Research Article

Observer-Based Fuzzy Control for Memristive Circuit Systems

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This paper proposes an observer-based fuzzy control scheme for a class of memristive chaotic circuit systems. First, the Takagi-Sugeno fuzzy model is adopted to reconstruct the nonlinear chaotic circuit system. Next, based on the proposed fuzzy model, an observer-based fuzzy controller is developed to estimate the states and stabilize the origin. Third, the results are extended to explore the L_∞ -gain observer-based fuzzy control for the chaotic system with disturbances. Finally, simulation results are also addressed to show the effectiveness of the proposed control scheme.

1. Introduction

In 1971, Chua postulated the existence of a fourth circuit element [1], called memristor, which was realized by Williams's group of HP Labs only 37 years later [2]. In recent years, the memristor has attracted much attention due to its potential application in associative memory [3], image processing [4], filter [5], programmable analog circuits [6], and so on. In particular, Pershin and Ventra experimentally demonstrated the formation of associative memory in a simple neural network, which consists of three electronic neurons using memristor-emulator synapses [3]. A new image encryption algorithm was presented in [4] based on chaos with the piecewise-linear memristor in Chua's circuit. The authors in [5] experimentally demonstrated an adaptive filter by introducing a memristor and using the memristive properties of vanadium dioxide. In [6], a memristor was designed for a pulse-programmable midband differential gain amplifier with fine resolution.

The HP memristor is described by a nonlinear constitutive relation as introduced in [7]

$$\begin{aligned} v &= M(q)i, \\ \text{or } i &= W(\varphi)v, \end{aligned} \quad (1)$$

between the device terminal voltage v and terminal current i , where $\varphi = \int v dt$, $M(q) = d\varphi(q)/dq$, and $W(\varphi) = dq(\varphi)/d\varphi$,

where $M(q)$ and $W(\varphi)$ are the memristance and memductance. Memristor-based systems may exhibit complex behaviors, such as chaotic and hyperchaotic dynamics. Recently, chaos control, hybrid control, and synchronization of memristor-based or memristive chaotic systems have received intensive investigation [8–14]. However, the “piecewise-linear” nonlinearity characterization by introducing a memristor may lead to challenges in dealing with the chaotic systems. Fuzzy modeling approaches result in a way that the original systems can be decomposed into a number of linear subsystems. In the context of the Takagi-Sugeno fuzzy models, Zhong et al. [8] addressed fuzzy modeling and impulsive control of the memristor-based Chua chaotic system. Cafagna and Grassi presented a novel fractional-order memristor-based chaotic system and carried out the theoretical analysis of the system dynamics [9]. The Takagi-Sugeno fuzzy method emerged as a promising approach for approximating nonlinear systems [8, 15, 16]. More recently, a new fuzzy model of the memristor-based Lorenz circuit, which was employed to synchronize with the memristor-based Chua circuit, was explored in [14].

Despite the rich achievements, most of the above results mainly focused on stability or synchronization of memristive chaotic circuit systems rather than state estimation [17, 18]. However, in real chaotic circuits, it is often the case that only partial information about the states (for instance, voltage) is

available in the system outputs. Therefore, in order to utilize the memristive chaotic circuit systems, one often needs to estimate the system state through available measurement, and then use the estimated system to achieve synchronization, optimal control [19], or tracking performances. In addition, general results on state estimation and observer-based control for such memristive systems do not seem to have received much attention so far. To the best of our knowledge, there is a lack of effort in the observer-based control and synchronization of memristive systems [18].

Inspired by [16], this paper aims at investigating the observer-based fuzzy control for the stabilization of the memristive Chua circuit systems with or without external disturbances. An observer-based fuzzy control scheme based on the Takagi-Sugeno fuzzy model of the Chua systems is proposed. The controller design based on linear matrix inequality (LMI) conditions is developed. The results are extended to explore the L_∞ -gain control problem for the chaotic system with disturbances using the observer-based fuzzy control approach. In addition, the nonlinear L_∞ -gain control problem is transformed into a suboptimal control problem, that is, to minimize the upper bound of the L_∞ -gain of the closed-loop system subject to LMI constraints.

Notations. $\mathbb{R}^n$ denotes the n -dimensional Euclidian space. Given a vector $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{x}^T$ denotes its transpose. $|\mathbf{x}| = (\sum_{i=1}^n x_i^2)^{1/2}$ denotes the Euclidean norm. For a function $s \in [0, +\infty) \rightarrow \mathbb{R}^n$, $\|s(t)\|_\infty \triangleq \sup_t |s(t)|$. A^T and A^{-1} represent the transpose of matrix $\mathbf{A}$ and the inverse of matrix $\mathbf{A}$, respectively. We use $\mathbf{A} > 0$ ($\mathbf{A} < 0$) to denote a positive- (negative-) definite matrix $\mathbf{A}$, and I (resp., 0) denotes the identity matrix (resp., zero matrix) of appropriate dimension. $\text{diag}(\cdot)$ denotes a block diagonal matrix. The symbol “ $\star$ ” within a matrix represents the symmetric term of the matrix. $\lambda_{\max}(\mathbf{P})$ and $\lambda_{\min}(\mathbf{P})$ represent the maximum and minimum eigenvalue of the real symmetric matrix $\mathbf{P}$, respectively. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

2. Modeling and Control of Memristive Chua Systems

2.1. Memristive System without Disturbances. Consider the memristive Chua circuit system [7]:

$$\begin{aligned}\dot{x}_1 &= \sigma_1(x_2 - W(x_4))x_1, \\ \dot{x}_2 &= x_3 - x_1, \\ \dot{x}_3 &= -\sigma_2 x_2 + \sigma_3 x_3, \\ \dot{x}_4 &= x_1,\end{aligned}\tag{2}$$

with the output $y = x_4$, and

$$\begin{aligned}W(x_4) &= \frac{dq(x_4)}{dx_4} = \begin{cases} a, & |x_4| < 1, \\ b, & |x_4| > 1, \end{cases} \\ q(x_4) &= bx_4 + 0.5(a-b)(|x_4+1| - |x_4-1|).\end{aligned}\tag{3}$$

When taking the parameters $\sigma_1 = 4$, $\sigma_2 = 1$, $\sigma_3 = 0.65$, $a = 0.2$, and $b = 10$, (2) exhibits the chaotic behavior in [8] as shown in Figure 1.

Our aim is to estimate the states of (2) and stabilize the origin of the system. To do this, by imposing a controller into the system, one gets

$$\begin{aligned}\dot{\mathbf{x}}(t) &= f(\mathbf{x}(t)) + \mathbf{B}\mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t),\end{aligned}\tag{4}$$

where $\mathbf{x}(t) = [x_1(t), \dots, x_4(t)]^T \in \mathbb{R}^4$ denotes the vector of the states, $\mathbf{u}(t) = [\mathbf{u}_1(t), \dots, \mathbf{u}_m(t)]^T \in \mathbb{R}^m$ denotes the vector of the control inputs, $\mathbf{C} = [0 \ 0 \ 0 \ 1]$, $\mathbf{B} \in \mathbb{R}^{4 \times m}$, and

$$f(\mathbf{x}) = \begin{bmatrix} \sigma_1(x_2 - W(x_4))x_1 \\ x_3 - x_1 \\ -\sigma_2 x_2 + \sigma_3 x_3 \\ x_1 \end{bmatrix}.\tag{5}$$

According to [19], we construct the Takagi-Sugeno fuzzy model for (2) as follows:

Rule 1. If $\mathbf{y}(t)$ is M_1 , then

$$\dot{\mathbf{x}}(t) = A_1 \mathbf{x} + \mathbf{B}\mathbf{u}.\tag{6}$$

Rule 2. If $\mathbf{y}(t)$ is M_2 , then

$$\dot{\mathbf{x}}(t) = A_2 \mathbf{x} + \mathbf{B}\mathbf{u},\tag{7}$$

where M_1 is $|\mathbf{y}(t)| < 1$, and M_2 is $|\mathbf{y}(t)| > 1$,

$$\begin{aligned}A_1 &= \begin{bmatrix} -a\sigma_1 & \sigma_1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -\sigma_2 & \sigma_3 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -b\sigma_1 & \sigma_1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -\sigma_2 & \sigma_3 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.\end{aligned}\tag{8}$$

Through a center-average defuzzifier, the overall fuzzy system is represented as

$$\dot{\mathbf{x}} = \sum_{i=1}^2 h_i(t)(A_i \mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)),\tag{9}$$

FIGURE 1: Memristive Chua system.

where the membership functions are

$$h_1(t) = \begin{cases} 1, & |\mathbf{y}(t)| < 1, \\ 0, & |\mathbf{y}(t)| > 1, \end{cases} \quad (10)$$

$$h_2(t) = \begin{cases} 0, & |\mathbf{y}(t)| < 1, \\ 1, & |\mathbf{y}(t)| > 1. \end{cases}$$

Suppose that the state variables are not fully measurable and only the flux of the capacitor (i.e., x_4) is measurable (when choosing other partial states, the results are also applicable), we propose the following fuzzy state observer for fuzzy model (9).

Observer rule i : if $\mathbf{y}(t)$ is M_i , $i \in \{1, 2\}$, then

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}_i \hat{\mathbf{x}}(t) + \mathbf{B} \mathbf{u}(t) + L_i (\mathbf{y}(t) - \mathbf{C} \hat{\mathbf{x}}(t)), \quad (11)$$

where $\hat{\mathbf{x}}$ is an estimate of $\mathbf{x}$ and L_i is the observer gain for the i th observer rule.

Note that $\mathbf{y}(t)$ is applied for the observer rule as it is available from the system (2). Similarly, the overall fuzzy observer is represented as follows:

$$\dot{\hat{\mathbf{x}}} = \sum_{i=1}^2 h_i(t) (\mathbf{A}_i \hat{\mathbf{x}}(t) + \mathbf{B} \mathbf{u}(t) + L_i (\mathbf{y}(t) - \mathbf{C} \hat{\mathbf{x}}(t))). \quad (12)$$

To stabilize the memristive chaotic system (2), the observer-based fuzzy controller is given by

$$\mathbf{u}(t) = \sum_{j=1}^2 h_j(t) \mathbf{K}_j \hat{\mathbf{x}}(t), \quad (13)$$

where $j \in \{1, 2\}$ is the observer rule index, $\mathbf{K}_j$ is the control gain for the j th controller rule.

The proposed observer-based fuzzy control scheme is depicted in Figure 2. The fuzzy observer estimates the states of the memristive chaotic systems based on the measurable output. Then, the estimated states are utilized in the observer-based fuzzy controller and the output of the controller will be imposed on the chaotic systems.

FIGURE 2: Observer-based fuzzy control structure diagram.

We denote the estimation error as $e(t) = \mathbf{x}(t) - \hat{\mathbf{x}}$. Hence, (9) is equivalent to

$$\begin{aligned} \dot{\mathbf{x}} &= \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{B} \mathbf{K}_j \hat{\mathbf{x}}(t)) \\ &= \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) [(\mathbf{A}_i + \mathbf{B} \mathbf{K}_j) \mathbf{x}(t) - \mathbf{B} \mathbf{K}_j e(t)]. \end{aligned} \quad (14)$$

By differentiating $e(t)$, one can readily obtain

$$\dot{e} = \sum_{i=1}^2 h_i(t) (\mathbf{A}_i - L_i C) e(t) = \sum_{i=1}^2 h_i(t) \sum_{j=1}^2 h_j(t) (\mathbf{A}_i - L_i C) e(t), \quad (15)$$

where we used the fact $\sum_{j=1}^2 h_j(t) = 1$.

Combining with (14)-(15) yields

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{e}(t) \end{bmatrix} = \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) \mathcal{A}_{ij} \begin{bmatrix} \mathbf{x}(t) \\ e(t) \end{bmatrix}, \quad (16)$$

$$\text{where } \mathcal{A}_{ij} = \begin{bmatrix} \mathbf{A}_i + \mathbf{B} \mathbf{K}_j & -\mathbf{B} \mathbf{K}_j \\ 0 & \mathbf{A}_i - L_i C \end{bmatrix}, \quad i, j \in \{1, 2\}.$$

Let us define

$$\boldsymbol{\eta} = \begin{bmatrix} \mathbf{x}(t) \\ e(t) \end{bmatrix}, \quad (17)$$

then the augmented system in (16) can be rewritten as

$$\dot{\boldsymbol{\eta}} = \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) \mathcal{A}_{ij} \boldsymbol{\eta}. \quad (18)$$

Proposition 1. For the augmented system (16), if there exists a symmetric positive-definite matrix $\mathbf{P} > 0$ and a positive scalar $\alpha > 0$ such that the following matrix inequality

$$\mathbf{P} \left(\frac{\mathcal{A}_{ij} + \mathcal{A}_{ji}}{2} \right)^T + \mathbf{P} \left(\frac{\mathcal{A}_{ij} + \mathcal{A}_{ji}}{2} \right) + \alpha \mathbf{P} < 0, \quad (19)$$

holds for all $i \leq j$ ($i, j \in \{1, 2\}$), then the augmented system (16) is asymptotically stable.

Proof. Consider the Lyapunov function candidate

$$V(t) = \boldsymbol{\eta}^T(t) \mathbf{P} \boldsymbol{\eta}(t), \quad (20)$$

where $\boldsymbol{\eta} = [\mathbf{x}^T(t) e^T(t)]^T$, $\mathbf{P} > 0$ is symmetric. After calculating the derivative of V along the trajectories of (18), we have

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) \boldsymbol{\eta}^T(t) (\mathcal{A}_{ij}^T \mathbf{P} + \mathbf{P} \mathcal{A}_{ij}) \boldsymbol{\eta}(t) \\ &= \sum_{i=1}^2 h_i^2(t) \boldsymbol{\eta}^T(t) \left[\mathbf{P} \left(\frac{\mathcal{A}_{ii} + \mathcal{A}_{ii}}{2} \right)^T + \mathbf{P} \left(\frac{\mathcal{A}_{ii} + \mathcal{A}_{ii}}{2} \right) + \alpha \mathbf{P} \right] \boldsymbol{\eta}(t) \\ &\quad + \sum_{i=1}^2 h_i(t) \sum_{i < j}^2 h_j(t) \boldsymbol{\eta}^T(t) \cdot \left[\mathbf{P} \left(\frac{\mathcal{A}_{ij} + \mathcal{A}_{ji}}{2} \right)^T + \mathbf{P} \left(\frac{\mathcal{A}_{ij} + \mathcal{A}_{ji}}{2} \right) + \alpha \mathbf{P} \right] \boldsymbol{\eta} - \alpha \boldsymbol{\eta}^T \mathbf{P} \boldsymbol{\eta}(t), \end{aligned} \quad (21)$$

where $\alpha > 0$ is an arbitrary scalar. Using (19), we get

$$\dot{V} \leq -\alpha \boldsymbol{\eta}^T \mathbf{P} \boldsymbol{\eta}(t). \quad (22)$$

It is obvious that $V(t)$ is positive definite and $\dot{V}(t) < 0$, $\forall \mathbf{x} \neq 0$, and $\forall e \neq 0$. According to the Lyapunov stability theorem [20], Theorem 5.16, the origin of (16) is globally asymptotically stable. This completes the proof.

Remark 1. It is worth pointing out that this matrix inequality (19) is hardly tractable numerically. Moreover, linearizing the matrix inequality is also a very difficult task due to the presence of the coupling term in $\mathbf{P} \mathcal{A}_{ij}$.

$$\text{For the convenience of design, let } \mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & 0 \\ 0 & \mathbf{P}_2 \end{bmatrix}.$$

Hence, the matrix inequalities in (19) are equivalent to the following matrix inequalities:

$$\begin{bmatrix} \Phi_{11} + \alpha \mathbf{P}_1 & -\Phi_{12} \\ * & \Phi_2 + \alpha \mathbf{P}_2 \end{bmatrix} < 0, \quad (23)$$

where

$$\begin{aligned} \Phi_{11} &= \Omega_{11} + \Omega_{11}^T, \\ \Omega_{11} &= \frac{\mathbf{P}_1 (\mathbf{A}_i + \mathbf{B} \mathbf{K}_j) + \mathbf{P}_1 (\mathbf{A}_j + \mathbf{B} \mathbf{K}_i)}{2}, \\ \Phi_{12} &= \frac{\mathbf{P}_1 \mathbf{B} \mathbf{K}_j + \mathbf{P}_1 \mathbf{B} \mathbf{K}_i}{2}, \\ \Phi_{22} &= \Omega_{22} + \Omega_{22}^T, \\ \Omega_{22} &= \frac{(\mathbf{P}_2 \mathbf{A}_i - \mathbf{Y}_i C) + (\mathbf{P}_2 \mathbf{A}_j - \mathbf{Y}_j C)}{2}, \\ \mathbf{Y}_i &= \mathbf{P}_2 L_i, \end{aligned} \quad (24)$$

for all $i \leq j$, $i, j \in \{1, 2\}$.

Since there are no effective algorithms for solving $\mathbf{P}_1$, $\mathbf{K}_j$, $\mathbf{P}_2$, and $\mathbf{Y}_i$ simultaneously, we use the separation method to solve the problem. Note that (23) can be decoupled as follows:

$$\begin{bmatrix} \Phi_{11} + \alpha \mathbf{P}_1 & -\Phi_{12} \\ * & \Phi_{22} + \alpha \mathbf{P}_2 \end{bmatrix} = \begin{bmatrix} \Phi_{11} + \alpha \mathbf{P}_1 & -\Phi_{12} \\ * & -\beta \mathbf{P}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ * & \Phi_{22} + \alpha \mathbf{P}_2 + \beta \mathbf{P}_1 \end{bmatrix}, \quad (25)$$

where β is a positive scalar. It is obvious that if

$$\begin{bmatrix} \Phi_{11} + \alpha \mathbf{P}_1 & -\Phi_{12} \\ * & -\beta \mathbf{P}_1 \end{bmatrix} < 0, \quad (26)$$

and

$$\begin{bmatrix} 0 & 0 \\ * & \Phi_{22} + \alpha \mathbf{P}_2 + \beta \mathbf{P}_1 \end{bmatrix} < 0, \quad (27)$$

then (23) holds.

Note that (26) is related to the controller part (the parameters are $\mathbf{P}_1$ and $\mathbf{K}_i$) and (27) is related to the observer part (the parameters are $\mathbf{P}_1$, $\mathbf{P}_2$ and L_i), respectively. Now we can determine $\mathbf{P}_1$, $\mathbf{K}_i$, $\mathbf{P}_2$, and $\mathbf{Y}_i$ simultaneously by the following arrangement.

It follows by pre- and postmultiplying the inequality (26) by the matrix $\text{diag}(\mathbf{P}_1^{-1}, \mathbf{P}_1^{-1})$

$$\begin{bmatrix} \Xi_{11} + \alpha \mathbf{X}_1 & -\Xi_{12} \\ * & -\beta \mathbf{X}_1 \end{bmatrix} < 0, \quad (28)$$

where

$$\begin{aligned} \mathbf{X}_1 &= \mathbf{P}_1^{-1}, \\ \Xi_{11} &= \Theta_{11} + \Theta_{11}^T, \\ \Theta_{11} &= \frac{(\mathbf{A}_i \mathbf{X}_1 + \mathbf{B} \mathbf{W}_j) + (\mathbf{A}_j \mathbf{X}_1 + \mathbf{B} \mathbf{W}_i)}{2}, \\ \Xi_{12} &= \frac{\mathbf{B} \mathbf{W}_j + \mathbf{B} \mathbf{W}_i}{2}, \quad \mathbf{W}_i = \mathbf{K}_i \mathbf{X}_1. \end{aligned} \quad (29)$$

By the well-known Schur complement [21], the inequality (28) is equivalent to

$$\begin{bmatrix} \Phi_{22} + \alpha \mathbf{P}_2 & I \\ * & -\beta^{-1} \mathbf{X}_1 \end{bmatrix} < 0. \quad (30)$$

Based on the above analysis, we can obtain the following observer-based control theorem.

Theorem 1. *System (2) is asymptotically stabilizable by (13) if for fixed scalars $\alpha > 0$ and $\beta > 0$, there exist two positive-definite matrices $\mathbf{X}_1 \in \mathbb{R}^{4 \times 4}$ and $\mathbf{P}_2 \in \mathbb{R}^{4 \times 4}$ and two matrices $\mathbf{W}_i$ and $\mathbf{Y}_i$, so that the following two LMI conditions are feasible:*

$$\begin{bmatrix} \Xi_{11} + \alpha \mathbf{X}_1 & -\Xi_{12} \\ * & -\beta \mathbf{X}_1 \end{bmatrix} < 0, \quad (31)$$

$$\begin{bmatrix} \Phi_{22} + \alpha \mathbf{P}_2 & I \\ * & -\beta^{-1} \mathbf{X}_1 \end{bmatrix} < 0, \quad (32)$$

for all $i \leq j, i, j \in \{1, 2\}$, where

$$\begin{aligned} \Xi_{11} &= \Omega_{11} + \Omega_{11}^T, \\ \Omega_{11} &= \frac{(\mathbf{A}_i \mathbf{X}_1 + \mathbf{B} \mathbf{W}_j) + (\mathbf{A}_j \mathbf{X}_1 + \mathbf{B} \mathbf{W}_i)}{2}, \\ \Xi_{12} &= \frac{\mathbf{B} \mathbf{W}_j + \mathbf{B} \mathbf{W}_i}{2}, \\ \Phi_{22} &= \Omega_{22} + \Omega_{22}^T, \\ \Omega_{22} &= \frac{(\mathbf{P}_2 \mathbf{A}_i - \mathbf{Y}_i \mathbf{C}) + (\mathbf{P}_2 \mathbf{A}_j - \mathbf{Y}_j \mathbf{C})}{2}. \end{aligned} \quad (33)$$

Moreover, the stabilizing observer-based control gains are given by $\mathbf{K}_i = \mathbf{W}_i \mathbf{X}_1^{-1}$, $L_i = \mathbf{P}_2^{-1} \mathbf{Y}_i$.

According to the analysis above, the design procedure is summarized as follows:

Design procedure:

Step 1. Construct the fuzzy plant rules in (9) and the observer rules in (12).

Step 2. Take a set of positive scalars $\alpha > 0$ and $\beta > 0$ iteratively.

Step 3. Solve the following linear matrix inequality problem

$$\begin{bmatrix} \Xi_{11} + \alpha \mathbf{X}_1 & -\Xi_{12} \\ * & -\beta \mathbf{X}_1 \end{bmatrix} < 0, \quad (34)$$

$$\begin{bmatrix} \Phi_{22} + \alpha \mathbf{P}_2 & I \\ * & -\beta^{-1} \mathbf{X}_1 \end{bmatrix} < 0,$$

to obtain $\mathbf{X}_1$, $\mathbf{P}_2$, $\mathbf{W}_i$, and $\mathbf{Y}_i$ (thus $\mathbf{K}_i = \mathbf{W}_i \mathbf{X}_1^{-1}$, $L_i = \mathbf{P}_2^{-1} \mathbf{Y}_i$ can also be derived).

Step 4. If $\mathbf{X}_1 > 0$ and $\mathbf{P}_2 > 0$ cannot be found, try another set of $\alpha > 0$, $\beta > 0$, and $c > 0$ iteratively and repeat Steps 3-4.

Step 5. Construct the fuzzy observer (12).

Step 6. Construct the fuzzy controller (13).

2.2. Memristive System with Disturbances. In this subsection, we will extend the above results to deal with the L_∞ -gain control problem for the memristive chaotic system with external disturbances.

Consider the following the memristive Chua system with external disturbances

$$\dot{\mathbf{x}} = \sum_{i=1}^2 h_i(t) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + D_i w(t)), \quad (35)$$

where $D_i \in \mathbb{R}^{4 \times p}$ and $w(t) = [w_1(t), \dots, w_p(t)]^T \in \mathbb{R}^p$ denotes the vector of the bounded external disturbances.

Applying the same fuzzy observer (12) and the observer-based fuzzy control law (13), we have

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{e}(t) \end{bmatrix} = \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) \mathcal{A}_{ij} \begin{bmatrix} \mathbf{x}(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} D_i & 0 \\ 0 & D_i \end{bmatrix} \begin{bmatrix} w(t) \\ w(t) \end{bmatrix}, \quad (36)$$

where $\mathcal{A}_{ij} = \begin{bmatrix} \mathbf{A}_i + \mathbf{B}\mathbf{K}_j & -\mathbf{B}\mathbf{K}_j \\ 0 & \mathbf{A}_i - \mathbf{L}_i\mathbf{C} \end{bmatrix}$.
Denote

$$\begin{aligned} \eta &= \begin{bmatrix} \mathbf{x}(t) \\ e(t) \end{bmatrix}, \\ \bar{w}(t) &= \begin{bmatrix} w(t) \\ w(t) \end{bmatrix}, \\ \bar{D}_i &= \begin{bmatrix} D_i & 0 \\ 0 & D_i \end{bmatrix}. \end{aligned} \quad (37)$$

Then (36) can be expressed as

$$\dot{\eta} = \sum_{i=1}^2 \sum_{j=1}^2 h_i(t) h_j(t) [\mathcal{A}_{ij} \eta(t) + \bar{D}_i \bar{w}(t)]. \quad (38)$$

The objective is to determine a fuzzy controller for the closed-loop system (38) with the following L_∞ -gain performance as small as possible. Given a disturbance attenuation ρ , we need to achieve

$$\|\eta(t)\|_\infty \leq \gamma \|\eta(0)\| + \rho \|\bar{w}(t)\|_\infty, \quad (39)$$

where γ and ρ are some positive scalars.

Following [16], to extend the results in the previous subsection, the L_∞ -gain control performance in (39) is guaranteed for an attenuated level $\rho = \sqrt{c/(\alpha \lambda_{\min}(\mathbf{P}))}$, and $\beta = \lambda_{\max}(\mathbf{P})/\lambda_{\min}(\mathbf{P})$.

The L_∞ control problem can be transformed to solve the following minimization problem:

$$\begin{aligned} \min \quad & \frac{c}{\alpha \lambda_{\min}(\mathbf{P})} \\ \text{s.t.} \quad & \mathbf{X}_1 = \mathbf{X}_1^T > 0, \\ & \mathbf{P}_2 = \mathbf{P}_2^T > 0, \\ & \mathbf{W}_i, \\ & \mathbf{Y}_i, \\ & \beta > 0, \\ & \alpha > 0, \\ & c > 0, \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} \Xi_{11} + \alpha \mathbf{X}_1 & -\Xi_{12} & \Xi_{13} \\ * & -\beta \mathbf{X}_1 & 0 \\ * & * & -\frac{c}{2} \mathbf{I} \end{bmatrix} < 0, \\ & \begin{bmatrix} \Phi_{22} + \alpha \mathbf{P}_2 & \mathbf{I} & \Phi_{23} \\ * & -\beta^{-1} \mathbf{X}_1 & 0 \\ * & * & -\frac{c}{2} \mathbf{I} \end{bmatrix} < 0, \end{aligned} \quad (40)$$

for all $i \leq j$, $i, j \in \{1, 2\}$, where

$$\begin{aligned} \Xi_{11} &= \Omega_{11} + \Omega_{11}^T, \\ \Omega_{11} &= \frac{(\mathbf{A}_i \mathbf{X}_1 + \mathbf{B} \mathbf{W}_j) + (\mathbf{A}_j \mathbf{X}_1 + \mathbf{B} \mathbf{W}_i)}{2}, \\ \Xi_{12} &= \frac{\mathbf{B} \mathbf{W}_j + \mathbf{B} \mathbf{W}_i}{2}, \\ \Phi_{22} &= \Omega_{22} + \Omega_{22}^T, \\ \Omega_{22} &= \frac{(\mathbf{P}_2 \mathbf{A}_i - \mathbf{Y}_i \mathbf{C}) + (\mathbf{P}_2 \mathbf{A}_j - \mathbf{Y}_j \mathbf{C})}{2}, \\ \Xi_{13} &= \frac{D_i + D_j}{2}, \\ \Phi_{23} &= \frac{\mathbf{P}_2 D_i + \mathbf{P}_2 D_j}{2}. \end{aligned} \quad (41)$$

Note that $\mathbf{X}_1$ instead of $\mathbf{P}_1$ is formulated in the constraint conditions, thus, the term $c/\alpha \lambda_{\min}(\mathbf{P})$ should be modified as follows to solve the minimization problem. It is observed that $c/\alpha \lambda_{\min}(\mathbf{P}) < \varepsilon^2$ if $(c/\alpha) \mathbf{I} < \lambda_{\min}(\mathbf{P}) \varepsilon^2 \mathbf{I} < \varepsilon^2 \mathbf{P}$, that is,

$$\begin{bmatrix} \frac{c}{\alpha} \mathbf{I} & 0 \\ 0 & \frac{c}{\alpha} \mathbf{I} \end{bmatrix} < \begin{bmatrix} \varepsilon^2 \mathbf{P}_1 & 0 \\ 0 & \varepsilon^2 \mathbf{P}_2 \end{bmatrix}, \quad (42)$$

which is equivalent to

$$\mathbf{X}_1 < \frac{\varepsilon^2 \alpha}{c} \mathbf{I}, \quad (43)$$

and

$$\frac{c}{\varepsilon^2 \alpha} \mathbf{I} < \mathbf{P}_2. \quad (44)$$

Therefore, we can summarize the theorem for the L_∞ -gain performance as follows.

Theorem 2. *The closed-loop system (38) has the L_∞ -gain performance*

$$\|\eta(t)\|_\infty < \gamma \|\eta(0)\| + \varepsilon \|\bar{w}(t)\|_\infty, \quad (45)$$

if for fixed scalars $\alpha > 0$ and $\beta > 0$, there exist two positive-definite matrices $\mathbf{X}_1 \in \mathbb{R}^{4 \times 4}$ and $\mathbf{P}_2 \in \mathbb{R}^{4 \times 4}$, and two matrices $\mathbf{W}_i, \mathbf{Y}_i$ and $i \in \{1, 2\}$, the following minimization problem is solved,

$$\begin{aligned}
& \min_{\mathbf{X}_1, \mathbf{P}_2, \mathbf{W}_i, \mathbf{Y}_i, \beta, \alpha, c} \quad \varepsilon^2 \\
& \text{s.t.} \quad \mathbf{X}_1 = \mathbf{X}_1^T > 0, \\
& \quad \mathbf{P}_2 = \mathbf{P}_2^T > 0, \\
& \quad \mathbf{W}_i, \\
& \quad \mathbf{Y}_i, \\
& \quad \beta > 0 \\
& \quad \alpha > 0, \\
& \quad c > 0, \\
& \quad \begin{bmatrix} \Xi_{11} + \alpha \mathbf{X}_1 & -\Xi_{12} & \Xi_{13} \\ * & -\beta \mathbf{X}_1 & 0 \\ * & * & -\frac{c}{2} I \end{bmatrix} < 0, \\
& \quad \begin{bmatrix} \Phi_{22} + \alpha \mathbf{P}_2 & I & \Phi_{23} \\ * & -\beta^{-1} \mathbf{X}_1 & 0 \\ * & * & -\frac{c}{2} I \end{bmatrix} < 0, \\
& \quad \mathbf{X}_1 < \frac{\varepsilon^2 \alpha}{c} I, \\
& \quad \frac{c}{\varepsilon^2 a} I < \mathbf{P}_2.
\end{aligned} \tag{46}$$

Design procedure:

Step 1. Construct the fuzzy plant rules in (9) and the observer rules in (12).

Step 2. Take a set of positive scalars $\alpha > 0$, $\beta > 0$, and $c > 0$ iteratively.

Step 3. Given an initial ε^2 .

Step 4. Solve the following linear matrix inequality problem

$$\begin{aligned}
& \begin{bmatrix} \Xi_{11} + \alpha \mathbf{X}_1 & -\Xi_{12} & \Xi_{13} \\ * & -\beta \mathbf{X}_1 & 0 \\ * & * & -\frac{c}{2} I \end{bmatrix} < 0, \\
& \begin{bmatrix} \Phi_{22} + \alpha \mathbf{P}_2 & I & \Phi_{23} \\ * & -\beta^{-1} \mathbf{X}_1 & 0 \\ * & * & -\frac{c}{2} I \end{bmatrix} < 0, \\
& \mathbf{X}_1 < \frac{\varepsilon^2 \alpha}{c} I, \quad \frac{c}{\varepsilon^2 a} I < \mathbf{P}_2,
\end{aligned} \tag{47}$$

to obtain $\mathbf{X}_1$, $\mathbf{P}_2$, $\mathbf{W}_i$, and $\mathbf{Y}_i$ (thus $\mathbf{K}_i = \mathbf{W}_i \mathbf{X}_1^{-1}$ and $\mathbf{L}_i = \mathbf{P}_2^{-1} \mathbf{Y}_i$ can also be derived).

FIGURE 3: Time response of the control input under observer-based fuzzy control.

Step 5. Decrease ε^2 and repeat Steps 4-5 until $\mathbf{X}_1 > 0$ and $\mathbf{P}_2 > 0$ cannot be found. If $\mathbf{X}_1 > 0$ and $\mathbf{P}_2 > 0$ cannot be found for all possible ε^2 , try another set of $\alpha > 0$, $\beta > 0$, and $c > 0$ iteratively and repeat Steps 3-5.

Step 6. Construct the fuzzy observer (12).

Step 7. Construct the fuzzy controller (13).

3. Numerical Simulations

The numerical simulations are carried out using the fourth-order Runge-Kutta method (via ode45 in MATLAB). Consider the memristive Chua circuit system (2) with the parameters $\sigma_1 = 4$, $\sigma_2 = 1$, $\sigma_3 = 0.65$, $a = 0.2$ and $b = 10$. Then, the overall fuzzy system to be stabilized can be represented as (9) and the corresponding fuzzy observer is given by (12). By Theorem 1, we need to verify (31) and (32). Take $\alpha = 0.0003 > 0$ and $\beta = 11.48 > 0$. Using MATLAB to solve the LMIs (31) and (32), we can obtain a feasible solution as follows

$$\mathbf{X}_1 = \begin{bmatrix} 136.9592 & 55.5789 & 1.1645 & -19.9132 \\ 55.5789 & 74.5034 & 38.5401 & -46.3594 \\ 1.1645 & 38.5401 & 44.8794 & -62.5622 \\ -19.9132 & -46.3594 & -62.5622 & 155.0652 \end{bmatrix} > 0, \tag{48}$$

and

$$\mathbf{P}_2 = \begin{bmatrix} 13.9891 & -3.4107 & 0.2092 & -26.6232 \\ -3.4107 & 11.5482 & -5.6816 & -58.0015 \\ 0.2092 & -5.6816 & 6.3545 & 0.0344 \\ -26.6232 & -58.0015 & 0.0344 & 950.4500 \end{bmatrix} > 0. \tag{49}$$

FIGURE 4: Time responses of the state variables under observer-based fuzzy control.

Consequently, we can obtain the gains

$$\begin{aligned}
 K_1 &= W_1 X_1^{-1} = [-2.6536 \quad 3.1573 \quad -8.4596 \quad -2.6001], \\
 K_2 &= W_2 X_1^{-1} = [36.5464 \quad 3.1573 \quad -8.4596 \quad -2.6001], \\
 L_1 &= P_2^{-1} Y_1 = [150.7265 \quad 197.9643 \quad 162.8277 \quad 16.4382]^T, \\
 L_2 &= P_2^{-1} Y_2 = [350.5547 \quad 528.3090 \quad 451.4756 \quad 42.1846]^T.
 \end{aligned} \tag{50}$$

Therefore, all conditions of Theorem 1 are satisfied, which means that the closed-loop system (16) asymptotically converges to the origin. By observation, the initial states of the original Chua system and the observer are set as $[0.1, 0.2, 0.3, 0.4]^T$ and $[0, 0, 0, 0]^T$. Figure 3 shows the time evolution of the control input $\mathbf{u}(t)$. Figure 4 shows the

FIGURE 5: Time response of the control input for (35).

FIGURE 6: Time responses of the state variables in (35) under observer-based fuzzy control.

time evolutions between the state $x_i(t)$ and its estimation $\hat{x}_i(t)$, $i=1, 2, 3, 4$. It is revealed from Figure 4 that the states $x_i(t)$ ($i=1, 2, 3, 4$) are estimated, and converge to the origin, which implies that the memristive Chua system is stabilized.

Next, we consider the memristive Chua system (35) with external disturbances with $w(t)$ as the bounded external disturbances with zero mean and variance 0.5^2 . The disturbance matrices are taken as $D_1 = D_2 = [0.11 \ 0.12 \ 0.13 \ 0.14]^T$ and apply the same fuzzy observer in (12). Set $\alpha = 10 > 0$, $\beta = 13 > 0$, and $c = 1$. Then, using MATLAB to solve the LMIs in Theorem 2, we can solve the minimization problem in Theorem 2 and obtain the

minimum value $\varepsilon = 7.99$. Consequently, we can obtain the gains

$$\begin{aligned}
 K_1 &= W_1 X_1^{-1} = [-12.8045 \quad 7.2009 \quad -8.1336 \quad -1.5516], \\
 K_2 &= W_2 X_1^{-1} = [-12.0583 \quad 34.1065 \quad -27.8910 \quad -2.8660], \\
 L_1 &= P_2^{-1} Y_1 = 10^3 \times [0.7551 \quad 3.2292 \quad 4.6485 \quad 0.0185]^T, \\
 L_2 &= P_2^{-1} Y_2 = 10^3 \times [0.9316 \quad 4.0538 \quad 5.8366 \quad 0.0200]^T.
 \end{aligned} \tag{51}$$

Therefore, all conditions of Theorem 2 are satisfied, which means the closed-loop system (38) with external disturbances asymptotically converges to the origin. To

see the simulation easily, the initial states of the original Chua system and the observer-based control are set as $[-1.0, 0.8, 2.0, 0.4]^T$ and $(0, 0, 0, 0)$. Figure 5 shows the time evolution of the control input $\mathbf{u}(t)$. Figure 6 shows the time evolutions between the state $x_i(t)$ and its estimation $\hat{x}_i(t)$, $i = 1, 2, 3, 4$. Initial time evolutions are shown in the subfigures. It is revealed from Figure 6 that the states $x_i(t)$ ($i = 1, 2, 3, 4$) are estimated, and converge to the origin, which implies that the memristive Chua system with external disturbances is stabilized.

4. Conclusion

In summary, we provide the fuzzy model for the memristive Chua system and an observer-based fuzzy control scheme to stabilize the system. Sufficient conditions that are easily verified based on LMI have been derived, with which the observer-based fuzzy controllers can be designed conveniently. The L_∞ -gain observer-based fuzzy control design for the chaotic system with disturbances has also been discussed. Finally, numerical simulations have been carried out to show the effectiveness of the proposed method for one system with chaotic behavior used as an example.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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