Weapon-Target Assignment Problem by Multiobjective Evolutionary Algorithm Based on Decomposition

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The weapon-target assignment (WTA) problem is a key issue in Command & Control (C2) [1]. It has attracted a great attention in recent years [2–6]. The WTA problem can be considered as a kind of resource allocation problem [7]. There are two versions of the WTA problem: the static weapon-target assignment (SWTA) problem [8, 9] and the dynamic weapon-target assignment (DWTA) problem [10]. In the SWTA problem, the weapons are launched at the same time, while they are launched asynchronously in the DWTA problem. Therefore, the outcomes of one stage of the engagement will affect the decision-making process of the next stage in DWTA problems. In addition, considering the different missions, each version includes the asset-based [11] problem and the target-based [12, 13] problem. In the asset-based problem, the task is to maximize the expected total value of assets which are defended by the defensive weapons. In the target-based problem, the task is to minimize the expected total value of targets which are not destroyed by the defensive weapons after the engagement. The target-based problem can be proven to be a special case of the asset-based problem.

Lloyd and Witsenhausen [14] proved that the WTA is an NP-complete problem [15]. Hence, when the total number of weapons and targets is large, it will have prohibitive computational complexity for traditional optimization methods. An evolutionary algorithm (EA) [16, 17] can solve the complex NP problems without restrictions on the type of problem so that it has a low computational cost for solving these problems. EA has been used for NP-complete problems for many years [18, 19]. All these algorithms have showed a promising performance. In view of the huge advantages of EA, many scholars tried to adopt it to solve the WTA problem in recent years. Lee et al. [20] proposed a genetic algorithm (GA) with greedy eugenics for the target-based SWTA. Wang et al. [21] proposed an improved target-based SWTA model and used the ant colony algorithm to optimize the target-based SWTA problem. Xue et al. [22] showed an improved discrete differential evolution (DE) algorithm for a cooperative jamming

1. Introduction

Weapon-target assignment (WTA) problem is one of the most important problems in Command & Control (C2) [1]. It has attracted a great attention in recent years [2–6]. The WTA problem can be considered as a kind of resource allocation problem [7]. There are two versions of the WTA problem: the static weapon-target assignment (SWTA) problem [8, 9] and the dynamic weapon-target assignment (DWTA) problem [10]. In the SWTA problem, the weapons are launched at the same time, while they are launched asynchronously in the DWTA problem. Therefore, the outcomes of one stage of the engagement will affect the decision-making process of the next stage in DWTA problems. In addition, considering the different missions, each version includes the asset-based [11] problem and the target-based [12, 13] problem. In the asset-based problem, the task is to maximize the expected total value of assets which are defended by the defensive weapons. In the target-based problem, the task is to minimize the expected total value of targets which are not destroyed by the defensive weapons after the engagement. The target-based problem can be proven to be a special case of the asset-based problem.
WTA problem. Hongtao and Fengju [23] proposed a new clonal selection algorithm for WTA problem. Chen et al. [24] used memetic algorithms for DWT. Xin et al. [25] used the virtual permutation and tabu search heuristics for DWT. Based on the WTA problem, the sensor-WTA problem is proposed [26, 27] to extend the WTA problem into a more comprehensive application area. All models adopted in these algorithms above are scalar optimization problems which have only one objective to be optimized. However, in some circumstances, we not only expect to achieve the desirable evaluation objectives mentioned above but also expect to reduce the consumption of weapons as more as possible. Therefore, the optimization of these two conflicting objectives could be considered as a multiobjective optimization problem (MOP). Thus, the multiobjective optimization weapon-target assignment (MOWTA) problem should be studied to provide more comprehensive information for the decision makers. Many multiobjective optimization techniques have been developed to solve the MOWTA over the last decade. Li et al. [28] used the NSGA-II to solve the target-based multiobjective optimization dynamic weapon-target assignment (MODWTA) problem. Peng et al. [29] adopted a hybrid multiobjective discrete particle swarm optimization to solve the target-based MODWTA problem. Li et al. [30] proposed an adaptive NSGA-II and an adaptive MOEA/D to solve the multistage weapon-target assignment problem, and they compared some efficient MOEAs for solving this problem [31]. Li et al. [32] introduced a modified Pareto ant colony optimization approach to solve the target-based multiobjective optimization static weapon-target assignment (MOSWTA). All these works have showed the advantage of solving MOWTA by using MOEA.

MOEA can optimize several objectives simultaneously and produce a set of optimal solutions which are nondominated for each other. It has attracted a massive interest of researchers for decades years. Currently, the well-known multiobjective evolutionary algorithm is SPEA-II [33], PESA-II [34], NSGA-II [35], MOEA/D [36], MOPSO [37], etc. Among these algorithms, the MOEA/D is a hot topic in recent years [38–41]. It can decompose the MOP into a number of scalar optimization subproblems by using some preference directions. It has a lower computational complexity over traditional MOEAs and a good performance in dealing with disparately scaled objectives. As the MOEA/D has many advantages to solve MOP, it has been applied to many real-world problems [42–45]. However, when the Pareto front (PF) is complex (discontinuous PF or PF with special shape), the performance of original MOEA/D will be declined. Some research efforts have been devoted to solving MOPs which have a complex PF. Qi et al. [46] proposed a MOEA/D with adaptive weight adjustment (MOEA/D-AWA) to improve the performance of MOEA/D in solving MOPs where the PF is discontinuous or with sharp peak or low tail. Zhang et al. [47] proposed an improved adaptive weighting method for biobjective optimization problems with discontinuous Pareto fronts. The above-mentioned works have demonstrated that the problem-specific multiobjective optimization techniques can greatly improve the performance of MOEA/D. Due to the MOWTA problem that has a particular multiobjective optimization objective, the PF of MOWTA is discrete and it has few Pareto optimal solutions. We notice that the multiobjective vehicle routing problem (MOVRP) have a very similar PF with MOWTA when the optimization objective is composed by the number of used vehicles and some continuous objective such as the total traveled distance, the total cost of routings, and traveling time of the longest route. Therefore, the multiobjective optimization techniques used in MOVRP is suitable for MOWTA. Qi et al. [48] proposed a novel selection operator and used the local search methods to enhance MOEA/D in solving MOVRP. These techniques can well improve the performance of original MOEA/D. However, the decomposition method used in this algorithm is the Tchebycheff method which is the most commonly used decomposition method in solving MOP. It is based on mathematical theory so that it does not take into account of the characteristics of the problem sufficiently. In this paper, in order to enhance the performance of MOEA/D in solving MOWTA, a novel framework of MOEA/D which is named as a multiobjective evolutionary algorithm based on decomposition for WTA (MOEA/D-WTA) is proposed, the contributions of this algorithm are described as follows:

(1) A novel multiobjective optimization framework is proposed. In MOEA/D-WTA, a novel decomposition mechanism is designed to decompose the population into several subpopulations. It decomposes the MOSWTA problem into M scalar optimization subproblems (M is the total number of weapons). Meanwhile, the mating restriction mechanism and the selection operation are redesigned

(2) A problem-specific population initialization method is presented to improve the proposed algorithm

(3) An improved nondominated solution-selection method is presented to handle the constraints of PF in the problem

The rest of this paper is organized as follows. In Section 2, the model of the asset-based MOSWTA problem is illustrated, and the characteristics of this problem are summarized and analyzed. In Section 3, the framework of MOEA/D-WTA is presented and the new mechanisms are described. In Section 4, MOEA/D-WTA is compared with four MOEA variants and the sensitivity is analyzed. The experimental studies on some generated cases are presented to demonstrate the performances of our proposed algorithm. The conclusion and future work are described in Section 5.

2. Problem Analysis

2.1. Some Basic Definitions of MOP. A multiobjective optimization problem (MOP) can be defined as follows:

\[
\begin{align*}
\text{minimize} & \quad F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^T, \\
\text{subject to} & \quad x \in \Omega,
\end{align*}
\]

(1)
where $\Omega$ is the decision space, $x$ is the decision vector, $x = [x_1, x_2, \ldots, x_n]$, $n$ is the total number of decision variables. $f_m(x)$ denotes the $m$th objective function. $F(x): \Omega \rightarrow \mathbb{R}^m$ is composed by $m$ objective functions.

Let $v, u \in \mathbb{R}^m$ are any two objective vectors of a MOP, $v$ is said to dominate $u$ (denoted as $v \prec u$) if and only if $f_i(v) \leq f_i(u)$ for every $i = 1, 2 \ldots m$ and there exists at least one $f_j(v) < f_j(u)$, $j \in \{1, 2 \ldots m\}$. Let objective vectors $x^* \in \Omega$, if there is no vector in $\Omega$ that dominates $x^*$; $x^*$ is said to be a Pareto optimal solution, and $F(x^*)$ is called a Pareto optimal vector. The set of all the Pareto optimal solutions is called the Pareto set (PS). The set of all the Pareto optimal vectors is called the Pareto front (PF).

### 2.2. The Asset-Based MOSWTA Problem

In a military conflict, we assume that all the weapons are launched at the same time and the evaluation is implemented after all engagement. The asset-based SWTA problem can be depicted in Figure 1.

In Figure 1, asset is the valuable resources that should be protected and each asset has a different value; target is the opponent which is aimed at the asset and each target has different probability to destroy different asset; weapon can destroy the target and each weapon has different kill probability (if a weapon is assigned to a target, kill probability means the probability of the weapon destroys the target) to different target; weapon platform can be regarded as a weapon storehouse, in which all the weapons have the same kill probability to a specific target.

Figure 1 is a simple scenario of asset-based SWTA problem. Each asset can be attacked by one or multiple targets, even by no target. Each weapon could be assigned to only one target. The purpose of this problem is to assign weapons to the appropriate targets to maximize the expected total value of assets. If the consumption of weapons is increased, there is a greater probability of destroying the target, which means that more assets will be retained and the expected total value of assets will be improved. On the contrary, if the consumption of weapons is reduced, the targets have a greater probability of surviving in the engagement and the expected total value of assets will be abated. Obviously, the consumption of weapons and the expected total value of assets are two conflicting objectives. However, sometimes, when we have already achieved the desirable total value of assets, we do not need to assign the remainder weapons to targets in order to save weapons for the next engagement. In order to optimize these two conflicting objectives simultaneously, the asset-based SWTA problem should be converted into an asset-based MOSWTA problem.

The model of asset-based MOSWTA problem can be written as

\[
\begin{align*}
\text{max} & \quad f = \sum_{k=1}^{K} \omega_k \prod_{j \in G_k} \left[ 1 - \pi_{jk} \prod_{i=1}^{M} (1 - p_{ij})^{x_{ij}} \right], \\
\text{min} & \quad g = \sum_{j=1}^{N} \sum_{i=1}^{M} x_{ij}, \\
\text{s.t.} & \quad \sum_{j=1}^{N} x_{ij} = 1, \quad i = 1, 2, \ldots, M,
\end{align*}
\]  

where $f$ is an evaluation objective which denotes the expected total value of assets; $g$ is the other evaluation objective which represents the consumption of weapons; $M$ is the total number of weapons; $N$ is the total number of targets; $K$ is the total number of assets; $G_k$ is the set of targets aimed for asset $k$; $\omega_k$ is the value of asset $k$ and $\omega_k \geq 0$; $p_{ij}$ is the kill probability that the weapons $i$ against the target $j$; $\pi_{jk}$ is the probability that target $j$ destroys the asset $k$; $x_{ij}$ is the assignment
 Complexity

variables which denotes that weapon $i$ is assigned to target $j$; $x_{ij}$ is represented by

$$x_{ij} = \begin{cases} 
1, & \text{if weapon } i \text{ is assigned to target } j, \\
0, & \text{otherwise}. 
\end{cases} \quad (3)$$

In equation (2), the survival probability of target $j$ is given by $\prod_{i=1}^{M} (1 - p_{ij})^{x_{ij}}$. Therefore, the probability that asset $k$ is destroyed by target $j$ is given by $\pi_k \prod_{i=1}^{M} (1 - p_{ij})^{x_{ij}}$. Hence, $\prod_{i \in G_k} [1 - \pi_k \prod_{i=1}^{M} (1 - p_{ij})^{x_{ij}}]$ is the survival probability of asset $k$. As a consequence, the total value of assets is given by $\sum_{k=1}^{N} \omega_k \prod_{i \in G_k} [1 - \pi_k \prod_{i=1}^{M} (1 - p_{ij})^{x_{ij}}]$. $\sum_{i=1}^{M} x_{ij}$ is the total number of weapons which are assigned to target $j$. As a result, the consumption of weapons is given by $\sum_{j=1}^{N} \sum_{i=1}^{M} x_{ij}$, and the constraint which means that each weapon can be assigned to only one target.

2.3. The Characteristics of Asset-Based MOSWTA Problem. In order to design appropriate mechanisms to solve the asset-based MOSWTA problem, the characteristics of this problem should be analyzed. As the asset-based MOSWTA problem is an NP-complete problem, the increase of targets or weapons will cause an exponential increase in the total number of solutions. For example, if there are 5 targets and 8 weapons, the total number of solutions is only 1,679,616. If there are 5 targets and 9 weapons, the total number of solutions is even as high as 40,353,607. However, no matter how many targets and weapons, the nature of this problem is the same. Hence, a small-scale scenario of this problem is adopted which have 4 targets and 8 weapons, and the enumeration method is employed to calculate all the solutions. All these solutions and the Pareto optimal solutions of this scenario is shown in Figure 2.

In Figure 2, $f$ and $g$ are the two evaluation objectives which are denoted as the expected total value of assets and the consumption of weapons, respectively. All these points in Figure 2 represent the feasible solutions of this scenario which are calculated by the enumeration method. All the Pareto optimal solutions are circled on the rightmost side of these solutions. As can be seen from Figure 2, these solutions have some special characteristics which can be summarized as follows.

1. The PF of asset-based MOSWTA problem is discrete, and the distribution is fixed
2. The solutions of asset-based MOSWTA problem could be divided into $M$ groups, and all the solutions in each group have the same objective $g$
3. The number of Pareto optimal solution is the same as the total number of weapons, and each solution has a different objective $g$
4. The direction of the convergence of all the solutions is the same as the direction of the increasing of objective $f$

It should be noted that all these characteristics can also be applied to the target-based MOSWTA problem. In addition, if we limit the consumption of weapons, such as a lower and upper bound on the number of the consumption of weapons assigned to the targets, all these characteristics are still available.

According to these characteristics, some appropriate techniques can be designed to enhance the MOEA/D in solving MOWTA. Therefore, our motivations to propose a new framework of MOEA/D in this paper is based on these characteristics. The interpretations of these characteristics are as follows:

1. The first characteristic describes the distribution of PF. The distribution of PF is a key factor which can greatly affect the performance of MOEA. This characteristic can guide us to design the framework and the appropriate mechanisms of MOEA to improve the performance.
2. The second characteristic describes the distribution of feasible solutions. According to this characteristic, the consumption of weapons is remaining unchanged in each group. It means that the problem can be decomposed into $M$ scalar optimization subproblems. Therefore, the asset-based MOSWTA problem can be considered as a composition of several scalar optimization problems which make the decomposition method feasible. Based on this characteristic, the appropriate decomposition method could be conceived and make the problem simple and efficient.
3. The third characteristic describes the constraints of PF. Due to the characteristic of PF, the total number of Pareto optimal solutions achieved by the nondominated solution-selection method must be a fixed number. The evaluation objective $g$ of each Pareto optimal solutions is mutually different. Hence, an appropriate
nondominated solution-selection method should be proposed to handle these constraints.

(4) The fourth characteristic describes the direction of convergence of all the feasible solutions. Owing to the direction of convergence of all the solutions is unique and unalterable, this characteristic could help us to evaluate the convergence of the algorithm especially for a large-scale asset-based MOSWTA problem.

3. The Proposed Framework and Its Implementation

3.1. The Framework of MOEA/D-WTA. In this paper, a new multiobjective evolutionary algorithm based on decomposition is proposed for asset-based MOSWTA problem, which is denoted as MOEA/D-WTA. The main framework of MOEA/D-WTA can be described as follows. MOEA/D-WTA begins with the initialization which can generate the population. Then, an improved nondominated solution-selection method is performed, and the nondominated solutions are reserved in an external population. Thereafter, a novel decomposition method is adopted to decompose the population into \( M \) subpopulations. Moreover, the neighborhood relationships among different subpopulations are redesigned, and the neighboring subpopulations are computed for mating restriction. Afterwards, the algorithm goes into the loop phase. At the beginning of the loop, the population is decomposed into a number of subpopulations by the proposed decomposition approach. Subsequently, the recombination is performed in which the evolutionary operations are used to generate the new individuals. Additionally, the nondominated solutions are selected to update the external population. Finally, the potential individuals are selected to the next loop. The loop is continued until the maximum number of generations is reached. Algorithm 1 shows the framework of MOEA/D-WTA.

In this framework, some novel multiobjective optimization techniques have been used to improve the performance of the MOEA/D in solving MOWTA, including initialization, nondominated solution selection, decomposition method, mating restriction, and selection operation. The details of these techniques are illustrated in the following sections.

3.2. The Individual Encoding and Constraint Handling. According to the model of asset-based MOSWTA problem, the individual of this problem has many constraints. Liang and Kang [23] designed a decimal encoding which can handle the constraints of this problem effectively. The illustration of this encoding is as follows.

For the asset-based MOSWTA problem, the permutation of these targets which are attacked by weapons in weapon platform \( i \) is defined as \( x_i = \{t_{i1}, \ldots, t_{ij}, \ldots, t_{iL}\} \), where \( c_i \) is the number of weapons in platform \( i \) and \( t_{ij} \) is the target label which is attacked by weapon \( j \) in weapon platform \( i \). \( t_{i1}, \ldots, t_{ij}, \ldots, t_{iL} \) are random integers between 0 and \( N \). The encoding of individual can be defined as \( X = \{x_1, x_2, \ldots, x_L\} \), where \( L \) is the total number of weapon platforms. Figure 3 is the illustration of the encoding.

According to the definition of the individual encoding, it can handle the constraints of the asset-based MOSWTA problem. The total number of bits is the same as the total number of weapons. Therefore, the number of weapons is limited within the designated scope. Afterwards, each bit of the encoding represents a different weapon, so that each weapon can be assigned to only one target.

3.3. Initialization. In the asset-based MOSWTA problem, there is a mass of priori knowledge, such as the characteristics in Section 2.3. If the individuals are initialized randomly, it...
would have a low fitness and the population would have a lot of redundancy. Meanwhile, the efficiency of the algorithm will be reduced inevitably. Hence, this priori knowledge should be fully used to improve the performance of the proposed algorithm. In this paper, a problem-specific initialization method is proposed to initialize the population. Each individual in this proposed method is initialized by the priori knowledge, so that the initial population will have a better fitness and the efficiency of the algorithm will be improved.

For the asset-based MOSWTA problem, it can be easily proved that if each weapon in an individual attacks different target, it will have a better fitness than the one in which some weapons have the same target. Therefore, the key point of the proposed initialization method is to make each weapon attack different target as far as possible. The pseudocode of the proposed initialization method is given in Algorithm 2.

Algorithm 2: Initialization.

In Algorithm 2, Step 4 is used to determine whether the number of weapons (each bit represents a weapon) that need to be initialized is less than the total number of targets. If it is true, means that we can assign these weapons to mutually different targets. If the number of weapons that need to be initialized is more than the total number of targets, we should assign these weapons to targets uniformly. Step 7 calculates the number of weapons which are uniformly assigned to each target. After the assignment of these weapons uniformly, the number of remaining weapons is calculated in Step 8. By this way, the priori knowledge of WTA can be considered into the initialization method to make the initialized individual achieve a preferable fitness.

3.4. Decomposition. The MOP can be decomposed into a number of scalar optimization problems by decomposition method. Although the ordinary decomposition methods, such as the Tchebycheff method, work very well in solving comprehensive MOPs, they do not fully consider the characteristics of a particular problem. Therefore, if the decomposition method can be designed according to the characteristics of the problem, the performance may be greatly improved. On the basis of the second characteristics of the problem in Section 2.3, the solutions could be divided into some groups. The optimization of the solutions in each group can be regarded as a scalar optimization problem. Based on these priori knowledge, a novel decomposition method is proposed to handle this problem. In the proposed decomposition method, the population is decomposed into $M$ subpopulations. All the individuals have the same value of objective $g$ in one subpopulation. This proposed decomposition method can be further illustrated in Figure 4.

The scenario used in Figure 5 is the same as Figure 2. The population has 300 individuals. As shown in Figure 5, because the total number of weapons is 8, all the solutions are decomposed into 8 subpopulations. Meanwhile, the solutions in each subpopulation have the same objective $g$. Therefore, the asset-based MOSWTA problem can be decomposed into a known number of scalar optimization problems. According to the above analysis, the proposed decomposition method can make full use of the characteristics of this problem.

3.5. Mating Restriction and the Neighboring Subpopulations. In the framework of original MOEA/D, each solution has $T$
closet neighbors. The mating restriction in this framework can make two solutions mate which are selected in two neighbors, respectively. This mating restriction contribute to maintain the diversity of the population and improve the convergence of the algorithm. However, in the proposed framework, the population is decomposed into a number of subpopulations. Therefore, the traditional mating restriction is no longer adapted to MOEA/D-WTA. In order to select subpopulations, Therefore, the traditional mating restriction is proposed. We use the neighborhood list level to describe the scale of the neighboring subpopulations set. \( T \) is the absolute value of the maximum difference in the consumption of weapons between the neighbor subpopulations and the selected subpopulation. Assume that the population is decomposed into \( M \) subpopulations as \( X = \{ s_1, s_2, \ldots, s_M \} \). The neighboring subpopulation set of the \( i \)th subpopulation is defined as follows:

\[
B(i) = \begin{cases} 
    s_1, \ldots, s_i, \ldots, s_{i+T}, & i - T < 1, \\
    s_{i-T}, \ldots, s_i, \ldots, s_M, & i + T > M, \\
    s_{i-T}, \ldots, s_i, \ldots, s_{i+T}, & \text{else}.
\end{cases}
\]

Here is an example of the proposed mating restriction for the population which is the same as Figure 4. Let \( T = 2 \), the neighboring subpopulation can be illustrated in Figure 5.

As shown in Figure 5, \( s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \) are the subpopulations achieved by the proposed decomposition method. \( B(1), B(2), B(3), B(4), B(5), B(6), B(7), \) and \( B(8) \) are the neighboring subpopulation sets of the subpopulation \( s_1, s_2, s_3, s_4, s_5, s_6, s_7, \) and \( s_8 \), respectively. Each transverse line represents a subpopulation which is pointing to. Each vertical line represents a neighboring subpopulation set. The transverse line crosses over a vertical line means that the subpopulation belongs to the neighboring subpopulation set. Therefore, the neighboring subpopulation sets of the subpopulation \( s_1, s_2, s_3, s_4, s_5, s_6, s_7, \) and \( s_8 \) are \( B(1) = \{ s_1, s_2, s_3 \} \), \( B(2) = \{ s_1, s_2, s_3, s_4 \} \), \( B(3) = \{ s_1, s_2, s_3, s_4, s_5 \} \), \( B(4) = \{ s_2, s_3, s_4, s_5, s_6, s_7 \} \), \( B(5) = \{ s_3, s_4, s_5, s_6, s_7 \} \), \( B(6) = \{ s_4, s_5, s_6, s_7, s_8 \} \), \( B(7) = \{ s_5, s_6, s_7, s_8 \} \), and \( B(8) = \{ s_6, s_7, s_8 \} \), respectively.

According to the example of the neighboring subpopulation set, we note that the neighborhood list level \( T \) is not the size of the neighboring subpopulation set. It is a range in which two individuals can be mate. When \( i - T < 1 \) or \( i + T > M \), the size of neighboring subpopulation set is different with each other. We do not add other subpopulations into these neighboring subpopulation sets to make all the subpopulation sets have the same size. It is because if the consumption of weapons of the added subpopulation has a large difference with the selected subpopulation, the efficiency of the algorithm may be reduced. Meanwhile, each subpopulation has many individuals so that all the neighboring subpopulation sets have enough individuals to perform the evolutionary operation efficiently. Based on the above analysis, we believe that the redesigned mating restriction is effective and feasible.

3.6. Nondominated Solution Selection. In the framework of original MOEA/D, PF is selected by comparing the dominance relationship of the solutions. However, the traditional nondominated solution-selection method could not take into account the constraints of PF in the asset-based MOSWTA problem. According to the third characteristic of this problem in Section 2.3, the PF has two constraints which are the constraint of quantity and the constraint of objective \( g \). For the constraint of quantity, the total number of solutions of PF, which is equal to the total number of weapons, is fixed. Meanwhile, for the constraint of objective \( g \), each solution of PF has the mutually different objective \( g \). Although the constraint of quantity can be easily handled, the constraint of objective \( g \) is difficult to be handled by comparing the dominance relationship of the solutions. This problem can be further illustrated in Figure 6.

Figure 6 shows a possible PF for an asset-based MOSWTA problem with the weapon number of 6. A and B are two solutions of the PF, and their values are \( (f_2, g_2) \) and \( (f_1, g_1) \), respectively. It can be seen that \( g_1 < g_2 \) and \( f_1 > f_2 \). The solution A is dominated by B, i.e., A > B. Therefore, if the nondominated solution selection is performed by comparing the dominance relationship of the solutions, the solution A will not be selected. As a result, the PF in Figure 6 will be a lack of the solution A, which means that the decision-maker is unable to get the solution in case the weapon consumption is \( g_2 \). However, if this situation happens, the constraint of quantity and the constraint of objective \( g \) would be broke. In addition, when the population is large, the method of comparing the dominance relationship of the solutions is time consuming. As a consequence, an improved nondominated solution-selection method is proposed in this paper. The pseudocode of the proposed nondominated solution selection is given in Algorithm 3.

In Algorithm 3, Step 3 means that the individual can be selected as the nondominated solution which has the maximum total value of assets in its subpopulation. The procedures between Step 4 and Step 6 are the method to update EP. Each individual in EP is updated only if there is an individual that dominate it which has the same consumption of
3.7. Selection Operation. In the framework of original MOEA/D, each individual is regarded as a scalar optimization problem. It can be updated by the generated new individuals according to the value of the objective functions. However, in MOEA/D-WTA, the problem is decomposed into $M$ scalar optimization problems; the traditional selection method is no longer available. As a result, the selection operation is redesigned to select the potential individuals and maintain the diversity of the population. Two important problems should be considered in the selection which are the diversity of the population and the fitness of an individual. According to the analysis of the first characteristic of the asset-based MOSWTA problem in Section 2.3, the distribution of PF is fixed. Therefore, the individuals should be selected uniformly from the subpopulations so that the distribution of PF can be sustained. At the same time, as the individuals are selected uniformly, the diversity of population can be also maintained.

The procedure of the proposed selection operation is illustrated in Figure 7, where newpop is the population which is composed by the selected individuals, $p$ is the size of new pop, $M$ is the total number of weapons, $n$ is the remainder of $p$ divided by $M$, and $x_1, x_2, \ldots, x_p$ are the selected individuals. In the first stage, the nondominated solutions of the population are selected, then all these selected individuals are removed from the population. After this stage, $M$ individuals are selected into newpop. In the second stage, the procedure in the first stage is employed again to select the second part of the newpop. This procedure will be repeated $h$ times; $h$ is the quotient of $p$ divided by $M$. The penultimate stage is the $h$th time of the repetition. After this stage, $h \times M$ individuals are selected into newpop. If $p$ is divisible by $M$, this stage will become the last stage. However, if $p$ is not evenly divisible by $M$, there are $n$ individuals that need to be selected. In the last stage, $n$ nondominated solutions are selected as the final part of newpop.

Since the proposed selection method selects the nondominated solutions in each stage, the expected total value of assets of the selected individuals can be guaranteed to be the largest one of the current population. At the same time, in most instances, the selected nondominated solutions includes all possible weapons consumption. Therefore, the constraint of the diversity of the population and the fitness of an individual can be fulfilled. In addition, our proposed selection operation has some similar behaviors with the Pareto-based MOEAs. Therefore, it is an important distinction between the proposed MOEA/D-WTA and the traditional MOEA/D. By this selection operation, the advantages of the two state-of-the-art frameworks can be integrated into our proposed algorithm. All these advantages of the selection operation lead to a suitable solution of the MOSWTA.
3.8. Computational Complexity. Assuming that the population size is $n$ and the number of weapons is $M$, the time complexity of MOEA/D-WTA can be calculated as follows: the time complexity for initialization is $O(nM)$. The time complexity for compute neighboring subpopulations is $O(M)$. The time complexity for decomposition is $O(M)$. The time complexity for nondominated solution selection is $O(nM)$. The time complexity for evolution is $O(n)$. The time complexity for selection is $O(nM)$. Based on the above analysis, the total time complexity of MOEA/D-WTA is $O(nM) + 2O(M) + O(n)$, so the worst time complexity of one generation is $O(nM)$.

4. Experimental Studies

4.1. Test Case Generation. In order to test the performance of the proposed algorithm comprehensively, some test cases are generated. In this paper, nine test cases are generated including the small-scale, the medium-scale, and the large-scale asset-based MOSWTA problem scenarios. Because the major
goal of the problem is to find a solution which has less consumption of weapons and a good expected total value of assets, we only generate the case in which the total number of weapons is larger than the total number of targets. The details of these cases are presented in Table 1 (all these data are available from the corresponding author upon request).

Since the individual encoding is composed of the weapon and target, the most important factors affecting the performance of the algorithm are the total number of weapons and the total number of targets. Therefore, these cases are generated mainly on the basis of the two factors. According to the definition of encoding, the dimension of the individual is based on the total number of weapons. Hence, it is the major factor of the scale of the problem. Case 1, case 2, and case 3 are the small-scale scenarios. Case 4, case 5, and case 6 are the medium-scale scenarios. Case 7, case 8, and case 9 are the large-scale scenarios. In addition, the difference of the gap between the total number of targets and the total number of weapons can also affect the performance of the algorithm. Therefore, the gap is also taken into account to generate these cases. Case 1, case 4, and case 7 are the scenarios which have a large gap. Case 2, case 5, and case 8 are the scenarios which have a medium gap. Case 3, case 6, and case 9 are the scenarios which have a small gap. As the total number of assets and the total number of weapons platforms are only used to calculate the value of the objective function, they are not the important factors for the algorithm. Thus, they are only designed based on the scale of the problem. Based on the above analysis, there is a good reason to believe that these cases can fully present the performance of the algorithm.

### 4.2. Comparative Metrics

In MOWTA, since the total number of the nondominated solutions are finite and the distribution of the PF is fixed, we do not need to consider the distribution of the nondominant solutions. Therefore, for the MOWTA problem, we only pay attention to evaluate how far the nondominated solutions found so far from the solutions in the Pareto optimal set. According to the above analysis, the generational distance (GD) [49] metric is adopted in this paper to compare the performance of the algorithms. Assume $P$ is the Pareto optimal set and $A$ is the set of the nondominated solutions found so far, the GD metric is defined as follows:

$$
\text{GD}(A, P) = \sqrt{\frac{\sum_{i=1}^{\left|\mathcal{A}\right|} d_i^2}{\left|\mathcal{A}\right|}},
$$

where $\left|\mathcal{A}\right|$ is the number of vectors of $A$; $v_i$ is the $i$th solution of $A$; $F(v_i)$ and $F(P)$ are the value of evaluation objectives of $v_i$ and the solutions of $P$, respectively; $d_i$ is the Euclidean distance between each of the nondominated solutions found so far and the nearest member of the Pareto optimal set. According to the GD metric, the lower the GD metric of the algorithm, the better the performance of the algorithm. $\text{GD}(A, P) = 0$, indicates that the nondominated solutions found so far are in the Pareto optimal set.

If the scale of the problem is small, the Pareto optimal set can be obtained by the exhaustion method, so the GD metric can be used to evaluate the performance of the algorithms. However, the number of the solutions of the problem increases exponentially with the increase of the number of weapons and targets. If the scale of the problem is large, the Pareto optimal set can hardly be achieved. As a result, the GD metric is unable to use. In order to evaluate the performance of the algorithms, a generational distance approximation (GDA) metric is adopted. The GDA metric is defined as follows:

$$
\text{GDA}(A, P) = \sqrt{\frac{\sum_{i=1}^{\left|\mathcal{A}\right|} d_i^2}{\left|\mathcal{A}\right|}},
$$

where $d_i$ should be defined as $d_i = \sqrt{(f(v_i) - 1)^2}$, $f$ is the value of evaluation objective $f$ of $v_i$. Based on the definition of GD metric, for the asset-based MOSWTA problem, $d_i$ should be defined as $d_i = \sqrt{\left( (f(v_i) - f(P))^2 + (g(v_i) - g(P))^2 \right)}$, where $(f(v_i), g(v_i))$ and $(f(P), g(P))$ are the value of evaluation objectives of $v_i$ and the solutions of $P$, respectively. According to the second characteristic of the problem in Section 2.3, each solution of the nondominated solutions found so far and the nearest solution of the Pareto optimal set have the same value of evaluation objective $g$. Therefore, $d_i$ is reduced to $d_i = \sqrt{\left( f(v_i) - 1 \right)^2}$. In addition, on the basis of the fourth characteristic of the problem, the direction of convergence of the solutions is the same so that the nearest solution of the Pareto optimal set can be easily determined. Hence, if we take the set (in this set, the objective $f$ of each solution is equal to 1) as the Pareto optimal set, $d_i$ is reduced to $d_i = \sqrt{f(v_i) - 1^2}$.

<table>
<thead>
<tr>
<th>Case name</th>
<th>The total number of weapons</th>
<th>The total number of targets</th>
<th>The total number of assets</th>
<th>The total number of weapon platforms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>10</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Case 2</td>
<td>20</td>
<td>10</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Case 3</td>
<td>30</td>
<td>20</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Case 4</td>
<td>40</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Case 5</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Case 6</td>
<td>60</td>
<td>50</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Case 7</td>
<td>70</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Case 8</td>
<td>80</td>
<td>40</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Case 9</td>
<td>90</td>
<td>80</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

**Table 1: The generated cases.**
According to equation (2), if \( \sum_{k=1}^{K} \omega_k = 1 \), there is \( f \leq 1 \), which means the value 1 is the maximum value of objective \( f \). Therefore, the GDA metric can evaluate how far the nondominated solutions found so far from \( f = 1 \). Obviously, the algorithm with a smaller GDA metric is better than others.

### 4.3. Comparison Algorithms

As the proposed MOEA/D-WTA is an EA-based algorithm, four EA-based algorithms named as NSGA2-V, MOEA/D-TC, MOEA/D-WS, and M-MOEA/D-V are chosen to compare with this algorithm.

NSGA2-V is a variant based on the NSGA2 framework [35]. The proposed initialization method and the WTA evolutionary operators are added into NSGA2 in order to solve the problem. NSGA2 is selected as our comparison algorithm because it is the state-of-the-art MOEA in which there is no decomposition involved. In addition, we want to show that the MOEA based on decomposition can perform better in this problem.

MOEA/D-TC and MOEA/D-WS are two improved algorithms based on MOEA/D framework [36], in which the Tchebycheff approach and the weighted sum approach are used, respectively. In order to make a fair comparison, the nondominated solution-selection method proposed in this paper is used in these two algorithms. We choose the algorithms as the comparison algorithms because our proposed algorithm is based on the decomposition. Moreover, the Tchebycheff approach and the weighted-sum approach are the most popular decomposition methods. We want to show that the proposed decomposition method is more suitable for the asset-based MOSWTA problem.

M-MOEA/D-V is a variant of M-MOEA/D [48] which has a very good performance in solving MOVPR. As M-MOEA/D is originally designed to solve the MOVPR, we use some techniques of MOWTA to improve this algorithm so that it can be used in solving MOWTA. This algorithm is chosen as the comparison algorithm because the MOVPR has a very similar behavior with MOWTA when the multiobjective optimization objective is composed of the number of used vehicles and some continuous objective. It means that the M-MOEA/D is suitable for solving MOWTA.

In addition, all these algorithms adopt the same initialization method proposed in this paper. Meanwhile, the EX operator [20] is adopted as the crossover operator, and the mutation operator (randomly change a gene) used in [20] is also employed. The main features of these MOEAs are shown in Table 2.

### 4.4. Experiments on MOEAs

The experiments are performed on a personal computer with i7-2.5GHz CPU and 8GB memory. All the algorithms are written by Matlab. In this experimental studies, for all the compared algorithms, the population size \( pop = M \times 10 \), the crossover probability \( pc = 0.8 \), the mutation probability \( pm = 0.4 \), the neighborhood list level \( T = 3 \). The crossover probability and the mutation probability are the recommended settings in [20]. We stop the algorithms until the nondominated solutions found so far remain unchanged in 50 generations. We performed 30 independent runs on each algorithm for each case. For each case, we use \( M \) points in the approximate PF to calculate the GDA metric. The expected total value of assets of each point is set to 1 and each point has different consumption of weapons. The results of the GDA metric for the comparison algorithms are presented in Table 3, the best results obtained are shown in boldface.

As shown in Table 3, MOEA/D-WTA performs better than the other four algorithms for the majority of the generated cases on these performance indexes. For case 1, all the algorithms have a very similar performance. For case 2, case 3, and case 4, M-MOEA/D-V perform remarkably better than MOEA/D-TC and MOEA/D-WS. However, for case 5–case 9, the average of GDA of MOEA/D-TC outperforms the M-MOEA/D-V. For the majority of the generated cases, MOEA/D-WS has the worst result of these algorithms. In addition, although MOEA/D-WTA performs better than NSGA2-V, the differences between these performance indexes obtained by MOEA/D-WTA and NSGA2-V are very small.

Based on analyzing the experimental results in Table 3, MOEA/D-WTA outperforms other algorithms in all the...
| Case name | Best | Worst | Average | Median | Std. dev. | Best | Worst | Average | Median | Std. dev. | Best | Worst | Average | Median | Std. dev. | Best | Worst | Average | Median | Std. dev. | Best | Worst | Average | Median | Std. dev. | Best | Worst | Average | Median | Std. dev. |
|-----------|------|-------|---------|--------|-----------|------|-------|---------|--------|-----------|------|-------|---------|--------|-----------|------|-------|---------|--------|-----------|------|-------|---------|--------|-----------|------|-------|---------|--------|-----------|------|-------|---------|--------|-----------|
| Case 3 | 1.265325e-02 | 1.265326e-02 | 1.265326e-02 | 1.265326e-02 | 1.265326e-02 | 4.141252e-02 | 4.142086e-02 | 4.236031e-02 | 4.144052e-02 | 4.144052e-02 | 4.141179e-02 | 4.141179e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 | 1.265325e-02 |
| Case 5 | 1.957724e-02 | 1.957990e-02 | 1.957990e-02 | 1.957990e-02 | 1.957990e-02 | 6.901256e-02 | 6.902019e-02 | 6.925883e-02 | 6.963906e-02 | 6.963906e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 | 1.958006e-02 |

**Complexity**
generated cases. All these algorithms can work well in very small-scale problem. M-MOEA/D-V performs better than MOEA/D-TC and MOEA/D-WS in the small-scale problem. MOEA/D-TC performs better than M-MOEA/D-V in the large-scale problem. Furthermore, although MOEA/D-WTA performs better than NSGA2-V for the majority of the generated cases, the performance of it is very similar to NSGA2-V, which means that the framework of NSGA2 also has a well convergence in this problem.

In order to compare the efficiency of the algorithms, the convergence rate of the algorithms is compared in this paper. We assume that if the nondominated solutions found so far remain unchanged in 50 generations, the algorithm is convergent. Therefore, we evaluate the efficiency according to the number of generations and the time consumption after the algorithms are converged. We performed 30 independent runs on each algorithm for each case. The results of the average number of convergent generations (AG) and the average time consumption (AT) for comparison algorithms are shown in Table 4.

As shown in Table 4, MOEA/D-WTA performs remarkably better than other algorithms with respect to the average number of convergent generations and the average time consumption. It should be noted that MOEA/D-WS has the best value of AT and AG in case 5–case 9. It does not mean the efficiency of MOEA/D-WS is better than others. According to the GDA metric of it in Table 3, MOEA/D-WS has the worst results of these cases. It means that MOEA/D-WS converges prematurely. In addition, NSGA2-V takes more time than other algorithms. It is because the fast nondominated sorting procedure ranks individuals according to their dominance relationships and crowding distances which is a time-consuming procedure. According to the above analysis, we can conclude that the proposed MOEA/D-WTA has a good convergence and efficiency in solving MOSWTA problem on both small-scale and large-scale problems.

4.5. Experiments on Initialization. In order to verify the performance of the proposed initialization method, the random initialization method is adopted to replace the proposed initialization method in MOEA/D-WTA. The random initialization method means that the encoding of an individual is generated randomly without any prior knowledge. The variant is called MOEA/D-WTA-r1. The operation parameters of MOEA/D-WTA-r1 and MOEA/D-WTA in this section are the same as MOEA/D-WTA in Section 4.4. We performed 30 independent runs on each algorithm for each case. Table 5 shows the GDA metric of the two algorithms.

As shown in Table 5, for case 1 ~ case 4, the two algorithms have a very similar performance. For case 5 ~ case 9, the average of GDA of MOEA/D-WTA is remarkably better than MOEA/D-WTA-r1. This is means that the proposed initialization method can significantly improve the convergence of the algorithm for large-scale problem. It is because when the scale of the problem is large, there are many feasible solutions. The proposed initialization method can greatly reduce the initial search space as well as improve the solution performance. On the contrary, if the scale of the problem is small, there are fewer feasible solutions. The effect of the proposed initialization method is not obvious.

Table 6 shows the AG and AT indexes of the two algorithms. As shown in Table 6, for case 1 ~ case 4, the two algorithms have a very similar efficiency. For case 5 ~ case 9, the AT and AG of MOEA/D-WTA are better than MOEA/D-WTA-r1. These experimental results support the same conclusions we made above. It can be
seen that the proposed initialization method can significantly enhance the convergence and the efficiency for large-scale MOSWTA problem.

In order to verify the proposed nondominated solution-selection method, the normal nondominated selection method [36] is adopted to replace the method in MOEA/D-WTA. The variant is called MOEA/D-WTA-r2. The operation parameters of MOEA/D-WTA-r2 and MOEA/D-WTA in this section are the same as MOEA/D-WTA in Section 4.4. We performed 30 independent runs on each algorithm for each case. Table 7 shows the GDA metric of the two algorithms.

4.6. Experiments on Nondominated Solution Selection. As shown in Table 7, the difference between the average of the GDA metric obtained by MOEA/D-WTA and MOEA/D-WTA-r2 is very small. This demonstrates that the proposed nondominated solution-selection method does not affect the convergence of the algorithm. This is because the proposed method is used to achieve the nondominated solutions from the population. Therefore, it does not affect the evolution of the individual as well as the convergence of the algorithm.

Table 8 shows the AG and AT indexes of the two algorithms. As shown in Table 8, MOEA/D-WTA outperforms MOEA/D-WTA-r2 in terms of the AT and AG metrics. Although the AG metric of MOEA/D-WTA-r2 is better than MOEA/D-WTA in case 1, case 2, and case 4, the AT metric of it is still worse than MOEA/D-WTA. The normal
nondominated selection method has a low-efficiency because it needs to calculate the dominance relationship between individuals. The proposed method only compares the dominance relationship between two nondominant solutions which have the same weapon consumption, so that it can reduce the time consumption of the algorithm. Therefore, the proposed nondominated solution selection can significantly enhance the efficiency of the algorithm.

4.7. Sensitivity Analysis. An extensive analysis of the impact of the parameters of MOEA/D-WTA on its performance is implemented. We use the cases generated before as well as the GDA metric described previously. In the proposed algorithm, there are five parameters, including the population size pop, the crossover probability pc, the mutation probability pm, the neighborhood list level T, and the maximum number of function evaluations. As pc and pm used in this paper are the recommended setting in [20], we no longer have to analyze their sensitivity. At the same time, all the algorithms are stopped until the nondominated solutions found so far remain unchanged in 50 generations. Therefore, the maximum number of function evaluations is self-adapting, so that we do not need to analyze its sensitivity. As a result, the sensitivity of the remainder two parameters, the population size pop and the neighborhood list level T, are analyzed in this section.

We performed two experiments as follows:

1. Experiments 1: we varied the population size. We performed runs using $M^5$, $M^{10}$, $M^{20}$, and $M^{50}$ ($M$ is the total number of weapons) populations. In addition, $pc = 0.8$, $pm = 0.4$, and $T = 3$

2. Experiments 2: we varied the neighborhood list level T. We performed runs using $T = 1$, $T = 2$, $T = 3$, $T = 4$, $T = 5$, $T = 7$, and $T = 10$ for case 2–case 9. Since case 1 has only 10 weapons, we performed runs using $T = 1$, $T = 2$, $T = 3$, $T = 4$, and $T = 5$. Moreover, $pc = 0.8$, $pm = 0.4$, and $pop = M^*10$

4.7.1. Experiments 1. This Experiments is designed to analyze the effect of population size on the proposed algorithm. We performed 30 independent runs on the algorithm regarding each case with several population sizes. For each case, we normalized the average of GDA metric from zero to one. The results are presented in Figure 8.

As shown in Figure 8, for case 1, the population size has no obvious influence on the convergence of the algorithm. It means that if the scale of the problem is very small, the algorithm is not sensitive to the setting of parameters. For other cases, when the population size is small ($pop = M^*10$ or $M^*20$ for the majority of the cases), the convergence of the algorithm is the worst for each case. With the increase of population, the convergence of the algorithm increases rapidly. In practical use, there should be a balance between convergence and time consumption. Although the convergence of the population size $M^*50$ is slightly better than the population size $M^*10$ or $M^*20$ for the majority of the cases, it has a very high time consumption than the two population size. Therefore, we recommend that the population size could be set as $M^*10$ or $M^*20$ to achieve the balance between time consumption and convergence.

4.7.2. Experiments 2. This Experiments is designed to investigate the effect of the neighborhood list level in the performance of MOEA/D-WTA. We performed 30 independent runs on the algorithm regarding each case with several neighborhood list levels. For each case, we normalize the average of GDA from zero to one. The results are presented in Figure 9.

As shown in Figure 9, for case 1, the neighborhood list level has no obvious influence on the convergence of the...
algorithm, which supports the same conclusions we have made above. For case 3, case 5, case 7, and case 8, the change of the average of GDA is stable when the neighborhood list level is larger than 3. For other cases, the average of GDA changes dramatically when the neighborhood list level is less than 3. Meanwhile, the average of GDA is increased when the neighborhood list level is larger than 5. Based on analyzing the experimental results in this paper, we recommend that the neighborhood list level could be set as 3, 4, or 5 to achieve a satisfying result.

4.8. Some Discussions. In this section, we mainly discuss two experiments which are very worthy of consideration.

(1) Comparisons between the best-known results and the best results obtained by MOEA/D-WTA. Although it is difficult to obtain the optimal solutions of large-scale MOSWTA problem, we can compare the achieved best results with the best-known results to conclude whether the proposed algorithm has a better performance, such as the experiments in [48]. The best-known results can be obtained by the single-objective optimization algorithm, such as the algorithms in [20–23]. However, most of the current WTA algorithms focus on the target-based SWTA problem, so that the asset-based MOSWTA problem is short of research. Therefore, we do not have enough data of the best-known results to verify the performance of the algorithm. As a result, in our future research, we will use the single-objective version of the problem to work out the solutions as the best-known results. We will make comprehensive comparisons between the best-known results and the best results obtained by our proposed algorithm achieved by MOEA/D-WTA. As the MOSWTA problem is a clearly decomposable problem, we can use $M$ local search procedures, such as the single-objective optimization algorithms, to work out all the expected total value of assets of the MOSTWA problem. Therefore, we can compare our proposed algorithms with $M$ local search procedures to conclude whether the proposed algorithm have a better performance. However, if the number of weapons is large, we need to perform the local search procedure many times which is a time-consuming process. Meanwhile, if we use the local search procedure, we have to set an upper bound on the number of the weapons assigned to the targets. As a consequence, we have to improve the mechanism of the algorithm to make the local search procedure to be able to handle this constraint. For example, we can design the repair mechanisms of an individual or we can improve the crossover operator and the mutation operator to handle this constraint. Nevertheless, all of these constraint handling methods need a reasonable and careful design so that this comparative experiment demand a huge amount of work that needs further study. In our future work, we will improve some of the single-objective optimization algorithms to make the local search procedure more efficient. As a result, we can compare the performance of our proposed algorithm with $M$ local search procedures to design better multiobjective optimization techniques solving the asset-based MOSWTA problem.

(2) Comparisons between the expected total value of assets obtained by $M$ local search procedures and the results.

5. Conclusions and Future Work

The WTA problem has been widely concerned and studied, but there is still less research on the problem of MOWTA. In this paper, the asset-based MOSWTA problem is studied,
which is a representative problem of MOWTA. This paper first illustrates the model of the asset-based MOSWTA problem. Furthermore, the four defining characteristics of this problem are analyzed. According to these characteristics, a novel MOEA framework named as MOEA/D-WTA is presented.

At the beginning of MOEA/D-WTA, a problem-specific population initialization method based on priori knowledge is designed to generate the population. Thereafter, a novel decomposition method is proposed to decompose the MOSWTA problem into a number of scalar problems. Afterwards, these scalar problems are optimized simultaneously by using MOEA/D-WTA where a new mating restriction, a problem-specific nondominated solution-selection method, and a novel selection operation are integrated.

The performances of the proposed algorithm are discussed in some generated cases. Extensive experiments with four improved algorithms of the state-of-the-art MOEA framework demonstrate that MOEA/D-WTA outperforms the other methods in terms of convergence and efficiency. In addition, the effect of the proposed initialization method and the nondominated solution-selection method are detailed, which shows that these mechanisms can enhance the performance of the problem.

Since MOEA/D-WTA is a multiobjective optimization framework, many good evolutionary mechanisms can be added to this framework to further improve the performance of the algorithm. One possible improvement is the local search methods, which is one of the most popular techniques to enhance the MOEAs for better convergence. Moreover, as all the comparison algorithms in our experiments are based on GA, we will further enhance our research by comparing with more algorithm frameworks in the future, such as the MOPSO and the MODE. In addition, our future work will focus on the MODWTA problem. As the MODWTA problem has multiple stages of the engagement, the problem will be extremely complex. Therefore, how to design an appropriate framework for the problem is a huge challenge for us.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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