An Integer-Order Memristive System with Two- to Four-Scroll Chaotic Attractors and Its Fractional-Order Version with a Coexisting Chaotic Attractor

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Received 19 April 2018; Accepted 28 June 2018; Published 30 July 2018

Abstract

First, based on a linear passive capacitor \( C \), a linear passive inductor \( L \), an active-charge-controlled memristor, and a fourth-degree polynomial function determined by charge, an integer-order memristive system is suggested. The proposed integer-order memristive system can generate two-scroll, three-scroll, and four-scroll chaotic attractors. The complex dynamics behaviors are investigated numerically. The Lyapunov exponent spectrum with respect to linear passive inductor \( L \) and the two-scroll, three-scroll, and four-scroll chaotic attractors are yielded by numerical calculation. Second, based on the integer-order memristive chaotic system with a four-scroll attractor, a fractional-order version memristive system is suggested. The complex dynamics behaviors of its fractional-order version are studied numerically. The largest Lyapunov exponent spectrum with respect to fractional-order \( p \) is yielded. The coexisting two kinds of three-scroll chaotic attractors and the coexisting three-scroll and four-scroll chaotic attractors can be found in its fractional-order version.

1. Introduction

Chaos is an interesting phenomenon in nonlinear systems. High irregularity, unpredictability, and complexity are the typical characteristics of chaotic systems [1, 2]. These typical characteristics have great applications in the following fields: data encryption [3], secure communication [4–7], power grid protection [8, 9], and so on [10–16]. Therefore, more and more attentions have been attracted on the study of chaotic systems in the last few decades [17–20]. In 1971, Chua reported the fourth circuit element named memristor [21], and a solid-state implementation of a memristor has been successfully realized in Hewlett-Packard in 2008 [22]. After then, the applications of a memristor have caught many attentions in nonlinear science [23–28]. Meanwhile, chaotic and hyperchaotic attractors have been found in many memristor-based circuits [21, 23–26]. For example, Muthuswamy and Chua provided a memristor-based circuit with a single-scroll chaotic attractor [24], Bao et al. reported a memristor-based circuit with a double-scroll chaotic attractor [25], Teng et al. found a memristor-based circuit with double-scroll and four-scroll chaotic attractors [26], and so on [27, 28]. On the other hand, many real physical systems such as electromagnetic wave propagation, dielectric polarization, and heat conduction can be described by fractional-order differential equations [29, 30]. Meanwhile, chaotic phenomenon has been discussed in many fractional-order nonlinear systems such as the fractional-order electronic circuits [31], the fractional-order gyroscopes [32], the fractional-order chaotic brushless DC motor [12], the fractional-order microelectromechanical system [33], and the fractional-order neural networks [34, 35]. So, more attentions have been paid to research the chaotic behaviors of fractional-order nonlinear systems.

Motivated by the above considerations, first, based on a memristor-based chaotic circuit reported by Muthuswamy...
and Chua [24], Bao et al. [25], and Teng et al. [26], an integer-order memristive chaotic system with two-scroll, three-scroll, and four-scroll chaotic attractors is provided in this paper. It is noticed that there is only a single-scroll chaotic attractor in [24], only a double-scroll chaotic attractor in [25], and only double-scroll and four-scroll chaotic attractors in [26]. However, there are two-scroll, three-scroll, and four-scroll chaotic attractors in our memristive system. Meanwhile, the Lyapunov exponent spectrum, and phase diagram for our memristive chaotic system are obtained. Second, based on the proposed integer-order memristive chaotic system with a four-scroll chaotic attractor, a fractional-order version chaotic system is suggested. We find that the coexisting three-scroll and four-scroll chaotic attractors and coexisting two kinds of three-scroll chaotic attractors are emerged in the fractional-order version. To the best of our knowledge, this result is rarely reported.

The outline of this paper is organized as follows. In Section 2, the concept of a memristor and some memristor-based system are briefly reviewed. Based on the review, we present an integer-order memristive chaotic system with two-scroll, three-scroll, and four-scroll chaotic attractors and some basic dynamics behaviors are obtained. In Section 3, based on the integer-order memristive chaotic system with a four-scroll chaotic attractor, we present its fractional-order version and we find that there are coexisting chaotic attractors in its fractional-order version. In Section 4, the conclusion is given.

2. An Integer-Order Memristive Chaotic System

The charge-controlled memristor [24, 26] is described by a nonlinear I-V characteristic as $V_M = M(q)I_M$ and $\dot{q} = F(q, I_M)$. Here, $V_M$, $I_M$, and $q$ are the voltage, current, and charge associated to the device, respectively. $M(q)$ is the memristance, and $F(q, I_M)$ is the internal state function. In [24, 26], two schematics of the simplest memristor-based chaotic circuit with a linear passive inductor, linear passive capacitor, and a nonlinear active memristor have been reported. The state equations represent the current-voltage relation for the linear passive capacitor, and the inductor is described as

$$\frac{CdV_C}{dt} = I_L,$$

$$L\frac{dI_L}{dt} = -(V_C + M(q)I_L),$$

where $V_C$ denotes the voltage of the linear passive capacitor $C$ and $I_L$ denotes the current of the linear passive inductor $L$.

In [24], the memristance $M(q)$ is defined as $M(q) = \beta(q^2 - 1)$, and the internal state function $F(q, I_M)$ is defined as $F(q, I_M) = I_M - (\alpha + I_M^2)q$, where $I_M = -I_L$. The memristor-based circuit in [24] has a single-scroll chaotic attractor (for more details, see [24]), and its dynamics are described by

$$\frac{CdV_C}{dt} = I_L,$$

$$L\frac{dI_L}{dt} = -(V_C + \beta(q^2 - 1)I_L),$$

$$\frac{dq}{dt} = -I_L - (\alpha - I_L^2)q.$$
By numerical calculation, the Lyapunov exponent spectrum of integer-order memristive system (4) with respect to linear passive inductor $L$ can be obtained and is displayed in Figure 1.

According to Figure 1, the maximum Lyapunov exponent $\lambda_1$ is positive for the suitable $L$. The positive Lyapunov exponent $\lambda_1$ indicates that the chaotic attractor is emerged in system (4). Next, some results are shown as follows:

2.1. Two Kinds of Three-Scroll Chaotic Attractors Are Emerged in System (4). Letting $L = 1.734$, the Lyapunov exponents are $\lambda_1 = 0.0168$, $\lambda_2 = 0$, and $\lambda_3 = -0.3275$. The Lyapunov dimension is $D_L = 2 + \lambda_1 / |\lambda_3| = 2.051$; so, system (4) is fractal. The chaotic attractor is shown in Figure 2. The result in Figure 2 indicates that the three-scroll chaotic attractor is emerged in system (4).

Letting $L = 1.8$, the Lyapunov exponents are $\lambda_1 = 0.0246$, $\lambda_2 = 0$, and $\lambda_3 = -0.3152$. The Lyapunov dimension is $D_L = 2 + \lambda_1 / |\lambda_3| = 2.078$; so, system (4) is fractal. The chaotic attractor is shown in Figure 3. The result in Figure 3 indicates that the three-scroll chaotic attractor is emerged in system (4).

According to Figures 2 and 3, we find that two kinds of three-scroll chaotic attractors are emerged in our integer-order memristive chaotic system.

2.2. The Four-Scroll Chaotic Attractor Is Emerged in System (4). Letting $L = 1.4$, the Lyapunov exponents are $\lambda_1 = 0.0663$, $\lambda_2 = 0$, and $\lambda_3 = -0.3593$. The Lyapunov dimension is $D_L = 2 + \lambda_1 / |\lambda_3| = 2.1845$; so, system (4) is fractal. The chaotic attractor is displayed in Figure 4. The result in Figure 4 indicates that the four-scroll chaotic attractor is emerged in system (4).

2.3. The Two-Scroll Chaotic Attractor Is Emerged in System (4). Letting $L = 4$, the Lyapunov exponents are $\lambda_1 = 0.0397$, $\lambda_2 = 0$, and $\lambda_3 = -0.3364$. The Lyapunov dimension is $D_L = 2 + \lambda_1 / |\lambda_3| = 2.1180$; so, system (4) is fractal. The chaotic attractor is displayed in Figure 5. The result in Figure 5 indicates that the two-scroll chaotic attractor is emerged in system (4).
According to the above results, the proposed integer-order memristive chaotic system (4) in this paper can generate two- to four-scroll chaotic attractors. This result is different with many previous results [21, 23–28].

3. A Fractional-Order Memristive Chaotic System with Coexisting Chaotic Attractors

In this section, based on integer-order memristive chaotic system (4), a fractional-order version with coexisting chaotic attractors is given.

According to Figure 4 in Section 2, the four-scroll chaotic attractor is emerged in integer-order memristive system (4) with $C = 1F$, $\delta = 0.5$, $\beta = 2.4$, $\alpha = 0.75$, and $L = 1.4$. Now, based on this case, a fractional-order version memristive system is suggested, which is shown as follows:

$$\frac{d^p V_C(t)}{dt^p} = I_L(t),$$
$$\frac{d^p I_L(t)}{dt^p} = -\frac{[V_C(t) + (0.5q^4(t) - 2.4)I_L(t)]}{1.4},$$
$$\frac{d^p q(t)}{dt^p} = -I_L(t) - (0.75 - I_L(t))q(t).$$

Here, $0.92 \leq p \leq 1$ is the fractional-order version and $d^p V_C(t)/dt^p = \int_0^t (t-\tau)^{p-1}dV_C(\tau)/\Gamma(1-p)$, $d^p I_L(t)/dt^p = \int_0^t (t-\tau)^{p-1}dI_L(\tau)/\Gamma(1-p)$.

Now, by the improved version of Adams-Bashforth-Moulton numerical algorithm [36], nonlinear fractional-order system (7) with initial condition $(I_L(0), V_C(0), q(0))$ can be discretized as follows:

$$V_C(n+1) = V_C(0) + \frac{\tau^p}{\Gamma(p+2)} \left[ I_L(n+1) + \sum_{j=0}^{n} \alpha_{j+1} I_L(j) \right],$$
$$I_L(n+1) = I_L(0) + \frac{\tau^p}{\Gamma(p+2)} \left[ -V_C(n+1) + (0.5q^4(n+1) - 2.4)I_L(n+1) \right],$$
$$q(n+1) = q(0) + \frac{\tau^p}{\Gamma(p+2)} \left[ I_L(n+1) - (0.75 - (I_L(n+1))^2)q(n+1) \right] + \sum_{j=0}^{n} \alpha_{j+1} (-I_L(n+1) - (0.75 - (I_L(n+1))^2)q(j)).$$

The approximation error is as follows:

$$|V_C(t_n) - V_C(n)| = o(t^{1+p}),$$
$$|I_L(t_n) - I_L(n)| = o(t^{1+p}),$$
$$|q(t_n) - q(n)| = o(t^{1+p}).$$

In this numerical algorithm, $T$ is the total time length of numerical calculation, $N$ is the iterative calculation time, and $\tau = T/N$ is the step length. So, $t_n = n\tau$ ($n = 0, 1, 2, \ldots, N$).

Next, we study the dynamical behaviors for fractional-order system (7) by the improved version of Adams-Bashforth-Moulton numerical algorithm [36]. First, using numerical calculation, the largest Lyapunov exponents (Largest LE) of fractional-order system (7) with respect to fractional-order $p$ can be obtained, which is shown in Figure 6.
According to Figure 6, the largest Lyapunov exponent is larger than zero for $0.92 \leq p \leq 1$. The positive largest Lyapunov exponent indicates that the chaotic attractor is emerged in fractional-order system (7). Next, some results are shown as follows:

3.1. Coexisting Three- and Four-Scroll Chaotic Attractors in System (7) for $p = 0.935$.

Letting $p = 0.935$, the Largest LE is 0.3251. Therefore, fractional-order system (7) has chaotic behavior. The chaotic attractor can be obtained by numerical calculation. Here, we find that there are coexisting three-scroll and four-scroll chaotic attractors which depend on the initial conditions. For example, let the initial condition be $(-2, -1, -1)$ and $(-2, 1, 1)$. The four-scroll chaotic attractor (black line) and three-scroll chaotic attractor (red line) are shown in Figure 7.

3.2. Coexisting Two Kinds of Three-Scroll Chaotic Attractors in System (7) for $p = 0.94$.

Letting $p = 0.94$, the Largest LE is 0.3864. Therefore, fractional-order system (7) has chaotic behavior. The chaotic attractor can be obtained by numerical calculation. Here, we find that there are coexisting two kinds of three-scroll chaotic attractors which depend on the initial conditions. For example, let the initial condition be $(-2, -1, -1)$ and $(-2, 1, 1)$. The two kinds of three-scroll chaotic attractors (black line, red line) are shown in Figure 8.

3.3. Four-Scroll Chaotic Attractor in System (7) for $p = 0.99$.

Letting $p = 0.99$, the Largest LE is 0.2247. Therefore, fractional-order system (7) has chaotic behavior. By numerical calculation, we find that the four-scroll chaotic attractor is emerged in fractional-order system (7). The four-scroll chaotic attractor is shown in Figure 9.

According to Figure 7, the coexisting three-scroll and four-scroll chaotic attractors are emerged in fractional-order system (7). But, there is only a four-scroll chaotic attractor in integer-order memristive chaotic system (4) with $L = 1.4$. So, the three-scroll chaotic attractor is newly produced.

According to Figure 8, the coexisting two kinds of three-scroll chaotic attractors are obtained in fractional-order system (7) and the two kinds of three-scroll chaotic attractors do not exist in integer-order memristive chaotic system (4) with $L = 1.4$. So, two kinds of three-scroll chaotic attractor are newly produced.

According to Figure 9, four-scroll chaotic attractor is emerged in fractional-order system (7). This result is just as that of integer-order memristive chaotic system (4) with $L = 1.4$.

In summary, for integer-order memristive chaotic system (4) with $L = 1.4$, there is only a four-scroll chaotic attractor. However, for its fractional-order version, it can produce two kinds of new three-scroll chaotic attractors and has coexisting three-scroll and four-scroll chaotic attractors. These results in Section 3 are rarely reported in the previous literature.

4. Conclusions

By a linear passive capacitor $C$, a linear passive inductor $L$, and an active-charge-controlled memristor, an integer-order
memristive system is devised in this paper. The memristance \( M(q) \) is defined as a fourth-degree polynomial function determined by charge, that is, \( M(q) = \delta q^4 - \beta \). By numerical calculation, the Lyapunov exponent spectrum of the proposed memristor-based chaotic circuit with respect to linear passive inductor \( L \) is yielded. The proposed integer-order memristive system can generate two-scroll, three-scroll, and four-scroll chaotic attractors for suitable linear passive inductor \( L \).

Furthermore, based on the proposed integer-order memristive system with a four-scroll chaotic attractor for \( L = 1, 4 \), a fractional-order version memristive system is given. By numerical calculation, we obtain the largest Lyapunov exponent with respect to fractional-order \( p \). This fractional-order version memristive system can newly produce two kinds of three-scroll chaotic attractors, and the coexisting three-scroll and four-scroll chaotic attractors are obtained.

**Disclosure**

The data used in our manuscript is obtained by MATLAB program.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

**References**


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