Adaptive Feedback Control for Synchronization of Chaotic Neural Systems with Parameter Mismatches

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This work pertains to the study of the synchronization problem of a class of coupled chaotic neural systems with parameter mismatches. By means of an invariance principle, a rigorous adaptive feedback method is explored for synchronization of a class of coupled chaotic delayed neural systems in the presence of parameter mismatches. Finally, the performance is illustrated with simulations in a two-order neural systems.

1. Introduction

Pecora and Carroll firstly addressed the chaotic synchronization in systems and used the drive-response conception in [1]. The idea is to control the response signal by using the output signal of drive system such that the two kinds of signals synchronize. The problems of synchronization in chaotic dynamical systems have received increasing attention in the control areas [2–6]. Different approaches including adaptive design control [2], intermittent control [3], adaptive-impulsive control [4], and sliding mode control [6] have been proposed. In particular, Liu et al. [3] obtained novel synchronization criteria for exponential synchronization of chaotic systems with time delays via periodically intermittent control. By an adaptive feedback control technique, the synchronization of a class of chaotic systems with unknown parameters is achieved via mimicking model reference adaptive control-like structure in [2]. Tam et al. [5] addressed adaptive synchronization of complicated chaotic systems with unknown parameters via a set of fuzzy modeling-based adaptive strategy. Chen et al. [6] designed a sliding mode control scheme for adaptive synchronization of multiple response systems under the effects of external disturbances.

Recently, there has sprung up hot research topics in the synchronization of chaotic neural systems (CNSs) due to possible chaotic behaviors in such systems [7–13]. For instance, synchronization of coupled delayed CNSs and applications to memristive CNSs in [12] have resulted in a theoretical condition under an irreducible assumption on coupling matrix. Cao and Lu [13] proposed a simple adaptive method for the synchronization of uncertain CNSs with or without variable delay via invariant principles. In particular, some efforts have been devoted to adaptive synchronization of CNSs [10–12, 14]. However, most existing works were applicable only for the CNSs with parameter matching. While in practical implementation of synchronized CNSs, it is well known that parameter mismatch in systems is generally inevitable [15–17], which will result in poor performance or loss of synchronization [17, 18]. For example, Zhang et al. [18] discussed asymptotical synchronization for delayed CNSs with fully unknown parameters based on the Lyapunov method and the inverse optimal method. Therefore, it is of importance to explore the effects of parameter mismatch in synchronization of CNSs.

In this paper, we present theoretical analysis and numerical simulations of the parameter mismatch effect on synchronization for a class of coupled CNSs. By using
adaptive control approaches [13, 19, 20] instead of traditional linear coupling scheme, and combining the invariance principle, we show that adaptive synchronization of such CNSs with parameter mismatches under loose conditions can be rapidly achieved. In addition, by adjusting the update gain of coupling strength introduced in this work, one can control the synchronization speed.

The organization of this work is as follows. In Section 2, we present needed formulation of synchronization of CNSs. Section 3 presents an adaptive control scheme in CNSs and provides two criteria for synchronization for CNSs. In Section 4, numerical simulations on a two-order CNS are provided to show the effectiveness of the proposed results. Section 5 concludes the paper.

Notation 1. Throughout the paper, we denote $A^T$ and $A^{-1}$ the transpose and the inverse of any square matrix $A$. $A > 0$ ($A < 0$) denotes a positive- (negative-) definite matrix $A$; and $I$ is used to denote the $n \times n$ identity matrix. $\|A\|$ denotes the spectral norm of matrix $A$. Let $\mathbb{R}$ denote the set of all real numbers, $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space, and $\mathbb{R}^{n \times m}$ denotes the set of all $n \times m$ real matrices. $\lambda_{\text{max}}(\cdot)$ or $\lambda_{\text{min}}(\cdot)$ denotes the largest or smallest eigenvalue of a matrix, respectively.

2. Formulation of Synchronization in Neural Networks

Consider the following CNS in a general form:

$$\dot{x}(t) = -C_1x(t) + A_1f(x(t)) + B_1f(x(t - \tau(t))) + J,$$  \hspace{1cm} (1)

where $x(t) = (x_1(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ denotes the state vector; $C_1$ represents a diagonal matrix with $c_{ij} > 0$, $i = 1, 2, \ldots, n$; $A_1 = (a_{ij})_{n \times n}$ denotes the weight matrix; $B_1 = (b_{ij})_{n \times n}$ denotes the delayed weight matrix; $J = (J_1, \ldots, J_n)^T \in \mathbb{R}^n$ is the input vector function; $\tau(t)$ represents the transmission variable delay; and $f(x(t)) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T$ represents the activation function.

Throughout the paper, we have the following two assumptions:

(A1) Each $f_j : \mathbb{R} \rightarrow \mathbb{R}$ satisfies the Lipschitz condition, that is, there exist positive scalars $k_j > 0$ such that

$$|f_j(x) - f_j(y)| \leq k_j|x - y|, j = 1, 2, \ldots, n,$$  \hspace{1cm} (2)

for any $x, y \in \mathbb{R}$.

(A2) $\tau(t) \geq 0$ is a function satisfying $\tau^* = \max \{\tau(t)\}$ and $0 \leq \tau(t) \leq \sigma < 1$, for all $t$.

$x_i(t) = \phi_i(t) \in \mathcal{C}([-\tau^*, 0], \mathbb{R})$ denotes initial conditions of (1), where $\mathcal{C}([-\tau^*, 0], \mathbb{R})$ represents the set of continuous functions from $[-\tau^*, 0]$ to $\mathbb{R}$.

To synchronize the drive (or master) system (1), the controlled response (or slave) system is given by

$$\dot{y}(t) = -C_2y(t) + A_2f(y(t)) + B_2f(y(t - \tau(t))) + J + u(t),$$  \hspace{1cm} (3)

where $u(t)$ is the driving signal, $y(t) = (y_1(t), \ldots, y_n(t))^T \in \mathbb{R}^n$, $C_2$, $A_2$, and $B_2$ are generally different from $C_1$, $A_1$, and $B_1$, respectively. Namely, parameter mismatches exist between the drive system and the response system. The initial conditions of system (3) denote $y_i(t) = \psi_i(t) \in \mathcal{C}([-\tau^*, 0], \mathbb{R})$ ($i = 1, 2, \ldots, n$). Denote the mismatch errors by $\Delta C = C_2 - C_1$, $\Delta A = A_2 - A_1$, and $\Delta B = B_2 - B_1$.

We aim to design an appropriate controller $u(t)$ in order to make the coupled CNSs remain synchronized in the presence of even large parameter mismatches. First, consider the feedback controller $u(t) = e \circ (y(t) - x(t))$, where $e = (e_1, \ldots, e_n)^T \in \mathbb{R}^n$ is the coupling strength, and the symbol $\circ$ is defined as

$$e \circ (y(t) - x(t)) \triangleq [e_1(y_1(t) - x_1(t)), \ldots, e_n(y_n(t) - x_n(t))]^T.$$  \hspace{1cm} (4)

Define the synchronization error as $e(t) = y(t) - x(t)$, which leads to the following synchronization error system:

$$\dot{e}(t) = -C_2e(t) + A_2g(e(t)) + B_2g(e(t - \tau(t)))$$
$$- (C_2 - C_1)x(t) + (A_2 - A_1)f(x(t))$$
$$+ (B_2 - B_1)f(x(t - \tau(t))) + e \circ e(t),$$  \hspace{1cm} (5)

or

$$\dot{e}(t) = -C_2e(t) + A_2g(e(t)) + B_2g(e(t - \tau(t))) - \Delta C x(t)$$
$$+ \Delta A f(x(t)) + \Delta B f(x(t - \tau(t))) + e \circ e(t),$$  \hspace{1cm} (6)

where

$$e(t) = (e_1(t), \ldots, e_n(t))^T,$$
$$g(e(t)) = (g(e_1(t)), \ldots, g(e_n(t)))^T,$$

with $g_i(e_i(t)) = f_j(e_i(t) + x_i(t)) - f_j(x_i(t))$, $i = 1, 2, \ldots, n$.

Obviously, using the assumption (A1), $g_i(\cdot)$ has the following properties:

$$|g_i(e_i)| \leq k_i |e_i|.$$  \hspace{1cm} (8)

3. Adaptive Control Scheme

In this section, based on Lyapunov function and an invariance principle by combining an adaptive control approach, we consider the adaptive synchronization for two CNSs with time-varying delay and parameter mismatches.
\textbf{Theorem 1.} Suppose that $\chi = \{x \in \mathbb{R}^n \mid ||x|| \leq \alpha_1\}$ and the parameter mismatches satisfy $\|\Delta C\| + (\|\Delta A\| + 1/1 - \sigma \|\Delta B\|) K^2 \leq \alpha_2$, where $K = \max_{\delta > 0} \{\delta_k\}$. Under the assumptions (A1) and (A2), let $\alpha = \alpha_1, \alpha_2$ and the controller $u(t) = e \circ (y(t) - x(t)) = e \circ e(t)$ with the following update law:

$$
i = -\delta i \left( \frac{\alpha}{\delta_i + 1} \chi_i^2 (t) \right),$$

(9)

where $\delta_i > 0$ $(i = 1, 2, \ldots, n)$ are arbitrary constants, and $l > 0$ is a constant to be determined. Then, the controlled uncertain response system (3) will globally synchronize with the drive system (1).

\textbf{Proof 1.} Consider the following Lyapunov function for the error dynamical system:

$$V(t) = e^T(t) e(t) + \sum_{i=1}^{n} \delta_{i} (\epsilon_i + l/2) + \frac{1}{1 - \sigma} \int_{t-\tau (t)}^{t} g^T (e(s)) g(e(s)) ds$$

$$+ \frac{1}{1 - \sigma} \int_{t-\tau (t)}^{t} f^T (x(s)) \Delta B^T \Delta B f(x(s)) ds.$$

(10)

Calculating the derivative of (10) along the trajectories of (6), we have

$$\dot{V}(t) = 2 e^T(t) \dot{e}(t) - 2 \sum_{i=1}^{n} (\epsilon_i + l/1) \left( \chi_i^2 + \frac{\alpha}{\delta_i + 1} \chi_i^2 (t) \right)$$

$$+ \frac{1}{1 - \sigma} g^T (e(t)) g(e(t)) + \frac{1}{1 - \sigma} f^T (x(t)) \Delta B^T \Delta B f(x(t))$$

$$- \frac{1 - \delta (t)}{1 - \sigma} g^T (e(t - \tau (t))) g(e(t - \tau (t)))$$

$$- \frac{1 - \delta (t)}{1 - \sigma} f^T (x(t - \tau (t))) \Delta B^T \Delta B f(x(t - \tau (t)))$$

$$\leq 2 e^T(t) (-C_2 \epsilon + A_2 g(e(t)) + B_2 g(e(t - \tau (t))))$$

$$- 2 e^T(t) \Delta C x(t) + 2 e^T(t) \Delta A f(x(t))$$

$$+ 2 e^T(t) \Delta B f(x(t - \tau (t))) + 2 e^T(t) \Delta B f(x(t))$$

$$- 2 \sum_{i=1}^{n} (\epsilon_i + l/1) \left( \chi_i^2 + \frac{\alpha}{\delta_i + 1} \chi_i^2 (t) \right) + \frac{1}{1 - \sigma} g^T (e(t)) g(e(t))$$

$$+ \frac{1}{1 - \sigma} f^T (x(t)) \Delta B^T \Delta B f(x(t))$$

$$- g^T (e(t - \tau (t))) g(e(t - \tau (t)))$$

$$- f^T (x(t - \tau (t))) \Delta B^T \Delta B f(x(t - \tau (t)))$$

$$\leq 2 e^T(t) (-C_2 \epsilon + A_2 g(e(t)) + B_2 g(e(t - \tau (t))))$$

$$- 2 e^T(t) \Delta C x(t) + 2 e^T(t) \Delta A f(x(t))$$

$$+ 2 e^T(t) \Delta B f(x(t - \tau (t))) - 2 \sum_{i=1}^{n} \chi_i^2 (t) - le^T(t) e(t)$$

$$+ \frac{1}{1 - \sigma} g^T (e(t)) g(e(t)) + \frac{1}{1 - \sigma} f^T (x(t)) \Delta B^T \Delta B f(x(t))$$

$$- g^T (e(t - \tau (t))) g(e(t - \tau (t)))$$

$$- f^T (x(t - \tau (t))) \Delta B^T \Delta B f(x(t - \tau (t))).$$

(11)

Using (8) and the Lemma 2 in [21], one can obtain

$$2 e^T(t) A_2 g(e(t)) \leq e^T(t) A_2 A_2^T e(t) + g^T (e(t)) g(e(t)),$$

(12)

$$2 e^T(t) B_2 g(e(t - \tau (t))) \leq e^T(t) B_2 B_2^T e(t) + g^T (e(t - \tau (t))) g(e(t - \tau (t))),$$

(13)

$$- 2 e^T(t) \Delta C x(t) \leq \|e(t)\| + \|\Delta C\| \|x(t)\|,$$

(14)

$$2 e^T(t) \Delta A f(x(t)) \leq \|e(t)\| + \|\Delta A\| K^2 \|x(t)\|,$$

(15)

$$2 e^T(t) \Delta B f(x(t - \tau (t))) \leq \|e(t)\| + f^T (x(t - \tau (t))) \Delta B^T \Delta B f,$$

(16)

$$\frac{1}{1 - \sigma} f^T (x(t)) \Delta B^T \Delta B f(x(t)) \leq \frac{1}{1 - \sigma} \|\Delta B\| K^2 \|x(t)\||,$$

(17)

Substituting (12), (13), (14), (15), (16), and (17) into (11) and combining

$$\left[ \|\Delta C\| + \left( \|\Delta A\| + \frac{1}{1 - \sigma} \|\Delta B\| \right) K^2 \right] \|x(t)\| \leq 2 \alpha$$

(18)

yield

$$\dot{V}(t) \leq e^T(t) (-2 C_2 + A_2 A_2^T + B_2 B_2^T + (3 - l) I) e(t)$$

$$+ g^T (e(t)) g(e(t)) + \frac{1}{1 - \sigma} g^T (e(t)) g(e(t))$$

$$\leq e^T(t) \left( 2 \lambda_{\max} (-C_2) + \lambda_{\max} (A_2 A_2^T) + \lambda_{\max} (B_2 B_2^T) \right)$$

$$+ K^2 + \frac{1}{1 - \sigma} K^2 + 3 - l \),

(19)

We properly choose the constant $l$ as

$$l = 2 \lambda_{\max} (-C_2) + \lambda_{\max} (A_2 A_2^T) + \lambda_{\max} (B_2 B_2^T)$$

$$+ K^2 + \frac{1}{1 - \sigma} K^2 + 4,$$

(20)

then we have $\dot{V} \leq - e^T(t) e(t)$.

It is obvious that $\dot{V} = 0$ if and only if $e_i = 0, i = 1, 2, \ldots, n$. It implies that the set $E = \{e(t), e^T \in \mathbb{R}^{2n} : e(t) = 0, e = e_0 \in \mathbb{R}^{n}\}$ is the largest invariant set included in $M = \{\dot{V} = 0\}$ for system (6). Then, using the well-known Lyapunov-LaSalle-type theorem, the error converges asymptotically to $E$, that is, $e(t) \to 0$ and $e \to e_0$ as $t \to \infty$. Therefore, the synchronization of the CNSs (1) and (3) is achieved under the coupling (9). The proof is completed.

For the coupled CNSs without time-varying delay (i.e., $B_1 = 0$ in (1) and $B_2$ in (3)), one can easily derive the
following corollary for two CNSs without delay (drive system and response system, resp.):

\[
\dot{x}(t) = -C_1 x(t) + A_1 f(x(t)) + J,
\]

and

\[
\dot{y}(t) = -C_2 y(t) + A_2 f(y(t)) + J + \epsilon \circ (y(t) - x(t)).
\]

**Corollary 1.** Suppose that \( \chi = \{ x \in \mathbb{R}^n \mid \| x \| \leq \alpha_1 \} \) and the parameter mismatches satisfy \( \| \Delta C \| + \| \Delta A \| K^2 \leq \alpha_2 \), where \( K = \max_{1 \leq i \leq n} \sigma_i(k_i) \). Under the assumptions (A1) and (A2), set \( \alpha = \alpha_1 \cdot \alpha_2 \) and the controller \( u(t) = \epsilon \circ (y(t) - x(t)) = \epsilon \circ e(t) \) with the following update law:

\[
\dot{\epsilon}_i = -\delta_i \left( \epsilon_i^2 + \frac{\alpha}{\epsilon_i + l} x_i^2(t) \right),
\]

where \( \delta_i > 0 \) \( (i = 1, 2, \ldots, n) \) are arbitrary constants; \( l \) is chosen as

\[
l = 2\lambda_{\max}(-C_2) + \lambda_{\max}(A_2 A_2^T) + K^2 + \frac{1}{1-\sigma} K^2 + 3.
\]

Then, the controlled uncertain response system (3) will globally synchronize with the drive system (1).

**Remark 1.** It is noted from Theorem 1 that one can choose the constant \( \delta_i \) properly to adjust the synchronization speed. Large adaptive gain \( \delta_i \) will lead to fast synchronization, while small adaptive gain \( \delta_i \) will result in slow synchronization. In addition, such a way is robust against the effect of noise. An extension effort that extends the results for systems with hybrid characteristics as in [7, 22–24] is possible, which remains an open problem.

**Remark 2.** In [9, 11, 13], the derived results are applicable for CNSs with parameter matches. While the results here are suitable for parameter mismatches. Therefore, our results have more expansive application foreground. In addition, using adaptive feedback method, the criterion obtained here improves and extends the results reported in [9, 11, 13].

### 4. Numerical Simulations

In this section, a numerical example is employed to illustrate our results. Simulation results show that the proposed adaptive synchronization scheme is valid.

**Example 1.** Consider the following two-order CNSs with time-varying delay:

\[
\dot{x}(t) = -C_1 x(t) + A_1 f(x(t)) + B_1 f(x(t - \tau(t))) + J,
\]

with

\[
C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 2.1 & -0.12 \\ -5.1 & 3.2 \end{bmatrix}, B_1 = \begin{bmatrix} -1.6 & -0.1 \\ -0.2 & -2.4 \end{bmatrix}, J = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
\]

and

\[
\tau(t) = \epsilon^2 / (1 + \epsilon^2), \quad \text{where} \quad x(t) = [x_1(t), x_2(t)]^T, f(x(t)) = [\tanh(x_1(t)), \tanh(x_2(t))]^T.
\]

It is seen that \( k_1 = k_2 = 1 \), and thus \( K = 1 \). Moreover,

\[
\tau^* = 1, \quad \dot{\epsilon}(t) = -\epsilon^2 / (1 + \epsilon^2)^2 \in [0, 0.5],
\]

that is, \( \sigma = 0.5 \). Therefore, (A1) and (A2) hold.

Note that the neural system in this example is chaotic. Figure 1 shows the phase plot of the CNS, and Figure 2 illustrates the power spectral plot of the CNS.
illustrates the power spectral with initial values $\phi_1(s) = -0.5$ and $\phi_2(s) = 0.3$, $\forall s \in [-1, 0]$. It is found in Figure 1 that the double-scroll attractor is confined within the set.

$$\chi = \left\{ x = (x_1, x_2)^T \mid -1 \leq x_1 \leq 1, -5 \leq x_2 \leq 5 \right\}, \quad (28)$$

In this case, it is verified that $a_1 = 5.0990$.

To verify the effectiveness of the proposed method, consider the output signals of drive system in CNS (25). Then, the controlled response system is given by

$$\dot{y}(t) = -\tilde{C}y(t) + \tilde{A}f(y(t)) + \tilde{B}f(y(t - \tilde{r}(t))) + J + u(t), \quad (29)$$

where $\tilde{r}(t) = 1 + 1/2 \sin (t)$, $y(t) = (y_1(t), y_2(t))^T$ and

$$C_2 = \begin{bmatrix} 1.1 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.2 & -0.1 \\ -5.0 & 3.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1.7 & -0.05 \\ -0.3 & -2.3 \end{bmatrix}, \quad J = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (30)$$

Therefore, we obtain

$$\Vert \Delta C \Vert + \left( \Vert \Delta A \Vert + \frac{1}{1 - \sigma} \Vert \Delta B \Vert \right) K^2 \leq a_2 = 0.7128, \quad (31)$$

then $\alpha = a_1 \cdot a_2 = 3.6345$.

By Theorem 1, since

$$2\lambda_{\text{max}}(-C_2) + \lambda_{\text{max}}(A_2 A_2^T) + \lambda_{\text{max}}(B_1 B_1^T) + K^2 + \frac{1}{1 - \sigma} K^2 + 4 = 48.9486, \quad (32)$$

take $l = 50$. Then, we can design the controller $u(t) = e \circ (y(t) - x(t))$ with the adaptive update laws

$$\dot{\epsilon}_1 = -0.5 \left( (y_1(t) - x_1(t))^2 + \frac{\alpha}{\epsilon_1 + 1} x_1^2(t) \right), \quad (33)$$

$$\dot{\epsilon}_2 = -0.5 \left( (y_2(t) - x_2(t))^2 + \frac{\alpha}{\epsilon_2 + 1} x_2^2(t) \right)$$

Here, the adaptive gains are taken as $\delta_1 = \delta_2 = 0.5$. Next, suppose the initial conditions are

$$(\phi_1(s), \phi_2(s))^T = (0.2, 0.5)^T$$

respectively, and $\epsilon_1(0) = \epsilon_2(0) = 0$. The simulation results are depicted in Figures 3–6. Figure 3 shows the temporal evolution of states and errors for $\delta_1 = \delta_2 = 0.5$. When $\delta_1 = \delta_2 = 0.3$, namely, decreasing the update gain of coupling strength, Figure 4 shows the corresponding simulation results and it is revealed that it takes longer to achieve synchronization. From Figures 3 and 4, we found that less time is needed to achieve synchronization when larger $\delta_1$ and $\delta_2$ are taken. When $\delta_1 = 0$ and $\delta_2 = 0.5$, Figure 5 shows the results for the case that only $x_2$ is chosen.
as the drive signal. When $\delta_1 = 0.5$ and $\delta_2 = 0$, Figure 6 shows the results for the case that only $x_1$ is chosen as the drive signal. It can be seen that the coupling between $x_1$ and $y_1$ would drive the two CNSs (25) and (29) synchronized, while the coupling between $x_2$ and $y_2$ is invalid.

5. Conclusions

This paper has analyzed the adaptive synchronization between two coupled CNSs with parameter mismatches by applying an invariance principle and a simple adaptive
feedback approach. Practical and less restrictive conditions have been presented for adaptive synchronization of CNSs. Numerical simulations of two-order-coupled CNSs have also been provided to verify the usefulness and practicality of proposed theoretical results.

**Data Availability**

The main results of our work are proved in detail, which can be seen in the context. All data related to the simulation part of our results are given in Section 4. The readers can replicate the analysis clearly.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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