned} \tag{14}$$

where $\bar{p}_F = [\bar{p}_1^T, \bar{p}_2^T, \dots, \bar{p}_M^T]^T$ and $\bar{q}_F = [\bar{q}_1^T, \bar{q}_2^T, \dots, \bar{q}_M^T]^T$.

Define $\bar{p}_i(t) = p_i(t) - g_i$, $i = M + 1, M + 2, \dots, N$ and $\bar{p}_L = [\bar{p}_{M+1}^T, \bar{p}_{M+2}^T, \dots, \bar{p}_N^T]^T$, and we rewrite the multiagent system (13) as

$$\begin{aligned} \dot{\bar{p}}_L(t) &= q_L(t), \\ \dot{q}_L(t) &= k_{21}(L_3 \otimes I_n) \bar{p}_L(t) + k_{22}(L_3 \otimes I_n) q_L(t), \\ t &\in [t_k, t_k + \theta_k), \\ \dot{q}_L(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}). \end{aligned} \quad (15)$$

The eigenvalues of the Laplacian matrix L_3 of $N - M$ leaders are defined as $\lambda_i^L, i \in \{M + 1, M + 2, \dots, N\}$, and the corresponding eigenvectors are u_i . We assume that $\lambda_{M+1}^L = 0$ is corresponding to associated eigenvector $u_{M+1} = \mathbf{1}_{N-M}$ and $0 < \text{Re}(\lambda_{M+2}^L) \leq \dots \leq \text{Re}(\lambda_N^L)$. Then, the Jordan canonical form J_3 of the Laplacian matrix L_3 is given by

$$\begin{aligned} J_3 &= U_L^{-1} L_3 U_L = \text{diag}\{0, \bar{J}_3\} \\ &= \text{diag}\{0, \lambda_{M+2}^L, \dots, \lambda_N^L\}, \end{aligned} \quad (16)$$

where $U_L = [u_{M+1}, u_{M+2}, \dots, u_N]$ and $U_L^{-1} = [\tilde{u}_{M+1}, \tilde{u}_{M+2}, \dots, \tilde{u}_N]^T$,

Defining $\varsigma(t) = (U_L^{-1} \otimes I_n)(\bar{p}_L^T, q_L^T)^T$ yields

$$\begin{aligned} \dot{\varsigma}(t) &= \left(\begin{bmatrix} 0 & I_{N-M} \\ k_{21}J_3 & k_{22}J_3 \end{bmatrix} \otimes I_n \right) \varsigma(t), \\ t &\in [t_k, t_k + \theta_k), \end{aligned} \quad (17)$$

$$\dot{\varsigma}(t) = \left(\begin{bmatrix} 0 & I_{N-M} \\ 0 & 0 \end{bmatrix} \otimes I_n \right) \varsigma(t), \quad t \in [t_k + \theta_k, t_{k+1}),$$

where $\varsigma = [\varsigma_{M+1}^T, \varsigma_{M+2}^T, \dots, \varsigma_N^T]^T$. Hence, the formation control problem of multiagent system (7) is solved if and only if $\varsigma_i(t), i = M + 2, M + 3, \dots, N$ converge to zero asymptotically.

Denoting $\tilde{\varsigma}(t) = (\varsigma_{M+2}^T(t), \dots, \varsigma_N^T(t))^T$ yields

$$\begin{aligned} \dot{\tilde{\varsigma}}(t) &= (E_1 \otimes I_n) \tilde{\varsigma}(t), \quad t \in [t_k, t_k + \theta_k), \\ \dot{\tilde{\varsigma}}(t) &= (E_2 \otimes I_n) \tilde{\varsigma}(t), \quad t \in [t_k + \theta_k, t_{k+1}), \end{aligned} \quad (18)$$

where $E_1 = \begin{bmatrix} 0 & I_{N-M-1} \\ k_{21}\bar{J}_3 & k_{22}\bar{J}_3 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 0 & I_{N-M-1} \\ 0 & 0 \end{bmatrix}$.

Lemma 7. The matrix E_1 is Hurwitz if and only if

$$k_{22}^2 \Re^3(\lambda_i^L) + k_{22}^2 \Re(\lambda_i^L) \Im^2(\lambda_i^L) > -k_{21} \Im^2(\lambda_i^L), \quad (19)$$

where $\Re(\cdot)$ and $\Im(\cdot)$ represent real and imaginary parts of complex number.

Proof. Characteristic polynomial of E_1 is given by

$$\begin{aligned} \det(\mu I_{2(N-M-1)} - E_1) &= \det \begin{pmatrix} \mu I_{N-M-1} & -I_{N-M-1} \\ -k_{21}\bar{J}_3 & \mu I_{N-M-1} - k_{22}\bar{J}_3 \end{pmatrix} \\ &= \det(\mu^2 I_{N-M-1} - k_{22}\mu \bar{J}_3 - k_{21}\bar{J}_3) = \prod_{i=M+2}^N (\mu^2 \\ &\quad - k_{22}\mu \lambda_i^L - k_{21}\lambda_i^L) = \prod_{i=M+2}^N (\mu^2 \\ &\quad + (-k_{22}\mu - k_{21})(\Re(\lambda_i^L) + \mathbf{i} * \Im(\lambda_i^L))). \end{aligned} \quad (20)$$

Using Generalized Routh Criterion, the eigenvalues of E_1 lie on the open left half complex plane if and only if $\Re(\lambda_i^L) > 0$ and (19) hold. If Assumption 2 holds, $\Re(\lambda_i^L) > 0, i = M + 2, M + 3, \dots, N$. Lemma 7 is proved. $\square$

Theorem 8. The multiagent systems (6) and (7) with Assumption 2 reach the desired formation-containment asymptotically, if the following conditions are satisfied simultaneously.

(i) $k_{22}^2 \Re^3(\lambda_i^L) + k_{22}^2 \Re(\lambda_i^L) \Im^2(\lambda_i^L) > -k_{21} \Im^2(\lambda_i^L), i = M + 2, M + 3, \dots, N$.

(ii) $\gamma_1/\gamma_2 > \bar{\theta}/\theta$, where $\bar{\theta} = \max\{t_{k+1} - t_k - \theta_k\}$ and $\theta = \min\{\theta_k\}$ with $k = 0, 1, \dots, n$, and $\gamma_1 = \lambda_{\min}(P_L^{-1}Q_L)$ and $\gamma_2 = \lambda_{\max}(P_L^{-1}(E_2P_L + P_LE_2^T))$ with positive-definite matrices $P_L > 0$ and $Q_L > 0$ satisfying $E_1P_L + P_LE_1^T = -Q_L$.

(iii) $k_{12}^2 \Re^3(\lambda_i^F) + k_{12}^2 \Re(\lambda_i^F) \Im^2(\lambda_i^F) > -k_{11} \Im^2(\lambda_i^F)$, where λ_i^F are the eigenvalues of $L_1, i = 1, 2, \dots, M$.

(iv) $\gamma_3/\gamma_4 > \bar{\theta}/\theta$, where $\gamma_3 = \lambda_{\min}(P_F^{-1}Q_F)$ and $\gamma_4 = \lambda_{\max}(P_F^{-1}(F_2P_F + P_FF_2^T))$ with positive-definite matrices P_F and Q_F satisfying $F_1P_F + P_FF_1^T = -Q_F, F_1 = \begin{bmatrix} 0 & I_M \\ k_{11}L_1 & k_{12}L_1 \end{bmatrix}$ and $F_2 = \begin{bmatrix} 0 & I_M \\ 0 & 0 \end{bmatrix}$.

Proof.

Step 1. Asymptotic convergence of leaders' formation is equivalent to the asymptotical stability of system (18).

The matrix E_1 is Hurwitz from condition (i) and Lemma 7, so there exists a unique $P_L = P_L^T > 0$ satisfying $E_1P_L + P_LE_1^T = -Q_L$ with a given positive-definite matrix Q_L . Choose a Lyapunov function for (18) as follows:

$$V(t) = \frac{1}{2} \tilde{\varsigma}^T(t) (P_L \otimes I_n) \tilde{\varsigma}(t). \quad (21)$$

It follows from Lemma 4 that the time derivative of $V(t)$ along the trajectory of system (18) with $t \in [t_k, t_k + \theta_k), k = 0, 1, \dots, n$ is

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2} \tilde{\varsigma}^T(t) [(E_1P_L + P_LE_1^T) \otimes I_n] \tilde{\varsigma}(t) \\ &= -\frac{1}{2} \tilde{\varsigma}^T(t) (Q_L \otimes I_n) \tilde{\varsigma}(t) \end{aligned}$$

$$\begin{aligned}
&\leq -\frac{1}{2}\lambda_{\min}(P_L^{-1}Q_L)\tilde{\zeta}^T(t)(P_L \otimes I_n)\tilde{\zeta}(t) \\
&= -\gamma_1 V(t),
\end{aligned} \tag{22}$$

where $\gamma_1 = \lambda_{\min}(P_L^{-1}Q_L)$. Meanwhile, the time derivative of $V(t)$ along the trajectory of the system (18) with $t \in [t_k + \theta_k, t_{k+1})$, $k = 0, 1, \dots, n$ is

$$\begin{aligned}
\dot{V}(t) &= \frac{1}{2}\tilde{\zeta}^T(t) \left[(E_2 P_L + P_L E_2^T) \otimes I_n \right] \tilde{\zeta}(t) \\
&\leq \frac{1}{2}\lambda_{\max}(E_2 P_L + P_L E_2^T) \tilde{\zeta}^T(t) \tilde{\zeta}(t) \\
&\leq \frac{1}{2}\lambda_{\max}(P_L^{-1}(E_2 P_L + P_L E_2^T)) \tilde{\zeta}^T(t) (P_L \otimes I_n) \tilde{\zeta}(t) \\
&= \gamma_2 V(t),
\end{aligned} \tag{23}$$

where $\gamma_2 = \lambda_{\max}(P_L^{-1}(E_2 P_L + P_L E_2^T))$.

It follows from (22) that

$$\begin{aligned}
V(t) &\leq V(0) e^{-\gamma_1(\theta_0 + \theta_1 + \dots + \theta_{k-1} + t - t_k) + \gamma_2(\bar{\theta}_0 + \bar{\theta}_1 + \dots + \bar{\theta}_{k-1})} \\
&\leq V(0) e^{-\gamma_1 k \theta + \gamma_2 k \bar{\theta}}
\end{aligned} \tag{24}$$

holds with $t \in [t_k, t_k + \theta_k)$, where θ and $\bar{\theta}$ are defined in condition (ii) of Theorem 8. In (24), θ and $\bar{\theta}$ are the minimum control time and the maximum communication loss time, respectively. Meanwhile, we obtain from (23) that

$$\begin{aligned}
V(t) &\leq V(0) e^{-\gamma_1(\theta_0 + \theta_1 + \dots + \theta_k) + \gamma_2(\bar{\theta}_0 + \bar{\theta}_1 + \dots + \bar{\theta}_{k-1} + t - t_k - \theta_k)} \\
&\leq V(0) e^{-\gamma_1(k+1)\theta + \gamma_2(k+1)\bar{\theta}}
\end{aligned} \tag{25}$$

holds with $t \in [t_k + \theta_k, t_{k+1})$.

We conclude from the above analysis that

$$V(t) \leq V(0) e^{-\gamma_1 k \theta + \gamma_2 (k+1) \bar{\theta}} \tag{26}$$

holds for arbitrary $t \in [t_k, t_{k+1})$, where $\gamma_1 \theta > \gamma_2 \bar{\theta} > 0$ for the condition (ii). Hence, the leaders reach the desired formation asymptotically.

Step 2. Containment convergence analysis of the followers.

If the multiagent systems (13) achieve the desired formation, we obtain $\lim_{t \rightarrow \infty} \dot{q}_L = 0$. Then, the system (14) becomes

$$\begin{aligned}
\begin{bmatrix} \dot{\bar{p}}_F(t) \\ \dot{\bar{q}}_F(t) \end{bmatrix} &= (F_1 \otimes I_n) \begin{bmatrix} \bar{p}_F(t) \\ \bar{q}_F(t) \end{bmatrix}, \quad t \in [t_k, t_k + \theta_k), \\
\begin{bmatrix} \dot{\bar{p}}_F(t) \\ \dot{\bar{q}}_F(t) \end{bmatrix} &= (F_2 \otimes I_n) \begin{bmatrix} \bar{p}_F(t) \\ \bar{q}_F(t) \end{bmatrix}, \quad t \in [t_k + \theta_k, t_{k+1}),
\end{aligned} \tag{27}$$

where F_1 and F_2 are defined in the condition (iv). Similar to Step 1, we analyze the asymptotic stability of system (27) and prove that the followers converge to the convex hull of the leaders if the condition (iii) and (iv) hold. Theorem 8 is proved. $\square$

Remark 9. Different from the references [27, 33, 34] that investigated the multiagent systems with periodical intermittent communication, Theorem 8 provides the conditions for the agents (6) and (7) reaching the desired formation-containment asymptotically within aperiodic intervals.

3.2. Intermittent Communication with Time-Varying Delay.

In this subsection, the formation-containment control is studied for the second-order multiagent systems with completely intermittent communication and time-varying communication delay.

For the followers, the containment control algorithms under time-varying communication delay become

$$u_i(t) = \begin{cases} k_{11} \sum_{j \in N_i} a_{ij} (p_i(t - \tau_1(t)) - p_j(t - \tau_1(t))) + k_{12} \sum_{j \in N_i} a_{ij} (q_i(t - \tau_1(t)) - q_j(t - \tau_1(t))), & t \in [t_k, t_k + \theta_k), \\ 0, & t \in [t_k + \theta_k, t_{k+1}), \end{cases} \tag{28}$$

where $i = 1, 2, \dots, M$ and $\tau_1(t) > 0$ is the time-varying communication delay. For the leaders, the formation control

algorithms under time-varying communication delay turn to be

$$\begin{aligned}
&u_i(t) \\
&= \begin{cases} k_{21} \sum_{j \in N_i} a_{ij} ((p_i(t - \tau_2(t)) - g_i) - (p_j(t - \tau_2(t)) - g_j)) + k_{22} \sum_{j \in N_i} a_{ij} (q_i(t - \tau_2(t)) - q_j(t - \tau_2(t))), & t \in [t_k, t_k + \theta_k), \\ 0, & t \in [t_k + \theta_k, t_{k+1}), \end{cases} \tag{29}
\end{aligned}$$

where $i = M + 1, M + 2, \dots, N$ and $\tau_2(t) > 0$ is the time-varying communication delay.

The closed-loop forms of agents (1) with (28) and (29) are given by

$$\begin{aligned}\dot{p}_i(t) &= q_i(t), \\ \dot{q}_i(t) &= k_{11} \sum_{j \in N_i} a_{ij} (p_i(t - \tau_1(t)) - p_j(t - \tau_1(t))) \\ &\quad + k_{12} \sum_{j \in N_i} a_{ij} (q_i(t - \tau_1(t)) - q_j(t - \tau_1(t))), \quad (30) \\ t &\in [t_k, t_k + \theta_k), \\ \dot{q}_i(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}),\end{aligned}$$

and

$$\begin{aligned}\dot{p}_i(t) &= q_i(t), \\ \dot{q}_i(t) &= k_{21} \sum_{j \in N_i} a_{ij} \\ &\quad \cdot ((p_i(t - \tau_2(t)) - g_i) - (p_j(t - \tau_2(t)) - g_j)) \\ &\quad + k_{22} \sum_{j \in N_i} a_{ij} (q_i(t - \tau_2(t)) - q_j(t - \tau_2(t))), \quad (31) \\ t &\in [t_k, t_k + \theta_k), \\ \dot{q}_i(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}).\end{aligned}$$

Similar to the above subsection, we define $\bar{p}_F(t) = p_F(t) - ((-L_1^{-1}L_2) \otimes I_n)p_L(t)$, $\bar{q}_F(t) = q_F(t) - ((-L_1^{-1}L_2) \otimes I_n)q_L(t)$, where $p_F = [p_1^T, p_2^T, \dots, p_M^T]^T$, $p_L = [p_{M+1}^T, p_{M+2}^T, \dots, p_N^T]^T$, $q_F = [q_1^T, q_2^T, \dots, q_M^T]^T$, $q_L = [q_{M+1}^T, q_{M+2}^T, \dots, q_N^T]^T$, and $g_L = [g_{M+1}^T, g_{M+2}^T, \dots, g_N^T]^T$. Then, we get

$$\begin{aligned}\dot{\bar{p}}_F(t) &= \bar{q}_F(t), \\ \dot{\bar{q}}_F(t) &= k_{11} (L_1 \otimes I_n) \bar{p}_F(t - \tau_1(t)) \\ &\quad + k_{12} (L_1 \otimes I_n) \bar{q}_L(t - \tau_1(t)) \\ &\quad - ((-L_1^{-1})L_2 \otimes I_n) \dot{q}_L(t - \tau_1(t)), \quad (32) \\ t &\in [t_k, t_k + \theta_k), \\ \dot{\bar{q}}_F(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}),\end{aligned}$$

where $\bar{p}_F = [\bar{p}_1^T, \bar{p}_2^T, \dots, \bar{p}_M^T]^T$ and $\bar{q}_F = [\bar{q}_1^T, \bar{q}_2^T, \dots, \bar{q}_M^T]^T$.

Define $\bar{p}_i(t) = p_i(t) - g_i$, $i = M + 1, M + 2, \dots, N$, and $\bar{p}_L = [\bar{p}_{M+1}^T, \bar{p}_{M+2}^T, \dots, \bar{p}_N^T]^T$, and we rewrite multiagent system (31) as

$$\begin{aligned}\dot{\bar{p}}_L(t) &= q_L(t), \\ \dot{q}_L(t) &= k_{21} (L_3 \otimes I_n) \bar{p}_L(t - \tau_2(t)) \\ &\quad + k_{22} (L_3 \otimes I_n) q_L(t - \tau_2(t)), \quad (33) \\ t &\in [t_k, t_k + \theta_k), \\ \dot{q}_L(t) &= 0, \quad t \in [t_k + \theta_k, t_{k+1}).\end{aligned}$$

Denote $\varsigma(t) = (U_L^{-1} \otimes I_n)(\bar{p}_L^T, q_L^T)^T$, and we get

$$\begin{aligned}\dot{\varsigma}(t) &= \left(\begin{bmatrix} 0 & I_{N-M} \\ 0 & 0 \end{bmatrix} \otimes I_n \right) \varsigma(t) \\ &\quad + \left(\begin{bmatrix} 0 & 0 \\ k_{21}J_3 & k_{22}J_3 \end{bmatrix} \otimes I_n \right) \varsigma(t - \tau_2(t)), \quad (34) \\ t &\in [t_k, t_k + \theta_k),\end{aligned}$$

$$\dot{\varsigma}(t) = \left(\begin{bmatrix} 0 & I_{N-M} \\ 0 & 0 \end{bmatrix} \otimes I_n \right) \varsigma(t), \quad t \in [t_k + \theta_k, t_{k+1}),$$

where $\varsigma = [\varsigma_{M+1}^T, \varsigma_{M+2}^T, \dots, \varsigma_N^T]^T$. Hence, the formation control problem of multiagent system (31) is solved if and only if $\lim_{t \rightarrow \infty} \varsigma_i(t) = 0$, $i = M + 2, M + 3, \dots, N$.

Use the Newton-Raphson formula: $\varsigma(t - \tau_2(t)) = \varsigma(t) - \int_{t-\tau_2(t)}^t \dot{\varsigma}(s) ds$, and denoting $\tilde{\varsigma}(t) = (\varsigma_{M+2}^T(t), \dots, \varsigma_N^T(t))^T$ yields

$$\begin{aligned}\dot{\tilde{\varsigma}}(t) &= (E_3 \otimes I_n) \tilde{\varsigma}(t) - \int_{t-\tau_2(t)}^t (E_4 \otimes I_n) \dot{\tilde{\varsigma}}(s) ds, \\ t &\in [t_k, t_k + \theta_k), \quad (35) \\ \dot{\tilde{\varsigma}}(t) &= (E_5 \otimes I_n) \tilde{\varsigma}(t), \quad t \in [t_k + \theta_k, t_{k+1}),\end{aligned}$$

where $E_3 = \begin{bmatrix} 0 & I_{N-M-1} \\ k_{21}\bar{J}_3 & k_{22}\bar{J}_3 \end{bmatrix}$, $E_4 = \begin{bmatrix} 0 & 0 \\ k_{21}\bar{J}_3 & k_{22}\bar{J}_3 \end{bmatrix}$, and $E_5 = \begin{bmatrix} 0 & I_{N-M-1} \\ 0 & 0 \end{bmatrix}$.

Before presenting the result, we make the following assumption on the time-varying delay.

Assumption 10. $0 \leq \tau_1(t) \leq \bar{\tau}_0$ and $0 \leq \tau_2(t) \leq \tau_0$, where $\bar{\tau}_0$ and τ_0 are known constants.

Theorem 11. The multiagent systems (30) and (31) with Assumptions 2 and 10 reach the desired formation-containment asymptotically, if the following conditions are satisfied simultaneously:

(i) $k_{22}^2 \Re^3(\lambda_i^L) + k_{22}^2 \Re(\lambda_i^L) \Im^2(\lambda_i^L) > -k_{21} \Im^2(\lambda_i^L)$, $i = M + 2, M + 3, \dots, N$.

(ii)

$$\min \left\{ \frac{\theta}{2}, \frac{\gamma_0 \theta - \gamma_3 \bar{\theta}}{2\gamma_0}, \sqrt{\frac{h(\lambda_{\min}(Q_L) - h)}{2\sigma^2(P_L E_3)(\sigma^2(E_3) + \sigma^2(E_4))\lambda_{\max}(P_L)\lambda_{\max}(P_L^{-1})}} \right\} > \tau_0, \quad (36)$$

where $h > 0$, $\theta = \min\{\theta_k\}$ and $\bar{\theta} = \max\{t_{k+1} - t_k - \theta_k\}$ with $k = 0, 1, \dots, n$, $\gamma_0 > 0$ is the unique real root of the equation $\gamma_1 - \gamma_0 - \gamma_2 e^{2\gamma_0 \tau_0} = 0$, $\gamma_1 = (\lambda_{\min}(Q_L) - h)/\lambda_{\max}(P_L) > 0$, $\gamma_2 = 2\tau_0^2 \sigma^2(P_L E_3)(\sigma^2(E_3) + \sigma^2(E_4))\lambda_{\max}(P_L^{-1})/h$, and $\gamma_3 = \lambda_{\max}(P_L^{-1}(E_5 P_L + P_L E_5^T))$ with positive-definite matrices P_L and Q_L satisfying the Lyapunov equation $E_1 P_L + P_L E_1^T = -Q_L$,
 (iii) $k_{12}^2 \mathfrak{R}^3(\lambda_i^F) + k_{12}^2 \mathfrak{R}(\lambda_i^F) \mathfrak{I}^2(\lambda_i^F) > -k_{11} \mathfrak{I}^2(\lambda_i^F)$, $i = 1, 2, \dots, M$.

(iv)

$$\min \left\{ \frac{\theta}{2}, \frac{\gamma_4 \theta - \gamma_7 \bar{\theta}}{2\gamma_4}, \sqrt{\frac{h(\lambda_{\min}(P_F) - h)}{2\sigma^2(P_F F_3)(\sigma^2(F_3) + \sigma^2(F_4))\lambda_{\max}(P_F)\lambda_{\max}(P_F^{-1})}} \right\} > \bar{\tau}_0, \quad (37)$$

where $\gamma_4 > 0$ is the unique real root of the equation $\gamma_5 - \gamma_4 - \gamma_6 e^{2\gamma_4 \bar{\tau}_0} = 0$, $\gamma_5 = (\lambda_{\min}(Q_F) - h)/\lambda_{\max}(P_F) > 0$, $\gamma_6 = 2\tau_0^2 \sigma^2(P_F F_3)(\sigma^2(F_3) + \sigma^2(F_4))\lambda_{\max}(P_F^{-1})/h$, and $\gamma_7 = \lambda_{\max}(P_F^{-1}(F_5 P_F + P_F F_5^T))$, with positive-definite matrix P_F and Q_F satisfying Lyapunov equation

$$\begin{aligned} F_1 P_F + P_F F_1^T &= -Q_F, \\ F_3 &= \begin{bmatrix} 0 & I_M \\ k_{11} L_1 & k_{12} L_1 \end{bmatrix}, \\ F_4 &= \begin{bmatrix} 0 & 0 \\ k_{11} L_3 & k_{12} L_3 \end{bmatrix} \\ \text{and } F_5 &= \begin{bmatrix} 0 & I_M \\ 0 & 0 \end{bmatrix}. \end{aligned} \quad (38)$$

Proof.

Step 1. Asymptotic convergence of leaders' formation is equivalent to the asymptotical stability of the system (35).

The matrix E_3 is Hurwitz from condition (i) and Lemma 7, so there exists a unique $P_L = P_L^T > 0$ satisfying $E_3 P_L + P_L E_3^T = -Q_L$ with a given positive-definite matrix $Q_L > 0$. Choose a Lyapunov function for (35) as follows:

$$V(t) = \frac{1}{2} \tilde{\zeta}^T(t) (P_L \otimes I_n) \tilde{\zeta}(t). \quad (39)$$

For $t \in [t_k, t_k + \theta_k)$, $k = 0, 1, \dots, n$, the time derivative of $V(t)$ along the trajectory of system (35) is

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2} \tilde{\zeta}^T(t) \left[(E_3 P_L + P_L E_3^T) \otimes I_n \right] \tilde{\zeta}(t) - \frac{1}{2} \tilde{\zeta}^T(t) \\ &\quad \cdot \left[((P_L E_4) \otimes I_n) \int_{t-\tau_2(t)}^t \dot{\tilde{\zeta}}(s) ds \right] \\ &\quad - \frac{1}{2} \left[((P_L E_4) \otimes I_n) \int_{t-\tau_2(t)}^t \dot{\tilde{\zeta}}(s) ds \right]^T \tilde{\zeta}(t) \\ &\leq -\frac{1}{2} (\lambda_{\min}(Q_L) - h) \tilde{\zeta}^T(t) \tilde{\zeta}(t) \\ &\quad + \frac{1}{2h} \left[((P_L E_4) \otimes I_n) \int_{t-\tau_2(t)}^t \dot{\tilde{\zeta}}(s) ds \right]^T \\ &\quad \cdot \left[((P_L E_4) \otimes I_n) \int_{t-\tau_2(t)}^t \dot{\tilde{\zeta}}(s) ds \right] \\ &\leq -\frac{1}{2} (\lambda_{\min}(Q_L) - h) \tilde{\zeta}^T(t) \tilde{\zeta}(t) \\ &\quad + \frac{\tau_0 \sigma^2(P_L E_4)}{2h} \left(\int_{t-\tau_2(t)}^t \dot{\tilde{\zeta}}^T(s) \dot{\tilde{\zeta}}(s) ds \right) \\ &\leq -\frac{1}{2} (\lambda_{\min}(Q_L) - h) \tilde{\zeta}^T(t) \tilde{\zeta}(t) \\ &\quad + \frac{\tau_0^2 \sigma^2(P_L E_4)(\sigma^2(E_3) + \sigma^2(E_4))}{h} \sup_{t-2\tau_0 \leq \theta \leq t} \tilde{\zeta}^T(\theta) \\ &\quad \cdot \zeta(\theta) \leq -\gamma_1 V(t) + \gamma_2 \sup_{t-2\tau_0 \leq \theta \leq t} V(\theta), \end{aligned} \quad (40)$$

where $\gamma_1 > \gamma_2 > 0$ holds for the condition (ii). In the light of Lemma 5, we obtain that

$$V(t) \leq \sup_{t_k - 2\tau_0 \leq \theta \leq t_k} V(\theta) e^{-\gamma_0(t-t_k)} \quad (41)$$

holds with $t \in [t_k, t_k + \theta_k)$, where $\gamma_0 > 0$ is the unique real root of the equation $\gamma_1 - \gamma_0 - \gamma_2 e^{2\gamma_0 \tau_0} = 0$.

For $t \in [t_k + \theta_k, t_{k+1})$, $k = 0, 1, \dots, N$, the time derivative of $V(t)$ along the trajectory of system (35) is

$$\begin{aligned} \dot{V}(t) &= \frac{1}{2} \tilde{\zeta}^T(t) \left[(E_5 P_L + P_L E_5^T) \otimes I_n \right] \tilde{\zeta}(t) \\ &\leq \frac{1}{2} \lambda_{\max}(E_5 P_L + P_L E_5^T) \tilde{\zeta}^T(t) \tilde{\zeta}(t) \\ &\leq \frac{1}{2} \lambda_{\max}(P_L^{-1}(E_5 P_L + P_L E_5^T)) \tilde{\zeta}^T(t) (P_L \otimes I_n) \tilde{\zeta}(t) \\ &= \gamma_3 V(t), \end{aligned} \quad (42)$$

where $\gamma_3 = \lambda_{\max}(P_L^{-1}(E_5 P_L + P_L E_5^T))$. Then, we get

$$\begin{aligned} V(t) &\leq V(t_k + \theta_k) e^{\gamma_3(t-t_k-\theta_k)} \\ &\leq \sup_{t_k + \theta_k - 2\tau_0 \leq \theta_k \leq t_k + \theta_k} V(\theta) e^{\gamma_3(t-t_k-\theta_k)}, \end{aligned} \quad (43)$$

holds with $t \in [t_k + \theta_k, t_{k+1})$.

Therefore, we come to the conclusion that

$$V(t) \leq \begin{cases} \sup_{t_k - 2\tau_0 \leq \theta \leq t_k} V(\vartheta) e^{-\gamma_0(t-t_k)}, & t \in [t_k, t_k + \theta_k), \\ \sup_{t_k + \theta_k - 2\tau_0 \leq \theta \leq t_k + \theta_k} V(\vartheta) e^{\gamma_3(t-t_k - \theta_k)}, & t \in [t_k + \theta_k, t_{k+1}). \end{cases} \quad (44)$$

It follows from (44) that

$$\begin{aligned} V(t) &\leq \sup_{-2\tau_0 \leq \theta \leq 0} V(\vartheta) \\ &\cdot e^{-\gamma_0(\theta_0 - 2\tau_0 + \theta_1 - 2\tau_0 + \dots + \theta_{k-1} - 2\tau_0 + t - t_k) + \gamma_3(\bar{\theta}_0 + \bar{\theta}_1 + \dots + \bar{\theta}_{k-1})} \\ &\leq \sup_{-2\tau_0 \leq \theta \leq 0} V(\vartheta) e^{-\gamma_0 k(\theta - 2\tau_0) + \gamma_3 k \bar{\theta}}, \end{aligned} \quad (45)$$

holds with $t \in [t_k, t_k + \theta_k)$, where θ and $\bar{\theta}$ are defined in the condition (ii) of Theorem 11. For $t \in [t_k + \theta_k, t_{k+1})$,

$$\begin{aligned} V(t) &\leq \sup_{-2\tau_0 \leq \theta \leq 0} V(\vartheta) \\ &\cdot e^{-\gamma_0(\theta_0 - 2\tau_0 + \theta_1 - 2\tau_0 + \dots + \theta_k - 2\tau_0) + \gamma_3(\bar{\theta}_0 + \bar{\theta}_1 + \dots + \bar{\theta}_{k-1} + t - t_k - \theta_k)} \\ &\leq \sup_{-2\tau_0 \leq \theta \leq 0} V(\vartheta) e^{-\gamma_0(k+1)(\theta - 2\tau_0) + \gamma_3(k+1)\bar{\theta}}. \end{aligned} \quad (46)$$

In conclusion, we get

$$V(t) \leq \sup_{-2\tau_0 \leq \theta \leq 0} V(\vartheta) e^{-\gamma_0 k(\theta - 2\tau_0) + \gamma_3(k+1)\bar{\theta}}, \quad t \in (0, +\infty), \quad (47)$$

where $\gamma_0(\theta - 2\tau_0) > \gamma_3 \bar{\theta} > 0$ holds for the condition (ii). Therefore, the leaders reach the desired formation.

Step 2. The followers' containment convergence analysis.

If the multiagent system (31) achieves the desired formation, we obtain $\lim_{t \rightarrow \infty} \dot{q}_L(t) = 0$. Then, the system (32) becomes

$$\begin{aligned} \begin{bmatrix} \dot{\bar{p}}_F(t) \\ \dot{\bar{q}}_F(t) \end{bmatrix} &= (F_3 \otimes I_n) \begin{bmatrix} \bar{p}_F(t) \\ \bar{q}_F(t) \end{bmatrix} \\ &- \int_{t-\tau_1(t)}^t (F_4 \otimes I_n) \begin{bmatrix} \dot{\bar{p}}_F(s) \\ \dot{\bar{q}}_F(s) \end{bmatrix} ds, \\ &t \in [t_k, t_k + \theta_k), \end{aligned} \quad (48)$$

$$\begin{bmatrix} \dot{\bar{p}}_F(t) \\ \dot{\bar{q}}_F(t) \end{bmatrix} = (F_5 \otimes I_n) \begin{bmatrix} \bar{p}_F(t) \\ \bar{q}_F(t) \end{bmatrix}, \quad t \in [t_k + \theta_k, t_{k+1}).$$

where F_3 , F_4 , and F_5 are defined in the condition (iv). Similar to Step 1, we analyze the asymptotic stability of system (48) and prove that the followers converge to the convex hull of the leaders if the conditions (iii) and (iv) hold. Theorem 11 is proved. $\square$

Remark 12. In [25], Dong et al. discussed the formation-containment control with identical time-invariant delay in which agents received information all the time. In addition, Wang et al. [39] investigated the containment control of second-order multiagent systems with multiple leaders and obtained the conditions for the agents under symmetric topologies and time-varying delay. Different above references, we consider the formation-containment control of second-order multiagent systems with different time-varying delays and gain the convergence conditions under general directed topology.

4. Numerical Simulations

In this section, two examples are given to verify the effectiveness of our theoretical analysis.

Example 1. Formation-containment control without communication delay.

Consider a second-order multiagent system composed of four followers labeled as 1, 2, 3, 4 and four leaders labeled as 5, 6, 7, 8. The interconnection topology is shown in Figure 2. Obviously, the topology G_L of leaders has a spanning tree, and each follower has at least one leader. For simplicity, the coupling weights are all chosen as 1 for the interconnection topology. Hence, the matrices L_1 , L_2 and L_3 are

$$\begin{aligned} L_1 &= \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ -1 & 0 & 0 & 2 \end{bmatrix}, \\ L_2 &= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \\ L_3 &= \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}. \end{aligned} \quad (49)$$

We assume that the agents move in the two-dimensional space, i.e., $p(t) \in R^2, q(t) \in R^2$. The initial positions and velocities of leaders and following agents are chosen between -5 and 5 randomly. The formation reference function is shown as follows: $g_5 = [10 \ 10]^T, g_6 = [-10 \ 10]^T, g_7 = [-10 \ -10]^T, g_8 = [10 \ -10]^T$. Based on the above matrices L_1 and L_3 , eigenvalues are $\lambda_1^F = 1, \lambda_2^F = 2+i, \lambda_3^F = 2-i, \lambda_4^F = 3, \lambda_5^L = 0, \lambda_6^L = 1+i, \lambda_7^L = 1-i, \lambda_8^L = 2$. According to condition (i) of Theorem 8, the control gains are chosen as $k_{11} = -2, k_{12} = -2, k_{21} = -2, k_{22} = -2$. Choose $\gamma_1 = 0.6907, \gamma_2 = 1.5617, \gamma_3 = 0.7597, \gamma_4 = 1.6387, \theta = 0.23$, and $\bar{\theta} = 0.1$, and the conditions in Theorem 8 are satisfied. Then, the communication intervals

FIGURE 2: Network of eight agents.

are set from 0.23 to 0.40 and the communication loss intervals are assigned from 0 to 0.1 randomly. The leaders reach the desired formation asymptotically, and the following agents converge to the convex hull generated by the leaders in Figure 3. In Figure 3, the initial states of all agents are described as diamonds, and it is obvious that the agents are not all in the convex hull formed by leaders. The final states of leaders and followers are described by the shape of circles and five-pointed stars, respectively. In addition, Figures 4 and 5 show the agents' positions and velocities, respectively, and the variation of control inputs is demonstrated in Figure 6. Therefore, the multiagent system (1) under the algorithms (4) and (5) achieve the desired formation-containment control asymptotically.

Example 2. Formation-containment control with time-varying communication delay.

Choose the same interaction topology, formation reference function, and initial state values as those in Example 1. According to condition (i) in Theorem 11, we choose $k_{11} = -2, k_{12} = -2, k_{21} = -2, k_{22} = -2$. By computation, $\gamma_0 = 0.3727, \gamma_3 = 1.5617, \gamma_4 = 0.2818, \gamma_7 = 1.6387, \theta = 0.43, \bar{\theta} = 0.1$, and $\tau_1 = 0.0002 + 0.0002 \sin(t), \tau_2 = 0.0004 + 0.0004 \cos(t)$ are set in order to make the conditions in Theorem 11 hold. Therefore, we take the communication intervals from 0.43 to 0.70 and communication loss intervals from 0 to 0.1 randomly. Then, the leaders reach the desired formation asymptotically, and the followers converge to the convex hull generated by the leaders (see Figure 7). Moreover, the trajectories of positions and velocities are shown in Figures 8 and 9, and the variation of control inputs is demonstrated in Figure 10.

FIGURE 3: The positions of eight agents.

FIGURE 4: The moving trajectories of eight agents.

5. Conclusion

In this paper, we propose the formation-containment algorithms for second-order multiagent systems with intermittent communication. By variable transformation, the formation-containment problems are equivalent to the asymptotic stability problems. With the help of Lyapunov function method, sufficient conditions, which depend on the communication intervals and communication loss intervals, are obtained for the leaders' formation convergence and following agents' containment convergence, respectively. Besides, the delay-dependent sufficient conditions are also obtained for second-order multiagent systems converging to the

FIGURE 5: The velocity states of eight agents.

FIGURE 6: The inputs of eight agents.

desired formation-containment under time-varying communication delay. However, this paper does not take into account the collision problem, which is nonnegligible in practical engineering. Thus, our further investigation will focus on the collision-avoidance strategy, which can be combined with the formation-containment control algorithm in this paper.

Data Availability

The simulation data generated during this study have been deposited with figshare (<https://figshare.com/s/b06cd2ff526aac7a1f69>). All other data arising from this study are contained within the manuscript.

FIGURE 7: The positions of eight agents.

FIGURE 8: The moving trajectories of eight agents.

b06cd2ff526aac7a1f69). All other data arising from this study are contained within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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FIGURE 9: The positions of eight agents.

FIGURE 10: The inputs of eight agents.

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