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Research Article

Boundary Robust Adaptive Control of a Flexible Timoshenko Manipulator

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In this study, we aim to construct the boundary robust adaptive control for weakening the vibration of the flexible Timoshenko manipulator in the presence of unknown disturbances. By Lyapunov's direct method, the adaptive controllers and disturbance observers are exploited to achieve the angle tracking and handle the external disturbances. With the suggested adaptive laws and disturbance observers, the controlled system with both parametric and disturbance uncertainties is guaranteed to be uniformly bounded. Finally, simulation results are provided to illustrate the applicability and effectiveness of the proposed control.

1. Introduction

With the development of the robotics and space technology, the flexible manipulator plays an essential part in the industrial production, aerospace, marine engineering, and so forth in recent years due to the advantages of light weight, lower energy consumption, and high speed operation [1, 2]. However, the external unknown disturbances exerted on the flexible structures will cause vibration, which will give rise to performance reduction of the system [3, 4]. As a consequence, it is necessary to design effective vibration control approaches for vibration abatement and performance improvement.

In order to prevent damage and enhance operational performance, a great deal of effort has been paid to develop effective control schemes [5–15]. These methods including modal control, distributed control, and boundary control make great contributions to the control problems of flexible structures. Modal control is a commonly used method which is based on truncated finite-dimensional modes, and this theory is well documented in [6, 7, 14]. Nevertheless, the truncated models are obtained via neglecting high-frequency modes, which can result in unstability of the system. For the sake of avoiding the spillover phenomenon, it can obtain better control performance by means of increasing the number of sensors and actuators, which is called distributed control [8]. In reality, the distributed control is hard to implement since it requires many sensors and controllers, and the distributed system becomes uncontrollable and unobservable when the point actuators and sensors are located at nodal points. For boundary control [10–12], actuators only need to be installed at the boundaries of the system and it has been proved to be a valid method which can attenuate the vibration. Compared with modal control and distributed control, the boundary control is more effective for infinite dimensional system dynamical model since boundary controller can avoid spillover phenomenon and it is considered to be physically more realistic due to nonintrusive actuation and sensing [16, 17]. For instance, in [18], the boundary control and disturbance observer are developed for vibrating flexible manipulator system under external disturbances. In [19], the boundary control for a flexible manipulator with disturbances and input backlash is introduced to globally stabilize the vibration. To simplify the system analysis, the above-mentioned research usually ignores the shear deformation of the beam. In fact, the effect of the axial deformation and shear deformation should not be ignored when the diameter-to-length rate of the flexible beam is not small, and in this case, the manipulator system can be described as a Timoshenko manipulator. Namely, the shear deformation should be taken into account in the process of modeling a Timoshenko manipulator, which

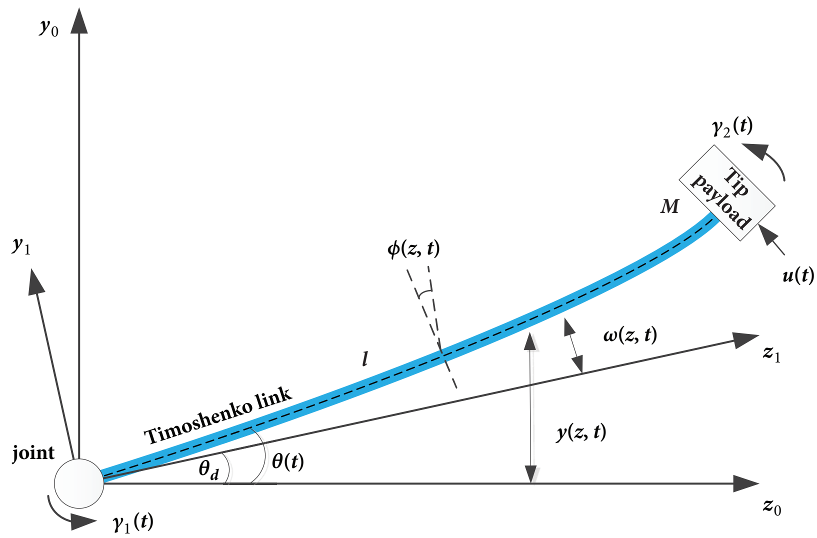

Figure 1: Flexible Timoshenko manipulator.

is usually ignored for the Euler-Bernoulli manipulator. It will be a more sophisticated and challenging problem to consider the nonlinear coupling of elastic deflection, shear deformation, and angular position in the control. For those cases, the control design is documented in [20, 21] to suppress the vibration and rotation of the system. In [20], for the flexible manipulator modeled as the Timoshenko beam, the boundary controllers are designed to eliminate the vibration and rotation of the system under external disturbances. However, in the aforesaid papers, there is no research taking into account the case that the parameters of the system and time-varying disturbances are unknown, which motivates us for this research.

In this paper, our interest lies in the boundary robust adaptive control of Timoshenko manipulator in the presence of both parametric and disturbance uncertainties. The adaptive laws and disturbance observers are put forward to compensate for the effect of parametric and disturbance uncertainties and achieve ultimate boundedness of the system. Compared with the existing work, the main contributions of this paper can be summarized as follows:

(i) Boundary robust adaptive controllers with disturbance observers are proposed when all the parameters and time-varying disturbances are unknown, by which the vibration is also reduced effectively.

(ii) With the proposed controllers, the uniform convergence of the Timoshenko manipulator system is obtained via Lyapunov's direct method. By choosing appropriate parameters, the closed-loop system states eventually converge to a compact and the effectiveness of the system is guaranteed.

2. Problem Statement

2.1. Nomenclature

l: length of the manipulator.

M: mass of the tip payload.

ρ: mass per unit length of the manipulator.

EI: bending stiffness of the manipulator.

J: inertia of the payload.

k: a positive constant related to cross-sectional area of the manipulator.

G: modulus of elasticity in shear.

A: cross-sectional area of the manipulator.

I_h: inertia of the hub.

I_ρ: distributed inertia of the manipulator.

Figure 1 depicts a flexible Timoshenko manipulator system, in which $K = kGA$, $\omega(z,t)$ denotes the elastic deflection, the displacement $y(z,t)$ is given as $y(z,t) = \omega(z,t) + z\theta(t)$ in the position z for the small deformation, $\phi(z,t)$ denotes the rotation of the beam's cross-section owing to bending, $\theta(t)$ denotes the angular position of the hub, $u(t)$ denotes the control input, $\gamma_1(t)$ denotes the control input torque on the hub, and $\gamma_2(t)$ denotes the control input torque on the tip payload.

Assumption 1. For the Timoshenko manipulator system, considering the finite energies of the boundary disturbances, we assume that there exist three constants $D_1 \in \mathbb{R}^+, D_2 \in \mathbb{R}^+, D_3 \in \mathbb{R}^+$, such that $|d_1(t)| \le D_1, |d_2(t)| \le D_2, |d_3(t)| \le D_3$. It is a reasonable assumption as the disturbances have finite energy and hence are bounded [22, 23].

Remark 2. We define notations as follows: $(\star)(t) = (\star)$, $(\star)' = \partial(\star)/\partial z$, $(\dot{\star}) = \partial(\star)/\partial t$, $(\star)'' = \partial^2(\star)/\partial z^2$, and $(\ddot{\star}) = \partial^2(\star)/\partial t^2$.

Remark 3. For analyzing the system stability and deriving our main results, we present the following inequality:

$$\left|y(l,t)\, d(t)\right| \le y(l,t) \tanh\left(y(l,t)\right) D \tag{1}$$

where $d(t) = [d_1(t), d_2(t), d_3(t)]$ represents the boundary disturbances bounded with the unknown positive constants $D = [D_1, D_2, D_3]$ to be estimated by developing a set of disturbance observers $\widehat{D}(t) = [\widehat{D}_1(t), \widehat{D}_2(t), \widehat{D}_3(t)]$.

In this study, the dynamical equation of the considered Timoshenko manipulator [20] is presented as follows:

$$\rho \ddot{y}(z,t) + K\left[\phi'(z,t) - \omega''(z,t)\right] = 0 \quad (2)$$

$$I_\rho \ddot{\phi}(z,t) - EI\phi''(z,t) + K\left[\phi(z,t) - \omega'(z,t)\right] = 0 \quad (3)$$

$\forall (z,t) \in (0,l) \times [0,\infty)$, with the following boundary conditions

$$\omega(0,t) = \phi(0,t) = 0 \quad (4)$$

$$M\ddot{y}(l,t) - K\left[\phi(l,t) - \omega'(l,t)\right] = d_1(t) + u(t) \quad (5)$$

$$J\ddot{\phi}(l,t) + EI\phi'(l,t) = d_2(t) + \gamma_2(t) \quad (6)$$

$$I_h\ddot{\theta}(t) - K\left[\omega(l,t) - \int_0^l \phi(z,t)\,dz\right] = d_3(t) + \gamma_1(t) \quad (7)$$

$\forall t \in [0,\infty)$.

3. Controller Design

In this section, we investigate the boundary adaptive control and angular position tracking problem for the flexible Timoshenko manipulator. Our aim lies in developing boundary control schemes to weaken the vibration and shear deformation, handle system uncertainties, and guarantee the angular in the desired position.

Define intermediate variables $\zeta_1(t), \zeta_2(t), \zeta_3(t)$ as

$$\zeta_1(t) = \dot{y}(l,t) + \omega'(l,t) - \phi(l,t) \quad (8)$$

$$\zeta_2(t) = \phi'(l,t) + \dot{\phi}(l,t) \quad (9)$$

$$\zeta_3(t) = \dot{\theta}(t) + e(t) \quad (10)$$

For stabilizing the presented system given by (2)-(7), the following boundary adaptive control yields

$$u(t) = -\Phi_1(t)\widehat{P_1}(t) - k_1\zeta_1(t) - \tanh\left(\zeta_1(t)\right)\widehat{D}_1(t) \quad (11)$$

$$\begin{aligned}\gamma_1(t) = &-k_2\zeta_3(t) - \widehat{I_h}\dot{\theta}(t) - k_3 e(t) - k_4\dot{\theta}(t) \\ &- \tanh\left(\zeta_3(t)\right)\widehat{D}_3(t)\end{aligned} \quad (12)$$

$$\gamma_2(t) = -\Phi_2(t)\widehat{P_2}(t) - k_5\zeta_2(t) - \tanh\left(\zeta_2(t)\right)\widehat{D}_2(t) \quad (13)$$

where $k_1, k_2, k_3, k_4, k_5 > 0$ and $e(t)$ denotes the angle error defined as

$$e(t) = \theta(t) - \theta_d \quad (14)$$

with θ_d being desired angle position, and vectors

$$\Phi_1(t) = \left[\phi(l,t) - \omega'(l,t) \quad \dot{\omega}'(l,t) - \dot{\phi}(l,t)\right] \quad (15)$$

$$\Phi_2(t) = \left[-\phi'(l,t) \quad \dot{\phi}'(l,t)\right] \quad (16)$$

and parameter estimate vectors are defined as

$$\widehat{P_1}(t) = \left[\widehat{K}(t) \quad \widehat{M}(t)\right]^T \quad (17)$$

$$\widehat{P_2}(t) = \left[\widehat{EI}(t) \quad \widehat{J}(t)\right]^T \quad (18)$$

We define parameter vectors P_1, P_2 and parameter estimate error vectors $\widetilde{P_1}(t), \widetilde{P_2}(t)$ as

$$\begin{aligned}P_1 &= [K \quad M]^T, \\ P_2 &= [EI \quad J]^T\end{aligned} \quad (19)$$

$$\widetilde{P_1}(t) = P_1 - \widehat{P_1}(t) = \left[\widetilde{K}(t) \quad \widetilde{M}(t)\right]^T \quad (20)$$

$$\widetilde{P_2}(t) = P_2 - \widehat{P_2}(t) = \left[\widetilde{EI}(t) \quad \widetilde{J}(t)\right]^T \quad (21)$$

Design the adaptive laws of the parameter estimate values and disturbance observers as

$$\dot{\widehat{P_1}}(t) = \Gamma_1\Phi_1^T(t)\zeta_1(t) - \chi_1\Gamma_1\widehat{P_1}(t) \quad (22)$$

$$\dot{\widehat{I_h}}(t) = \dot{\theta}(t)\zeta_3(t) - \chi_3\widehat{I_h}(t) \quad (23)$$

$$\dot{\widehat{P_2}}(t) = \Gamma_2\Phi_2^T(t)\zeta_2(t) - \chi_2\Gamma_2\widehat{P_2}(t) \quad (24)$$

$$\dot{\widehat{D_1}}(t) = -\iota_1\widehat{D}_1(t) + \zeta_1(t)\tanh\left(\zeta_1(t)\right) \quad (25)$$

$$\dot{\widehat{D_2}}(t) = -\iota_2\widehat{D}_2(t) + \zeta_2(t)\tanh\left(\zeta_2(t)\right) \quad (26)$$

$$\dot{\widehat{D_3}}(t) = -\iota_3\widehat{D}_3(t) + \zeta_3(t)\tanh\left(\zeta_3(t)\right) \quad (27)$$

with $\chi_1, \chi_2, \chi_3, \iota_1, \iota_2, \iota_3$ being positive constants and $\Gamma_1, \Gamma_2 \in R^{2\times 2}$ being diagonal positive-definite matrices.

Given the Lyapunov candidate function as

$$V(t) = V_e(t) + V_f(t) + V_g(t) \quad (28)$$

where

$$V_e(t) = \frac{1}{2}\rho \int_0^l [\dot{y}(z,t)]^2 dz + \frac{1}{2}I_\rho \int_0^l \left[\dot{\phi}(z,t)\right]^2 dz + \frac{1}{2}EI \int_0^l \left[\phi'(z,t)\right]^2 dz + \frac{1}{2}K \int_0^l \left[\phi(z,t) - \omega'(z,t)\right]^2 dz \tag{29}$$

$$V_f(t) = \frac{1}{2}M\zeta_1^2(t) + \frac{1}{2}J\zeta_2^2(t) + \frac{1}{2}I_h\zeta_3^2(t) + \frac{1}{2}(k_3 + k_4)e^2(t) + \frac{1}{2}\left[\widetilde{I}_h(t)\right]^2 + \frac{1}{2}\widetilde{P}_1^T(t)\Gamma_1^{-1}\widetilde{P}_1(t) + \frac{1}{2}\widetilde{P}_2^T(t)\Gamma_2^{-1}\widetilde{P}_2(t) + \frac{1}{2}\left[\widetilde{D}_1(t)\right]^2 + \frac{1}{2}\left[\widetilde{D}_2(t)\right]^2 + \frac{1}{2}\left[\widetilde{D}_3(t)\right]^2 \tag{30}$$

$$V_g(t) = \beta\rho \int_0^l z\dot{y}(z,t)\omega'(z,t)\,dz + \beta I_\rho \int_0^l z\dot{\phi}(z,t)\phi'(z,t)\,dz + \mu\beta I_\rho \int_0^l \dot{\phi}(z,t)\phi(z,t)\,dz \tag{31}$$

with $\beta, \mu > 0$.

Lemma 4. *The constructed Lyapunov candidate function presented in (28) is a positive-definite function*

$$0 \le \lambda_1\left[V_e(t) + V_f(t)\right] \le V(t) \le \lambda_2\left[V_e(t) + V_f(t)\right] \tag{32}$$

Invoking Young's inequality, it can be obtained from (31)

$$V_g(t) \le \beta\rho l \int_0^l [\dot{y}(z,t)]^2 dz + \beta I_\rho l \int_0^l \left[\phi'(z,t)\right]^2 dz - 2\beta\rho l \int_0^l [\phi(z,t)]^2 dz + \mu\beta I_\rho \int_0^l [\phi(z,t)]^2 dz + 2\beta\rho l \int_0^l \left[\phi(z,t) - \omega'(z,t)\right]^2 dz + \beta\rho l \int_0^l \omega'(z,t)\left[4\phi(z,t) - \omega'(z,t)\right] dz + \left(\beta I_\rho l + \mu\beta I_\rho\right)\int_0^l \left[\dot{\phi}(z,t)\right]^2 dz \tag{33}$$

Then, we further have

$$\left|V_g(t)\right| \le \beta\rho l \int_0^l [\dot{y}(z,t)]^2 dz + \left[\beta I_\rho l + \left(\mu\beta I_\rho + 14\beta\rho l\right)l^2\right]\int_0^l \left[\phi'(z,t)\right]^2 dz + 2\beta\rho l \int_0^l \left[\phi(z,t) - \omega'(z,t)\right]^2 dz + \left(\beta I_\rho l + \mu\beta I_\rho\right)\int_0^l \left[\dot{\phi}(z,t)\right]^2 dz \le \beta_1 V_e(t) \tag{34}$$

where $\beta_1 = 2\beta\max(2\rho l, (l+\mu)I_\rho, [I_\rho l + (\mu I_\rho + 14\rho l)l^2])/\min(\rho, I_\rho, EI, K)$

We thus get

$$-\beta_1 V_e(t) \le V_g(t) \le \beta_1 V_e(t) \tag{35}$$

Then, we further derive

$$0 \le \lambda_1\left[V_e(t) + V_f(t)\right] \le V(t) \le \lambda_2\left[V_e(t) + V_f(t)\right] \tag{36}$$

where $\lambda_1 = 1 - \beta_1 > 0, \lambda_2 = 1 + \beta_1 > 0$.

Lemma 5. *The time derivative of the Lyapunov candidate function (28) is upper bounded as*

$$\dot{V}(t) \le -\lambda V(t) + c \tag{37}$$

where $\lambda > 0$.

Differentiating (28) leads to

$$\dot{V}(t) = \dot{V}_e(t) + \dot{V}_f(t) + \dot{V}_g(t) \tag{38}$$

$\dot{V}_e(t)$ *is given as*

$$\dot{V}_e(t) = EI\phi'(l,t)\dot{\phi}(l,t) - K\omega(l,t)\dot{\theta}(t) + K\dot{y}(l,t)\left[\omega'(l,t) - \phi(l,t)\right] + K\int_0^l \phi(z,t)\dot{\theta}(t)\,dz \tag{39}$$

Then, we have

$$\dot{V}_e(t) \le \frac{K}{2}\zeta_1^2(t) - \frac{K}{2}[\dot{y}(l,t)]^2 - K\omega(l,t)\dot{\theta}(t) - \frac{K}{2}\left[\omega'(l,t) - \phi(l,t)\right]^2 - \frac{EI}{2}\left[\phi'(l,t)\right]^2 + \frac{EI}{2}\zeta_2^2(t) - \frac{EI}{2}\left[\dot{\phi}(l,t)\right]^2 + K\int_0^l \phi(z,t)\dot{\theta}(t)\,dz \tag{40}$$

Differentiating (30) and substituting the boundary conditions, we obtain

$$\begin{aligned}\dot{V}_f(t) \le{}& -k_1\zeta_1^2(t) - k_5\zeta_2^2(t) - k_2\zeta_3^2(t) \\ &+ \chi_1\widetilde{P}_1^T(t)\widehat{P}_1(t) + \chi_2\widetilde{P}_2^T(t)\widehat{P}_2(t) \\ &+ \chi_3\widetilde{I}_h(t)\widehat{I}_h(t) - \frac{\iota_1}{2}\left[\widetilde{D}_1(t)\right]^2 + \frac{\iota_1}{2}D_1^2 \\ &- \frac{\iota_2}{2}\left[\widetilde{D}_2(t)\right]^2 + \frac{\iota_2}{2}D_2^2 - \frac{\iota_3}{2}\left[\widetilde{D}_3(t)\right]^2 + \frac{\iota_3}{2}D_3^2 \\ &- k_3e^2(t) - k_4\dot{\theta}^2(t) \\ &+ K\zeta_3(t)\left[\omega(l,t) - \int_0^l \phi(z,t)\,dz\right]\end{aligned} \tag{41}$$

The third term of (38) is given as

$$\begin{aligned}\dot{V}_g(t) \le{}& \frac{\beta\rho l}{2}\left[\dot{y}(l,t)\right]^2 - \frac{\beta EI}{2}\int_0^l\left[\phi'(z,t)\right]^2 dz \\ &- \frac{\beta\rho}{2}\int_0^l\left[\dot{y}(z,t)\right]^2 dz \\ &- \mu\beta EI\int_0^l\left[\phi'(z,t)\right]^2 dz + \frac{\beta EIl}{2}\left[\phi'(l,t)\right]^2 \\ &+ \frac{\beta\rho l}{\delta_2}\int_0^l\left[\dot{y}(z,t)\right]^2 dz \\ &+ \beta Kl^2\delta_3\int_0^l\left[\phi'(z,t)\right]^2 dz + \frac{\beta I_\rho l}{2}\left[\dot{\phi}(l,t)\right]^2 \\ &+ \frac{\beta Kl}{\delta_3}\left[\phi(l,t) - \omega'(l,t)\right]^2 \\ &+ \beta K\left|1-\mu\right| l^2\delta_4\int_0^l\left[\phi'(z,t)\right]^2 dz \\ &+ \frac{\beta Kl}{2}\left[\phi(l,t) - \omega'(l,t)\right]^2 \\ &+ \frac{\beta K\left|1-\mu\right|}{\delta_4}\int_0^l\left[\phi(z,t) - \omega'(z,t)\right]^2 dz \\ &- \frac{\beta K}{2}\int_0^l\left[\phi(z,t) - \omega'(z,t)\right]^2 dz \\ &- \frac{\beta I_\rho}{2}\int_0^l\left[\dot{\phi}(z,t)\right]^2 dz + \frac{\mu\beta EI}{\delta_5}\left[\phi'(l,t)\right]^2 \\ &+ \mu\beta EIl\delta_5\int_0^l\left[\phi'(z,t)\right]^2 dz \\ &+ \mu\beta I_\rho\int_0^l\left[\dot{\phi}(z,t)\right]^2 dz + \beta\rho l\delta_2\dot{\theta}^2(t)\end{aligned} \tag{42}$$

Invoking (40)-(42), we can obtain

$$\begin{aligned}\dot{V}(t) \le{}& -\left(k_1 - \frac{K}{2}\right)\zeta_1^2(t) - \left(k_5 - \frac{EI}{2}\right)\zeta_2^2(t) \\ &- \frac{\iota_1}{2}\left[\widetilde{D}_1(t)\right]^2 + \frac{\iota_1}{2}D_1^2 - \frac{\iota_2}{2}\left[\widetilde{D}_2(t)\right]^2 + \frac{\iota_2}{2}D_2^2 \\ &- k_2\zeta_3^2(t) - \frac{\iota_3}{2}\left[\widetilde{D}_3(t)\right]^2 + \frac{\iota_3}{2}D_3^2 \\ &- \frac{\chi_3}{2}\left[\widetilde{I}_h(t)\right]^2 + \frac{\chi_3}{2}I_h^2 - \frac{\chi_1}{2}\left[\widetilde{P}_1(t)\right]^2 + \frac{\chi_1}{2}P_1^2 \\ &- \frac{\chi_2}{2}\left[\widetilde{P}_2(t)\right]^2 + \frac{\chi_2}{2}P_2^2 - \sigma_5\int_0^l\left[\dot{y}(z,t)\right]^2 dz \\ &- \left(k_4 - \beta\rho l\delta_2\right)\dot{\theta}^2(t) - \sigma_6\int_0^l\left[\dot{\phi}(z,t)\right]^2 dz \\ &- \sigma_7\int_0^l\left[\phi'(z,t)\right]^2 dz \\ &- \sigma_8\int_0^l\left[\omega'(z,t) - \phi(z,t)\right]^2 dz \\ &- \left(k_3 - K\delta_1\right)e^2(t) - \sigma_1\left[\dot{y}(l,t)\right]^2 \\ &- \sigma_2\left[\omega'(l,t) - \phi(l,t)\right]^2 - \sigma_3\left[\dot{\phi}(l,t)\right]^2 \\ &- \sigma_4\left[\phi'(l,t)\right]^2 + c\end{aligned} \tag{43}$$

where the intermediate parameters are selected such that

$$\begin{aligned}k_1 - \frac{K}{2} &> 0, \\ k_5 - \frac{EI}{2} &> 0,\end{aligned} \tag{44}$$

$$\begin{aligned}k_4 - \beta\rho l\delta_2 &> 0, \\ k_3 - K\delta_1 &> 0,\end{aligned} \tag{45}$$

$$\sigma_1 = \frac{K}{2} - \frac{\beta\rho l}{2} > 0, \tag{46}$$

$$\sigma_2 = \frac{K}{2} - \frac{\beta Kl}{\delta_3} - \frac{\beta Kl}{2} > 0, \tag{47}$$

$$\sigma_3 = \frac{EI}{2} - \frac{\beta I_\rho l}{2} > 0, \tag{48}$$

$$\sigma_4 = \frac{EI}{2} - \frac{\beta EIl}{2} - \frac{\mu\beta EI}{\delta_5} > 0, \tag{49}$$

$$\begin{aligned}\sigma_5 &= \frac{\beta\rho}{2} - \frac{\beta\rho l}{\delta_2} > 0, \\ \sigma_6 &= \frac{\beta I_\rho}{2} - \mu\beta I_\rho > 0,\end{aligned} \tag{50}$$

$$\sigma_7 = \frac{\beta EI}{2} - \beta Kl^2\delta_3 - \beta K\left|1-\mu\right| l^2\delta_4 \tag{51}$$

$$+\mu\beta EI-\mu\beta EIl\delta_5>0, \quad (52)$$

$$\sigma_8=\frac{\beta K}{2}-\frac{\beta\left|1-\mu\right|}{\delta_4}-\frac{K}{\delta_1}>0 \quad (53)$$

Then, we have

$$\dot V(t)\le-\lambda_3\left[V_e(t)+V_f(t)\right]+c\le-\lambda V(t)+c \quad (54)$$

where $\lambda_3=\min(2\sigma_5/\rho, 2\sigma_6/I_\rho, 2\sigma_7/EI, 2\sigma_8/K, (2k_1-K)/M, (2k_5-EI)/J, 2k_2/I_h, (2k_3-2K\delta_1)/(k_3+k_4), \chi_1, \chi_2, \chi_3, \iota_1, \iota_2, \iota_3)$, $\lambda=\lambda_3/\lambda_2>0$, *and* $c=(\iota_1/2)D_1^2+(\iota_2/2)D_2^2+(\iota_3/2)D_3^2+(\chi_1/2)P_1^2+(\chi_2/2)P_2^2+(\chi_3/2)I_h^2$.

Theorem 6. *For the system dynamics described by (2)-(7), under the developed control laws (11)-(13), if the initial conditions are bounded, we can conclude that the closed-loop states* $\omega(z,t)$, $\phi(z,t)$, *and* $e(t)$ *are convergent.*

Multiplying (37) by $e^{\lambda t}$ *yields*

$$\dot V(t)e^{\lambda t}\le-\lambda e^{\lambda t}V(t)+ce^{\lambda t} \quad (55)$$

$$V(t)\le V(0)e^{-\lambda t}+\frac{c}{\lambda} \quad (56)$$

Then, we have

$$\begin{aligned}\frac{K}{4l}\omega^2(z,t)&\le\frac{K}{4}\int_0^l\left[\omega'(z,t)\right]^2dz\\&\le\frac{K}{2}\int_0^l\left[\phi(z,t)-\omega'(z,t)\right]^2dz\\&\quad+\frac{7Kl^2}{2}\int_0^l\left[\phi'(z,t)\right]^2dz\le\frac{7Kl^2}{EI}V_1(t)\\&\le\frac{7Kl^2}{EI\lambda_1}V(t)\le\frac{7Kl^2}{EI\lambda_1}V(0)\left[e^{-\lambda t}+\frac{c}{\lambda}\right]\end{aligned} \quad (57)$$

Finally, we get

$$|\omega(z,t)|\le\sqrt{\frac{28l^3}{EI\lambda_1}V(0)\left[e^{-\lambda t}+\frac{c}{\lambda}\right]} \quad (58)$$

Similarly, we can obtain

$$|\phi(z,t)|\le\sqrt{\frac{2l}{EI\lambda_1}V(0)\left[e^{-\lambda t}+\frac{c}{\lambda}\right]} \quad (59)$$

$$|e(t)|\le\sqrt{\frac{2}{(k_3+k_4)\lambda_1}V(0)\left[e^{-\lambda t}+\frac{c}{\lambda}\right]} \quad (60)$$

Remark 7. In this paper, the designs of controllers and observers are conducted on the basis of the infinite dimensional partial differential dynamics; hence there is absence of control spillover issue [24]. In future studies, we will exploit modal method to conduct neural network based control design for achieving the transient performance regulation [25–28].

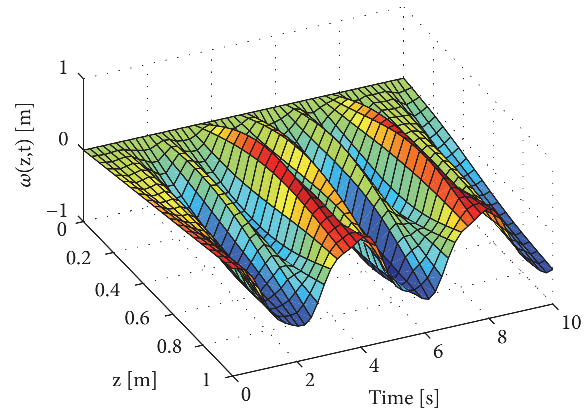

Figure 2: Deflection of the flexible link without control.

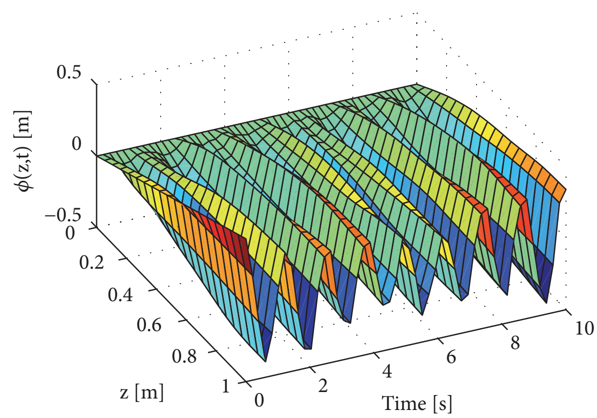

Figure 3: Rotation of the flexible link without control.

4. Simulations

In this article, the finite difference method [29–31] is introduced for simulating the performance of the controllers proposed. The space step is chosen as 20, and the time step is chosen as 38000. The parameters of the system are given as ρ=0.1 kg/m, M=2.0 kg, l=1.0 m, EI=10 Nm^2, I_ρ=1.0 kgm, J=0.1 kgm^2, I_h=1.0 kgm^2, K=2.0 N, and $\theta_d=\pi/3$ rad. The initial conditions are $\omega(z,0)=\phi(z,0)=z/2l$, $\dot\omega(z,0)=\dot\phi(z,0)=0$, and $\theta(0)=\pi/3+0.1$ rad.

The disturbances are given as

$$d_1(t)=d_2(t)=\frac{\sin(10\pi t)+\cos(3\pi t)}{4} \quad (61)$$

$$d_3(t)=\cos(3\pi t) \quad (62)$$

Figures 2 and 3 show the spatiotemporal response of the Timoshenko manipulator system under no control, that is, $u(t)=\gamma_1(t)=\gamma_2(t)=0$, influenced by external disturbances. Figures 4 and 5 display the spatiotemporal representation of the manipulator system under the proposed controllers by selecting the control design parameters $k_1=50$, $k_2=80$, $k_3=200$, $k_4=k_5=1$, $k_6=k_7=k_8=0.01$, $k_{10}=k_{20}=k_{30}=1$, $u_m=20$, and $\gamma_{1m}=\gamma_{2m}=1$. The angle position under proposed controllers is depicted in Figure 6.

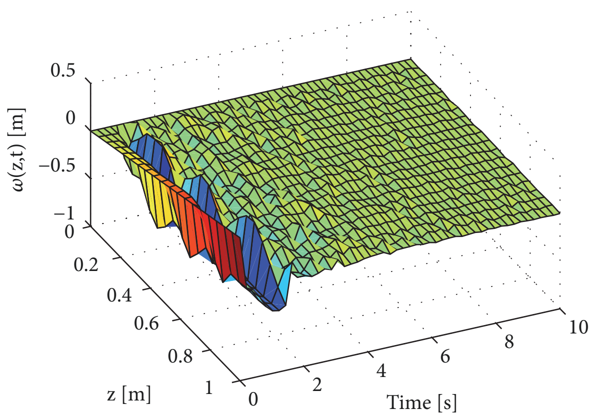

FIGURE 4: Deflection of the flexible link with proposed control.

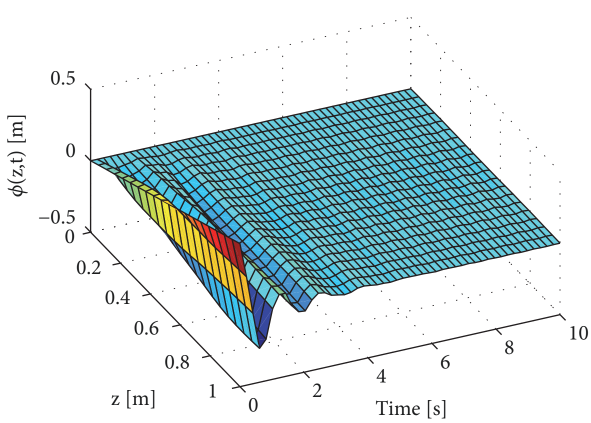

FIGURE 5: Rotation of the flexile link with proposed control.

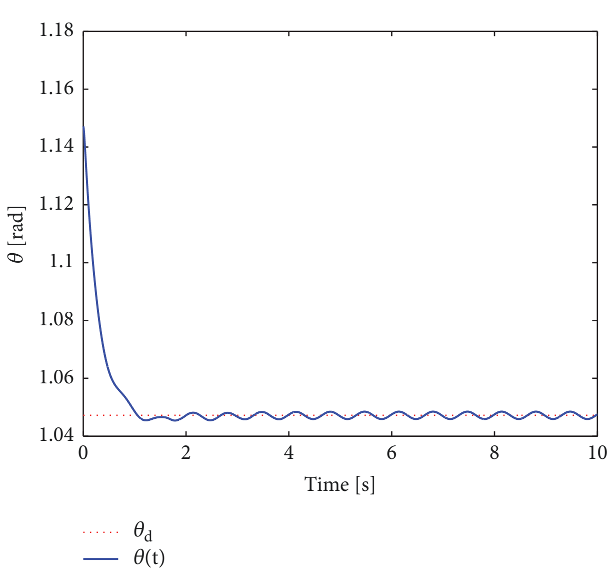

FIGURE 6: Angular position of the flexible link with proposed control.

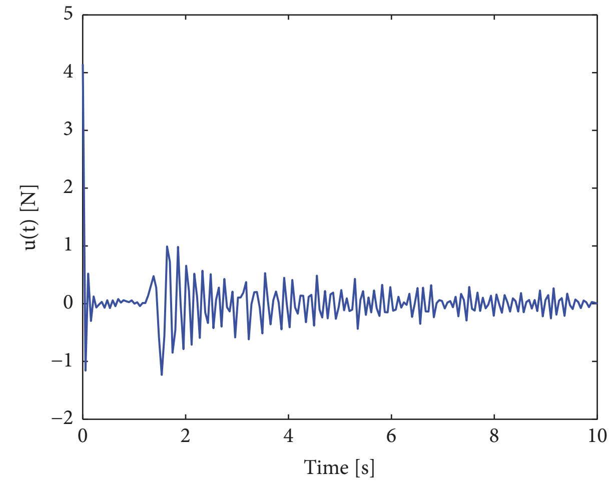

FIGURE 7: Control input force $u(t)$.

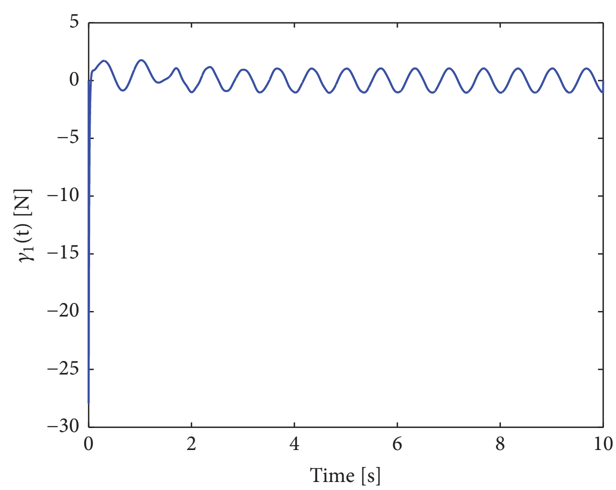

FIGURE 8: Control input torque $\gamma_1(t)$.

The two-dimensional responses of the designed control input $u(t)$ and control input torque $\gamma_1(t)$ and $\gamma_2(t)$ are, respectively, shown in Figures 7–9. By employing the proposed adaptive updated laws, the estimation values of the parameters about EI, J, K, M, I_h are presented in Figures 10–12.

From Figures 2–9, we can conclude that the developed boundary adaptive robust control laws can regulate the vibration and shear deformation of the manipulator system and position the manipulator in the desired angle with a better performance despite the existence of system uncertainties. From Figures 10–12, we can learn that although the parameter estimation values are not able to converge to actual values quite precisely, the suggested control in this paper can stabilize the system with a better performance.

5. Conclusion

This study was concerned with boundary adaptive robust control scheme design for a flexible Timoshenko manipulator

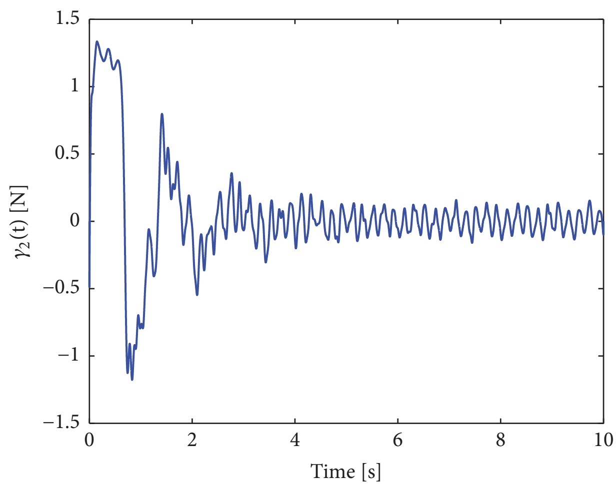

Figure 9: Control input torque $\gamma_2(t)$.

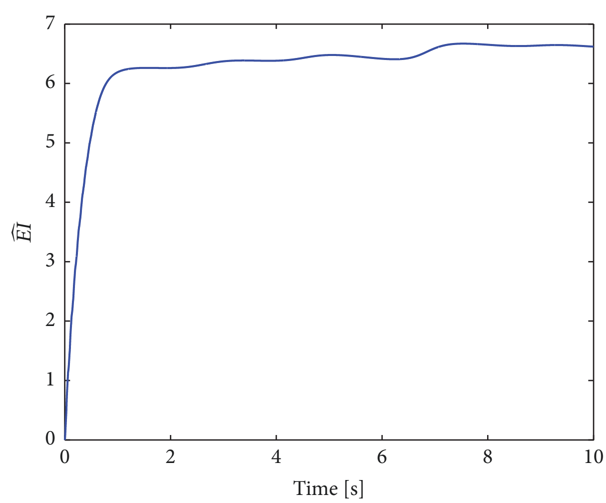

Figure 10: Estimation value of EI.

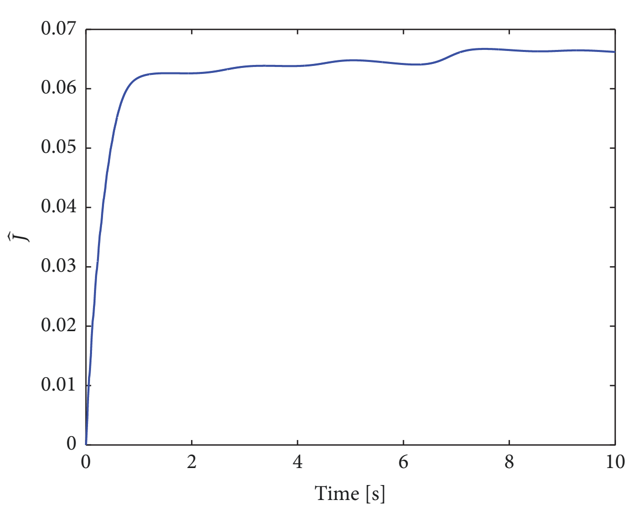

Figure 11: Estimation value of J.

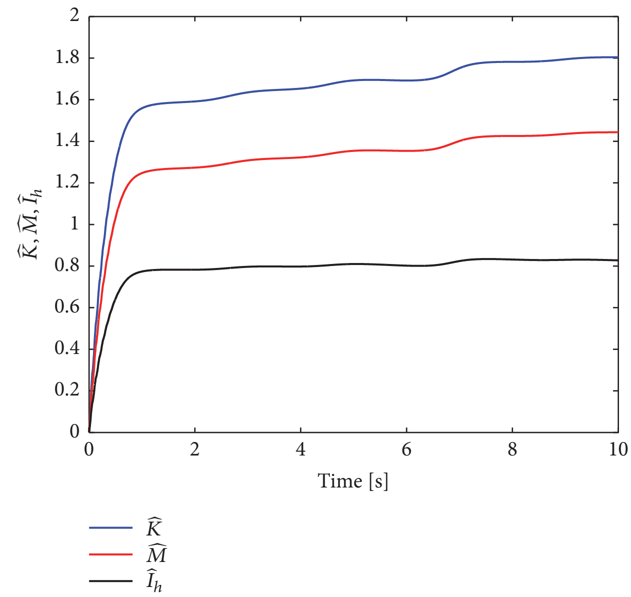

Figure 12: Estimation values of K, M and I_h.

influenced by external disturbances. To suppress the vibration and shear deformation, realize the desired angular positioning, and cope with system uncertainties effect, three boundary adaptive robust controllers with disturbance observers were developed. Under the control schemes proposed, the bounded stability of the closed-loop system was guaranteed without simplification of PDE dynamics. In the end, simulations were executed to show the performance of the control schemes developed. Future interests lie in learning approaches [32–38] for flexible manipulator system. Moreover, the implementation of the proposed control will be researched, and how to overcome the nonlinearities of the actuators is also a meaningful topic.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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