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Research Article

Couple-Group Consensus: A Class of Delayed Heterogeneous Multiagent Systems in Competitive Networks

Lianghao Ji, Yue Zhang, and Yiliu Jiang

Chongqing Key Laboratory of Computational Intelligence, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

Correspondence should be addressed to Lianghao Ji; lianghao.ji@gmail.com

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This paper discusses the couple-group consensus issues of a class of heterogeneous multiagent systems containing first-order and second-order dynamic agents under the influence of both input and communication delays. In distinction to the existing works, a novel distributed coordination control protocol is proposed which is not only on the foundation of the competitive interaction between the agents but also has no virtual velocity estimation in the first-order dynamics. Furthermore, without the restrictive assumptions existing commonly in the related works, several sufficient algebraic criteria are established for the heterogeneous systems to realize couple-group consensus asymptotically. The obtained conclusions show that the achievement of the systems' couple-group consensus intimately relates to the coupling weights between the agents, the systems control parameters, and the input time delays of the agents, while communication time delays between the agents are irrelevant to it. Finally, several simulations are illustrated to verify the effectiveness of the obtained theoretical results.

1. Introduction

As a fundamental issue of the coordinated control of multiagent systems (MASs), consensus problem of MASs has attracted wide attention in recent years due to its wide applications in various fields, including the mobile robot systems, distributed target tracking, and group decision-making. So far, lots of research works on various consensus problems have been constantly reported, such as in [1–5] and the refs. therein.

Group consensus, as an extension of consensus issue, implies that distinctive subconsensus states can be achieved in complex systems while separate subgroups cannot reach agreement. In the last few years, lots of research works on group consensus have widely emerged, such as in [6–14].

1.1. Related Works. Note that all the works mentioned above mainly focus on the MASs which constructed by multiple agents with homogeneous dynamics. Namely, all agents of the entire complex systems featured the identical dynamics. In fact, this case is very restricted because in the real world, the difference existed in the dynamics among agents is unavoidable. Meanwhile, we usually have such applications using multiple agents with different dynamics to achieve the desired goals for the case of reducing control cost. Therefore, more and more concerns are devoted by the researchers to heterogeneous MASs' consensus problems, such as the [15], in which the necessary and sufficient criteria were proposed for the synchronization of the heterogeneous networks. Meanwhile, Wang et al. [16] also investigated the heterogeneous systems and addressed certain sufficient conditions to ensure the achievement of consensus. With regard of the time delays, Cui et al. discussed the consensus problem for the heterogeneous chaotic systems in [17]. Based on undirected topology, Goldin and Raisch addressed the consensus of the heterogeneous networks in [18]. In [19], Wang et al. studied the globally limited consensus of the heterogeneous systems and put forward certain criteria to guarantee the consensus. The consensus issues for the heterogeneous systems with time delays were also studied in [20, 21]. As for the discrete-time heterogeneous systems, Kim et al. [22] and Li et al. [23], respectively, discussed the consensus problems for the systems with accidental connection failures. The second-order consensus for the systems with Euler-Lagrange networks was investigated in [24], where the multiple consensus states were achieved. In [25], the effective protocols for achieving consensus of the heterogeneous systems were designed and certain conditions based on the state transformation method were obtained. Based on certain hypothesis, Liu et al. also investigated the heterogeneous MASs and its group consensus problems in [26]. Considering the influence of input delays or not, Wen et al. studied the group consensus of the systems with heterogeneous dynamics [27, 28], respectively. By utilizing the Lyapunov method, in [29], Qin et al. studied both linear and nonlinear heterogeneous systems as well as their group consensus issues.

1.2. Primary Motivations. Note that most of the research works mentioned above have been paid attention to consensus problems of the heterogeneous MASs. As we know, group consensus owns an important practical significance in the coordinated control of large scale and complex tasks. Hence, many further works for group consensus of heterogeneous MASs need to be concerned with. Meanwhile, it is known that communication and input time delays existing in the complex systems can usually affect and even destroy the stability of the system. Therefore, one motivation of this paper is to discuss the group consensus problems of a class of heterogeneous MASs with time delays.

The main contributions of this paper are summarized in the following three aspects. First, to our knowledge, there are rare works on the group consensus issue of heterogeneous MASs which considered both input and communication delays. Second, from a different point of view, we propose an original couple-group consensus protocols which established on the agents' competitive interaction. It is distinguished with the aforementioned works which mainly modeled by the agents' cooperative relationship [15–28]. It is known that competitive interaction is also an important relationship in complex systems, such as in ecology, the problems of predator-prey on food chain. Meanwhile, in our control protocol, the first-order agents' dynamics possess no virtual velocity, which been included in many related works (e.g., in [20, 26–28]) for simplifying the process of analysis. Third, our results relax the following two conservative prerequisites existed in [20, 25–29]: in-degree balance and the geometric multiplicity of the zero eigenvalue of the systems' Laplacian matrix have no less than 2. Both of them limit the communication of the agents and the topology of the system. Based on matrix theory and frequency domain analysis, some criteria for ensuring the achievement of couple-group consensus are effectively proposed. The combination of the above innovative points makes the application scope of our results more general.

Notation. Throughout this paper, $\mathbb{R}$ and $\mathbb{C}$ indicate the real and complex numbers sets, respectively. $\forall z \in \mathbb{C}$, then its real part and modulus are defined as $\mathrm{Re}\,(z)$ and $|z|$. I_N stands for the identity matrix with N-dimension. $\det(A)$ and $\lambda_i(A)$ mean the determinant and the ith eigenvalue of the matrix A, individually.

2. Preliminaries and Problem Statements

2.1. Graph Theory. Using the graph theory, the agents and their information exchange in a MAS with N agents can be represented by a digraph $G = (V, E, A)$, where $V = \{v_1, v_2, \ldots, v_N\}$, $E \subseteq V \times V$, and $A = (a_{ij})_{N\times N} \in \mathbb{R}^{N\times N}$ denote the node set, the edge set, and the adjacency matrix, respectively. Noting that an undirected graph can be seen as a special digraph, we assume $a_{ij} > 0$ if $e_{ij} \in E$ throughout this paper. Namely, $a_{ij} > 0$ if and only if the node (agent) v_i can receive the information from the node (agent) v_j; otherwise, $a_{ij} = 0$. Meanwhile, the neighbour sets and the in-degree of node i are defined as $N_i = \{j \in V : e_{ij} \in E\}$ and $D_i = \deg_{\mathrm{in}}(i) = \sum_{j=1}^{N} a_{ij}$, individually. Design in-degree matrix as $D = \mathrm{diag}\,\{d_1, d_2, \ldots, d_N\}$, so $L = D - A$ is defined to be the Laplacian matrix.

2.2. Problem Statement. In this paper, we suppose a heterogeneous multiagent system consisting of $n + m$ agents which contained first-order and second-order dynamics. For convenience, assume the first n and the remaining m agents own second-order and first-order dynamics, respectively, then the system can be described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \\ \qquad i \in g_1, \end{cases}$$
$$\dot{x}_i(t) = u_i(t), \quad i \in g_2, \tag{1}$$

where $g_1 = \{1, 2, \ldots, n\}$, $g_2 = \{n+1, n+2, \ldots, n+m\}$, and $g = g_1 \cup g_2$. $x_i(t)$, $v_i(t)$, and $u_i(t) \in \mathbb{R}$ represent position, velocity, and input control of agent i, individually.

Regard a heterogeneous MAS, to each agent, its neighbor may has second-order and first-order dynamics, which may be distinguishingly denoted as $N_{i,s}$ and $N_{i,f}$. Thus, the neighbors set of agent i can be distinguished as $N_i = N_{i,f} \cup N_{i,s}$. With regard that the agents' dynamics in the systems are heterogeneous, then adjacency matrix A can be denoted as

$$A = \begin{bmatrix} A_s & A_{sf} \\ A_{fs} & A_f \end{bmatrix}, \tag{2}$$

where $A_s \in \mathbb{R}^{n\times n}$ and $A_f \in \mathbb{R}^{m\times m}$ represent the adjacency matrix consisting of all second-order or first-order agents, A_{fs} composes of coupling weights from first-order agents to second-order ones, and A_{sf} is the opposite of A_{fs}. The systems' Laplacian matrix can be modified as

$$L = D - A = \begin{bmatrix} L_s + D_{sf} & -A_{sf} \\ -A_{fs} & L_f + D_{fs} \end{bmatrix}, \tag{3}$$

in the Laplacian matrix L_s and L_f, the interactions of only second-order agents or first-order agents are included; $D_{sf} = \mathrm{diag}\,\{\sum_{j\in N_{i,f}} a_{ij}, i \in g_1\}$ and $D_{fs} = \mathrm{diag}\,\{\sum_{j\in N_{i,s}} a_{ij}, i \in g_2\}$

are in-degree matrix of agent i, which includes information received from the neighbors of different orders.

Firstly, some fundamental definitions and lemma are introduced as below.

Definition 1. To a heterogeneous MASs, such as (1), which can be said to achieve couple-group consensus asymptotically when and only when the following two conditions are satisfied:

$$\begin{aligned} &\lim_{t\to+\infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad \text{if } i, j \in g_k,\ k = 1, 2, \\ &\lim_{t\to+\infty} \left\| v_i(t) - v_j(t) \right\| = 0, \quad \text{if } i, j \in g_k,\ k = 1. \end{aligned} \tag{4}$$

Definition 2. For a bipartite graph $G = (V, E)$, where E and V denote its edge and vertex sets, it has the following two properties: (1) $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \varnothing$ and (2) $\forall e = (p, q) \in E$, where $p \in V_1$ and $q \in V_2$.

Lemma 1 [10]. *Regarding a directed bipartite graph which contains a directed spanning tree, it then possesses the following two properties:* rank $(L) = n - 1$ *and* Re $(\lambda_i(L)) > 0$ *when* $\lambda_i(L) \neq 0$. *Meanwhile, for a undirected bipartite graph,* $\lambda_i(L) \in R$. *Where n is the number of agents of systems, matrix* $L = D + A$.

3. Main Results

In [18], the authors investigated the group consensus of the heterogeneous systems with identical input time delay. The systems are described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = \sum_{j\in g_1} a_{ij}\left[x_j(t-\tau) - x_i(t-\tau)\right] + \sum_{j\in g_2} a_{ij}x_j(t-\tau) + \sum_{j\in g_1} a_{ij}\left[v_j(t-\tau) - v_i(t-\tau)\right] + \sum_{j\in g_2} a_{ij}v_j(t-\tau), \\ \qquad i \in g_1, \end{cases} \tag{5}$$

$$\begin{cases} \dot{x}_i(t) = v_i(t-\tau) + \sum_{j\in g_2} a_{ij}\left[x_j(t-\tau) - x_i(t-\tau)\right] + \sum_{j\in g_1} a_{ij}x_j(t-\tau), \\ \dot{v}_i(t) = \sum_{j\in g_2} a_{ij}\left[x_j(t) - x_i(t)\right] + \sum_{j\in g_1} a_{ij}x_j(t), \\ \qquad i \in g_2. \end{cases} \tag{6}$$

For the sake of analysis, the velocity estimation is added to the first-order agents in (6). Meanwhile, the systems (5) and (6) are modeled by the cooperative relationship among the agents.

To realize group consensus and be distinguished to (5) and (6), we design an original distributed control protocol which utilizes the agents' competitive interactions. It is showed as follows:

$$\begin{aligned} &\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = -\alpha\left[\sum_{j\in N_i} a_{ij}\left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right]\right] - \beta v_i(t-\tau), \\ \qquad i \in g_1, \end{cases} \\ &\dot{x}_i(t) = -\gamma\left[\sum_{j\in N_i} a_{ij}\left[x_j(t-\tau_{ij}) + x_i(t-\tau)\right]\right], \quad i \in g_2, \end{aligned} \tag{7}$$

where τ_{ij} indicates communication delay between agent j and agent i and τ represents the identical input delay of the agents. Meanwhile, we suppose the control parameters α, β, and $\gamma > 0$.

Theorem 1. *Consider the MASs (7) and suppose the systems' topology is an undirected bipartite graph, the systems' couple-group consensus can be realized asymptotically if these conditions hold:* $\beta^2 > 2\alpha D_i$ *and* $\tau \in [1/2\beta, 1/2\gamma \max\{\tilde{D}_i\}]$, *where* $D_i = \sum_{j\in N_i} a_{ij}$, $i \in g_1$ *and* $\tilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in g_2$.

Proof 1. Do Laplace transforms to (7) firstly, we get

$$\begin{cases} sx_i(s) = v_i(s), \\ sv_i(s) = -\alpha\left[\sum_{j\in N_i} a_{ij}\left[e^{-\tau_{ij}s}x_j(s) + e^{-\tau s}x_i(s)\right]\right] - \beta e^{-\tau s}v_i(s), \\ \qquad i \in g_1, \end{cases} \tag{8}$$

$$sx_i(s) = -\gamma\left[\sum_{j\in N_i} a_{ij}\left[e^{-\tau_{ij}s}x_j(s) + e^{-\tau s}x_i(s)\right]\right], \quad i \in g_2, \tag{9}$$

where $x_i(s)$ and $v_i(s)$ represent the Laplace transform of $x_i(t)$ and $v_i(t)$, respectively.

From the (8), we have

$$s^2x_i(s) = -\alpha\left[\sum_{j\in N_i} a_{ij}\left[e^{-\tau_{ij}s}x_j(s) + e^{-\tau s}x_i(s)\right]\right] - \beta s e^{-\tau s}x_i(s), \quad i \in g_1. \tag{10}$$

After a simple manipulation, it yields that

$$sx_i(s) = \frac{-s^2x_i(s) - \alpha\left[\sum_{j\in N_i} a_{ij}\left[e^{-\tau_{ij}s}x_j(s) + e^{-\tau s}x_i(s)\right]\right]}{\beta e^{-\tau s}}, \quad i \in g_1. \tag{11}$$

Define $x_s(s) = [x_1(s), x_2(s), \dots, x_n(s)]^T$, $x_f(s) = [x_{n+1}(s), x_{n+2}(s), \dots, x_{n+m}(s)]^T$, and

$$\widehat{L} = \left(\widehat{l}_{ij}\right)_{(n+m)\times(n+m)} = \begin{cases} e^{-\tau_{ij}s}a_{ij}, & i \neq j, \\ \sum_{j\in N_i} a_{ij}e^{-\tau s}, & i = j, \end{cases} \tag{12}$$

then from the (9) and (11), we have

$$\begin{aligned} sx_s(s) &= \frac{\left[-s^2x_s(s) - \alpha\left(\widehat{L}_s + \widehat{D}_{sf}\right)x_s(s) - \alpha\widehat{A}_{sf}x_f(s)\right]}{\beta e^{-\tau s}}, \\ sx_f(s) &= -\gamma\widehat{A}_{fs}x_s(s) - \gamma\left(\widehat{L}_f + \widehat{D}_{fs}\right)x_f(s). \end{aligned} \tag{13}$$

Define $y(s) = [x_s^T(s), x_f^T(s)]^T$, then (13) can be rewritten as

$$sy(s) = \widetilde{\Psi}(s)y(s), \tag{14}$$

where

$$\widetilde{\Psi}(s) = \begin{bmatrix} \dfrac{-s^2I_n - \alpha\left(\widehat{L}_s + \widehat{D}_{sf}\right)}{\beta e^{-\tau s}} & \dfrac{-\alpha\widehat{A}_{sf}}{\beta e^{-\tau s}} \\ -\gamma\widehat{A}_{fs} & -\gamma\left(\widehat{L}_f + \widehat{D}_{fs}\right) \end{bmatrix}. \tag{15}$$

Define $\widetilde{\Gamma}(s) = \det\left(sI - \widetilde{\Psi}(s)\right)$. Based on the Lyapunov stability criterion, the systems can reach group consensus if the roots of $\widetilde{\Gamma}(s)$ are at $s = 0$ or $\operatorname{Re}\left(\lambda_i(\widetilde{\Gamma}(s))\right) < 0$. Subsequently, we will discuss these two cases, respectively, according to the general Nyquist criterion.

When $s = 0$, $\widetilde{\Gamma}(0) = \det\,(D + A)(-\alpha/\beta)^n(-\gamma)^m$. According to Lemma 1, one can know that zero is the simple eigenvalue of the matrix $(D + A)$. Thus, the roots of $\widetilde{\Gamma}(0)$ are at the point $s = 0$.

When $s \neq 0$, set $\widetilde{\Gamma}(s) = \det\,(\Phi(s) + I)$ and

$$\Phi(s) = \begin{bmatrix} \dfrac{s^2I_n + \alpha\left(\widehat{L}_s + \widehat{D}_{sf}\right)}{s\beta e^{-\tau s}} & \dfrac{\alpha\widehat{A}_{sf}}{s\beta e^{-\tau s}} \\ \dfrac{\gamma\widehat{A}_{fs}}{s} & \dfrac{\gamma\left(\widehat{L}_f + \widehat{D}_{fs}\right)}{s} \end{bmatrix}. \tag{16}$$

Set $s = j\omega$, according to the general Nyquist criterion, when and only when the point $(-1, j0)$ is not enclosed by the Nyquist curve of $\Phi(j\omega)$; $\widetilde{\Gamma}(s)$'s roots are located in the open left-half plane of the complex field. In other words, the group consensus can be reached in this situation. By the Gerschgorin disk theorem, one can obtain

$$\lambda(\Phi(j\omega)) \in \{\Phi_i, i \in g_1\} \bigcup \{\Phi_i, i \in g_2\}. \tag{17}$$

When $i \in g_1$, it follows

$$\Phi_i = \left\{x : x \in \left\| x - \frac{\alpha}{j\omega\beta}\sum_{j\in N_i} a_{ij} - \frac{j\omega}{\beta}e^{j\omega\tau}\right| \leq \sum_{j\in N_i}\left|\frac{\alpha a_{ij}}{j\omega\beta}e^{-j\omega\left(\tau_{ij}-\tau\right)}\right|\right\}. \tag{18}$$

Let $\sum_{j\in N_i} a_{ij} = D_i$, $i \in g_1$. Since the point $(-a, j0)$, $a \geq 1$ cannot be encircled in Φ_i, $i \in g_1$, then the following inequality is obtained:

$$\left|-a - \frac{\alpha D_i}{j\omega\beta} - \frac{j\omega}{\beta}e^{j\omega\tau}\right| > \sum_{j\in N_i}\left|\frac{\alpha a_{ij}}{j\omega\beta}e^{-j\omega\left(\tau_{ij}-\tau\right)}\right|. \tag{19}$$

Based on the Euler formula and from (19), we have

$$\begin{aligned} &\left|-a + \frac{\alpha D_i}{\omega\beta}j - \frac{j\omega}{\beta}(\cos\omega\tau + j\sin\omega\tau)\right| \\ &\quad > \left|\frac{\alpha D_i}{j\omega\beta}\left(\cos\left(\tau_{ij} - \tau\right) - j\sin\left(\tau_{ij} - \tau\right)\right)\right|. \end{aligned} \tag{20}$$

Thus, we can get

$$a^2 - \frac{2a\omega}{\beta}\sin\omega\tau + \frac{\omega^2}{\beta^2} - \frac{2\alpha D_i}{\beta^2}\cos\omega\tau > 0. \tag{21}$$

It is easy to know that $a^2 - (2a\omega/\beta)\sin\omega\tau$ is monotonically increasing for $a \geq 1$. Thus, we obtain

$$1 - \frac{2\omega}{\beta}\sin\omega\tau + \frac{\omega^2}{\beta^2} - \frac{2\alpha D_i}{\beta^2}\cos\omega\tau > 0. \tag{22}$$

Here, as the control parameter β is positive, the following inequation can be derived:

$$\beta^2 - 2\omega\beta\sin\omega\tau + \omega^2 - 2\alpha D_i\cos\omega\tau > 0. \tag{23}$$

Obviously, (23) can hold when these two inequations are satisfied:

$$2\alpha D_i \cos \omega\tau - \beta^2 < 0, \tag{24}$$

$$2\omega\beta \sin \omega\tau - \omega^2 < 0. \tag{25}$$

Based on (24), we can get $\beta^2 > 2\alpha D_i$ because $\cos \omega\tau \leq 1$. According to (25), it follows that $1 - 2\beta\tau(\sin \omega\tau/\omega\tau) > 0$. Since $(\sin \omega\tau/\omega\tau) \leq 1$, $\tau \leq (1/2\beta)$ can be established.

Similarly, when $i \in g_2$, we can get the following inequation:

$$\Phi_i = \left\{ x : x \in \left\| x - \frac{\gamma}{j\omega} \sum_{j\in N_i} a_{ij} e^{-j\omega\tau} \right| \leq \sum_{j\in N_i} \left| \frac{\gamma a_{ij}}{j\omega} e^{-j\omega\tau_{ij}} \right\| \right\}. \tag{26}$$

Similarly, the point $(-a, j0)$, $a \geq 1$ cannot be encircled in Φ_i, $i \in g_2$, thus it follows that

$$\left| -a - \frac{\gamma}{j\omega} \sum_{j\in N_i} a_{ij} e^{-j\omega\tau} \right| > \sum_{j\in N_i} \left| \frac{\gamma a_{ij}}{j\omega} e^{-j\omega\tau_{ij}} \right|. \tag{27}$$

Define $\sum_{j\in N_i} a_{ij} = \tilde{D}_i$, $i \in g_2$, then from (27), which leads to

$$\left| -a + \frac{\gamma\tilde{D}_i}{\omega} (j \cos \omega\tau + \sin \omega\tau) \right| > \left| \frac{\gamma\tilde{D}_i}{\omega} \left(-\sin \tau_{ij} - j \cos \omega\tau_{ij} \right) \right|. \tag{28}$$

After some manipulations, we have

$$a^2 - \frac{2a\gamma\tilde{D}_i}{\omega} \sin \omega\tau > 0. \tag{29}$$

From (29), we know that $a^2 - (2a\gamma\tilde{D}_i/\omega) \sin \omega\tau$ will gradually increase as a increases. Thus, $1 - (2\gamma\tilde{D}_i/\omega) \sin \omega\tau > 0$ is obtained. As $(\sin \omega\tau/\omega\tau) \leq 1$, $\tau \leq (1/2\gamma\tilde{D}_i)$ can be established.

In summary, that finishes the Proof 1 of Theorem 1.

Corollary 1. *To the systems (7), if its topology is a bipartite digraph and contains a directed spanning tree, the systems' couple-group consensus is said to be realized asymptotically if this two conditions hold:* $\beta^2 > 2\alpha D_i$ *and* $\tau \in [1/2\beta, 1/2\gamma \max\{\tilde{D}_i\}]$, *where* $D_i = \sum_{j\in N_i} a_{ij}$, $i \in g_1$ *and* $\tilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in g_2$.

By Lemma 1 and combining the Proof 1 of Theorem 1, we can get the results in Corollary 1.

Remark 1. From the results in Theorem 1, they reveal that the coupling weights and the control parameters play key roles in systems' couple-group consensus. However, communication delays have no effect on the realization of group consensus. Furthermore, they also imply that the smaller either the coupling weights between the agents or the control parameters, the greater the input time delays of the system can be tolerated.

Remark 2. The proposed protocol (7) is constructed by employing the competitive interaction among the agents. As nearly all of the relevant works rely on the collaborative relationship, such as in [15–28], this paper investigates the group consensus of the heterogeneous complex systems from a new perspective. At the same time, the protocol (7) has no virtual velocity in first-order agents' dynamics, which is included in some existing articles (e.g., in [20, 26–28]) for the convenience of analysis, while it will cause extra calculations and degrade the flexibility of the systems. As known, relaxing some conditions may lead to a more challenging job.

Remark 3. Different from the works in [20, 25–29], we further relax the following two conservative prerequisites: in-degree balance and the geometric multiplicity of the zero eigenvalue of the systems' Laplacian matrix have no less than 2. As known, in-degree balance means that the interaction among the agents in different clusters is offset. That is to say, there is no actual communication between the subsystems [8]. It is a very restrictive condition. Meanwhile, the second assumption also makes some limitations on the topology of the systems. Furthermore, the existing researches either took no time delay into consideration, such as in [24–27, 29], or merely consider the influence of input delay [28].

Next, the case of different input and communication delays will be discussed.

On the basis of (7), the systems models can be extended as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = -\alpha \left[\sum_{j\in N_i} a_{ij} \left[x_j\left(t - \tau_{ij}\right) + x_i(t - \tau_i) \right] \right] - \beta v_i(t - \tau_i), \\ \qquad\qquad\qquad\qquad i \in g_1, \end{cases}$$
$$\dot{x}_i(t) = -\gamma \left[\sum_{j\in N_i} a_{ij} \left[x_j\left(t - \tau_{ij}\right) + x_i(t - \tau_i) \right] \right], \quad i \in g_2, \tag{30}$$

where τ_i is the input delay of agent i and τ_{ij} is the communication delay between the agents j and i.

Theorem 2. *To the MASs (30), if systems' topology is an undirected bipartite graph, couple-group consensus can be realized if the following two conditions hold:* $\beta^2 > 2\alpha D_i$ *and if* $i \in g_1$, $\tau_i \in [0, 1/2\beta]$; *otherwise,* $\tau_i \in [0, 1/2\gamma\tilde{D}_i]$, $i \in g_2$, *where* $D_i = \sum_{j\in N_i} a_{ij}$, $i \in g_1$ *and* $\tilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in g_2$.

Corollary 2. *To the MASs (30) with a directed bipartite topology and which contains a spanning tree, systems' couple-group consensus can be realized asymptotically when these two*

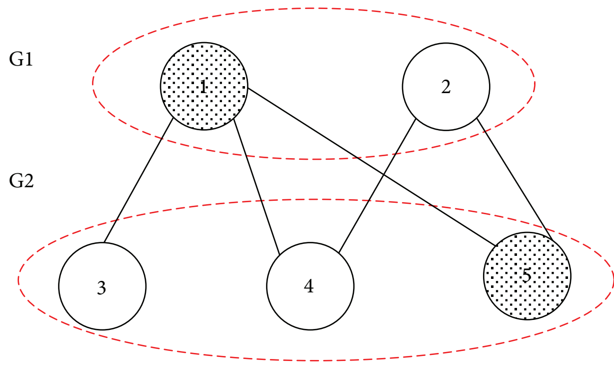


Figure 1: The undirected bipartite topology.

conditions hold: $\beta^2 > 2\alpha D_i$ *and if* $i \in g_1$, $\tau_i \in [0, (1/2\beta)]$; *otherwise,* $\tau_i \in [0, 1/2\gamma\tilde{D}_i]$, $i \in g_2$, *where* $D_i = \sum_{j\in N_i} a_{ij}$, $i \in g_1$ *and* $\tilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in g_2$.

The proof is omitted here due to the limitation of space.

Remark 4. The results in Theorem 2 show that the upper bound of the input time delay will vary with the different dynamics of the agents and decided by the control parameter and coupling weights between the agents with same dynamic. Similarly, communication delays between agents have no effect on the achievement of systems' couple-group consensus.

Remark 5. The system topologies we considered in this paper are either an undirected bipartite graph or a directed bipartite graph containing a directed spanning tree. They seem to be a specific topology. But as a matter of fact, the systems' group consensus can usually be reached with the help of some stronger conditions. For instance, in [20, 25–29], the systems' topology is also an undirected graph or a digraph containing a spanning tree. At the same time, for the purpose of realizing the group consensus, some extra assumptions are needed, which have been mentioned in Remarks 1 and 2. It will be a challenge work to investigate the group consensus problem of a class of heterogeneous MASs in a more general condition.

4. Simulation

In this section, the correctness of the obtained results will be illustrated by several simulations.

Regard the heterogeneous systems with bipartite topology as Figure 1. Hereinto, agents 1 and 2 and 3, 4, and 5 belong to the subgroups G_1 and G_2, individually. For the generality, agents 1 and 5 are represented as second-order agents and the first-order agents include the remaining agents 2, 3, and 4. Therefore, each subgroup in Figure 1 is designed as heterogeneous.

Remark 6. Distinguish from the cases in [24–26, 28], the dynamics of the agents in this paper are not restricted to be homogeneous within the same subgroup. Obviously, that is the special case of ours.

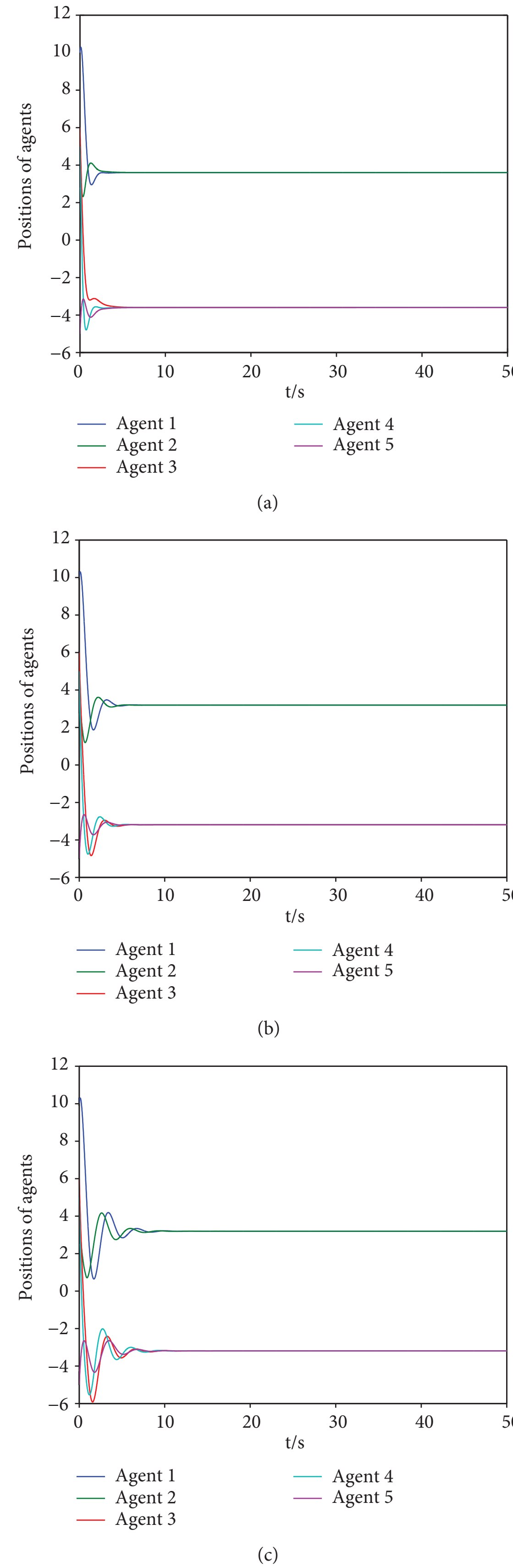
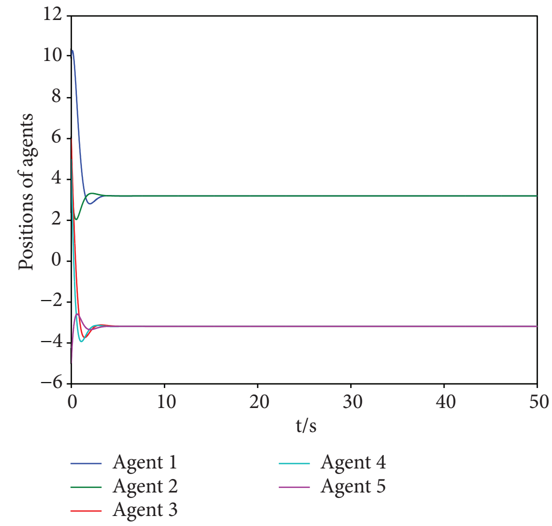
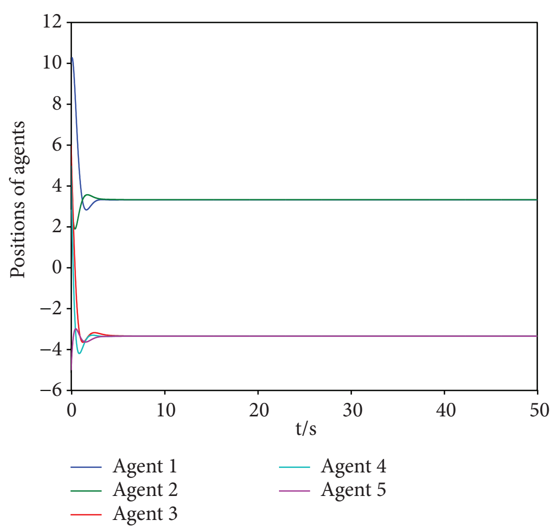


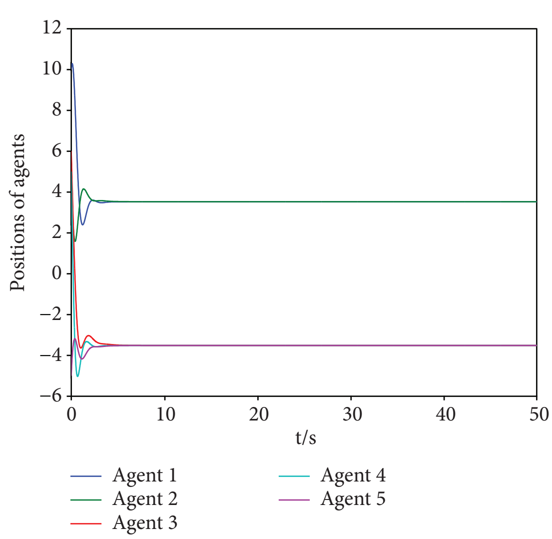
Figure 2: The agents' position trajectories, where $\tau = 0.15$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.8$.



(a)

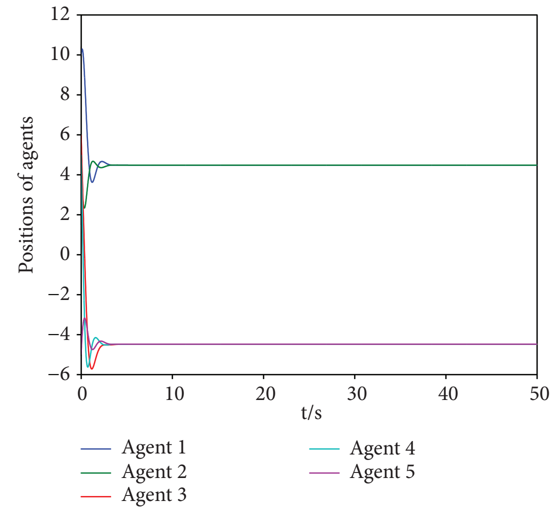


(b)

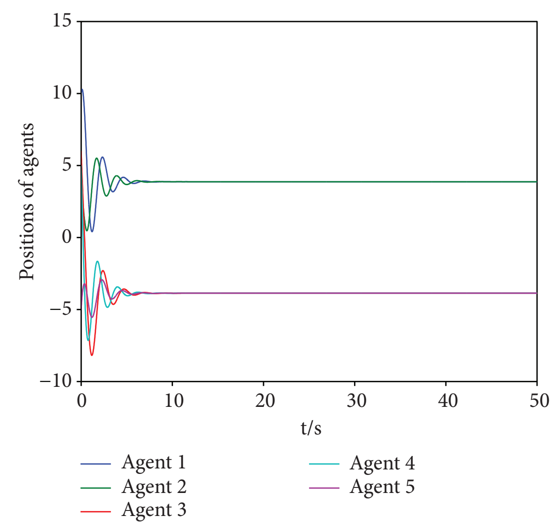


(c)

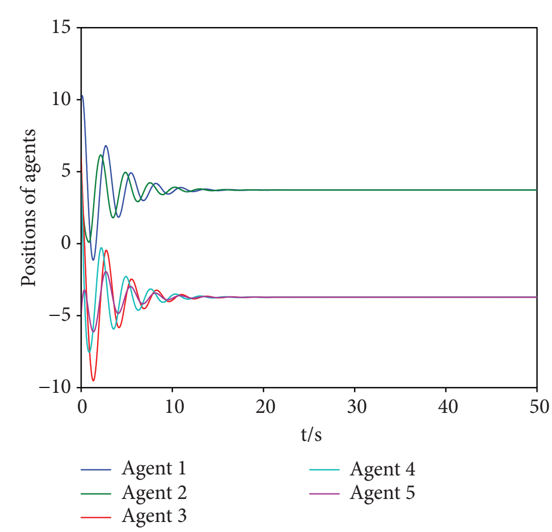
FIGURE 3: The agents' position trajectories, where $\tau_{ij} = 0.15$. (a) $\tau = 0$, (b) $\tau = 0.1$, and (c) $\tau = 0.15$.



(a)



(b)



(c)

FIGURE 4: The agents' position trajectories, where $\tau_1 = 0.15$, $\tau_2 = 0.2$, $\tau_3 = 0.5$, $\tau_4 = 0.25$, and $\tau_5 = 0.15$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.8$.

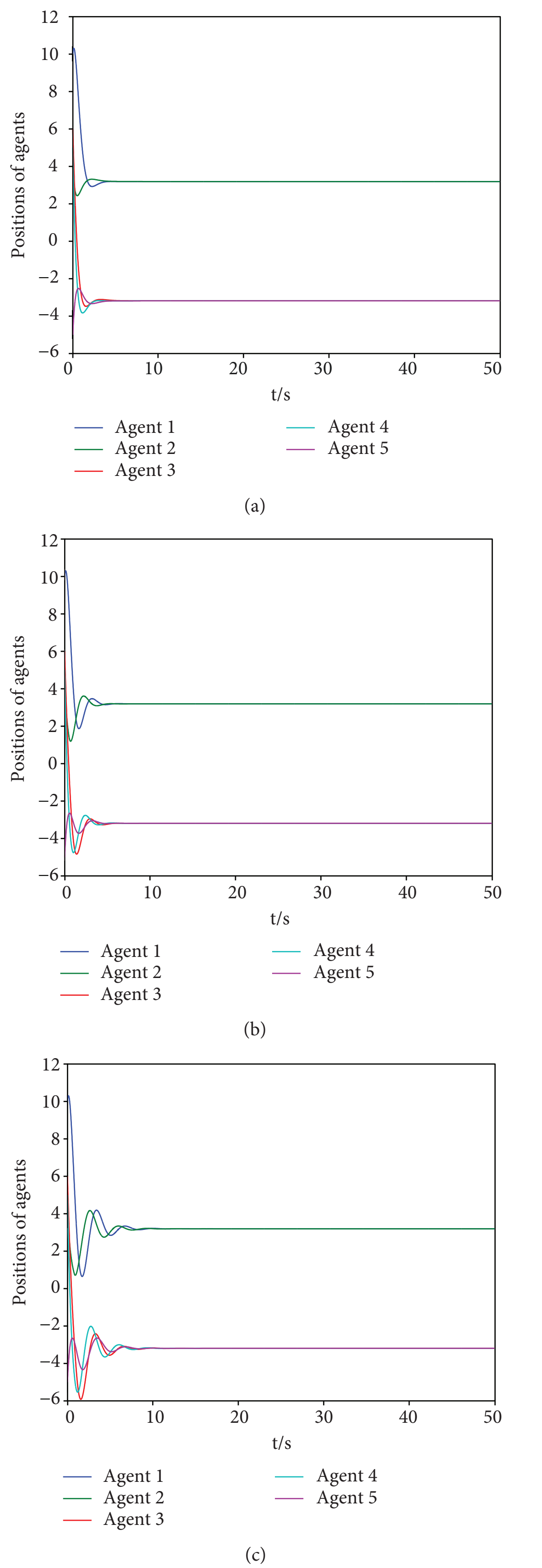


Figure 5: The agents' position trajectories, where $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.8$.

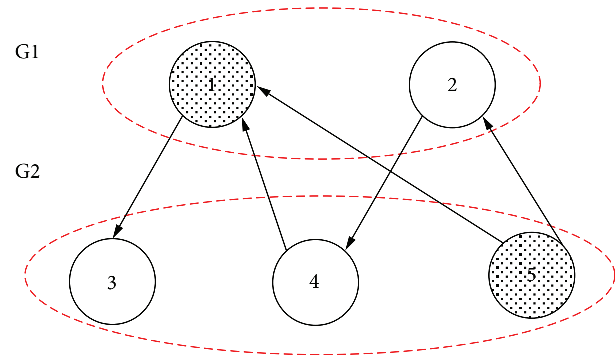


Figure 6: The directed bipartite topology.

For simplicity, we set $a_{ij} = 1, i, j \in [1, 5]$ and select $\alpha = 1$, $\beta = 3$, and $\gamma = 1$. From the systems' topology shown in Figure 1, it is easy to know that $d_1 = 3$, $d_2 = 2$, $d_3 = 1$, $d_4 = 2$, and $d_5 = 2$.

Example 1. According to the group consensus criteria proposed in Theorem 1, the range of input time delay is calculated as $\tau \in [0, \min\{1/6, 1/4\}]$. In this case, we select the identical input time delay as $\tau = 0.15$ s. Here, the conditions of Theorem 1 are now all satisfied. From the results in Theorem 1, we know that communication delay does not affect the systems' couple-group consensus. Hence, for convenience, their values are set the same. Meanwhile, several different cases are considered to discover the impact of the delays including the communication and input delays on the systems' convergence rate. Figures 2 and 3 show the trajectories of the agents in the system (7). They indicate that all agents of the heterogeneous system converge to two reverse subgroups, that is, the couple-group consensus is realized.

Remark 7. The results in Figures 2 and 3 reveal that the communication and input time delays are both capable to affect the trajectories of the agents. To illustrate, the shorter the delays, the faster the system converges. Hence, we can improve the convergence rate by reducing the time delays including communication time delays, the input ones, or both of them.

Example 2. From Theorem 2 and according to the given parameters, it has $\tau_i \leq 1/6$, $i \in \sigma_1$, $\tau_3 \leq 0.5$, $\tau_4 \leq 0.25$, and $\tau_5 \leq 0.25$. In the simulation, we choose $\tau_1 = 0.15$ s, $\tau_2 = 0.2$ s, $\tau_3 = 0.5$ s, $\tau_4 = 0.25$ s, and $\tau_5 = 0.15$ s. Clearly, the conditions in Theorem 2 are all hold. Similar to the Example 1, we also set several sets of time delays, then the agents' trajectories in the systems (30) under the influence of different input and communication time delays are illustrated as Figures 4 and 5. From them, we know that the couple-group consensus of each case is realized, respectively.

Example 3. Suppose the topology of the heterogeneous systems (7) is shown in Figure 6, which contains a directed spanning tree. We set the input time delays as $\tau = 0.15$ s that satisfies all the conditions proposed in Corollary 1.

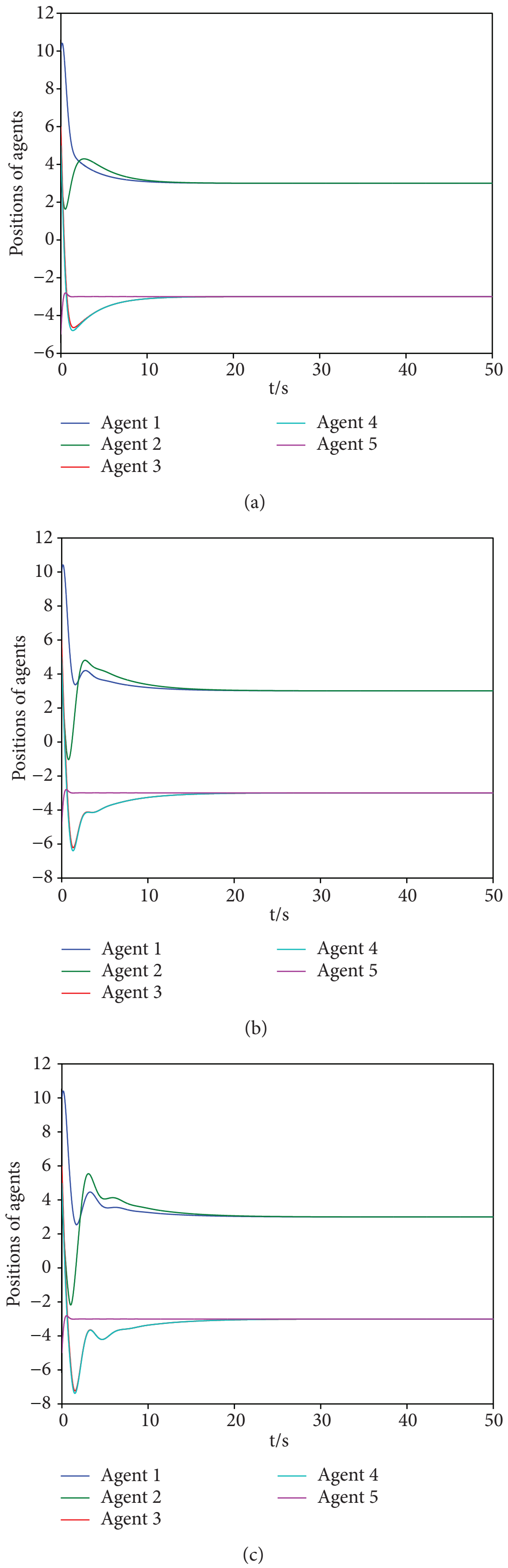


FIGURE 7: The agents' position trajectories, where $\tau = 0.15$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.8$ s.

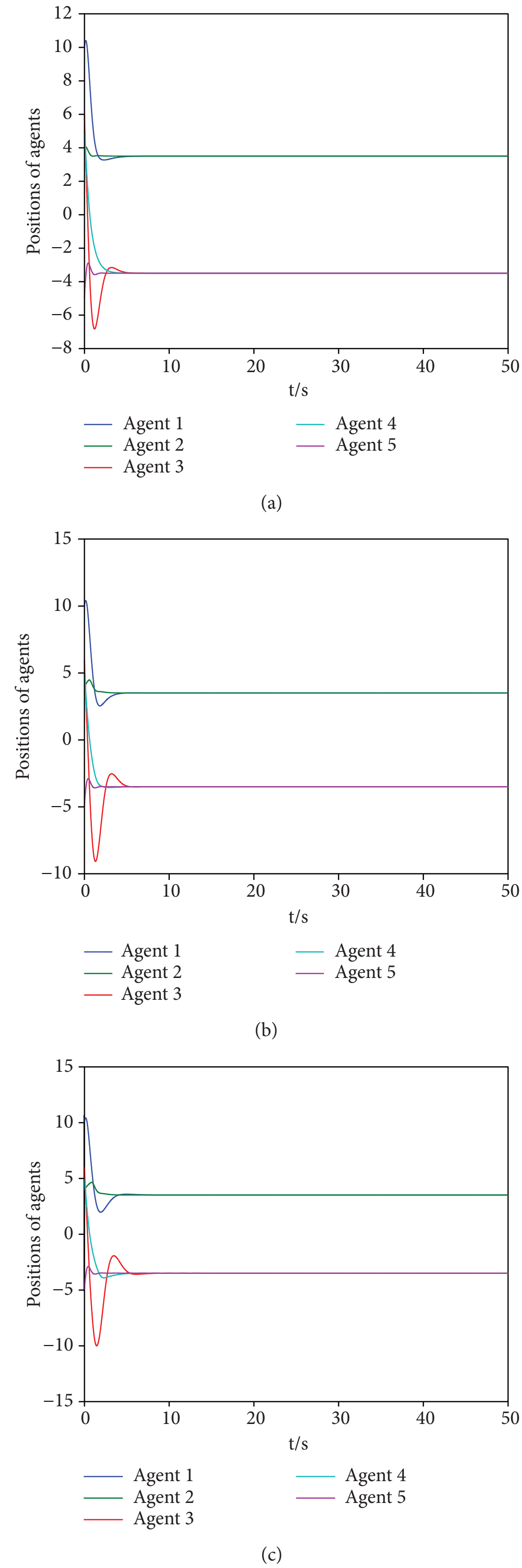


FIGURE 8: The agents' position trajectories, where $\tau_1 = 0.15$, $\tau_2 = 0.2$, $\tau_3 = 0.5$, $\tau_4 = 0.25$, and $\tau_5 = 0.15$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.8$.

According to the trajectories of the agents shown in Figure 7, one can know that the couple-group consensus is achieved asymptotically. Similarly, regard Figure 6 as the topology of the systems (30). And set $\tau_1 = 0.15$ s, $\tau_2 = 0.2$ s, $\tau_3 = 0.5$ s, $\tau_4 = 0.25$ s, and $\tau_5 = 0.15$ s that follow the conditions in Corollary 2. From the trajectories of the agents shown in Figure 8, the couple-group consensus with the influence of different input time delays is also achieved.

5. Conclusion

Considering the influence of both communication and input delays, an original distributed coordination control protocol based on agents' competitive interaction is designed for realizing the consensus of heterogeneous systems. By utilizing matrix theory and general Nyquist criteria, some sufficient criteria as well as the input delays' upper bound are theoretically presented. From the results, we find that the control parameters of systems, the coupling weights between the agents, and the input delay play key roles on the achievement of couple-group consensus of the heterogeneous systems, while communication delays are independent with it. Furthermore, the simulation results show that the systems' convergence rate can be improved by reducing the time delays including communication, input, or both of them. In the future, this issues for the heterogeneous and/or hybrid MASs with switching interaction topology can be extended.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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