Study of the Bullwhip Effect in a Multistage Supply Chain with Callback Structure considering Two Retailers

Junhai Ma, Liqing Zhu, Ye Yuan, and Shunqi Hou

College of Management and Economics, Tianjin University, Tianjin 300072, China

Correspondence should be addressed to Ye Yuan; yyuan20160531@126.com and Shunqi Hou; sqhou70228@126.com

Received 6 May 2017; Revised 4 July 2017; Accepted 8 January 2018; Published 23 April 2018

Academic Editor: Dimitri Volchenkov

Copyright © 2018 Junhai Ma et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

With the purpose of researching the bullwhip effect when there is a callback center in the supply chain system, this paper establishes a new supply chain model with callback structure, which has a material supplier, a manufacturer, and two retailers. The manufacture and retailers all employ AR(1) demand processes and use order-up-to inventory policy when they make order decisions. Moving average forecasting method is used to measure the bullwhip effect of each retailer and manufacture. We investigate the impact of lead-times of retailers and manufacture, forecasting precision, callback index, and marketing share on the bullwhip effect of both retailers and manufacture. Then we use the method of numerical simulation to indicate the different parameters in this supply chain. Furthermore, this paper puts forward some suggestions to help the enterprises to control the bullwhip effect in the supply chain with callback structure.

1. Introduction

Bullwhip effect is a major issue that threatens the stable and smooth performance of inventory, cost, and information of different layers of participants. Generated from the changes of orders, bullwhip effect manifests the information delivery of initial demand through supply chain. Due to the failure of information sharing, lead-time, batch order, and predictive information which is not consistent with actual need, upstream enterprises cannot receive correct signals of demand. Bullwhip effect greatly influences the inventory and causes the increase of inventory cost and the cost of marketing. Value is decreased in excessive inventory, which even affects the smooth performance of supply chain. Currently, the research of bullwhip effect mainly focuses on theoretical study of general model of supply chain, while some supply chain models with practical significance have not been explored yet. This paper builds a multilevel supply chain model with callback center and quantifies the model according to the influence of callback mechanism in actual supply chain system, which innovatively introduces callback system to the supply chain model. The callback center recalls, classifies, and sends rejected products to retailers and producers. Those rejected products go back to the supply chain after being handled. In the field of reverse supply chains, many scholars have carried out relevant researches from different aspects.

Angelus and Özalp studied a supply chain where 4 kinds of ordering decisions: regular, reverse, expedited, and secondary market selling can be made by each location. They developed a new method based on short and long purchases of inventory, which enables them to find the conditions for an optimal solution to be a nested echelon base stock policy [1]. Sobotka et al. took repair and demolishing works as background and studied reverse logistics of construction products and materials. The results reveal that reverse supply chains can bring significant added value to a business project [2]. Jena et al. took various scenarios for the government subsidies and fees into consideration and established four closed-loop supply chain models. The results show that an increase in subsidy will efficiently lift both total surplus and channel profit [3]. Huang and Wang analyzed the impacts of information sharing an a closed-loop supply chain and found that information sharing always lifts the profits of the manufacturer and the third party regardless the existence of technology licensing [4].

Ma and Lou consider a two-level household appliance supply chain system consisting of a manufacturer with an
Internet channel and a retailer with a traditional channel and an Internet channel [5]. Ma and Bao investigated the energy-efficient air conditioning supply chain, which is composed of one supplier (MIDEA) and two retailers [6]. Chen et al. quantify this effect for simple, two-stage supply chains consisting of a single retailer and a single manufacturer [7–9]. Ma and Xie discuss the impact of fluctuated demand triggered by price-discount promotion policy on the comprehensive inventory control system in which the final inventory and supply line stock are considered and weighted differently according to corresponding practical operation situations [10]. Ma and Xie consider two scenarios in which the manufacturer holds either asymmetric or symmetric channel power as the retailer and develop the dynamic game models for the two scenarios and analyze the models’ dynamic behavior [11]. Duc et al. quantify the impact of the bullwhip effect—the phenomenon in which information on demand is distorted as moving up a supply chain—for a simple two-stage supply chain with one supplier and one retailer [12].

J. Ma and X. Ma show that, in order to consider the market competition, a new supply chain with one supplier and two retailers is established in this paper. Two retailers employ different AR(1) demand processes, respectively, and an order-up-to inventory policy characterizes the inventory decisions [13]. Luong develops a measure of bullwhip effect for a simple two-stage supply chain that includes only one retailer and one supplier in the environment where the retailer employs base stock policy for their inventory and demand forecast is performed through the first-order autoregressive model, AR(1) [14]. Zhang considers the impact of forecasting methods on the bullwhip effect for a simple replenishment system in which a first-order autoregressive process AR(1) describes the customer demand and an order-up-to inventory policy characterizes the replenishment decision [15]. Caplin develops a general theory of the aggregate implications of (S, s) inventory policies. It is shown that (S, s) policies add to the variability of demand, with the variance of orders exceeding the variance of sales [16]. Kahn presents a model of production decisions with demand uncertainty that incorporates nonnegativity constraints on inventories. Even with no productivity shocks, optimal behavior by the firm is consistent with this stylized fact, either if demand exhibits positive serial correlation, or if the firm can backlog excess demand [17]. Towill et al. review the dynamic operation of supply chains and reach some simple conclusions for reducing demand amplification, which consequently attenuates swings in both production rates and stock levels. The results are based on one particular supply chain, for which the use of systems simplification techniques has generated valuable insight into supply chain design [18–20].

At the same time, this paper studies callback system’s impact on bullwhip effect from the aspect of the order of retailer and manufacturer and offers theoretical support to detailed control measures in multilevel supply chain. It divides the traditional demand market in line with certain market share, to probe into the mutual influence and differentiation among several retailers. The order-up-to inventory strategy and MA forecasting method are used in AR(1) and ARMA(1, 1) demand model for the quantification and analysis of bullwhip effect.

The following notation will be used in the paper:

- \( D_i \): total demand quantity at period \( t \)
- \( D_{ij} \): demand quantity of retailer \( i \) at period \( t \)
- \( \delta \): constant of autoregressive in demand model
- \( a \): first-order autocorrelation coefficient in demand model
- \( \epsilon_i \): independent and identically distributed in demand model
- \( \alpha \): marketing share index of Retailer 1
- \( \theta_1 \): callback index of retailer
- \( \theta_2 \): callback index of manufacture
- \( q_i \): order of total retailer at period \( t \)
- \( q_{ij} \): order of retailer \( i \) at period \( t \)
- \( l_i \): lead-time of retailer \( i \)
- \( L \): lead-time of manufacture
- \( k \): forecasting precision of retailers
- \( K \): forecasting precision of manufacture
- \( \sigma^2 \): variance of demand quantity
- \( S_{ij} \): order-up-to level of retailer \( i \) at period \( t \)
- \( \hat{D}_{ij} \): forecast of the lead-time demand of retailer \( i \) at period \( t \)
- \( \hat{\epsilon}_{ij} \): standard deviation of forecasting errors on the lead-time demand of retailer \( i \) at period \( t \)

2. Supply Chain Model

2.1. Demand Process. In this research, we consider a three-stage supply chain with one supplier, one manufacture, two retailers, and a callback center as shown in Figure 1. The two retailers face the same customer demand at a certain rate \( \alpha \) and send orders to manufacture. We assume that the customer demand process conforms to AR(1) process. The callback center collects the rejected products from customers and sends a part of them to Retailer 1 and the others to manufacture in Figure 1.

The customer demand conforms an AR(1) process as

\[
D_i = \delta + aD_{i-1} + \epsilon_i = D_{1i} + D_{2i}.
\]
In (1), \( a \) is the autoregressive coefficient of the customer demand process, while \(-1 < a < 1\). And \( \epsilon_t \) are independent and identically subject to a normal distribution, whose mean is 0 and variance is \( \sigma^2 \).

As we assume that the two retailers share the customer demand and the rate is \( \alpha \), we can derive each process of Retailer 1 and Retailer 2:

\[
D_{1,t} = aD_{t-1} + \alpha \epsilon_t, \\
D_{2,t} = (1 - \alpha)D_{t-1} + \alpha \epsilon_t.
\]

Furthermore, we get

\[
\frac{D_{1,t}}{D_{2,t}} = \frac{\alpha}{1 - \alpha}.
\]

And the demand process of Retailer 1 is

\[
D_{1,t} = a\delta + aD_{1,t-1} + \alpha \epsilon_t.
\]

So we know that \( a \) is the autoregressive coefficient of the customer demand process faced by Retailer 1 and \( \epsilon_t \) are independent and identically subject to a normal distribution, whose mean is 0 and variance is \( \sigma^2 \). And we can get

\[
E(D_{1,t}) = E(D_{1,t-1}) = \mu_{1,d} = \frac{a\delta}{1 - a}, \\
\var(D_{1,t}) = \var(D_{1,t-1}) = \sigma_{1,d}^2 = \frac{\alpha^2 \sigma^2}{1 - a^2}.
\]

Similarly, Retailer 2 faces the same demand process and its model is

\[
D_{2,t} = (1 - \alpha)\delta + aD_{2,t-1} + (1 - \alpha) \epsilon_t.
\]

And we have

\[
E(D_{2,t}) = E(D_{2,t-1}) = \mu_{2,d} = \frac{(1 - \alpha)\delta}{1 - a}, \\
\var(D_{2,t}) = \var(D_{2,t-1}) = \sigma_{2,d}^2 = \frac{(1 - \alpha)^2 \sigma^2}{1 - a^2}.
\]

As callback center sends the rejected products to Retailer 1 and manufacture at a certain rate, we get the callback model as

\[
W_t = (\theta_1 + \theta_2)D_{1,t-1} = (\theta_1 + \theta_2)(D_{1,t-1} + D_{2,t-1}).
\]

Here, the part \( \theta_1D_{1,t-1} \) is sent to Retailer 1 for being renewed and sold. The other part \( \theta_2D_{1,t-1} \) is sent to manufacture; these rejected products can be fixed and then sold to retailers.

2.2. Inventory Policy. Order-up-to inventory policy is widely used in many researches, so we assist using this policy in this inventory management issue. The two retailers employ the OUT inventory policy: at the beginning of period \( t \), Retailer 1 knows the customer demand of period \( t - 1 \) as \( D_{1,t-1} \). So Retailer 1 can calculate the OUT policy point \( S_{1,t} \) of period \( t \) and send an order \( q_{1,t} \) to manufacture after calculating the order. After a lead-time \( l_1 \), Retailer 1 receives the products from the manufacture at the starting of period \( t + l_1 \). And the OUT level \( S_{1,t} \) can be determined as

\[
S_{1,t} = \hat{D}_{1,t} + z\hat{\delta}_{1,t},
\]

In this formula, \( \hat{D}_{1,t} \) is the forecast demand of Retailer 1 in the lead-time \( l_1 \) which depends on forecasting method and lead-time. \( \hat{\delta}_{1,t} \) is the standard deviation of demand forecast error, and \( z \) is a given service level. Although it is difficult to accurately estimate the inventory holding costs and shortage costs in practice, the method using service level, which is defined as the probability for the first inventory fulfilling lead-time demand requirement to determine OUT level, is usually more feasible. The OUT level \( S_{1,t} \) shown in (9) is optimal, depending on total inventory cost, for inventory system, while there is no fixed ordering cost or holding costs and shortage costs are proportional to the volume of first inventory or shortage (Luong [14]).

Then, Retailer 1 sends an order of quantity \( q_{1,t} \) to the manufacture at the start of period \( t \). The order quantity can be given as

\[
q_{1,t} = S_{1,t} - S_{1,t-1} + D_{1,t-1} - \theta_1D_{1,t-1}.
\]

Here, \( \theta_1W_t \) is the rejected products received from callback center, and \( S_{1,t} - S_{1,t-1} + D_{1,t-1} \) is the normal quantity depending on OUT level.

Similarly, the OUT level of Retailer 2 at period \( t \) is

\[
S_{2,t} = \hat{D}_{2,t} + z\hat{\delta}_{2,t}.
\]

As Retailer 2 do not have rejected products, so its order quantity is

\[
q_{2,t} = S_{2,t} - S_{2,t-1} + D_{2,t-1}.
\]

2.3. Forecasting Method. In this paper, we assume the two retailers both use the MA forecasting method to forecast the demand in each lead-time. In the next section, the bullwhip effect is measured via the MA forecasting technique.

According to the MA method, we can establish the demand forecast model of Retailer 1 as

\[
\hat{D}_{1,t} = \frac{1}{k} \sum_{i=1}^{k} D_{1,t-i}.
\]

Here, \( k \) is the number of date samples for MA forecasting method.

As Retailer 2 faces the same consumer and sells the same products, we can consider that the two retailers use the same parameter \( k \).

So, the forecasted demand of Retailer 2 can be established as

\[
\hat{D}_{2,t} = \frac{1}{k} \sum_{i=1}^{k} D_{2,t-i}.
\]
3. Measure of the Bullwhip Effect

In this section, we will measure the bullwhip effect of the order which manufacture receives from the retailers under the MA forecasting method. Furthermore, we will measure each order sent by Retailer 1 and Retailer 2 to compare their orders’ bullwhip effect.

3.1. The Bullwhip Effect of Retailer 1. The order quantity of Retailer 1 is

\[ q_{1,t} = S_{1,t} - S_{1,t-1} + D_{1,t-1} - \theta_1 D_{1,t-1} \]

\[ = (\tilde{D}_{1,t} + z\tilde{\sigma}_{1,t}) - (\tilde{D}_{1,t-1} + z\tilde{\sigma}_{1,t-1}) + D_{1,t-1} \]

\[ - \theta_1 D_{1,t-1}. \]  

(15)

Here we can prove that

\[ (\tilde{\sigma}_{1,t})^2 = \left\{ \left[ \frac{l_1 (1 + a)}{(1 - a)} - \frac{2a (1 - d^l)}{(1 - a)^2} \right] \right\} \text{ var}(D_{1,t}). \]

(16)

The proof of (15) can be seen in Appendix.

The variance of order quantity that Retailer 1 sends to manufacture and the variance of demand that Retailer 1 faces, which does not change via \( \tau \) and has no influence on the bullwhip effect. So (15) equals

\[ q_{1,t} = \tilde{D}_{1,t} - \tilde{D}_{1,t-1} + D_{1,t-1} - \theta_1 D_{1,t-1}. \]  

(17)

Putting (13) into (17), we get

\[ q_{1,t} = \left( 1 + \frac{l_1}{K} - \theta_1 \right) D_{1,t-1} - \frac{l_1}{K} D_{1,t-2} + \theta_1 D_{2,t-1}. \]  

(18)

3.2. The Bullwhip Effect of Retailer 2. Similarly, we get the order quantity of Retailer 1 sends to manufacture as

\[ \text{BWE}_{R1} = \frac{\text{var}(q_{1,t})}{\text{var}(D_{1,t})} \]

\[ = \left( 1 + \frac{l_1}{K} - \theta_1 \right)^2 + \left( \frac{l_1}{K} \right)^2 - 2 \left( 1 + \frac{l_1}{K} - \theta_1 \right) \frac{l_1}{K} \text{ cov}(D_{1,t-1}, D_{1,t-2}) \]

\[ - 2 \left( 1 + \frac{l_1}{K} - \theta_1 \right) \theta_1 \text{ cov}(D_{1,t-1}, D_{2,t-1}) \]

\[ + 2 \frac{l_1}{K} \theta_1 \text{ cov}(D_{1,t-1}, D_{2,t-1}). \]  

(22)

The proof of (20) can be seen in Appendix. So we can get the variance of order quantity of Retailer 1 as

\[ \text{var}(q_{1,t}) = \left[ \left( 1 + \frac{l_1}{K} - \theta_1 \right)^2 + \left( \frac{l_1}{K} \right)^2 \right. \]

\[ - 2 \left( 1 + \frac{l_1}{K} - \theta_1 \right) \frac{l_1}{K} \text{ cov}(D_{1,t-1}, D_{1,t-2}) \]

\[ + 2 \left( 1 + \frac{l_1}{K} - \theta_1 \right) \theta_1 \text{ cov}(D_{1,t-1}, D_{2,t-1}) \]

\[ + 2 \frac{l_1}{K} \theta_1 \text{ cov}(D_{1,t-1}, D_{2,t-1}). \]  

(21)

We measure the bullwhip effect by the ratio of the variance of order quantity that Retailer 1 sends to manufacture and the variance of demand that Retailer 1 faces, which has been used by many researchers. So the bullwhip effect, denoted as BWE, can be determined as

\[ \text{BWE}_{R1} = \frac{\text{var}(q_{1,t})}{\text{var}(D_{1,t})} \]
The variance of order quantity of Retailer 2 is
\[
\text{var}(q_{2,t}) = \left( 1 + \frac{l_2}{k} \right)^2 \text{var}(D_{2,t-1}) + \left( \frac{l_2}{k} \right)^2 \\
\cdot \text{var}(D_{2,t-k-1}) - 2 \left( 1 + \frac{l_2}{k} \right) \frac{l_2}{k} \\
\cdot \text{cov}(D_{2,t-1}, D_{2,t-k-1}) \tag{25}
\]
\[
= \left( 1 + \frac{l_2}{k} \right)^2 + \left( \frac{l_2}{k} \right)^2 - 2 \left( 1 + \frac{l_2}{k} \right) \frac{l_2}{k} \text{a}^k.
\]

The bullwhip effect of Retailer 2 is
\[
\text{BWE}_{R_2} = \frac{\text{var}(q_{2,t})}{\text{var}(D_{2,t})} \tag{26}
\]
\[
= \left( 1 + \frac{l_2}{k} \right)^2 + \left( \frac{l_2}{k} \right)^2 - 2 \left( 1 + \frac{l_2}{k} \right) \frac{l_2}{k} \text{a}^k.
\]

3.3. The Bullwhip Effect of Total Order Received by Manufacturer. As we know the total order is
\[
q_t = q_{1,t} + q_{2,t}. \tag{27}
\]
In the derivation above, we have got the order quantity of each retailer, so we get the quantity of total order as
\[
q_t = q_{1,t} + q_{2,t} = \left( 1 + \frac{l_1}{k} - \theta_1 \right) D_{1,t-1} - \frac{l_1}{k} D_{1,t-k-1} \tag{28}
\]
\[
+ \left( 1 + \frac{l_2}{k} - \theta_1 \right) D_{2,t-1} - \frac{l_2}{k} D_{2,t-k-1}.
\]
Taking the variance of this formula, we get
\[
\text{var}(q_t) = \left( 1 + \frac{l_1}{k} - \theta_1 \right)^2 \text{var}(D_{1,t-1}) + \left( \frac{l_1}{k} \right)^2 \\
\cdot \text{var}(D_{1,t-k-1}) + \left( 1 + \frac{l_2}{k} - \theta_1 \right)^2 \\
\cdot \text{var}(D_{2,t-1}) + \left( \frac{l_2}{k} \right)^2 \text{var}(D_{2,t-k-1}) - 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \frac{l_1}{k} \text{cov}(D_{1,t-1}, D_{1,t-k-1}) \\
+ 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \left( 1 + \frac{l_2}{k} - \theta_1 \right) \frac{l_2}{k} \\
\cdot \text{cov}(D_{1,t-1}, D_{2,t-1}) - 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \frac{l_2}{k} \\
\cdot \text{cov}(D_{1,t-1}, D_{2,t-k-1}) - 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \frac{l_2}{k} \text{cov}(D_{1,t-k-1}, D_{2,t-k-1}) \\
+ 2 \frac{l_1^2}{k} \text{cov}(D_{1,t-k-1}, D_{2,t-k-1}) \\
- 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \frac{l_2}{k} \text{cov}(D_{2,t-1}, D_{2,t-k-1}). \tag{29}
\]

Here, we can prove
\[
\text{cov}(D_{1,t-1}, D_{1,t-k-1}) = \alpha^2 \text{a}^k \text{var}(D_t), \tag{30}
\]
\[
\text{cov}(D_{1,t-1}, D_{2,t-1}) = \alpha (1 - \alpha) \text{var}(D_t), \tag{31}
\]
\[
\text{cov}(D_{1,t-k-1}, D_{2,t-1}) = \alpha^k \alpha (1 - \alpha) \text{var}(D_t), \tag{32}
\]
\[
\text{cov}(D_{2,t-1}, D_{2,t-k-1}) = (1 - \alpha)^2 \text{a}^k \text{var}(D_t).
\]

The proof of (30) can be seen in Appendix. So we can get the variance of total order quantity
\[
\text{var}(q_t) = \left[ \left( 1 + \frac{l_1}{k} - \theta_1 \right)^2 + \left( \frac{l_1}{k} \right)^2 \right] \\
- 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \frac{l_1}{k} \text{a}^k \tag{31}
\]
\[
\cdot \text{var}(D_t) + \left[ \left( 1 + \frac{l_2}{k} - \theta_1 \right)^2 + \left( \frac{l_2}{k} \right)^2 \right] \\
- 2 \left( 1 + \frac{l_2}{k} - \theta_1 \right) \frac{l_2}{k} \text{a}^k \tag{32}
\]
\[
\cdot (1 - \alpha) \text{var}(D_t).
\]

We use the same formula to quantify bullwhip effect as above
\[
\text{BWE}_R = \frac{\text{var}(q_t)}{\text{var}(D_t)} = \left[ \left( 1 + \frac{l_1}{k} - \theta_1 \right)^2 + \left( \frac{l_1}{k} \right)^2 \right] \\
- 2 \left( 1 + \frac{l_1}{k} - \theta_1 \right) \frac{l_1}{k} \text{a}^k \tag{32}
\]
\[
\cdot \text{var}(D_t) + \left[ \left( 1 + \frac{l_2}{k} - \theta_1 \right)^2 + \left( \frac{l_2}{k} \right)^2 \right] \\
- 2 \left( 1 + \frac{l_2}{k} - \theta_1 \right) \frac{l_2}{k} \text{a}^k \cdot (1 - \alpha) \tag{33}
\]
\[
\cdot \text{var}(D_t) + 2 \frac{l_1^2}{k} \alpha (1 - \alpha).
\]
3.4. The Bullwhip Effect of Order Sent to Supplier by Manufacture. In this section, we will find what influences the bullwhip effect of supplier. The order is sent from manufacture to supplier, which depends on the demand and forecasting method of the Manufacture. Similar to previous research on the bullwhip effect of supplier, we use the similar technique to derive the BWE of supplier.

As we know, the demand of manufacture is the total quantity of orders from the retailers, so we can get the demand of manufacture from (28) as

\[
D_{M,t} = q_{R,t} = q_{1,t} + q_{2,t} = \left(1 + \frac{l_1}{k} - \theta_1\right)D_{1,t-1} - \frac{l_1}{k}D_{1,t-k-1} + \left(1 + \frac{l_2}{k} - \theta_1\right)D_{2,t-1} - \frac{l_2}{k}D_{2,t-k-1}.
\]

(33)

Using the OUT inventory policy, we can also get the inventory level of manufacture as

\[
S_{M,t} = \bar{D}_{M,t}^L + z\sigma_{M,t}^L.
\]

(34)

So, the order that manufacture sends to supplier is

\[
q_{M,t} = S_{M,t} - S_{M,t-1} + D_{M,t-1} - \theta_2D_{t-1}.
\]

(35)

Similarly, we know \(\sigma_{M,t}^L\) does not change via \(t\) and has no influence on the bullwhip effect. So (35) can be simplified as

\[
q_{M,t} = S_{M,t} - S_{M,t-1} + D_{M,t-1} - \theta_2D_{t-1}.
\]

(36)

We assume the forecasting method of manufacture is also MA forecasting technique, so we have

\[
\bar{D}_{M,t}^L = \sum_{j=1}^{K} \frac{L}{K} D_{M,j-1}.
\]

(37)

Now we know the order is

\[
q_{M,t} = S_{M,t} - S_{M,t-1} + D_{M,t-1} - \theta_2D_{t-1}
\]

\[
= \left( \frac{L}{K} \sum_{j=1}^{K} D_{M,j-1} - \frac{L}{K} \sum_{j=1}^{K} D_{M,j-1} \right) + D_{M,t-1}
\]

\[
= \left( \frac{L}{K} D_{M,t-1} - \frac{L}{K} D_{M,t-k-1} + D_{M,t-1} - \theta_2D_{t-1} \right)
\]

\[
= \left( \frac{L}{K} + 1 \right) D_{M,t-1} - \frac{L}{K} D_{M,t-k-1} - \theta_2D_{t-1}.
\]

We can get the variance of order quantity of Supplier as

\[
\text{var}(q_{M,t}) = \text{var}\left(\left( \frac{L}{K} + 1 \right) D_{M,t-1} - \frac{L}{K} D_{M,t-k-1} - \theta_2D_{t-1}\right)
\]

\[
= \left( \frac{L}{K} + 1 \right)^2 \text{BWE}_{R,MA} \cdot \text{var}(D_t) + \frac{L}{K} \sigma_{MA}^2 + 2 \left( \frac{L}{K} + 1 \right) \theta_2 \text{BWE}_{R,MA} + 2 \frac{L}{K} \theta_2 \sigma_{MA}^2
\]

(38)

So the order of manufacture can be determined as

\[
\text{var}(q_{M,t}) = \left( \frac{L}{K} + 1 \right)^2 \text{BWE}_{R,MA} \cdot \text{var}(D_t)
\]

\[
+ \left( \frac{L}{K} \right)^2 \text{BWE}_{R,MA} \cdot \text{var}(D_t) + 2 \left( \frac{L}{K} + 1 \right) \theta_2 \text{BWE}_{R,MA} + 2 \frac{L}{K} \theta_2 \text{var}(D_t)
\]

(39)

Here we have

\[
\text{var}(D_{M,t-1}) = \text{BWE}_{R,MA} \cdot \text{var}(D_t),
\]

\[
\text{var}(D_{M,t-K-1}) = \text{BWE}_{R,MA} \cdot \text{var}(D_t),
\]

\[
\text{var}(D_{t-1}) = \text{var}(D_t)
\]

(40)

\[
\text{cov}(D_{M,t-1}, D_{M,t-K-1}) = a^K \cdot \text{BWE}_{R,MA} \cdot \text{var}(D_t),
\]

\[
\text{cov}(D_{M,t-1}, D_{t-1}) = a \cdot \sqrt{\text{BWE}_{R,MA}} \cdot \text{var}(D_t).
\]

(41)

The proof of (40) can be seen in Appendix. So we can get the variance of order quantity of Supplier as

\[
\text{var}(q_{M,t}) = \left[ \left( \frac{L}{K} + 1 \right)^2 \text{BWE}_{R,MA} + \left( \frac{L}{K} \right)^2 \text{BWE}_{R,MA} + 2 \left( \frac{L}{K} + 1 \right) \theta_2 \text{BWE}_{R,MA} + 2 \frac{L}{K} \theta_2 \text{var}(D_t) \right] \cdot \text{var}(D_t)
\]

Similarly, we derive the BWE of supplier as

\[
\text{BWE}_M = \frac{\text{var}(q_{M,t})}{\text{var}(D_t)}
\]

\[
= \left( \frac{L}{K} + 1 \right)^2 \text{BWE}_{R,MA} + \left( \frac{L}{K} \right)^2 \text{BWE}_{R,MA} + 2 \left( \frac{L}{K} + 1 \right) \theta_2 \text{BWE}_{R,MA} + 2 \frac{L}{K} \theta_2 \text{var}(D_t) \cdot \sqrt{\text{BWE}_{R,MA}}.
\]

(42)
4. The Behavior of the Bullwhip Effect
Measure and Numerical Simulation

From the last section we have derived the formulation of bullwhip effect of each retailer and manufacturer; we realize that it is a function of the lead-time, forecasting precision, callback index, and marketing share rate. In the following part, we use method of numerical analysis to indicate the impact of all the parameters on the bullwhip effect of retailers and manufacturer in Figure 2.

Figures 2 and 3 show the behavior of BWE$_{R1}$ with respect to various values of $l_1$ or $k$ on the autocorrelation coefficient. It is observed from the presentation that the bullwhip effect of Retailer 1 decreases when the autocorrelation coefficient increases, which is a general conclusion. In Figure 2, we use $\alpha$ as an argument, fixed other variables, and plotted other variables in the case of $l_1 = 1$, $l_1 = 2$, and $l_1 = 3$. It shows that we can make the bullwhip decrease by cutting down the lead-time of Retailer 1.

4.1. The Impact of Parameters on the Bullwhip Effect of Retailer 1.

Similarly shifting the parameter $k$ while fixing the others in Figure 3, we can observe that if we improve forecasting precision of Retailer 1, the bullwhip effect will also decrease. And the decrease rate of bullwhip effect via $k$ slows down when the forecasting precision increases.

Figures 4 and 5 show the different behaviors of bullwhip effect via various $\alpha$ when we shift $l_1$ or $a$. From the two pictures, we can have a general conclusion about the market share index that bullwhip effect decreases fast and then increases when the market share changes from 0 to 0.7. As we observed in the presentation, bullwhip effect always has a minimum when the value of $\alpha$ varies between 0.1 and 0.2. In BWE$_{R1}$’s decreasing area, it reduces very fast while changing $\alpha$, but it increases slowly in the following area.

Figure 6 shows the variation of bullwhip effect when the callback index of Retailer 1 changes from 0 to 0.7. A general conclusion can be observed that when the callback index improves, BWE$_{R1}$ of Retailer 1 will decrease, and the rate
will slow down. The same conclusion can be observed when we change other parameters such as lead-time or forecasting precision.

4.2. The Impact of Parameters on the Bullwhip Effect of Manufacture. In this section, we will analyze the impact of lead-time, forecasting precision, callback index, and market share rate on the bullwhip effect of the order of manufacture.

(a) The Impact of Retailers’ Lead-Time on Bullwhip Effect. When we research the impact of retailers’ lead-time on bullwhip effect of manufacture, we consider two situations:

(1) When the lead-time satisfies \( l_1 = l_2 \), the impact of retailers’ lead-time of bullwhip when we set \( k = 3, L = 4, K = 4, \theta_1 = 0.3, \theta_2 = 0.2 \) is presented by Figure 7. We must point out that market share has no influence on bullwhip effect of manufacturer in this situation. Hence, market share can get any number between 0 and 1. We can see that the BWE of the manufacture goes up at first when \(-1 < a < -0.6\). And then it falls down slowly when \(-0.6 < a < 0.4\). Finally, the BWE decreases fast when \(0.4 < a < 1\). We can also investigate that the BWE becomes grater when the lead-time of the retailers becomes greater.

Figure 7 shows the bullwhip effect on different lead-time \((l_1 = l_2)\).

(b) The Impact of Manufacture’s Lead-Time on Bullwhip Effect. Figure 9 shows the impact of manufacture’s lead-time on bullwhip when the values of \( l_1, l_2, k, a, \theta_1, \theta_2, \) and \( \alpha \) are fixed. It can be observed that the bullwhip effect will increase fast when the manufacture has a larger lead-time.

(c) The Impact of Forecasting Precision on Bullwhip Effect. Figure 10 shows different bullwhip effect curves with various values of forecasting index of retailers. When the forecasting index is even, the bullwhip effect is an increasing function in the area \(-1 < a < 0\) but changes to a decreasing function in the area \(0 < a < 1\). But if the forecasting index is odd, bullwhip effect is always a decreasing function when \( a \) changes from \(-1\) to 1.

So when we draw the curves of bullwhip effect with different forecasting indexes of manufacture, we have to draw different pictures.
Figure 9: The impact of manufacture’s lead-time on bullwhip effect.

Figure 10: The impact of \( k \) on bullwhip effect.

Figure 11: The impact of \( K \) on bullwhip effect (\( k \) is even).

Figure 12: The impact of \( K \) on bullwhip effect (\( k \) is odd).

Figure 13: The impact of \( K \) on bullwhip effect when \( K \) is odd too and \( a \) is negative. But the bullwhip effect is always a decreasing function when \( a \) changes from \(-1\) to \(1\).

If we fix the other parameters as shown in Figure 13 and just increase the value of forecasting index of retailers or manufacture, we find that the bullwhip effect decreases soon when the forecasting index increases from \(1\) to \(4\). The decreasing rate becomes smaller when it enlarges \( k \) or \( K \) continuously. Finally, the BWE is stable near a value around \(0\). So, the span of the parameters \( k \) and \( K \) is really counts for the forecasting. The BWE would become smaller when the parameters \( k \) and \( K \) becomes greater.

(d) The Impact of Callback Index on Bullwhip Effect. First we simulate different behaviors of the bullwhip effect when \( \theta_1 \) or \( \theta_2 \) changes from \(0\) to \(0.8\). The presentation is shown in Figures 14 and 15. It is observed that the bullwhip effect decreases when the callback index increases for both \( \theta_1 \) and \( \theta_2 \). But \( \theta_1 \) has a much bigger effect than \( \theta_2 \).

And if we consider the situation where the sum of callback index is a fixed value, for example, \( \theta_1 + \theta_2 = 0.5 \), the bullwhip effect curve is shown as in Figure 16.
\( l_1 = l_2 = 2, L = 1, a = 0.5, \theta_1 = 0.3, \theta_2 = 0.2, \alpha = 0.5 \)

Figure 13: The impact of \( K \) and \( k \) on bullwhip effect.

\( l_1 = l_2 = 2, L = 1, k = 3, K = 4, a = 0.5, \alpha = 0.4 \)

Figure 14: The impact of \( \theta_1 \) on bullwhip effect.

\( l_1 = l_2 = 2, L = 1, k = 3, K = 4, a = 0.5, \alpha = 0.4, \theta_1 + \theta_2 = 0.5 \)

Figure 15: The impact of \( \theta_2 \) on bullwhip effect.

\( l_1 = l_2 = 2, L = 1, k = 3, K = 4, a = 0.5, \alpha = 0.4, \theta_1 + \theta_2 = 0.5 \)

Figure 16: The impact of callback index on bullwhip effect (\( \theta \) is fixed).

\[
\text{BWE}_{R,MA} = \frac{\text{var}(q_t)}{\text{var}(D_t)}
\]

Figure 17: The impact of \( \theta_1 \) on bullwhip effect.

Figure 16 shows that the bullwhip effect decreases when \( \theta_1 \) enlarges from 0 to 0.5, which means the increase of \( \theta_2 \) makes the bullwhip effect bigger. So, if the total callback index is a constant number, we shall make \( \theta_1 \) larger.

\( l_1 = l_2 = 2, L = 1, a = 0.5, \alpha = 0.4 \)

(e) The Impact of Marketing Share Index on Bullwhip Effect.

When we research the impact of marketing share rate on the bullwhip effect, we must consider the lead-time of retailers. If the retailers’ lead-time is a constant, the expression of retailers’ bullwhip effect is

\[
\text{BWE}_{R,MA} = \frac{\text{var}(q_t)}{\text{var}(D_t)}
\]

We further probe into the case when \( l_1 \neq l_2 \); the curve of bullwhip effect is shown in Figure 17.
Figure 7 shows the impact of marketing share rate on the bullwhip effect of manufacture. The bullwhip effect increases as \( \alpha \) increases if the lead-time of Retailer 1 is bigger than Retailer 2. But if the lead-time of Retailer 1 is smaller, the bullwhip effect will be different. It shows the two situations can be described as a symmetric relation.

5. Conclusion

This paper draws the following conclusions: the callback system and the market share will affect the whole supply chain and will especially influence the bullwhip effect of different layers of participants. We can see that the call back factor is very important for the bullwhip effect on a reverse supply chain. For single retailer or the total order, the decrease of lead-time and the increase of demand forecasting precision will reduce bullwhip effect effectively. In the meantime, the introduction of callback system can cut down the bullwhip effect of retailer demand, and a higher callback index relates to a lower bullwhip effect. From the perspective of market share, retailers should take action to improve their market shares in order to ensure the drop of bullwhip effect results from introducing the callback system. As for manufacturer, retailers’ callback index and manufacturer’s callback information will decrease the bullwhip effect to some extent. The higher the callback index is, the lower the manufacturer’s bullwhip effect will be. If the comprehensive callback index is fixed, the larger the retailers’ callback amount is, the lower the bullwhip effect will be. This paper also explores the market share of retailers and finds that only if the lead-times of retailers are different, manufacturer’s bullwhip effect is affected by market share. And a bigger share has more positive significance to retailer whom has a shorter lead-time. This research may be valuable in the perspective of management.

Appendix

Proof of (20). From (1) to (3), we know that

\[
D_2 = \frac{1}{\alpha} D_{1,2} = \frac{1}{1-\alpha} D_{2,2}.
\]

(A.1)

Iteration computation for (2), we have

\[
D_{1,t-1} = \left(1 + a + \cdots + a^k\right)a\delta + a^k D_{1,t-k}\n + \alpha \left(\epsilon_{t-1} + a\epsilon_{t-2} + \cdots + a^k \epsilon_{t-k}\right).
\]

(A.2)

So we can get

\[
\text{cov}(D_{1,t-1}, D_{1,t-k}) = \text{cov}\left(\left(1 + a + \cdots + a^k\right)a\delta \n + a^k D_{1,t-k}\n + \alpha \left(\epsilon_{t-1} + a\epsilon_{t-2} + \cdots + a^k \epsilon_{t-k}\right), D_{1,t-k}\right) = a^k \n \cdot \frac{1-\alpha}{\alpha} \text{var}(D_{1,t}).
\]

Likewise

\[
\text{cov}(D_{1,t-1}, D_{2,t-k}) = \text{cov}\left(\left(1 + a + \cdots + a^k\right)a\delta \n + a^k D_{1,t-k}\n + \alpha \left(\epsilon_{t-1} + a\epsilon_{t-2} + \cdots + a^k \epsilon_{t-k}\right), D_{2,t-k}\right) = a^k \n \cdot \frac{1-\alpha}{\alpha} \text{var}(D_{1,t}).
\]

(A.3)

\[
\text{cov}(D_{1,t-1}, D_{2,t-1}) = \frac{1-\alpha}{\alpha} \text{var}(D_{1,t}).
\]

(A.4)

Proof of (30). Using the similar process upside, we know

\[
\text{cov}(D_{1,t-1}, D_{2,t-1}) = \frac{1-\alpha}{\alpha} \text{var}(D_{1,t})
 = \alpha (1-\alpha) \text{var}(D_t).
\]

(A.5)

Likewise

\[
\text{cov}(D_{1,t-1}, D_{2,t-k-1}) = \text{cov}\left(\left(1 + a + \cdots + a^k\right)a\delta \n + a^k D_{1,t-k}\n + \alpha \left(\epsilon_{t-1} + a\epsilon_{t-2} + \cdots + a^k \epsilon_{t-k}\right), D_{2,t-k-1}\right) = a^k \n \cdot \frac{1-\alpha}{\alpha} \text{var}(D_{1,t}) = a^k \alpha (1-\alpha) \text{var}(D_t).
\]

(A.6)

The other equation can be derivate in same method.

\[\square\]
Proof of (40). From (32), we can know
\[
\text{var}(D_{M,t-1}) = \text{var}(q_{R,t-1}) = BWE_{R,MA} \cdot \text{var}(D_t),
\]
\[
\text{cov}(D_{M,t-1}, D_{M,t-K-1}) = \text{cov}(q_{R,t-1}, q_{R,t-K-1}) = a^K \cdot \text{var}(q_{R,t}) = a^K \cdot BWE_{R,MA} \cdot \text{var}(D_t). \tag{A.7}
\]
Similarly, we can get
\[
\text{cov}(D_{M,t-1}, D_{t-1}) = a \cdot BWE_{R,MA} \cdot \text{var}(D_t), \tag{A.8}
\]
\[
\text{cov}(D_{M,t-K-1}, D_{t-1}) = a \cdot BWE_{R,MA} \cdot \text{var}(D_t).
\]
\[
\Box
\]

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The research was supported by the National Natural Science Foundation of China (no. 71571131).

References
