Research Article

Permanence and Almost Periodic Solutions for N -Species Nonautonomous Lotka-Volterra Competitive Systems with Delays and Impulsive Perturbations on Time Scales

Xuxu Yu,¹ Qiru Wang¹ and Yuzhen Bai²

¹School of Mathematics, Sun Yat-sen University, Guangzhou 510275, China

²School of Mathematical Sciences, Qufu Normal University, Qufu 273165, China

Correspondence should be addressed to Yuzhen Bai; baiyu99@126.com

Received 6 February 2018; Accepted 28 June 2018; Published 9 October 2018

Academic Editor: Roberto Natella

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We investigate a class of nonautonomous N -species Lotka-Volterra-type competitive systems with time delays and impulsive perturbations on time scales. By using comparison theorems of impulsive dynamic equations on time scales, we obtain sufficient conditions to guarantee the permanence of the system. Then based on the Massera-type theorem for impulsive dynamic equations on time scales, we establish existence and uniformly asymptotic stability of the unique positive almost periodic solution of the system. Finally, an example is employed to illustrate our main results.

1. Introduction

The well-known Lotka-Volterra models concerning ecological population, epidemiology, economics modeling, or even neural networks have been extensively investigated in the literature. Many researchers have studied the dynamical characteristics such as stability, persistence, periodicity, or almost periodicity of various Lotka-Volterra systems (see [1–6]). In [6], the authors studied the existence of globally asymptotically stable periodic, quasiperiodic, or almost periodic solutions of the nonautonomous Lotka-Volterra system.

Impulsive differential equations have become important in recent years in mathematical models of real processes, and they rise in phenomena studied in physics, chemical technology, population dynamics, biotechnology, and economics. There have been significant developments in impulse theory in recent years (see [7–9]), and the fundamental theory of impulsive differential equations can be seen in the monographs [10–12].

In [7], the authors considered the following N -species nonautonomous competitive system with impulsive perturbations and time delays:

$$\begin{aligned} \dot{x}_i(t) &= x_i(t) \left[b_i(t) - a_i(t)x_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_{ij}(t)) - \sum_{j=1}^n \int_{-\sigma_{ij}}^0 c_{ij}(t, s)x_j(t+s)ds \right], \quad t \neq t_k, k \in \mathbb{N}, i = 1, 2, \dots, n, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad t = t_k, \end{aligned} \quad (1)$$

where $t \in \mathbb{R}_+ := [0, \infty)$, $x(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \mathbb{R}_+^n := \{x = (x_1, x_2, \dots, x_n) : x_i \geq 0, i = 1, 2, \dots, n\}$ represents the density vector of N -species at time t and h_{ik} for each $i = 1, 2, \dots, n$ and $k \in \mathbb{N}$ are positive constants.

The theory of calculus on time scales was initiated by Hilger [13] in 1988 to unify continuous and discrete analysis (see [14, 15] and references cited therein). Dynamic equations on time scales include differential and difference equations as special cases, and their qualitative analysis is of particular importance (see [16–22]). In [23, 24], the authors considered the existence and stability of periodic solutions

$$\begin{aligned} x_i^\Delta(t) &= x_i(t) \left[b_i(t) - d_i(t)x_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)x_j(t+s)\Delta s \right], \quad t \neq t_k, t \in [t_0, +\infty)_{\mathbb{T}}, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad t = t_k, k \in \mathbb{N}, i = 1, 2, \dots, n, \end{aligned} \quad (2)$$

where $\mathbb{T}$ is an almost periodic time scale, $t_0 \in \mathbb{T}$, and $t_0 > 0$; $x(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \mathbb{R}_+^n$, σ_j is positive constant, and time delays $\tau_j(t)$ ($j = 1, 2, \dots, n$) are bounded functions; $x_i(t_k^+) = \lim_{t \rightarrow t_k^+} x_i(t)$ represents the population density x_i after the impulse point t_k , and h_{ik} is the effect of the k th impulse on species x_i ; the other notations can be seen in Section 2.

The rest of this paper is organized as follows. In Section 2, some useful definitions and lemmas are presented. In Sections 3, we discuss the permanence of (2). In Section 4, we establish existence and uniformly asymptotic stability of the almost periodic solution of (2). In Section 5, a numerical example is given to illustrate the feasibility of our results. Finally, we provide a discussion section.

2. Preliminaries and Lemmas

In this section, we introduce some basic definitions and lemmas which are used in this paper.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of $\mathbb{R}$. For $t \in \mathbb{T}$, the forward and backward jump operators $\sigma, \rho : \mathbb{T} \rightarrow \mathbb{T}$ and the forward graininess $\mu : \mathbb{T} \rightarrow \mathbb{R}_+$ are defined, respectively, by

$$\begin{aligned} \sigma(t) &:= \inf \{s \in \mathbb{T} : s > t\}, \\ \rho(t) &:= \sup \{s \in \mathbb{T} : s < t\}, \\ \mu(t) &= \sigma(t) - t. \end{aligned} \quad (3)$$

A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m , then $\mathbb{T}^k := \mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}^k := \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m , then $\mathbb{T}_k := \mathbb{T} \setminus \{m\}$; otherwise, $\mathbb{T}_k := \mathbb{T}$.

for Lotka-Volterra systems. In [25], the authors investigated the permanence and almost periodic solutions for a single-species system with impulsive effects on time scales.

To the best of our knowledge, there is little work considering the permanence and stability of almost periodic solutions for the N -species nonautonomous Lotka-Volterra competitive system with delays and impulsive perturbations on time scales. Motivated by the above works, in this paper, we consider the following N -species nonautonomous Lotka-Volterra competitive system with delays and impulsive perturbations on time scales:

A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is called right-dense continuous or rd-continuous provided that it is continuous at right-dense points in $\mathbb{T}$ and its left-side limits exist (finite) at left-dense points in $\mathbb{T}$. A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is called continuous if and only if it is both left-dense continuous and right-dense continuous.

For a function $f : \mathbb{T} \rightarrow \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of f at t , denoted $f^\Delta(t)$, to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \leq \varepsilon |\sigma(t) - s|, \quad \text{for all } s \in U. \quad (4)$$

If f is rd-continuous, then there is a function F such that $F^\Delta(t) = f(t)$. In this case, we define

$$\int_a^b f(t)\Delta t = F(b) - F(a). \quad (5)$$

The function p is regressive if $1 + \mu(t)p(t) \neq 0$, for all $t \in \mathbb{T}^k$. Define the regressive class of functions on $\mathbb{T}^k$ to be $\mathcal{R} = \{p : \mathbb{T} \rightarrow \mathbb{R} \mid p \text{ is rd-continuous and regressive}\}$. We define the set $\mathcal{R}^+$ of all positively regressive elements by $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T}, \mathbb{R}) = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0 \text{ for all } t \in \mathbb{T}\}$.

If $p \in \mathcal{R}$, then we define the generalized exponential function by

$$e_p(t, s) = \exp \left\{ \int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau \right\}, \quad (6)$$

for $s, t \in \mathbb{T}$, where the cylinder transformation

$$\xi_h(z) = \begin{cases} \frac{\log(1 + hz)}{h}, & \text{if } h \neq 0, \\ z, & \text{if } h = 0. \end{cases} \quad (7)$$

Definition 1 (see [14, 15]). If $p, q \in \mathcal{R}$, then we define a circle plus addition by

$$(p \oplus q)(t) := p(t) + q(t) + p(t)q(t)\mu(t), \quad (8)$$

for all $t \in \mathbb{T}^k$. For $p \in \mathcal{R}$, define a circle minus p by

$$\ominus p := -\frac{p}{(1 + \mu p)}. \quad (9)$$

Lemma 1 (see [14, 15]). Let $p, q \in \mathcal{R}$ and $s, t, r \in \mathbb{T}$. Then

- (i) $e_0(t, s) \equiv 1$ and $e_p(t, t) \equiv 1$;
- (ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;
- (iii) $e_p(t, s) = 1/e_p(s, t) = e_p(s, t)$;
- (iv) $e_p(t, r)e_p(r, s) = e_p(t, s)$;
- (v) $(e_p(t, s))^\Delta = p(t)e_p(t, s)$;
- (vi) if $a, b, c \in \mathbb{T}$, then

$$\int_a^b e_p(c, \sigma(t))p(t)\Delta t = e_p(c, a) - e_p(c, b). \quad (10)$$

Lemma 2 (see [14, 15]). Let f, g be delta differentiable functions on $\mathbb{T}$. Then

- (i) $(\nu_1 f + \nu_2 g)^\Delta = \nu_1 f^\Delta + \nu_2 g^\Delta$, for any constants ν_1, ν_2 ;
- (ii) $(fg)^\Delta(t) = f^\Delta(t)g(t) + f(\sigma(t))g^\Delta(t) = f(t)g^\Delta(t) + f^\Delta(t)g(\sigma(t))$;
- (iii) if f and f^Δ are continuous, then $(\int_a^t f(t, s)\Delta s)^\Delta = f(\sigma(t), t) + \int_a^t f^\Delta(t, s)\Delta s$;
- (iv) if $f(t)f(\sigma(t)) \neq 0$, then $1/f$ is differentiable at t with

$$\left(\frac{1}{f}\right)^\Delta(t) = -\frac{f^\Delta(t)}{f(t)f(\sigma(t))}. \quad (11)$$

Lemma 3 (see [14, 15]). Assume $p \in \mathcal{R}$ and $t_0 \in \mathbb{T}$. If $1 + \mu(t)p(t) > 0$ for $t \in \mathbb{T}$, then $e_p(t, t_0) > 0$, for all $t \in \mathbb{T}$.

Throughout this paper, we use $\mathbb{E}^n$ to denote $\mathbb{R}^n$ or $\mathbb{C}^n$.

Definition 2 (see [26]). A time scale $\mathbb{T}$ is called an almost-periodic time scale if

$$\Pi := \{\tau \in \mathbb{R} : t \pm \tau \in \mathbb{T}, \quad \forall t \in \mathbb{T}\} \neq \{0\}. \quad (12)$$

Definition 3 (see [26]). Let $\mathbb{T}$ be an almost-periodic time scale. A function $f \in C(\mathbb{T}, \mathbb{E}^n)$ is called an almost periodic function if the ε -translation set of f ,

$$E\{\varepsilon, f\} = \{\tau \in \Pi : |f(t + \tau) - f(t)| < \varepsilon, \quad \forall t \in \mathbb{T}\} \quad (13)$$

is a relatively dense set in $\mathbb{T}$, for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists a constant $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains a $\tau(\varepsilon) \in E\{\varepsilon, f\}$ such that

$$|f(t + \tau) - f(t)| < \varepsilon, \quad \forall t \in \mathbb{T}. \quad (14)$$

τ is called the ε -translation number of f and $l(\varepsilon)$ is called the inclusion length of $E\{\varepsilon, f\}$.

Definition 4 (see [26]). Let $\mathbb{T}$ be an almost periodic time scale. A function $f(t, x) \in C(\mathbb{T} \times D, \mathbb{E}^n)$ is called an almost periodic function in t uniformly for $x \in D$, if the ε -translation set of f ,

$$E\{\varepsilon, f, S\} = \{\tau \in \Pi : |f(t + \tau, x) - f(t, x)| < \varepsilon, \quad \forall (t, x) \in \mathbb{T} \times S\} \quad (15)$$

is a relatively dense set in $\mathbb{T}$, for all $\varepsilon > 0$ and for each compact subset S of D ; that is, for any given $\varepsilon > 0$ and each compact set S in D , there exists a constant $l(\varepsilon, S) > 0$ such that each interval of length $l(\varepsilon, S)$ contains a $\tau(\varepsilon, S) \in E\{\varepsilon, f, S\}$ such that

$$|f(t + \tau, x) - f(t, x)| < \varepsilon, \quad \forall (t, x) \in \mathbb{T} \times S. \quad (16)$$

τ is called the ε -translation number of f and $l(\varepsilon, S)$ is called the inclusion length of $E\{\varepsilon, f, S\}$.

Lemma 4 (see [26]). (i) Let $f(t), g(t) \in C(\mathbb{T}, \mathbb{E}^n)$ be almost periodic functions, then $f(t) \pm g(t), f(t) \cdot g(t)$ are also almost periodic functions. If $\inf_{t \in \mathbb{T}} |g(t)| > 0$, then the quotient $f(t)/g(t)$ is almost periodic too.

(ii) Let $f(t, x), g(t, x) \in C(\mathbb{T} \times D, \mathbb{E}^n)$ be almost periodic in t uniformly for $x \in D$, then $f(t, x) \pm g(t, x), f(t, x) \cdot g(t, x)$ are also uniformly almost periodic functions. If $\inf_{(t, x) \in \mathbb{T} \times S} |g(t, x)| > 0$, where $S \subset D$ be any compact set, then the quotient $f(t, x)/g(t, x)$ is uniformly almost periodic too.

Definition 5 Species $x_i (i = 1, 2, \dots, n)$ of (2) is said to be permanent if there are positive constants m and M such that

$$m \leq \liminf_{t \rightarrow \infty} x_i(t) \leq \limsup_{t \rightarrow \infty} x_i(t) \leq M, \quad (17)$$

for any positive solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ of system (2).

Lemma 5 (see [9]). Assume that $x \in PC^1[\mathbb{T}, \mathbb{R}]$ and

$$\begin{aligned} x^\Delta(t) &\leq (\geq) p(t)x(t) + q(t), \quad t \neq t_k, t \in [t_0, +\infty)_{\mathbb{T}}, \\ x(t_k^+) &\leq (\geq) d_k x(t_k) + b_k, \quad t = t_k, k \in \mathbb{N}, \end{aligned} \quad (18)$$

then for $t \geq t_0 \geq 0$,

$$\begin{aligned} x(t) &\leq (\geq) x(t_0) \prod_{t_0 < t_k < t} d_k e_p(t, t_0) + \sum_{t_0 < t_k < t} \left(\prod_{t_0 < t_j < t} d_j e_p(t, t_k) \right) b_k \\ &\quad + \int_{t_0}^t \prod_{s < t_k < t} d_k e_p(t, \sigma(s)) q(s) \Delta s, \end{aligned} \quad (19)$$

where $PC^1 = \{y : [0, \infty)_{\mathbb{T}} \rightarrow \mathbb{R} \text{ which is rd-continuous except at } t_k, k = 1, 2, \dots, \text{ for which } y(t_k^-), y(t_k^+), y^\Delta(t_k^-), y^\Delta(t_k^+) \text{ exist with } y(t_k^-) = y(t_k), y^\Delta(t_k^-) = y^\Delta(t_k)\}$.

Throughout this paper, we let $\mathbb{T}$ be an almost periodic time scale. Denote $BC(\mathbb{R}, \mathbb{R}^n)$ the set of all bounded continuous functions from $\mathbb{R}$ to $\mathbb{R}^n$, with norm

$$\|f\| = \max_{1 \leq i \leq n} \sup_{t \in \mathbb{T}} |f_i(t)|. \quad (20)$$

We can see that $(BC(\mathbb{R}, \mathbb{R}^n), \|\cdot\|)$ is a Banach space. Consider the following equation:

$$\begin{aligned} x^\Delta(t) &= f(t, x), \quad t \neq t_k, t \in \mathbb{T}^+, \\ \Delta x(t_k) &= I_k(x(t_k)), \quad t = t_k, k \in \mathbb{N}, \end{aligned} \quad (21)$$

where $f : \mathbb{T}^+ \times S_B \rightarrow \mathbb{R}$, $S_B = \{x \in \mathbb{R} : \|x\| < B\}$, $\|x\| = \sup_{t \in \mathbb{T}} |x(t)|$; the functions $I_k \in C[\mathbb{R}, \mathbb{R}]$, $k \in \mathbb{N}$ are almost periodic uniformly with respects to $x \in S_B$ and are Lipschitz continuous in x , $f(t, x)$ is almost periodic in t uniformly for $x \in S_B$ and is continuous in x . The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $j \in \mathbb{N}$ is uniformly almost periodic and $\inf_k t_k^1 = \theta > 0$. The product systems of (21) are as follows:

$$\begin{aligned} x^\Delta(t) &= f(t, x), \quad t \neq t_k, t \in \mathbb{T}^+, \\ \Delta x(t_k) &= I_k(x(t_k)), \quad t = t_k, k \in \mathbb{N}, \\ y^\Delta(t) &= f(t, y), \quad t \neq t_k, t \in \mathbb{T}^+, \\ \Delta y(t_k) &= I_k(y(t_k)), \quad t = t_k, k \in \mathbb{N}. \end{aligned} \quad (22)$$

Define $V_1 = \{V : \mathbb{T}^+ \times S_B \times S_B \rightarrow \mathbb{R}^+, V \text{ is rd-continuous in } (t_{k-1}, t_k]_{\mathbb{T}^+} \times S_B \times S_B \text{ and } \lim_{(t,x,y) \rightarrow (t_k, x_0, y_0), t > t_k} V(t, x, y) = V(t_k^+, x_0, y_0)\}$.

Lemma 6 (see [25]). Suppose that there exists a Lyapunov functional $V(t, x, y) \in V_1$ satisfying the following conditions:

Condition 1. $a(\|x - y\|) \leq V(t, x, y) \leq b(\|x - y\|)$, where $(t, x, y) \in \mathbb{T}^+ \times S_B \times S_B$, $a, b \in \mathbb{R}^+$, $\kappa = \{a \in C(\mathbb{R}^+, \mathbb{R}^+) : a(0) = 0 \text{ and } a \text{ is increasing}\}$.

Condition 2. $|V(t, x, y) - V(t, x_1, y_1)| \leq L(\|x - x_1\| + \|y - y_1\|)$, where $(t, x, y) \in \mathbb{T}^+ \times S_B \times S_B$, $L > 0$ is a constant.

Condition 3. $V(t_k^+, x + I_k(x), y + I_k(y)) \leq V(t, x, y)$, $x, y \in S_B$, $t = t_k$, $k \in \mathbb{N}$.

Condition 4. $D^+ V_{(2,1)}^\Delta(t, x, y) \leq -cV(t, x, y)$, where $c > 0$, $-c \in \mathcal{R}^+$, $x, y \in S_B$, $t \neq t_k$, $k \in \mathbb{N}$.

Moreover, if there exists a solution $x(t) \in S$ of (21) for $t \in \mathbb{T}^+$, where $S \subset S_B$ is a compact set, then there exists a unique almost periodic solution $p(t) \in S$ of (21), which is uniformly asymptotically stable. In particular, if $f(t, x)$ is ω -periodic in t uniformly for $x \in S_B$ and there exists a positive integer q such that $t_{k+q} = t_k + \omega$, $I_{k+q}(x) = I_k(x)$ with $t_k \in \mathbb{T}^+$, then $p(t)$ is also periodic.

Let ϑ_1, ϑ_2 be the two positive constants and $\{t_k\}$ be a time sequence satisfying $-\vartheta_1 < t_1 < t_2 < \dots < t_n < \dots$ and $t_k \rightarrow +\infty$ as $k \rightarrow +\infty$; $\vartheta_2 = \max_{1 \leq j \leq n} \{\sup_{t \in [t_0, \infty)_{\mathbb{T}}} |\tau_j(t)|\}$ denote $\vartheta = \max \{\vartheta_1, \vartheta_2\}$. Let $\phi(s) : [t_0 - \vartheta, t_0]_{\mathbb{T}} \rightarrow \mathbb{R}_+^n$ be a piecewise continuous function satisfying that $\phi(s)$ is continuous for all $s \in [t_0 - \vartheta, t_0]_{\mathbb{T}}$ and $s \neq t_k$, for any $s = t_k \in [t_0 - \vartheta, t_0]_{\mathbb{T}}$, $\phi(t_k^-) = \lim_{s \rightarrow t_k^-} \phi(s) = \phi(t_k)$ and $\phi(t_k^+) = \lim_{s \rightarrow t_k^+} \phi(s)$ exist.

For system (2), we introduce the following assumptions:

(H₁) $b_i(t), d_i(t), a_{ij}(t), \tau_j(t)$ are almost periodic functions with positive constants $\underline{b}_i, \bar{b}_i, \underline{d}_i, \bar{d}_i$, and $\bar{a}_{ij}$ such that $\underline{b}_i \leq b_i(t) \leq \bar{b}_i$, $\underline{d}_i \leq d_i(t) \leq \bar{d}_i$, and $a_{ij}(t) \leq \bar{a}_{ij}$, for all $t \in [t_0, +\infty)_{\mathbb{T}}$, $i, j = 1, 2, \dots, n$.

(H₂) $\{h_{ik}\}$ is an almost periodic sequence, and there exist positive constants h and H such that $1 \leq h \leq \prod_{s < t_k < t} h_{ik} \leq H$ for all $s < t \in [t_0, +\infty)_{\mathbb{T}}$, $k \in \mathbb{N}$.

(H₃) The kernel $c_{ij}(t, s) (\geq 0) : [t_0, +\infty)_{\mathbb{T}} \times [-\sigma_j, 0]_{\mathbb{T}}$ is uniformly almost periodic in t for $s \in [-\sigma_j, 0]_{\mathbb{T}}$ and Δ -integrable on $[-\sigma_j, 0]_{\mathbb{T}}$ with $\int_{-\sigma_j}^0 c_{ij}(t, s) \Delta s \leq c_{ij}$, for all $t \in [t_0, +\infty)_{\mathbb{T}}$; here, c_{ij} is positive constant.

(H₄) The set of sequence $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k, j \in \mathbb{N}$, is uniformly almost periodic and $\inf_k t_k^1 = \theta > 0$.

3. Permanence

In this section, we will discuss the permanence of system (2). First, we present the existence of solutions of system (2).

Lemma 7. Assume that (H₁) and (H₂) hold, then system (2) has a solution for any $x(t_0) = (x_1(t_0), \dots, x_n(t_0)) \in \mathbb{R}_+^n$.

Proof. For any $x(t_0) = (x_1(t_0), \dots, x_n(t_0)) \in \mathbb{R}_+^n$, let

$$\begin{aligned} \varphi_i^0(t) &= x_i(t_0) \prod_{t_0 < t_k < t} h_{ik}, \\ \varphi_i^m(t) &= x_i(t_0) \prod_{t_0 < t_k < t} h_{ik} + \int_{t_0}^t \prod_{s < t_k < t} h_{ik} F_i(s, \varphi^{m-1}) \Delta s, \end{aligned} \quad (23)$$

$$m = 1, 2, \dots,$$

where $F(s, \varphi^{m-1}) = (F_1(s, \varphi^{m-1}), \dots, F_n(s, \varphi^{m-1}))$ is defined as

$$\begin{aligned} F_i(t, \varphi^{m-1}) &= \varphi_i^{m-1}(t) \left[b_i(t) - d_i(t) \varphi_i^{m-1}(t) \right. \\ &\quad - \sum_{j=1}^n a_{ij}(t) \varphi_j^{m-1}(t - \tau_j(t)) \\ &\quad \left. - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s) \varphi_j^{m-1}(t + s) \Delta s \right], \\ &\quad i = 1, 2, \dots, n. \end{aligned} \quad (24)$$

Clearly, $F(t, \varphi^{m-1})$ satisfies local Lipschitz conditions for variable φ^{m-1} . By calculation, we can get that

$$|F_i(t, \varphi^{m-1})| \leq \|\varphi^{m-1}\| \bar{b}_i. \quad (25)$$

By Lemma 5, we have that $\varphi^m(t) = (\varphi_1^m(t), \dots, \varphi_n^m(t))$ satisfies the following system.

$$\begin{aligned} x_i^\Delta(t) &= F_i(t, \varphi^{m-1}), \quad t \neq t_k, t \in [t_0, +\infty)_{\mathbb{T}}, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad t = t_k, i = 1, 2, \dots, n, k \in \mathbb{N}. \end{aligned} \quad (26)$$

Denote $K = I \times G$ be any compact subset of $[t_0, +\infty)_{\mathbb{T}} \times \mathbb{R}^n$, then we can choose positive constants ω, L_K , and M^G such that

$$\begin{aligned} |t - t_0| &\leq \omega, \\ \|F(t, \varphi) - F(t, \psi)\| &\leq L_K \|\varphi - \psi\|, \\ \|\varphi\| &\leq M^G, \end{aligned} \quad (27)$$

for all $(t, \varphi), (t, \psi) \in K$. Next, we will show the uniformly convergence of the sequence $\{\varphi^m(t)\}$.

Note that

$$\begin{aligned} |\varphi_i^1(t) - \varphi_i^0(t)| &= \left| \int_{t_0}^t \prod_{s < t_k < t} h_{ik} F_i(s, \varphi^0) \Delta s \right| \\ &\leq H \int_{t_0}^t \Delta s \leq HM^G \bar{b}_i |t - t_0| \leq HM^G \bar{b}_i \omega. \end{aligned} \quad (28)$$

Thus, $|\varphi_i^1(t) - \varphi_i^0(t)| \leq HM^G \bar{b} \omega (i = 1, 2, \dots, n)$, where $\bar{b} = \max_{1 \leq i \leq n} \bar{b}_i$.

Assume that when $m = n$, $|\varphi_i^n(t) - \varphi_i^{n-1}(t)| \leq H^n M^G \bar{b} L_K^{n-1} (|t - t_0|^n / n!) \leq H^n M^G \bar{b} L_K^{n-1} (\omega^n / n!)$ holds, then

$$\begin{aligned} |\varphi_i^{n+1}(t) - \varphi_i^n(t)| &= \left| \int_{t_0}^t \prod_{s < t_k < t} h_{ik} (F_i(s, \varphi^n) - F_i(s, \varphi^{n-1})) \Delta s \right| \\ &\leq H \int_{t_0}^t |F_i(s, \varphi^n) - F_i(s, \varphi^{n-1})| \Delta s \leq H \int_{t_0}^t L_K \|\varphi^n - \varphi^{n-1}\| \Delta s \\ &\leq H \int_{t_0}^t L_K H^n M^G \bar{b} L_K^{n-1} \frac{|s - t_0|^n}{n!} \Delta s \leq H^{n+1} M^G \bar{b} L_K^n \frac{|t - t_0|^{n+1}}{(n+1)!} \\ &\leq H^{n+1} M^G \bar{b} L_K^n \frac{\omega^{n+1}}{(n+1)!}. \end{aligned} \quad (29)$$

By mathematical induction method, we have that $\|\varphi^n(t) - \varphi^{n-1}(t)\| \leq H^n M^G \bar{b} L_K^{n-1} \omega^n / n!, n \in \mathbb{N}$.

Note that $\varphi^m(t) = \varphi^0(t) + \sum_{n=1}^m (\varphi^n(t) - \varphi^{n-1}(t))$. According to Weierstrass's judgment method, the sequence $\{\varphi^m(t)\}$ is uniformly convergent on K . Since $F(t, \varphi^{m-1})$ is Lipschitz continuous in φ^{m-1} , we have that the sequence $\{F(t, \varphi^m)\}$ is also uniformly convergent.

Let

$$\begin{aligned} \varphi(t) &:= \lim_{m \rightarrow +\infty} \varphi^m(t), \\ \Phi(t) &:= \lim_{m \rightarrow +\infty} F(t, \varphi^{m-1}) = \lim_{m \rightarrow +\infty} (\varphi^m(t))^\Delta, \end{aligned} \quad (30)$$

for all $t \neq t_k$.

If $\sigma(t) = t$, then

$$\begin{aligned} (\varphi_i(t))^\Delta &= (\varphi_i(t))' \\ &= \lim_{s \rightarrow t} \frac{\varphi_i(s) - \varphi_i(t)}{s - t} \\ &= \lim_{s \rightarrow t} \lim_{m \rightarrow +\infty} \frac{(\varphi_i(s) - \varphi_i^m(s)) + (\varphi_i^m(s) - \varphi_i^m(t)) + (\varphi_i^m(t) - \varphi_i(t))}{s - t} \\ &= \lim_{s \rightarrow t} \Phi_i(t + \theta(s - t)) \\ &= \Phi_i(t). \end{aligned} \quad (31)$$

Here, $\theta \in [0, 1]$ is a constant.

If $\sigma(t) > t$, then

$$\begin{aligned} (\varphi_i(t))^\Delta &= \frac{\varphi_i(\sigma(t)) - \varphi_i(t)}{\sigma(t) - t} \\ &= \lim_{m \rightarrow +\infty} \frac{(\varphi_i(\sigma(t)) - \varphi_i^m(\sigma(t))) + (\varphi_i^m(\sigma(t)) - \varphi_i^m(t)) + (\varphi_i^m(t) - \varphi_i(t))}{\sigma(t) - t} \\ &= \lim_{m \rightarrow +\infty} (\varphi_i^m(t))^\Delta \\ &= \Phi_i(t). \end{aligned} \quad (32)$$

From above, we have $\varphi(t)^\Delta = \lim_{m \rightarrow +\infty} (\varphi^m(t))^\Delta$. In view of (26) and the arbitrary K , we can see that $\varphi(t)$ is a solution of system (2). $\square$

Lemma 8. Assume that (H_1) and (H_2) hold. Let $x(t) = (x_1(t), \dots, x_n(t))$ be any given solution of system (2), then there exist positive constants x_i^* for $i = 1, 2, \dots, n$ such that $\limsup_{t \rightarrow +\infty} x_i(t) \leq x_i^*, i = 1, 2, \dots, n$.

Proof. From (H_1) , we can choose constants k_i for $i = 1, 2, \dots, n$, such that $\bar{b}_i - \underline{d}_i k_i < 0$ and $\bar{b}_i - \underline{d}_i k_i \in \mathcal{R}^+$. For any positive solution $x(t) = (x_1(t), \dots, x_n(t))$ of system (2) and each index $i \in \{1, 2, \dots, n\}$, we only need to consider the following three cases:

Case I. There is a $t_i' \geq t_0 (\in \mathbb{T})$ such that $x_i(t) > k_i$, for all $t \in [t_i', +\infty)_{\mathbb{T}}$.

Case II. There is a $t'_i \geq t_0$ ($\in \mathbb{T}$) such that $x_i(t) \leq k_i$, for all $t \in [t'_i, +\infty)_{\mathbb{T}}$.

Case III. $x_i(t)$ is oscillatory about k_i , for all $t \in [t_0, +\infty)_{\mathbb{T}}$.

We first consider Case I. In view of system (2), we have that

$$\begin{aligned} x_i^\Delta(t) &\leq x_i(t)[b_i(t) - d_i(t)k_i] \\ &\leq x_i(t)[\bar{b}_i - \underline{d}_i k_i], \quad t \neq t_k, t \in [t'_i, +\infty)_{\mathbb{T}}, \end{aligned} \quad (33)$$

$$x_i(t_k^+) = h_{ik}x_i(t_k), \quad t = t_k, k \in \mathbb{N}.$$

By Lemma 5 and (H₂), for all $t \in [t'_i, +\infty)_{\mathbb{T}}$, we obtain

$$x_i(t) \leq x_i \prod_{t'_i < t_k < t} h_{ik} e^{(\bar{b}_i - \underline{d}_i k_i)}(t, t'_i) \leq x_i H e^{(\bar{b}_i - \underline{d}_i k_i)}(t, t'_i). \quad (34)$$

Thus, $x_i(t) \rightarrow 0$ as $t \rightarrow +\infty$, which leads to a contradiction.

Next, we consider Case III. From the oscillation of $x_i(t)$ about k_i , we can choose two sequences $\{\xi_{in}\}$ and $\{\xi_{in}^*\}$ satisfying $t_0 < \xi_{i1} < \xi_{i1}^* < \dots < \xi_{in} < \xi_{in}^* < \dots$ and $\lim_{n \rightarrow +\infty} \xi_{in} = \lim_{n \rightarrow +\infty} \xi_{in}^* = +\infty$ such that

$$\begin{aligned} x_i(\xi_{in}) &\leq k_i, \\ x_i(\xi_{in}^+) &\geq k_i, \\ x_i(\xi_{in}^*) &\geq k_i, \\ x_i(\xi_{in}^{*+}) &\leq k_i, \\ x_i(t) &\geq k_i, \quad \text{for all } t \in (\xi_{in}, \xi_{in}^*]_{\mathbb{T}} \\ x_i(t) &\leq k_i, \quad \text{for all } t \in (\xi_{in}^*, \xi_{i(n+1)})_{\mathbb{T}}. \end{aligned} \quad (35)$$

For all $t \in [t_0, +\infty)_{\mathbb{T}}$, if $t \in (\xi_{in}, \xi_{in}^*]_{\mathbb{T}}$ for some integer n , then $x_i(t) \geq k_i$ and

$$\begin{aligned} x_i^\Delta(t) &\leq x_i(t)[b_i(t) - d_i(t)k_i] \leq x_i(t)[\bar{b}_i - \underline{d}_i k_i], \quad t \neq t_k, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad t = t_k, k \in \mathbb{N}. \end{aligned} \quad (36)$$

By Lemma 5, for all $t \in (\xi_{in}, \xi_{in}^*]_{\mathbb{T}}$, we have that

$$\begin{aligned} x_i(t) &\leq x_i(\xi_{in}) \prod_{\xi_{in} < t_k < t} h_{ik} e^{(\bar{b}_i - \underline{d}_i k_i)}(t, \xi_{in}) \\ &\leq x_i(\xi_{in}) H e^{(\bar{b}_i - \underline{d}_i k_i)}(t, \xi_{in}) \leq k_i H. \end{aligned} \quad (37)$$

Since $x_i(t) \leq k_i \leq H k_i$ for all $t \in (\xi_{in}, \xi_{i(n+1)})_{\mathbb{T}}$, together with Case II, we always have $\limsup_{t \rightarrow +\infty} x_i(t) \leq H k_i = x_i^*$, $i = 1, 2, \dots, n$. $\square$

We make another assumption.

$$(H_5) \quad \underline{b}_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_i^* > 0, \quad i = 1, 2, \dots, n.$$

Lemma 9. Assume that (H₁), (H₂), and (H₅) hold. Let $x(t) = (x_1(t), \dots, x_n(t))$ be any given solution of system (2), then there exist positive constants x_{i*} , for $i = 1, 2, \dots, n$, such that $\liminf_{t \rightarrow +\infty} x_i(t) \geq x_{i*}$, $i = 1, 2, \dots, n$.

Proof. By Lemma 8, there exists $t' \in [t_0, +\infty)_{\mathbb{T}}$ such that $x_i(t) \leq x_i^*$, $\forall t \in [t', +\infty)_{\mathbb{T}}$, $i = 1, 2, \dots, n$, for any solution $x(t) = (x_1(t), \dots, x_n(t))$ of system (2). From (H₅), we can choose positive constants k'_i for $i = 1, 2, \dots, n$, such that $\underline{b}_i - \bar{d}_i k'_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_i^* > 0$. In view of system (2), we have that, if $t \neq t_k$ and $t \in [t', +\infty)_{\mathbb{T}}$,

$$\begin{aligned} x_i^\Delta(t) &= x_i(t) \left[b_i(t) - d_i(t)x_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) \right. \\ &\quad \left. - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)x_j(t+s)\Delta s \right] \\ &\geq x_i(t) \left[\underline{b}_i - \bar{d}_i x_i(t) - \sum_{j=1}^n \bar{a}_{ij} x_j^* - \sum_{j=1}^n x_j^* \int_{-\sigma_j}^0 c_{ij}(t, s)\Delta s \right] \\ &\geq x_i(t) \left[\underline{b}_i - \bar{d}_i x_i(t) - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right], \quad i = 1, \dots, n. \end{aligned} \quad (38)$$

For each $i \in \{1, 2, \dots, n\}$, similarly, we only need to consider the following three cases:

Case I. There is a $t''_i \geq t' \in \mathbb{T}$ such that $x_i(t) \geq k'_i$ for all $t \in [t''_i, +\infty)_{\mathbb{T}}$.

Case II. There is a $t'_i \geq t' \in \mathbb{T}$ such that $x_i(t) < k'_i$ for all $t \in [t'_i, +\infty)_{\mathbb{T}}$.

Case III. $x_i(t)$ is oscillatory about k'_i for all $t \in [t', +\infty)_{\mathbb{T}}$.

We first consider Case II. In view of system (2), we have that for all $t \in [t'_i, +\infty)_{\mathbb{T}}$,

$$\begin{aligned} x_i^\Delta(t) &\geq x_i(t) \left[\underline{b}_i - \bar{d}_i x_i(t) - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right] \\ &\geq x_i(t) \left[\underline{b}_i - \bar{d}_i k'_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right], \quad t \neq t_k, \end{aligned} \quad (39)$$

$$x_i(t_k^+) = h_{ik}x_i(t_k), \quad t = t_k, k \in \mathbb{N}.$$

By Lemma 5 and (H₂), for all $t \in [t'_i, +\infty)_{\mathbb{T}}$, we obtain that

$$\begin{aligned} x_i(t) &\leq x_i(t'_i) \prod_{t'_i < t_k < t} h_{ik} e^{(\bar{b}_i - \underline{d}_i k_i)}(t, t'_i) \\ &\leq x_i(t'_i) H e^{(\bar{b}_i - \underline{d}_i k_i)}(t, t'_i) \end{aligned} \quad (40)$$

Thus, $x_i(t) \rightarrow +\infty$ as $t \rightarrow +\infty$, which leads to a contradiction.

Next, we consider Case III. From the oscillation of $x_i(t)$ about k'_p we can choose two sequences $\{\zeta_{in}\}$ and $\{\zeta_{in}^*\}$ satisfying $t' < \zeta_{i1} < \zeta_{i1}^* < \dots < \zeta_{in} < \zeta_{in}^* < \dots$ and $\lim_{n \rightarrow +\infty} \zeta_{in} = \lim_{n \rightarrow +\infty} \zeta_{in}^* = +\infty$ such that

$$\begin{aligned} x_i(\zeta_{in}) &\geq k'_p \\ x_i(\zeta_{in}^+) &\leq k'_p \\ x_i(\zeta_{in}^*) &\leq k'_p \\ x_i(\zeta_{in}^{*+}) &\geq k'_p \\ x_i(t) &\leq k'_p \quad \text{for all } t \in (\zeta_{in}, \zeta_{in}^*]_{\mathbb{T}}, \\ x_i(t) &\geq k'_p \quad \text{for all } t \in (\zeta_{in}^*, \zeta_{i(n+1)})_{\mathbb{T}}. \end{aligned} \quad (41)$$

For all $t \in [t', +\infty)_{\mathbb{T}}$, if $t \in (\zeta_{in}, \zeta_{in}^*]_{\mathbb{T}}$ for some integer n , $x_i(t) \leq k'_p$ then we have

$$x_i^\Delta(t) \geq x_i(t) \left[\underline{b}_i - \bar{d}_i k'_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right], \quad t \neq t_k, \quad (42)$$

$$x_i(t_k^+) = h_{ik} x_i(t_k), \quad t = t_k, k \in \mathbb{N}.$$

By Lemma 5, for all $t \in (\zeta_{in}, \zeta_{in}^*]_{\mathbb{T}}$, we have

$$\begin{aligned} x_i(t) &\geq x_i(\zeta_{in}) \prod_{\zeta_{in} < t_k < t} h_{ik} e \left(\underline{b}_i - \bar{d}_i k'_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right) (t, \zeta_{in}) \\ &\geq h x_i(\zeta_{in}) e \left(\underline{b}_i - \bar{d}_i k'_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right) (t, \zeta_{in}), \end{aligned} \quad (43)$$

from which we can see that $x_i(t) \rightarrow +\infty$ as $t \rightarrow +\infty$, which is a contradiction.

From the above, we see that only Case I holds and then we always have $\liminf_{t \rightarrow +\infty} x_i(t) \geq k'_i = x_{i*}$, $i = 1, 2, \dots, n$. $\square$

Remark 1. Through the proofs of Lemmas 8 and 9, we can see that the constants $x_i^* = H k_i$ and $x_{i*} = k'_i$ ($i = 1, 2, \dots, n$) are independent of the solutions of system (2).

From Lemmas 7, 8, and 9, we establish the permanence of system (2).

Theorem 1. Assume that (H_1) , (H_2) , and (H_5) hold, then system (2) is permanent.

Remark 2. In Lemma 7, we have proved the existence of solutions of system (2). However, in most of the previous similar results, the authors did not.

We denote by Λ the set of all solutions $x(t) = (x_1(t), \dots, x_n(t))$ of system (2) satisfying $x_{i*} \leq x_i(t) \leq x_i^*$, $\forall t \in [t_0, +\infty)_{\mathbb{T}}$.

We can see that $(\Lambda, \|\cdot\|)$ is a Banach space (where $\|\cdot\|$ is defined in Section 2).

Theorem 2. Assume that (H_1) , (H_2) , (H_3) , (H_4) , and (H_5) hold, then the set $\Lambda \neq \emptyset$.

Proof. By Lemma 7, let $x(t) = (x_1(t), \dots, x_n(t))$ be a solution of system (2). From Theorem 1, for any $\varepsilon > 0$, there exists a $t^* \geq t_0$ such that

$$x_{i*} - \varepsilon \leq x_i(t) \leq x_i^* + \varepsilon, \quad \forall t \in [t^*, +\infty)_{\mathbb{T}}, i = 1, 2, \dots, n. \quad (44)$$

By the almost periodicity of $b_i(t)$, $d_i(t)$, $a_{ij}(t)$, $\tau_j(t)$, and $c_{ij}(t, s)$, there exists a sequence $\eta = \{\eta_p\} \in \Pi$ with $\eta_p \rightarrow +\infty$ as $p \rightarrow +\infty$, such that for $t \neq t_k$, we have

$$\begin{aligned} b_i(t + \eta_p) &\rightarrow b_i(t), \\ d_i(t + \eta_p) &\rightarrow d_i(t), \\ a_{ij}(t + \eta_p) &\rightarrow a_{ij}(t), \\ c_{ij}(t + \eta_p, s) &\rightarrow c_{ij}(t, s), \\ \tau_j(t + \eta_p) &\rightarrow \tau_j(t), \\ &\text{as } p \rightarrow +\infty, \end{aligned} \quad (45)$$

and there exists a subsequence $\{k_l\}$ of $\{p\}$ with $k_l \rightarrow +\infty$ as $l \rightarrow +\infty$, such that $t_{k_l} \rightarrow t_k$, $h_{ik_l} \rightarrow h_{ik}$, as $l \rightarrow +\infty$, $i = 1, 2, \dots, n$.

Denote $x_{ip}(t) = x_i(t + \eta_p)$, for $t \geq t^* - \eta_p$. For any positive integer q , it is easy to see that there exists a sequence $\{x_{ip}(t) : p > q\}$ such that the sequence $\{x_{ip}(t)\}$ has subsequence, denote by $\{x_{ip}(t)\}$ again, converging on any finite interval of $t \in [t_0, +\infty)_{\mathbb{T}}$, as $p \rightarrow +\infty$. Thus, there is a function $y(t)$ defined on $\mathbb{T}$ such that

$$x_{ip}(t) \rightarrow y_i(t) \quad \text{as } p \rightarrow +\infty, \forall t \in [t_0, +\infty)_{\mathbb{T}}. \quad (46)$$

Since

$$\begin{aligned} x_{ip}^\Delta(t) &= x_{ip}(t) \left[b_i(t + \eta_p) - d_i(t + \eta_p) x_{ip}(t) \right. \\ &\quad - \sum_{j=1}^n a_{ij}(t + \eta_p) x_j(t + \eta_p - \tau_j(t + \eta_p)) \\ &\quad \left. - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t + \eta_p, s) x_j(t + \eta_p + s) \Delta s \right], \\ &\quad t \neq t_k - \eta_p, k \in \mathbb{N}, \end{aligned} \quad (47)$$

$$x_{ip}(t_k^+) = h_{ik_l} x_{ip}(t_k + \eta_p), \quad t = t_k - \eta_p, i = 1, 2, \dots, n.$$

Letting $p \rightarrow +\infty$, we have

$$\begin{aligned} y_i^\Delta(t) &= y_i(t) \left[b_i(t) - d_i(t)y_i(t) - \sum_{j=1}^n a_{ij}(t)y_j(t - \tau_j(t)) \right. \\ &\quad \left. - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)y_j(t+s)\Delta s \right], \quad t \neq t_k, k \in \mathbb{N}, \\ y_i(t_k^+) &= h_{ik}y_i(t_k), \quad t = t_k, i = 1, 2, \dots, n. \end{aligned} \quad (48)$$

Easily, we can see that $y(t) = (y_1(t), \dots, y_n(t))$ is a solution of system (2) and

$$x_{i*} - \varepsilon \leq y_i(t) \leq x_i^* + \varepsilon, \quad \forall t \in [t_0, +\infty)_{\mathbb{T}}, i = 1, 2, \dots, n. \quad (49)$$

Since ε is an arbitrary small positive number, it follows that $x_{i*} \leq y_i(t) \leq x_i^*$, for all $t \in [t_0, +\infty)_{\mathbb{T}}, i = 1, 2, \dots, n$. $\square$

4. Almost Periodic Solutions

In this section, we shall establish the existence and uniformly asymptotic stability of the unique almost periodic solution to system (2).

Theorem 3. Assume that (H_1) , (H_2) , (H_3) , (H_4) , and (H_5) hold and further that

(H_6) There are positive constants α_i , for $i = 1, 2, \dots, n$, such that $A < B$ with $-A, -B \in \mathcal{R}^+$, and

$$\begin{aligned} \frac{\alpha_i}{x_i^*} a_{ii}(t) - \sum_{j \neq i} \frac{\alpha_j}{x_{j*}} a_{ji}(t) &\geq 0, \quad \text{for all } t \in [t_0, +\infty)_{\mathbb{T}}, \\ \frac{\alpha_i}{x_i^*} c_{ii}(t, s) - \sum_{j \neq i} \frac{\alpha_j}{x_{j*}} c_{ji}(t, s) &\geq 0, \quad \text{for all } (t, s) \in [t_0, +\infty)_{\mathbb{T}} \times [-\sigma_i, 0]_{\mathbb{T}}, \end{aligned} \quad (50)$$

where $A = \max_{1 \leq i \leq n} \{\bar{d}_i x_i^* - \underline{b}_i + \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^*\}$, $B = \min_{1 \leq i \leq n} \{\underline{d}_i x_{i*}^2 / x_i^*\}$, $x_i^* = Hk_i$, $x_{i*} = k_i^l$ ($i = 1, 2, \dots, n$) are as in Remark 1.

System (2) has a unique almost periodic solution in the set Λ , which is uniformly asymptotically stable.

Proof. From Theorem 2, we can see that system (2) has a bounded solution $x^*(t) = (x_1^*(t), \dots, x_n^*(t))$ with $x_{i*} \leq x_i^*(t) \leq x_i^*$, $\forall t \in [t_0, +\infty)_{\mathbb{T}}, i = 1, 2, \dots, n$.

Consider the product systems of (2).

$$\begin{aligned} x_i^\Delta(t) &= x_i(t) \left[b_i(t) - d_i(t)x_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)x_j(t+s)\Delta s \right], \quad t \neq t_k, t \in [t_0, +\infty)_{\mathbb{T}}, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad t = t_k, k \in \mathbb{N}, i = 1, 2, \dots, n, \\ y_i^\Delta(t) &= y_i(t) \left[b_i(t) - d_i(t)y_i(t) - \sum_{j=1}^n a_{ij}(t)y_j(t - \tau_j(t)) - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)y_j(t+s)\Delta s \right], \quad t \neq t_k, t \in [t_0, +\infty)_{\mathbb{T}}, \\ y_i(t_k^+) &= h_{ik}y_i(t_k), \quad t = t_k, k \in \mathbb{N}, i = 1, 2, \dots, n. \end{aligned} \quad (51)$$

Define Lyapunov function $V(t, x, y)$ on $\mathbb{T}^+ \times \Lambda \times \Lambda$ by

$$V(t, x, y) = \sum_{i=1}^n \alpha_i \left| \frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right|. \quad (52)$$

It is easy to see that there exist two positive constants C_1 and C_2 such that

$$C_1 \|x - y\| \leq V(t, x, y) \leq C_2 \|x - y\|. \quad (53)$$

Let $a, b \in C(\mathbb{R}_+, \mathbb{R}_+)$, $a(x) = C_1 x$, $b(x) = C_2 x$, then Condition 1 of Lemma 6 is satisfied.

Noting that

$$\begin{aligned} &|V(t, x, y) - V(t, x', y')| \\ &= \left| \sum_{i=1}^n \alpha_i \left| \frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right| - \sum_{i=1}^n \alpha_i \left| \frac{1}{x'_i(t)} - \frac{1}{y'_i(t)} \right| \right| \\ &\leq \sum_{i=1}^n \alpha_i \left| \left(\frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right) - \left(\frac{1}{x'_i(t)} - \frac{1}{y'_i(t)} \right) \right| \\ &\leq \sum_{i=1}^n \alpha_i \left(\left| \frac{1}{x_i(t)} - \frac{1}{x'_i(t)} \right| + \left| \frac{1}{y_i(t)} - \frac{1}{y'_i(t)} \right| \right) \\ &= \sum_{i=1}^n \alpha_i \left(\frac{|x_i(t) - x'_i(t)|}{x_i(t)x'_i(t)} + \frac{|y_i(t) - y'_i(t)|}{y_i(t)y'_i(t)} \right) \\ &\leq \sum_{i=1}^n \frac{\alpha_i}{x_{i*}^2} \left(|x_i(t) - x'_i(t)| + |y_i(t) - y'_i(t)| \right) \\ &\leq L (\|x - x'\| + \|y - y'\|), \end{aligned} \quad (54)$$

where $x, y, x', y' \in \Lambda$, $L = \sum_{i=1}^n (\alpha_i/x_{i*}^2)$. Then Condition 2 of Lemma 6 is satisfied.

On the other hand, for $t = t_k (k \in \mathbb{N})$,

$$V(t_k^+, x(t_k^+), y(t_k^+)) = \sum_{i=1}^n \alpha_i \left| \frac{1}{x_i(t_k^+)} - \frac{1}{y_i(t_k^+)} \right| = \sum_{i=1}^n \frac{\alpha_i}{h_{ik}} \left| \frac{1}{x_i(t_k)} - \frac{1}{y_i(t_k)} \right| \leq \sum_{i=1}^n \alpha_i \left| \frac{1}{x_i(t_k)} - \frac{1}{y_i(t_k)} \right| = V(t_k, x(t_k), y(t_k)), \quad (55)$$

which means that Condition 3 of Lemma 6 is also satisfied.

In view of system (51), we have that, for $t \neq t_k (k \in \mathbb{N})$,

$$\begin{aligned} D_{(1,2)}^+ V^\Delta(t, x, y) &= \sum_{i=1}^n \left(\alpha_i \left| \frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right| \right)^\Delta \\ &= \sum_{i=1}^n \alpha_i \operatorname{sign}(y_i(t) - x_i(t)) \left[\frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right]^\Delta \\ &= \sum_{i=1}^n \alpha_i \operatorname{sign}(y_i(t) - x_i(t)) \left\{ -\frac{x_i^\Delta(t)}{x_i(t)x_i^\sigma(t)} + \frac{y_i^\Delta(t)}{y_i(t)y_i^\sigma(t)} \right\} \\ &= \sum_{i=1}^n \alpha_i \operatorname{sign}(y_i(t) - x_i(t)) \left\{ -\frac{1}{x_i^\sigma(t)} \left[b_i(t) - d_i(t)x_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)x_j(t+s)\Delta s \right] \right. \\ &\quad \left. + \frac{1}{y_i^\sigma(t)} \left[b_i(t) - d_i(t)y_i(t) - \sum_{j=1}^n a_{ij}(t)y_j(t - \tau_j(t)) - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)y_j(t+s)\Delta s \right] \right\} \\ &= \sum_{i=1}^n \frac{\alpha_i \operatorname{sign}(y_i(t) - x_i(t))}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ b_i(t)[x_i^\sigma(t) - y_i^\sigma(t)] - d_i(t)[x_i^\sigma(t)y_i(t) - x_i(t)y_i^\sigma(t)] \right. \\ &\quad \left. - \sum_{j=1}^n a_{ij}(t)[x_i^\sigma(t)y_j(t - \tau_j(t)) - x_j(t - \tau_j(t))y_i^\sigma(t)] - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)[x_i^\sigma(t)y_j(t+s) - x_j(t+s)y_i^\sigma(t)]\Delta s \right\} \\ &= \sum_{i=1}^n \frac{\alpha_i}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ -b_i(t)|x_i^\sigma(t) - y_i^\sigma(t)| - d_i(t)[x_i^\sigma(t)|y_i(t) - x_i(t)| - x_i(t)|x_i^\sigma(t) - y_i^\sigma(t)|] \right. \\ &\quad \left. - \sum_{j=1}^n a_{ij}(t)[\operatorname{sign}(y_i(t) - x_i(t))x_i^\sigma(t)[y_j(t - \tau_j(t)) - x_j(t - \tau_j(t))] - x_j(t - \tau_j(t))|x_i^\sigma(t) - y_i^\sigma(t)|] \right. \\ &\quad \left. - \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)[\operatorname{sign}(y_i(t) - x_i(t))x_i^\sigma(t)[y_j(t+s) - x_j(t+s)] - x_j(t+s)|x_i^\sigma(t) - y_i^\sigma(t)|]\Delta s \right\} \\ &\leq \sum_{i=1}^n \frac{\alpha_i}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ -b_i(t)|x_i^\sigma(t) - y_i^\sigma(t)| - d_i(t)[x_i^\sigma(t)|y_i(t) - x_i(t)| - x_i(t)|x_i^\sigma(t) - y_i^\sigma(t)|] \right. \\ &\quad \left. - a_{ii}(t)x_i^\sigma(t)|y_i(t - \tau_i(t)) - x_i(t - \tau_i(t))| + \sum_{j \neq i} a_{ij}(t)x_i^\sigma(t)|y_j(t - \tau_j(t)) - x_j(t - \tau_j(t))| \right. \\ &\quad \left. + \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t))|y_i^\sigma(t) - x_i^\sigma(t)| - \int_{-\sigma_i}^0 c_{ii}(t, s)x_i^\sigma(t)|y_i(t+s) - x_i(t+s)|\Delta s \right. \\ &\quad \left. + \sum_{j \neq i} \int_{-\sigma_j}^0 c_{ij}(t, s)x_i^\sigma(t)|y_j(t+s) - x_j(t+s)| + \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)x_j(t+s)\Delta s|x_i^\sigma(t) - y_i^\sigma(t)|\Delta s \right\} \\ &= \sum_{i=1}^n \frac{\alpha_i}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ \left(d_i(t)x_i(t) - b_i(t) + \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) + \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t, s)x_j(t+s)\Delta s \right) \right. \end{aligned}$$

$$\begin{aligned}
& \cdot |x_i^\sigma(t) - y_i^\sigma(t)| - d_i(t)x_i^\sigma(t)|y_i(t) - x_i(t)| - x_i^\sigma(t) \left(a_{ii}(t)|y_i(t - \tau_i(t)) - x_i(t - \tau_i(t))| - \sum_{j \neq i} a_{ij}(t)|y_j(t - \tau_j(t)) \right. \\
& \left. - x_j(t - \tau_j(t))| \right) - x_i^\sigma(t) \left(\int_{-\sigma_i}^0 c_{ii}(t)|y_i(t+s) - x_i(t+s)|\Delta s - \sum_{j \neq i} \int_{-\sigma_j}^0 c_{ij}(t,s)|y_j(t+s) - x_j(t+s)|\Delta s \right) \Big\} \\
& = \sum_{i=1}^n \frac{\alpha_i}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ \left(d_i(t)x_i(t) - b_i(t) + \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) + \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)x_j(t+s)\Delta s \right) |x_i^\sigma(t) - y_i^\sigma(t)| \right. \\
& \left. - d_i(t)x_i^\sigma(t)|y_i(t) - x_i(t)| \right\} - \sum_{i=1}^n \frac{\alpha_i x_i^\sigma(t)}{x_i^\sigma(t)y_i^\sigma(t)} \left(a_{ii}(t)|y_i(t - \tau_i(t)) - x_i(t - \tau_i(t))| - \sum_{j \neq i} a_{ij}(t)|y_j(t - \tau_j(t)) \right. \\
& \left. - x_j(t - \tau_j(t))| \right) - \sum_{i=1}^n \frac{\alpha_i x_i^\sigma(t)}{x_i^\sigma(t)y_i^\sigma(t)} \times \left(\int_{-\sigma_i}^0 c_{ii}(t)|y_i(t+s) - x_i(t+s)|\Delta s - \sum_{j \neq i} \int_{-\sigma_j}^0 c_{ij}(t,s)|y_j(t+s) \right. \\
& \left. - x_j(t+s)|\Delta s \right) = \sum_{i=1}^n \frac{\alpha_i}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ \left(d_i(t)x_i(t) - b_i(t) + \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) + \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)x_j(t+s)\Delta s \right) \right. \\
& \cdot |x_i^\sigma(t) - y_i^\sigma(t)| - d_i(t)x_i^\sigma(t)|y_i(t) - x_i(t)| \Big\} - \sum_{i=1}^n \left(\frac{\alpha_i}{y_i^\sigma(t)} a_{ii}(t)|y_i(t - \tau_i(t)) - x_i(t - \tau_i(t))| \right. \\
& \left. - \sum_{j \neq i} \frac{\alpha_j}{y_j^\sigma(t)} a_{ji}(t)|y_j(t - \tau_j(t)) - x_j(t - \tau_j(t))| \right) - \sum_{i=1}^n \left(\int_{-\sigma_i}^0 \frac{\alpha_i}{y_i^\sigma(t)} c_{ii}(t,s)|y_i(t+s) - x_i(t+s)|\Delta s \right. \\
& \left. - \sum_{j \neq i} \int_{-\sigma_j}^0 \frac{\alpha_j}{y_j^\sigma(t)} c_{ji}(t,s)|y_j(t+s) - x_j(t+s)|\Delta s \right) \leq \sum_{i=1}^n \frac{\alpha_i}{x_i^\sigma(t)y_i^\sigma(t)} \left\{ \left(d_i(t)x_i(t) - b_i(t) + \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) \right. \right. \\
& \left. \left. + \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)x_j(t+s)\Delta s \right) |x_i^\sigma(t) - y_i^\sigma(t)| - d_i(t)x_i^\sigma(t)|y_i(t) - x_i(t)| \right\} \\
& = \sum_{i=1}^n \alpha_i \left(d_i(t)x_i(t) - b_i(t) + \sum_{j=1}^n a_{ij}(t)x_j(t - \tau_j(t)) + \sum_{j=1}^n \int_{-\sigma_j}^0 c_{ij}(t,s)x_j(t+s)\Delta s \right) \left| \frac{1}{x_i^\sigma(t)} - \frac{1}{y_i^\sigma(t)} \right| \\
& - \sum_{i=1}^n \frac{\alpha_i x_i(t)y_i(t)d_i(t)}{y_i^\sigma(t)} \left| \frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right| \leq \sum_{i=1}^n \alpha_i \left(\bar{d}_i x_i^* - \underline{b}_i + \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right) \left| \frac{1}{x_i^\sigma(t)} - \frac{1}{y_i^\sigma(t)} \right| \quad (56) \\
& - \sum_{i=1}^n \frac{\alpha_i d_i x_{i*}^2}{x_i^*} \left| \frac{1}{x_i(t)} - \frac{1}{y_i(t)} \right| \leq AV(\sigma(t), x, y) - BV(t, x, y) = (A - B)V(t, x, y) + A\mu(t)D_{(1,2)}^+ V^\Delta(t, x, y).
\end{aligned}$$

It follows that

$$D_{(1,2)}^+ V^\Delta(t, x, y) \leq \frac{A - B}{1 - A\bar{\mu}} V(t, x, y) = -\eta V(t, x, y), \quad (57)$$

where constant $\bar{\mu} = \sup_{t \in [t_0, +\infty)} \mu(t)$. By (H_6) , we have that $\eta = (B - A)/(1 - A\bar{\mu}) > 0$ and $-\eta \in \mathcal{R}^+$. Then Condition 4 of Lemma 6 holds. Hence, according to Lemma 6, there exists a unique uniformly asymptotically stable almost periodic solution of system (2) in the set Λ . $\square$

5. Applications

In order to validate our results, we consider a kind of three-species system with delays and impulsive perturbations on time scales as a special situation of system (2), where $\mathbb{T} = (1/5)\mathbb{Z}$.

Choose $t_0 = 0, n = 3$, we let $\alpha_j = \sigma_j = 1, h = e^{1/\pi}, H = e^2, t_k = k$,

$$\begin{aligned}
h_{ik} &= \exp \left(\sin \left(\frac{1}{k^2} \right) \right), \\
\tau_j(t) &= 2 + \sin \left(\sqrt{j}t \right), \\
b_i(t) &= 6.6 + 0.1 \sin \left(\sqrt{7}it \right), \\
d_i(t) &= 1.6 + 0.1 \cos \left(\sqrt{5}it \right), \\
a_{ii}(t) &= \frac{2}{1000} + \frac{1}{1000} \cos \left(\sqrt{6}it \right), \\
c_{ii}(t, s) &= \frac{1}{1000} \left(1.8 + 0.2 \sin \left(\sqrt{7}it \right) \right) e_1(s, 0), \\
&\text{and for all } i \neq j,
\end{aligned} \quad (58)$$

$$\begin{aligned} a_{ij}(t) &= \frac{2}{10000} + \frac{1}{10000} \sin(\sqrt{i+j}t), \\ c_{ij}(t, s) &= \frac{1}{10000} \left(1.8 + 0.2 \cos(\sqrt{i+j}t) \right) e_1(s, 0). \end{aligned} \quad (59)$$

By calculating, we have that

$$\begin{aligned} 1 &< e^{1/\pi} \leq \prod_{t_0 < t_k < t} h_{ik} \leq e^2, \\ \bar{b}_i &= 6.7, \\ \underline{b}_i &= 6.5, \\ \bar{d}_i &= 1.7, \\ \underline{d}_i &= 1.5, \\ \bar{a}_{ii} &= \frac{3}{1000}, \\ \underline{a}_{ii} &= \frac{1}{1000}, \\ c_{ii} &= \frac{2}{1000}, \quad \text{for all } i = 1, 2, 3, \end{aligned} \quad (60)$$

and for all $i \neq j$, we have

$$\begin{aligned} \bar{a}_{ij} &= \frac{3}{10000}, \\ \underline{a}_{ij} &= \frac{1}{10000}, \\ c_{ij} &= \frac{2}{10000}. \end{aligned} \quad (61)$$

Easily, we can see that (H_1) , (H_2) , (H_3) , and (H_4) hold. Choosing constants $x_i^* = 4.5$, $x_{i*} = 3.8$ ($i = 1, 2, 3$), it follows that

$$\begin{aligned} \bar{b}_i - \underline{d}_i x_i^* &= -0.05 < 0 \quad (\in \mathcal{R}^+), \\ \underline{b}_i - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* &= 6.473 > 0, \\ \underline{b}_i - \bar{d}_i x_{i*} - \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* &= 0.013 > 0, \\ \frac{\alpha_i}{x_i^*} a_{ii}(t) - \sum_{j \neq i} \frac{\alpha_j}{x_{j*}} a_{ji}(t) &> 0.0005 > 0, \\ \frac{\alpha_i}{x_i^*} c_{ii}(t, s) - \sum_{j \neq i} \frac{\alpha_j}{x_{j*}} c_{ji}(t, s) &> 0.0001 > 0, \\ A &= \max_{1 \leq i \leq 3} \left\{ \bar{d}_i x_i^* - \underline{b}_i + \sum_{j=1}^n (\bar{a}_{ij} + c_{ij}) x_j^* \right\} = 1.177, \\ B &= \min_{1 \leq i \leq 3} \left\{ \frac{\underline{d}_i x_{i*}^2}{x_i^*} \right\} = 4.813, \quad A < B. \end{aligned} \quad (62)$$

That is, (H_5) and (H_6) hold. Then, all conditions in Theorem 3 are satisfied. Thus, system (2) has a unique

positive almost periodic solution, which is uniformly asymptotically stable.

6. Discussion

In this paper, we extend the N -species nonautonomous Lotka-Volterra competitive system in [7] to systems on time scales. By using comparison theorems of impulsive dynamic equations on time scales, we obtain sufficient conditions to guarantee the permanence of the system. Then based on the Massera-type theorem for impulsive dynamic equations on time scales, we establish existence and uniformly asymptotic stability of the unique positive almost periodic solution of the system. The method of constructing Lyapunov function is skillful. Finally, a difference system with $\mathbb{T} = (1/5)\mathbb{Z}$ is given to illustrate the feasibility of our results.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest

Acknowledgments

This project is supported by the National Natural Science Foundation of China (no. 11671406). The authors would like to thank the referees and the editors for their valuable comments and suggestions.

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