

Research Article

Coordinated Depth Control of Multiple Autonomous Underwater Vehicles by Using Theory of Adaptive Sliding Mode

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This paper addresses the coordinated depth control problem of multiple autonomous underwater vehicles, which means to maneuver a group of underwater vehicles which move at the same depth synchronously. Firstly, a coordinated error of depth between vehicles and the common desired depth is defined by using extended graph theory in a distributed manner; then a deep-pitch double loop control algorithm based on sliding mode is designed for each vehicle, by which each vehicle is driven to and move at the common depth coordinately. In particular, a pitch reference command is firstly calculated by the predefined coordinated depth error, which can be regarded as the outer loop control, and then, the input rudder angle for each vehicle is derived according to the pitch reference command being as the inner loop control. Considering the uncertainties of the model hydrodynamic parameters, an online parameter adaptive algorithm is introduced to improve the performance of the sliding mode control algorithm proposed. Simulations were performed to verify the theoretical results proposed.

1. Introduction

With the development of artificial intelligence, robots play more and more roles in our daily lives. The high demand of industrial applications and automation devices always needs the robot operate in a complex environment which is posing requirements for the system stability, safety, and strong challenges for the controller design [1–3]. As a typical underwater robot, Autonomous Underwater Vehicle (AUV) plays great roles in ocean exploring missions and even more in the field of militaries. In order to fulfill these applications, it is usually necessary to maneuver an AUV cruise at a fixed and expected depth; this task is always called the vertical plane control of AUV, which is a typical motion form in the field of marine crafts [4]. Many works have been done to develop strategies capable of depth control of a single AUV, mainly include fuzzy control [5], neural networks [6], and sliding mode [7, 8]. Although significant progress has been made in the area, however, much work remains to be done to develop strategies for multiple AUVs. Considering the fact that current missions are getting more complex and AUV technology matures, it is common to require multiple AUVs

to work cooperatively to solve these types of tasks with low cost, high adaptively, and easy maintenance.

Consensus theory and algorithms have proven to be effective tools to perform the network-wide distributed computation tasks such as computing aggregate quantities and functions over networks. There have been much works on consensus problems of first-order agents, such as consensus under time-varying topology [9], finite-time consensus [10], consensus over random networks [11], and asynchronous consensus [12]. Taking into account the fact that many vehicles such as AUVs considered in this paper and mobile robots are always controlled directly by their accelerations rather than by their velocities, hence it is also necessary to investigate consensus problems of second-order agents. In [13, 14], the authors studied conditions on the interaction graph and the control gains for two different consensus algorithms to ensure agreement on both positions and velocities. Despite significant progress has been made in aforementioned works, much work remains to be done for the AUV systems due to the fact that the dynamics of AUV are often complex and cannot be simply ignored or drastically simplified for control design purpose. In the field of marine vehicles, authors in

[15] proposed a synchronized path following controller for fully actuated surface ships by using passivity theory. A decentralized formation controller was derived in [16], which deals with the cooperative problem of fully actuated surface vehicles with considering the influencing of the sea currents and model uncertainty. Even all the results proposed in the above works are decentralized because each member of the group only needs to exchange the necessary information under local interactions, but all of them assumed that there is a global reference speed for the whole group which should be known to all the team members. In this sense, the controller proposed is not distributed because there is a global vector which should be known to all of them.

Motivated by the ideas of consensus tracking and aforementioned works, we consider the truly distributed depth coordinated control problem of AUVs in this proposal. By using extended graph theories, the coordinated depth error between each AUV and the common desired depth was defined firstly, in which we only need the common desired depth be available to one subset of AUVs by local interactions, so the strategy is truly distributed and this can be regarded as our first contribution. Moreover, a double loop based on adaptive sliding mode control algorithm for each AUV was derived with considering the model uncertainties of each AUV, and each AUV can move at the same depth coordinately by using the algorithm proposed. Furthermore, the control parameters used in the algorithm proposed are obtained from the kinematic model of the AUV and the expected dynamic characteristics of each link; except for the initial value of the model parameter vector, all other parameters are irrelevant to the AUV, so the strategy proposed has good application values.

The remainder of this paper is organized as follows: in Section 2, models of the vehicles considered in this work and extended graph theory which will be used throughout this paper are described. We also presented the basic principles of the synchronized depth control by defining a coordinated error vector novelty in this section. Sections 3 and 4 especially present the procedures of designing the sliding mode control for the depth tracking with and without considering the model uncertainties. In Section 5, simulation examples are executed to validate the effectiveness of the strategy proposed. Finally, conclusions and future works are summarized and discussed in Section 6.

2. Problem Formulation

2.1. AUV Model in Vertical Plane. The six degrees of freedom dynamic model of AUV can be decomposed into vertical plane motion and horizontal plane motion independently if some coupling constraints are satisfied. The depth control considered in this paper is a typical freedom in vertical plane, so before going on, we will give the AUV model in vertical plane borrowed from [4]

$$\begin{aligned} \dot{\boldsymbol{\eta}}_v &= \mathbf{R}_v(\theta) \mathbf{v}_v \\ \mathbf{M}_v \dot{\mathbf{v}}_v + \mathbf{C}_v(\mathbf{v}_v) \mathbf{v}_v + \mathbf{D}_v(\mathbf{v}_v) \mathbf{v}_v + \mathbf{g}_v(\theta) &= \boldsymbol{\tau}_v, \end{aligned} \quad (1)$$

where $\boldsymbol{\eta}_v = [x \ z \ \theta]^T$ denotes the position and pitch of AUV in earth-fixed frame $\{I\}$ and $\mathbf{v}_v = [u \ w \ q]^T$ represents the speed in body-fixed frame $\{b\}$ with $\mathbf{R}_v(\theta)$ being the transformation matrix from frame $\{b\}$ to $\{I\}$. \mathbf{M}_v , $\mathbf{C}_v(\mathbf{v}_v)$, $\mathbf{D}_v(\mathbf{v}_v)$, and $\mathbf{g}_v(\theta)$ are the inertial matrix, Coriolis force matrix, damping matrix and gravity, and buoyancy generalized force vector with the following form, respectively:

$$\mathbf{R}_v(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (2)$$

$$\mathbf{M}_v = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Z_{\dot{w}} & 0 \\ 0 & 0 & I_{yy} - M_{\dot{q}} \end{bmatrix}, \quad (3)$$

$$\begin{aligned} \mathbf{C}_v(\mathbf{v}_v) &= \begin{bmatrix} 0 & 0 & (m - Z_{\dot{w}})w \\ 0 & 0 & -(m - X_{\dot{u}})u \\ -(m - Z_{\dot{w}})w & (m - X_{\dot{u}})u & 0 \end{bmatrix}, \end{aligned} \quad (4)$$

$$\begin{aligned} \mathbf{D}_v(\mathbf{v}_v) &= \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Z_w & 0 \\ 0 & 0 & -M_q \end{bmatrix} \end{aligned} \quad (5)$$

$$\begin{aligned} &+ \begin{bmatrix} -X_{|u|u}|u|u & 0 & 0 \\ 0 & -Z_{|w|w}|w| & 0 \\ 0 & 0 & -M_{|q|q}|q| \end{bmatrix}, \\ \mathbf{g}_v(\theta) &= \begin{bmatrix} (W - B) \sin \theta \\ -(W - B) \cos \theta \\ (z_g W - z_b B) \sin \theta + (x_g W - x_b B) \cos \theta \end{bmatrix}, \end{aligned} \quad (6)$$

$$\boldsymbol{\tau}_v = [X \ Z \ M]^T = [T \ Z_\delta \delta_e \ M_\delta \delta_e]^T, \quad (7)$$

where $I_{yy} - M_{\dot{q}} > 0$ is the combined moment of inertia with additional mass produced and the rotation inertia of the y-axis and M_δ is the rotation torque coefficient produced by rudder angle δ_e . Due to the fact that when AUV makes down steering which will produce the negative pitch angular acceleration \dot{q} , therefore, $M_\delta < 0$. $\boldsymbol{\tau}_v$ denotes the control force and torque for AUV which will be designed in following sections to enable each AUV move at the desired depth.

From the above formulations, we can find that the diving equations of AUV motion should include the heave velocity w , the angular velocity in pitch q , the pitch angle θ , and the depth z . Assume that the forward speed is constant and that the sway and yaw modes can be neglected, this suggests

the following simplified equations of AUV motion in vertical plane as

$$\begin{aligned} \dot{z} &= -u \sin \theta + w \cos \theta \\ \dot{\theta} &= q \\ (I_{yy} - M_{\dot{q}}) \dot{q} &= (X_{\dot{u}} - Z_{\dot{w}}) uw + M_q q + M_{|q|q} |q| \\ &+ Gz_G \sin \theta + Gx_G \cos \theta + M_{\delta} \delta_e \\ &+ n'_{\theta}, \end{aligned} \quad (8)$$

where n'_{θ} denotes the bounded total modeling errors and external disturbance. Based on parameterized linearization techniques, the dynamic equation of pitch angular velocity q can be rewritten as

$$\begin{aligned} \dot{q} &= a_1 uw + a_2 q + a_3 |q| + a_4 \sin \theta + a_5 \cos \theta + b \delta_e \\ &+ n_{\theta}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} a_1 &= \frac{(X_{\dot{u}} - Z_{\dot{w}})}{(I_{yy} - M_{\dot{q}})}, \\ a_2 &= \frac{M_q}{(I_{yy} - M_{\dot{q}})}, \\ a_3 &= \frac{M_{|q|q}}{(I_{yy} - M_{\dot{q}})}, \\ a_4 &= \frac{Gz_G}{(I_{yy} - M_{\dot{q}})}, \\ a_5 &= \frac{Gx_G}{(I_{yy} - M_{\dot{q}})}, \\ b &= \frac{M_{\delta}}{(I_{yy} - M_{\dot{q}})}, \\ n_{\theta} &= \frac{n'_{\theta}}{(I_{yy} - M_{\dot{q}})}. \end{aligned} \quad (10)$$

To be noted that, all the AUVs considered will be isomorphic in this paper, so for the sake of convenience, we omitted the index of the AUV number in the presentation of the model, and furthermore, in the implementing of the controller designing, we will make the following assumptions.

Assumption 1. The depth z can be measured by a pressure meter and the pitch angle θ can be measured by an inclinometer while the pitch rate q requires a rate gyro or a rate sensor. We also assumed that the velocities in the vertical plane can be measured by using a Doppler log for directly obtaining velocity measurements.

Assumption 2. The external disturbance n'_{θ} is bounded by an unknown upper bound, and n_{θ} is bounded apparently. We

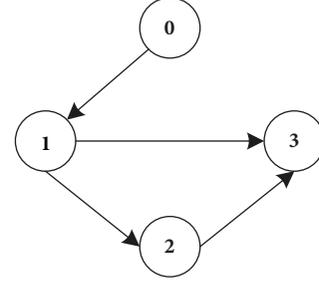


FIGURE 1: Example graph and the according Laplacian matrix.

assumed that $n_{\theta_i} \leq n_i$, where $N = [n_0, n_1, \dots, n_n]^T$ to be the unknown upper bound of disturbance [17].

2.2. Extended Communication Graph. Assumed that there are n AUVs in the group, it is a conventional way to model the interactions between them using directed graph adjacency matrix and topology graphs [18]. Firstly, we introduce a virtual AUV, denoted as v_0 , which specifically represents the desired state. For the n AUVs system, the graph $G = (V, A^*)$ contains a set of nodes $V = \{v_0, v_1, \dots, v_n\}$, and an adjacent matrix $A^* = [a_{ij}^*]$, where $a_{ij}^* = 1$ indicates that the i th AUV can obtain state information from j th, otherwise $a_{ij}^* = 0$. Then we define a degree matrix Δ with the elements $\delta_{ii} = \sum_j a_{ij}^*$. The Laplacian matrix of graph G is expressed as $\bar{L} = \Delta - A^*$ and the normalized Laplacian matrix of G is $L = I - A$, where A is the normalization of $a_{ij} = a_{ij}^* / \delta_{ii}$ when $\delta_{ii} \neq 0$; otherwise $a_{ij} = a_{ij}^*$ [19]. If we add one virtual AUV to the group, then we call the graph G as *extended communication graph*. An example of *extended communication graph* is given below; where Figure 1 is a topology graph with four nodes which contains three AUVs and one virtual AUV, its adjacency matrix, degree matrix, normalized adjacency matrix, Laplacian matrix, and normalized Laplacian matrix are, respectively,

$$\begin{aligned} A^* &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \\ \Delta &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \\ A &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}, \end{aligned}$$

$$\bar{L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 2 \end{bmatrix},$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}. \quad (11)$$

Because the virtual AUV can not get any information from others, the row sum of the matrix A is equal to 1, except for the first row. It is obvious that all the diagonal elements of L are 1; we can know that L is a diagonal dominant matrix, positive definite, and reversible; its inverse matrix will satisfy with $L^{-1} = \sum_{i=0}^n A^i$ [20].

2.3. Basic Idea of Sliding Mode Control. Consider the following second-order controlled object [21]:

$$\ddot{x}(t) = f(x, t) + bu(t) + d(t), \quad (12)$$

where $f(x, t)$ is nonlinear term, $d(t)$ is external disturbances, and $u(t)$ is the control input for the system.

Define tracking error as

$$e(t) = x(t) - x_d, \quad (13)$$

where x_d is desired state value.

Sliding surface vector is defined as follows:

$$s(t) = \lambda e(t) + \dot{e}(t), \quad (14)$$

where $\lambda > 0$ and satisfies Hurwitz condition. Then the basic idea of sliding mode control is how to derive the input $u(t)$ determined by $s(t)$ for the system to make $s(t)$ and $e(t)$ go to zero asymptotically.

2.4. Basic Principles of Coordinated Depth Control. Assume that there are n AUVs in the group, and in the following sections, we will use a subscript i to denote the index number of AUV. For multiple AUVs, we redefine each parameter as follows: $z = [z_0, z_1, \dots, z_n]^T$ is the depth of n AUVs, respectively, and z_0 means depth of the virtual AUV, $u = [u_0, u_1, \dots, u_n]^T$ is the velocity of n AUVs along x-axis, $w = [w_0, w_1, \dots, w_n]^T$ is the velocity of n AUVs along z-axis, $\theta = [\theta_0, \theta_1, \dots, \theta_n]^T$ is the pitch angle of n AUVs, $q = [q_0, q_1, \dots, q_n]^T$ is the angular velocity around y-axis, and $\delta_e = [\delta_{e0}, \delta_{e1}, \dots, \delta_{en}]^T$ is the rudder angle. To make each AUV track a common desired depth synchronously, the tracking error for i th vehicle will be defined as

$$\tilde{z}_i = \sum_{j=0}^n a_{ij} (z_i - z_j), \quad (15)$$

where a_{ij} is the ij th element of the normalized adjacent matrix A induced by the AUVs. According to the definition of normalized adjacent matrix in Section 2, the tracking error can also be described as

$$\tilde{z}_i = z_i - z_{di}, \quad (16)$$

where $z_{di} = \sum_{j=0}^n a_{ij} z_j$. Denote desired depth as $z_d = [z_{d0}, z_{d1}, \dots, z_{dn}]^T$; then the collective depth tracking error of n AUVs can be described as

$$\tilde{z} = z - z_d, \quad (17)$$

where $\tilde{z} = [\tilde{z}_0, \tilde{z}_1, \dots, \tilde{z}_n]^T$. It is known from (8) that the dynamics of the depth can be also described as

$$\dot{z} = -\nu \sin(\theta - \alpha), \quad (18)$$

where $\nu = \text{diag}(\nu_0, \nu_1, \dots, \nu_n)$ with $\nu_i = \sqrt{u_i^2 + w_i^2}$, ($i = 0, 1, \dots, n$), $\alpha = [\alpha_0, \alpha_1, \dots, \alpha_n]^T$ with $\alpha_i = \arctan(-w_i/u_i)$, ($i = 0, 1, \dots, n$), and $\sin(\theta - \alpha) = [\sin(\theta_0 - \alpha_0), \sin(\theta_1 - \alpha_1), \dots, \sin(\theta_n - \alpha_n)]^T$.

Assume that the depth tracking error will evolve with the following dynamics:

$$\dot{\tilde{z}} + \lambda_z \tilde{z} = 0, \quad (19)$$

where $\lambda_z = \text{diag}(\lambda_{z0}, \lambda_{z1}, \dots, \lambda_{zn})$ with λ_{zi} being the positive gains. Then the tracking error \tilde{z} will converge to zero exponentially at the rate of λ_z . This assumption is commonly used for error dynamic equations. Consider the following first-order homogeneous linear differential equation and its general solutions:

$$\frac{dy}{dx} + P(x)y = 0, \quad (20)$$

$$y = Ce^{-\int P(x)dx}, \quad (21)$$

where C is a constant. Then solution (21) will be determined by the initial condition of (20) and P .

That means

$$\tilde{z} = Ce^{-\int \lambda_z dt} = Ce^{-\lambda_z t}, \quad (22)$$

so the depth error \tilde{z} converges to zero when t tends to infinity.

Denote the desired pitch angle for each AUV as $\theta_f = [\theta_{f0}, \theta_{f1}, \dots, \theta_{fn}]^T$, then we can have

$$-\nu \sin(\theta_f - \alpha) = \dot{z}_d - \lambda_z (z - z_d). \quad (23)$$

By simply calculations we can get

$$\theta_f = -\arcsin(\nu^{-1}(\dot{z}_d - \lambda_z(z - z_d))) + \alpha, \quad (24)$$

assuming that $\nu^{-1}(\dot{z}_d - \lambda_z(z - z_d)) = \mu$, then $\arcsin(\mu) = [\arcsin(\mu_0), \arcsin(\mu_1), \dots, \arcsin(\mu_n)]^T$.

Here we require $\mu_i \leq 1$; otherwise, the objective of depth control cannot be achieved. In physical sense, this

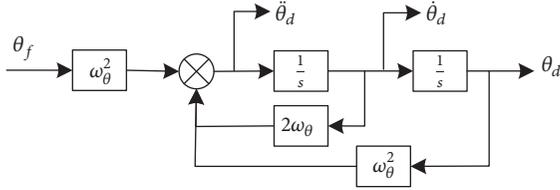


FIGURE 2: Second-order system structure block diagram.

inequality constraint means the expectation of the depth changing rate of each AUV can not be greater than its total speed; moreover, consider the fact that we have assumed a linearization hypothesis of a pitching angle being as a small angle before, then characteristic of $\arcsin x = x$ could be used here. Furthermore, by putting some saturation limit on $(z - z_d)$ we will have

$$\theta_f = -\nu^{-1} \left(\dot{z}_d - \lambda_z \Phi_z \text{sat} \left(\frac{(z - z_d)}{\Phi_z} \right) \right) + \alpha, \quad (25)$$

and the coefficient $\Phi_z = [\Phi_{z0}, \Phi_{z1}, \dots, \Phi_{zn}]^T$ determines the range of the depth deviation limit, that is, when the depth deviation of i th AUV is greater than Φ_{zi} , the pitch angle instruction will no longer be increased.

To make the desired pitch angle for each AUV to be more smooth, we will introduce a second-order filter to represent the dynamic relationship between θ_d and θ_f as

$$\omega_\theta^2 \theta_d + 2\omega_\theta \dot{\theta}_d + \ddot{\theta}_d = \omega_\theta^2 \theta_f, \quad (26)$$

where $\omega_\theta = \text{diag}(\omega_{\theta0}, \omega_{\theta1}, \dots, \omega_{\theta n})$ with $\omega_{\theta i} > 0$ and $\theta_d = [\theta_{d0}, \theta_{d1}, \dots, \theta_{dn}]^T$ will be the desired pitch angle output for each AUV. The block diagram of this filter was shown in Figure 2.

As so far, the structure of depth control considered in this paper can be shown in the Figure 3, where we put forward a deep-pitch double loop control strategy with pitch angle controller as the inner loop and depth controller as the outer loop [22]. The desired common depth z_d is initialized by the outer loop, and the according desired pitch angle θ_d is determined by the desired depth which means the input θ_d of the inner loop to be the output of the outer loop control, and the entire control system constitutes a closed-loop feedback structure.

3. Coordinated Depth Control without Model Uncertainties Based on Sliding Mode

To make the attitude of each AUV track the desired attitude θ_{d0} derived in Section 2 before synchronously, define the i th attitude tracking error as

$$e_i = \sum_{j=0}^n a_{ij} (\theta_i - \theta_j). \quad (27)$$

According to the definition of normalized adjacent matrix A , this attitude tracking error also can be described as

$$e_i = \theta_i - \theta_{di}, \quad (28)$$

where $\theta_{di} = \sum_{j=0}^n a_{ij} \theta_j$; as an example of Figure 1, the synchronized attitude error will be

$$\begin{aligned} e_0 &= 0 \\ e_1 &= \theta_1 - \theta_{d0} \\ e_2 &= \theta_2 - \theta_1 \\ e_3 &= \theta_3 - \frac{1}{2}\theta_2 - \frac{1}{2}\theta_1, \end{aligned} \quad (29)$$

so the attitude deviation of n AUVs can be defined as

$$e = \theta - \theta_d, \quad (30)$$

where $e = [e_0, e_1, \dots, e_n]$; according to the definition of normalized Laplacian L , we can obtain

$$e = L\theta. \quad (31)$$

Nextly, we will show how to derive the control input for each AUV by using the coordinated error defined by (30). Before going on, define the linearized parameter vector $\mathbf{a} = [a_1 \ a_2 \ a_3 \ a_4 \ a_5]$ and $\mathbf{Y} = [y_1 \ y_2 \ y_3 \ y_4 \ y_5] = [uw \ q \ q|q| \ \sin \theta \ \cos \theta]^T$, then (9) can be rewritten as

$$\dot{q} = \mathbf{a}\mathbf{Y} + b\delta_e + n_\theta. \quad (32)$$

According to the introduction of Section 2, define the sliding mode vector as

$$S = \dot{e} + \Lambda e, \quad (33)$$

where $S = [s_0, s_1, \dots, s_n]^T$ and $\Lambda = \text{diag}(\lambda_0, \lambda_1, \dots, \lambda_n)$ is a design parameter with $\lambda_i > 0$.

By simply calculations we will get the derivatives of (33):

$$\dot{S} = \dot{q} - \ddot{\theta}_d + \Lambda Lq. \quad (34)$$

Substituting (32) into (34), we can get

$$\dot{S} = \mathbf{a}\mathbf{Y} + b\delta_e + n_\theta - \ddot{\theta}_d + \Lambda Lq. \quad (35)$$

Choose the exponential reaching law as

$$\dot{S} = -TS - K \text{sgn}(S), \quad (36)$$

where $T = \text{diag}(t_0, t_1, \dots, t_n)$ and $K = \text{diag}(k_0, k_1, \dots, k_n)$. Then the control law for AUVs can be proposed as

$$\begin{aligned} \delta_e & \\ &= c \left[-TS - K \text{sgn}(S) - \mathbf{a}\mathbf{Y} - N \text{sgn}(S) + \ddot{\theta}_d - \Lambda Lq \right], \end{aligned} \quad (37)$$

where $c = \text{diag}(c_0, c_1, \dots, c_n) = b^{-1}$ with $c_i > 0$.

Theorem 3. Consider the system composed by a fleet of n AUVs of the form (8), guided by the slide mode control laws (37) with the sliding mode surface defined by (33). Then, the control system proposed solves the coordinated depth problem; that is, all the AUVs will move at the same common desired depth when this referred depth information can be available to at least one AUV and the graph induced by the AUVs has a directed spanning tree.

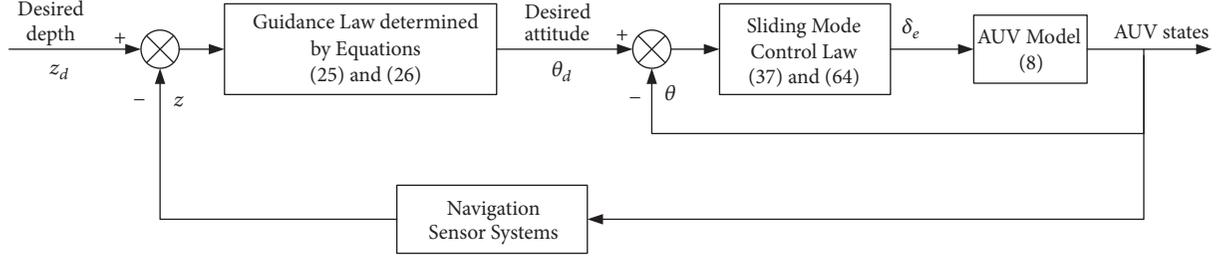


FIGURE 3: The structure of depth control system.

Proof. Consider the Lyapunov function candidate:

$$V = \frac{1}{2} S^T L^{-1} S. \quad (38)$$

The derivative of V is

$$\dot{V} = S^T L^{-1} \dot{S}. \quad (39)$$

Substituting (34) into (39), we will have

$$\dot{V} = S^T L^{-1} [\mathbf{a}\mathbf{Y} + b\delta_e + n_\theta - \ddot{\theta}_d + \Lambda Lq], \quad (40)$$

and continuously substituting control law (37) into (40), we will get

$$\begin{aligned} \dot{V} = & S^T L^{-1} [\mathbf{a}\mathbf{Y} \\ & + [-TS - K \operatorname{sgn}(S) - \mathbf{a}\mathbf{Y} - N \operatorname{sgn}(S) + \ddot{\theta}_d - \Lambda Lq] \\ & + n_\theta - \ddot{\theta}_d + \Lambda Lq]. \end{aligned} \quad (41)$$

Simply manipulations will yield

$$\dot{V} = S^T L^{-1} [-TS - K \operatorname{sgn}(S) - N \operatorname{sgn}(S) + n_\theta]. \quad (42)$$

According to Assumption 2, we know that $-N \operatorname{sgn}(S) + n_\theta < 0$, thus we can get

$$\dot{V} \leq -S^T L^{-1} TS - S^T L^{-1} K \operatorname{sgn}(S). \quad (43)$$

According to Lyapunov stability theorem, the following two stability conditions will be hold as

$$\lim_{|s_i| \rightarrow \infty} V = \infty, \quad (44)$$

$$\dot{V}_i < 0, \quad (s_i \neq 0). \quad (45)$$

So S and e will converge to zero asymptotically. Consequently, we will show that when the desired common depth information can only be obtained by at least one of the AUVs, all the AUVs can track this desired state coordinately; in order to do this, define the error between i th AUV and the desired attitude angle as

$$\bar{\theta}_i = \theta_i - \theta_{d0}, \quad (46)$$

and the i th sliding mode surface as

$$s_i = \dot{e}_i + \lambda_i e_i = \left(\dot{\theta}_i - \sum_{j=0}^n a_{ij} \dot{\theta}_j \right) + \lambda_i \left(\theta_i - \sum_{j=0}^n a_{ij} \theta_j \right), \quad (47)$$

consequently introducing the auxiliary variables as

$$\xi_i = \dot{\theta}_i + \lambda_i \theta_i. \quad (48)$$

Substituting (48) into (47), the sliding mode surface can be rewritten as

$$s_i = \xi_i - \sum_{j=0}^n a_{ij} \xi_j, \quad (49)$$

then we can have

$$\tilde{\xi}_i = \xi_i - \xi_0 = \sum_{j=0}^n a_{ij} \xi_j + s_i - \xi_0, \quad (50)$$

and rewrite (50) in a vector form with

$$\tilde{\xi} = A\xi + S - \xi_0 \mathbf{1}, \quad (51)$$

where $\mathbf{1} = [1, 1, \dots, 1]^T$ and A is the corresponding normalized weighted adjacency matrix. Considering the fact that the sum of first row elements of A is 0 and other rows is 1 which means $A[1, 1, \dots, 1]^T = [0, 1, 1, \dots, 1]^T$, then we will have

$$\begin{aligned} \tilde{\xi} &= A\xi + S - \xi_0 \mathbf{1} \\ &= A(\tilde{\xi} + \xi_0 \mathbf{1}) + S + [1, 0, 0, \dots, 0]^T \xi_0 - \xi_0 \mathbf{1} \\ &= A\tilde{\xi} + [0, 1, \dots, 1]^T \xi_0 + S + [1, 0, \dots, 0]^T \xi_0 - \xi_0 \mathbf{1} \\ &= A\tilde{\xi} + S, \end{aligned} \quad (52)$$

which means

$$\tilde{\xi} = (I - A)^{-1} S, \quad (53)$$

according to Section 2, we know that the Laplace matrix of the system is positive definite and reversible. So (53) will become

$$\tilde{\xi} = L^{-1} S. \quad (54)$$

According to (48), the time derivative of $\tilde{\theta}_i$ is

$$\dot{\tilde{\theta}}_i = \dot{\theta}_i - \dot{\theta}_{d0} = a_\theta \tilde{\theta}_i + \tilde{\xi}_i, \quad (55)$$

where $a_\theta = -\lambda_i$, and the vector form of (55) is

$$\dot{\tilde{\theta}} = \bar{a}_\theta \tilde{\theta} + \tilde{\xi}, \quad (56)$$

where $\bar{a}_\theta = -\Lambda$. By simply manipulations, we can get

$$\dot{\tilde{\theta}} = \bar{a}_\theta \tilde{\theta} + L^{-1}S, \quad (57)$$

Lemma 4. For the time constant T_1 , define $s_{\max,i}(t) = \max_i \sup_{0 \leq \tau \leq t} |s_i|$; the following inequalities will be hold:

$$\|\tilde{\theta}(t)\| \leq \bar{\kappa} \|\tilde{\theta}(0)\| + \frac{k_0}{\lambda_0} (\beta_s(T_1) + \beta_{sT_1}), \quad (58)$$

where λ_{\max} is the maximum eigenvalue of L^{-1} , $\beta_s(t) = N\lambda_{\max}s_{\max,i}(t)$, $\beta_{sT_1} = N\lambda_{\max}\sup_{T_1 < t} s_{\max,i}(t)$, and $\bar{\kappa} = k_0 e^{-\lambda_0 t} \geq \|e^{\bar{a}_\theta t}\|$, $k_0 > 0$, $\lambda_0 > 0$.

Proof. The solution of (57) can be derived as

$$\tilde{\theta}(t) = \tilde{\theta}(0) e^{\bar{a}_\theta t} + \int_0^t e^{\bar{a}_\theta(t-\tau)} L^{-1} S d\tau. \quad (59)$$

Considering the fact that the Laplace matrix of system L is positive definite and L^{-1} is positive definite too. According to triangle inequality axiom of the matrix norm

$$\|A + B\| \leq \|A\| + \|B\|, \quad (60)$$

we will have

$$\begin{aligned} \|\tilde{\theta}(t)\| &\leq \|\tilde{\theta}(0) e^{\bar{a}_\theta t}\| + \left\| \int_0^t e^{\bar{a}_\theta(t-\tau)} L^{-1} S d\tau \right\| \\ &\leq k_0 e^{-\lambda_0 t} \|\tilde{\theta}(0)\| \\ &\quad + k_0 e^{-\lambda_0 t} [N\lambda_{\max}] s_{\max,i}(t) \frac{e^{\lambda_0 t} - 1}{\lambda_0} \\ &\leq \bar{\kappa} \|\tilde{\theta}(0)\| + \frac{k_0}{\lambda_0} \beta_s(t). \end{aligned} \quad (61)$$

Considering the following equation consequently:

$$\begin{aligned} \int_0^t e^{\bar{a}_\theta(t-\tau)} L^{-1} S d\tau &= \int_0^{T_1} e^{\bar{a}_\theta(t-\tau)} L^{-1} S d\tau \\ &\quad + \int_{T_1}^t e^{\bar{a}_\theta(t-\tau)} L^{-1} S d\tau, \end{aligned} \quad (62)$$

and we will get

$$\begin{aligned} \|\tilde{\theta}(t)\| &\leq \bar{\kappa} \|\tilde{\theta}(0)\| + \int_0^{T_1} e^{\bar{a}_\theta(t-\tau)} \|L^{-1} S\| d\tau \\ &\quad + \int_{T_1}^t e^{\bar{a}_\theta(t-\tau)} \|L^{-1} S\| d\tau \\ &\leq \bar{\kappa} \|\tilde{\theta}(0)\| + \frac{k_0}{\lambda_0} e^{-\lambda_0 t} (e^{\lambda_0 T_1} - 1) \beta_s(T_1) \\ &\quad + \frac{k_0}{\lambda_0} e^{-\lambda_0 t} (e^{\lambda_0 t} - e^{\lambda_0 T_1}) \beta_{sT_1} \\ &\leq \bar{\kappa} \|\tilde{\theta}(0)\| + \frac{k_0}{\lambda_0} (\beta_s(T_1) + \beta_{sT_1}). \end{aligned} \quad (63)$$

This completes the proof, which indicates that the tracking error of the total system will converge to the neighborhood of the origin; that is to say, the pitch angle of each AUV can eventually reach the desired value, and simultaneously the each AUV will move at the common desired depth synchronously. \square

4. Coordinated Depth Control Design by Using Adaptive Sliding Mode with Model Uncertainties

In practical applications for AUVs, the parameters of the model always have uncertainties, which can affect the performance of the closed-loop system seriously. It is necessary to design control strategy to take into account these uncertainties which is the main contribution of this section.

Consider the control law described by (37) in the previous section; if the control gains c , N and the model parameters are not known, consider the following adaptive slide control law:

$$\begin{aligned} \delta_e &= \hat{c} [-TS - K \operatorname{sgn}(S) - \hat{\mathbf{a}}\mathbf{Y} - \hat{N} \operatorname{sgn}(S) + \ddot{\theta}_d - \Lambda Lq] \\ &= \hat{c} X, \end{aligned} \quad (64)$$

where $X = [-TS - K \operatorname{sgn}(S) - \hat{\mathbf{a}}\mathbf{Y} - \hat{N} \operatorname{sgn}(S) + \ddot{\theta}_d - \Lambda Lq]$ and

$$\begin{aligned} \hat{\mathbf{a}}_{ji} &= k_{1i} S^T L^{-1} y_j \\ \hat{c}_i &= -k_{2i} S^T L^{-1} X \\ \hat{n}_i &= k_{3i} S^T L^{-1} \operatorname{sgn}(S), \end{aligned} \quad (65)$$

where $i = 0, 1, \dots, n$, $j = 1, 2, \dots, 5$, $K_1 = \operatorname{diag}(k_{10}, k_{11}, \dots, k_{1n})$, $K_2 = \operatorname{diag}(k_{20}, k_{21}, \dots, k_{2n})$, and $K_3 = \operatorname{diag}(k_{30}, k_{31}, \dots, k_{3n})$ are design parameters and their diagonal elements are all positive.

Theorem 5. Consider the system composed by a fleet of n AUVs of the form (8), guided by the slide mode control laws (64) with the sliding mode surface defined by (33) and adaptation

law described by (65). Then, the control system proposed solves the coordinated depth problem; that is, all the AUVs will move at the same common desired depth when this referred depth information can be available to at least one AUV and the graph induced by the AUVs has a directed spanning tree.

Proof. Consider the following Lyapunov candidate function:

$$V = \frac{1}{2} S^T L^{-1} S + \sum_{j=1}^5 \sum_{i=0}^n \frac{1}{2k_{1i}} \tilde{a}_{ji}^2 + \sum_{i=0}^n \frac{1}{2k_{2i}c_i} \tilde{c}_i^2 + \sum_{i=0}^n \frac{1}{2k_{3i}} \tilde{n}_i^2, \quad (66)$$

where $\tilde{a}_{ji} = \hat{a}_{ji} - a_{ji}$, $\tilde{c}_i = \hat{c}_i - c_i$, $\tilde{n}_i = \hat{n}_i - n_i$ are estimation errors of a_{ji} , c_i , and n_i . The derivative of (66) can be derived as

$$\dot{V} = S^T L^{-1} \dot{S} + \sum_{j=1}^5 \sum_{i=0}^n \frac{1}{k_{1i}} \tilde{a}_{ji} \dot{\hat{a}}_{ji} + \sum_{i=0}^n \frac{1}{k_{2i}c_i} \tilde{c}_i \dot{\hat{c}}_i + \sum_{i=0}^n \frac{1}{k_{3i}} \tilde{n}_i \dot{\hat{n}}_i. \quad (67)$$

Substituting (34) and adaptation law (65) into (67), we will get

$$\begin{aligned} \dot{V} = & S^T L^{-1} [\mathbf{a}\mathbf{Y} + b\hat{c}X + n_\theta - \dot{\theta}_d + \Lambda Lq] \\ & + \sum_{j=1}^5 \sum_{i=0}^n \tilde{a}_{ji} S^T L^{-1} y_j - \sum_{i=0}^n \frac{\tilde{b}_i}{b_i} S^T L^{-1} X \\ & + \sum_{i=0}^n \tilde{n}_i S^T L^{-1} \text{sgn}(S) \end{aligned} \quad (68)$$

Consequently substituting control law (64) into (68), we can have

$$\begin{aligned} \dot{V} = & S^T L^{-1} [\mathbf{a}\mathbf{Y} - TS - K \text{sgn}(S) - \hat{\mathbf{a}}\mathbf{Y} - \hat{N} \text{sgn}(S) \\ & + \dot{\theta}_d - \Lambda Lq + n_\theta - \dot{\theta}_d + \Lambda Lq] + \sum_{j=1}^5 \sum_{i=0}^n \tilde{a}_{ji} S^T L^{-1} y_j \\ & + \sum_{i=0}^n \tilde{n}_i S^T L^{-1} \text{sgn}(S). \end{aligned} \quad (69)$$

Simply manipulations will yield

$$\begin{aligned} \dot{V} = & S^T L^{-1} [-TS - K \text{sgn}(S) - \hat{N} \text{sgn}(S) + n_\theta] \\ & + \sum_{i=0}^n \tilde{n}_i S^T L^{-1} \text{sgn}(S). \end{aligned} \quad (70)$$

According to Assumption 2, we can get

$$\dot{V} \leq -S^T L^{-1} TS - S^T L^{-1} K \text{sgn}(S), \quad (71)$$

and we can find $\dot{V} < 0$ for all $s_i \neq 0$. According to Lemma 4 consequently, we can know that there is a time constant T_2 which satisfies

$$\|\bar{\theta}(t)\| \leq \bar{\kappa} \|\bar{\theta}(0)\| + \frac{k_0}{\lambda_0} (\beta_s(T_2) + \beta_{sT_2}), \quad (72)$$

and this completes the proof, so the system state would converge to the sliding mode surface $S(t) = 0$ in finite time, all errors converge to zero, and the closed-loop system is stable. Then the pitch angle of each AUV can eventually reach the desired value; meanwhile, all the AUVs will move at the same desired depth synchronously. \square

5. Simulation Results

In this section, we present simulation examples to validate the theoretical results proposed in this work. We consider a group of AUVs modeled by (8) with hydrodynamic parameters shown in Table 1.

The control law designed did not consider the input saturation, while the actual AUV always has a rudder angle limit and in this paper we assumed that $\delta_{ei} < 15^\circ$. Choose the adaptive sliding mode controller control gains: $\lambda_{zi} = 0.05$, $\lambda_i = 1$, $k_i = 10$, and $t_i = 0.05$, the initial depth of the three AUVs was 10 meters, 12 meters, and 8 meters, respectively, and the common desired depth is set to 20 meters. The simulation results are shown from Figures 4–9.

From Figure 4 we can see that all AUVs will move at the same common desired depth; specifically, the AUVs move to the desired depth more quickly under the control of adaptive sliding mode strategy. In the meantime, we can also find that the pitch angles of all AUVs can reach synchronization from Figure 5; moreover, when the AUVs move to the common desired depth, all the pitch angles tend to 0 degrees. Compared to the sliding mode strategy, the pitch angle of each AUV has a more quick convergent speed by using the adaptive sliding mode method. Figure 6 depicts the according rudder inputs for each AUV, and we can find that they also achieved synchronization behaviors. Figure 7 shows the evolving of the sliding mode surface of each controller; we can find that it will converge to zero in 12 and 6 seconds under the sliding mode and adaptive sliding mode, respectively, which shows the effectiveness of adaptive sliding mode strategy proposed specifically. Figures 8 and 9 show the coordinated tracking error of depth and pitch attitudes separately; they all tend to zero eventually.

In summary, due to the model parameter error, although the sliding mode control without adaptive mechanism is stable, there is always a steady-state error, and the adjustment time is also longer. With the introducing of adaptive parameter estimation algorithms, the system can estimate the linearization parameters online and converge to zero quickly. At the same time, it should be noted that the introducing of adaptive also makes the jitter frequency of the AUV rudder angle higher. This shows that the adaptive algorithm increases the system's overshoot while improving the steady-state accuracy performance, but because the rudder angle converges quickly, it will hardly affect the stability of the system.

TABLE 1: The parameters of each AUV.

Parameter	Definition	Value
m	mass	1600kg
ρ	density	1019.2kg/m ³
T	thrust	1078N
G	gravity	15680N
L_a	length	7.738m
S_a	maximum cross-sectional area	0.224m ²
J_z	moment of inertia around the z-axis	6771.8kg · m ²
x_c	distance between center of mass and center of buoyancy	0m
y_c	distance of the center of mass move down	-0.03m
z_c	distance of the center of mass side shift	0m
K_{11}	vertical additional mass factor	31.588
K_{33}	lateral additional mass factor	1554.672
K_{35}	additional spacing factor	276.113
K_{55}	lateral additional moment of inertia factor	6905.1545
C_y^α	positional derivative of lift factor versus angle of attack	2.172
$C_y^{\delta_e}$	positional derivative of lift factor versus horizontal rudder angle	0.4994
$C_y^{\bar{\omega}_z}$	positional derivative of lift factor versus angular velocity	1.362
m_z^α	positional derivative of pitching moment factor versus angle of attack	0.5892
$m_z^{\delta_e}$	positional derivative of pitching moment factor versus horizontal rudder angle	-0.7339
$m_z^{\bar{\omega}_z}$	positional derivative of pitching moment factor versus angular velocity	-0.2659

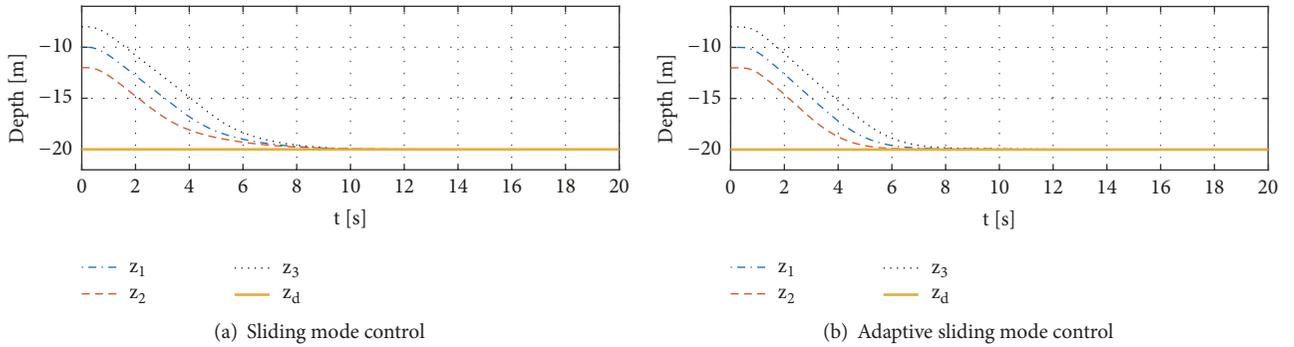


FIGURE 4: The depth evolving of each AUV.

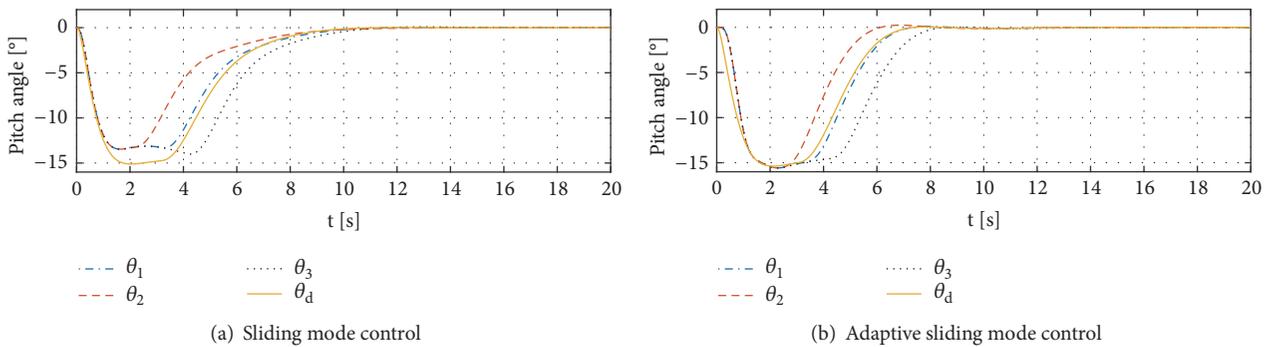


FIGURE 5: Synchronized attitude of pitch angles of each AUV.

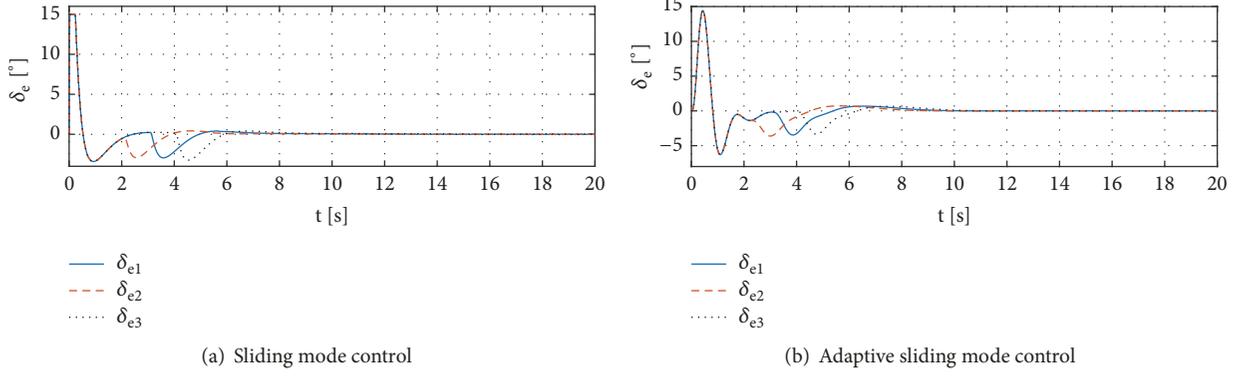


FIGURE 6: The rudder input of each AUV.

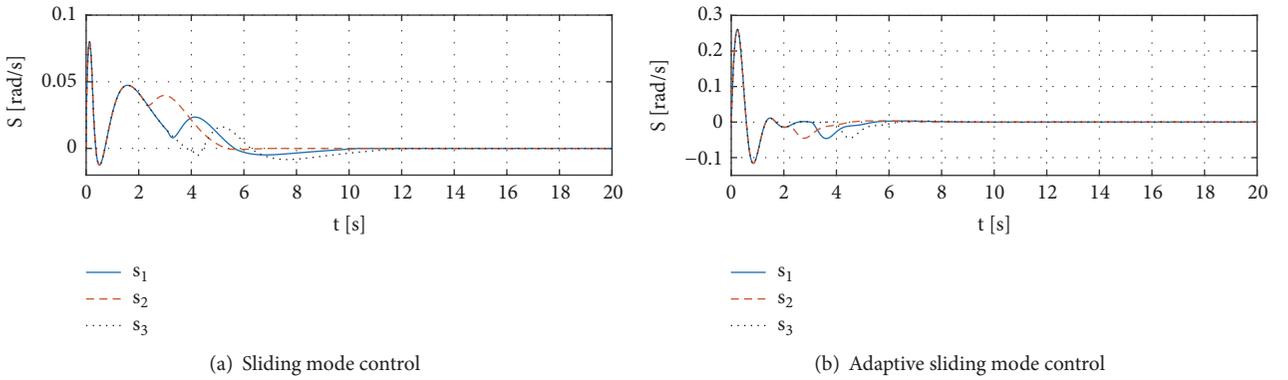


FIGURE 7: The sliding mode surface of each controller.

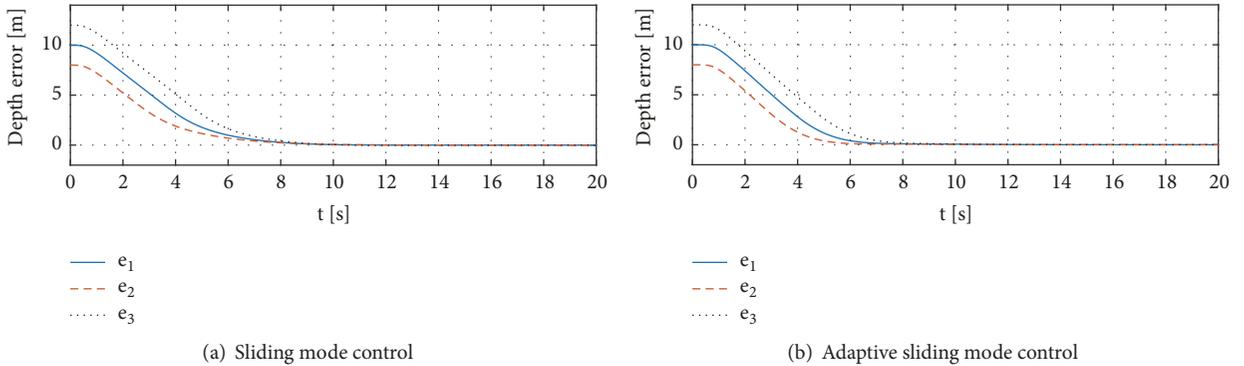


FIGURE 8: The coordinated errors of depth tracking.

6. Conclusion

In this article, we discuss the coordinated depth control problem of multiple AUVs. We propose a distributed adaptive control law using the coordinated depth error based on sliding method; each AUV adopts deep-pitch double loop control. Our strategy is fully distributed because each controller only needs information of its neighbors and the desired common depth information can only be available to one subset of the AUVs. The uncertainty of the models is considered by adaptive laws. The stability of the control law proposed is analyzed and proved theoretically. The simulation

results validate the effectiveness of algorithm proposed. We suppose that the topology between the AUVs is fixed and there are no communication delays. All the topics about multiple vehicles, such as dynamic changing topologies, time delays, and data dropouts, will warrant our future research.

Data Availability

The participant-level data of this work are the dynamic model parameters of AUV, which can be referred to Table 1 for details.

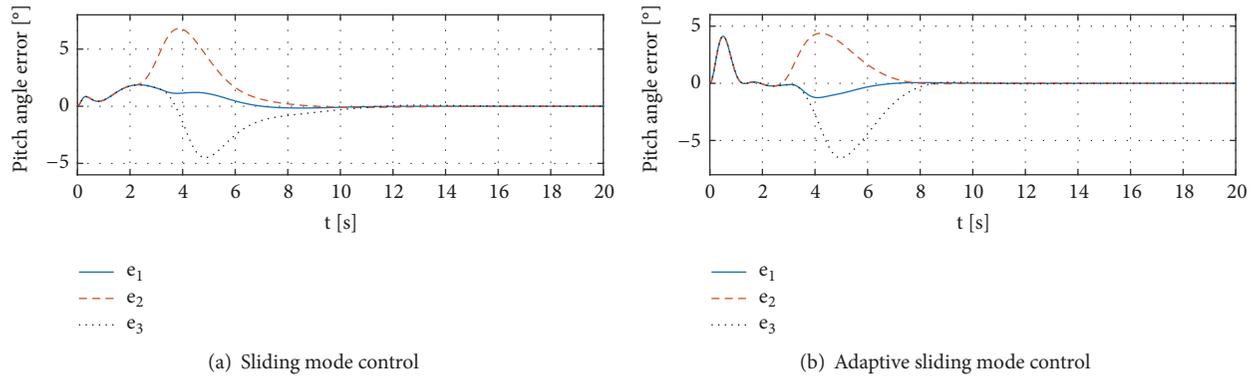


FIGURE 9: The coordinated errors of pitch tracking.

Disclosure

Yintao Wang and Yani Zhang are the co-first authors.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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