Research Article

Statistical Identification of Parameters for Damaged FGM Structures with Material Uncertainties in Thermal Environment

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Considering that the statistic numerical characteristics are often required in the probability-based damage identification and safety assessment of functionally graded material (FGM) structures, an stochastic model updating-based inverse computational method to identify the second-order statistics (means and variances) of material properties as well as distribution of constituents for damaged FGM structures with material uncertainties is presented by using measurable modal parameters of structures. The region truncation-based optimization method is employed to simplify the computational process in stochastic model updating. In order to implement the forward propagation of uncertainties required in the stochastic model updating and avoid large error resulting in the nonconvergence of the iteration process, an algorithm is proposed to compute the covariance between the modal parameters and the identified parameters for damaged FGM structures. The proposed method is illustrated by a numerically simulated damaged FGM beam with continuous spatial variation of material properties and verified by comparing with the Monte-Carlo simulation (MCS) method. The influences of the levels and sources of measured data uncertainties as well as the boundary conditions on the identification results are investigated. The numerical simulation results show the efficiency and effectiveness of the presented method for the identification of material parameter variability by using the measurable modal parameters of damaged FGM structures.

1. Introduction

Functionally graded materials (FGMs) have found wide application in modern industries including aerospace engineering, military application, and mechanical engineering, due to resistant high temperature gradient and strong mechanical performance [1], and a great deal of research has already been done into both deterministic [2–6] and stochastic [7–12] mechanical behavior of FGM structures. However, damage such as crack and fracture often occurs in FGM structures as most of FGM structures work in harsh environment such as high temperature gradient and corrosion. Damage problems in FGM structures have been widely investigated in literatures [13–19].

As is well-known, accurate and reliable mechanical behavior analysis of FGM structures with initial damage is vital for the early condition assessment and damage prognosis in order to guarantee the safe performance and prevent the possible disastrous failure of FGM structures [20, 21].

What is more, the reliable analysis of overall mechanical behavior of any FGM structure, undamaged or damaged, relies on a precise knowledge of the material properties and, especially, the distribution of constituents. Traditionally, mechanical properties and volume fraction distribution of heterogeneous materials can be determined by starting from indentation tests [22, 23]. Owing to the inhomogeneous nature of FGMs, however, experimental characterization of materials constants and volume distribution is cumbersome and time-consuming since a large number of property parameters need to be determined, and nondestructive techniques have been developed to evaluate the material property parameters and the volume fraction index for FGMs by utilizing complex relationships between the structural behavior and the material properties. Liu et al. [24] suggested a progressive neural network process for characterizing the material properties of functionally graded material (FGM) plate by using elastic wave. Han and Liu [25] presented an inverse method for determining the material properties and

distribution in the thickness direction of FGM plates by using uniform crossover microgenetic algorithm. Rahmani et al. [26] presented a regularized finite element model updating for identification of elastic constitutive parameters identification of 2D composites from full-field measured displacement data, in which mechanical constraints are used as regularization factors in the optimization algorithm. Sun et al. [27] developed a strategy to identify the temperature-dependent properties of a thermoelastic structure in thermal environment taking into time-varying material properties and thermal stresses account. Mishra and Chakraborty [28, 29] dealt with a modal analysis based inverse identification of material properties of fiber reinforced plastics composite plates with rotational flexibility at boundaries and panels having elastically restrained boundary from the experimental modal testing using finite element model updating.

All of the above studies focus on the identification of deterministic quantities of material constants or volume distribution of composite materials and structures. In reality, however, both constituent material properties and volume distribution in FGMs present inherent fluctuation due to the typicality and technical variety in the manufacturing and fabrication of FGMs and environmental temperature [30, 31]. From this point of view, therefore, the inherent uncertainties need to be incorporated in parameter identification, which is an important issue in mechanical behavior analysis and safety evaluation of FGM structures, especially in the early stage of damage.

In the present work, considering that the statistic numerical characteristics, for example, second-order statistics (means and variances) of material properties, are often required in evaluating the mechanical behavior and safety of FGM structures, an inverse computational procedure is presented to identify the second-order statistics (means and variances) of material properties and volume fraction index of FGM structures with initial damage by using stochastic finite element model updating [32], which is implemented by minimizing the differences between the analytical and actual structural modal parameters [33] easily obtained by measuring the structural response signal. It is worth noting that, as an inverse problem, both the efficient optimization method with good convergence and the forward uncertainty propagation with relatively small errors are two key issues for the statistical identification of FGM parameters based on stochastic model updating. Instead of utilizing the trust region method with expensive computation, the region truncation-based optimization method has been proposed to simplify the computational process without harming the convergence of the iteration process in this work. On the other hand, the forward uncertainty propagation needs to be carried out in every iteration step, in which not only the means and variances of the dynamic characteristics [7–12] but also the covariance between the dynamic characteristics and the identification parameters needs to be computed, and the computation of first-order derivatives is involved. Finite difference approximation is traditionally used for obtaining the involved derivatives for structures with a large number of degrees, and, however, it is obviously not suitable to be utilized in computing the derivatives with respect to

volume fraction index for FGM structures due to large errors which will accumulate in every iteration step and lead to the nonconvergences of the results, since the effective material properties of FGMs are assumed as power functions of volume fraction index. On account of the fact mentioned above, an algorithm is developed for computing the first derivative of dynamic characteristics with respect to random variables, which is employed to compute the covariance between the dynamic characteristics and the identification parameters. In addition, in consideration of the difficult of random experiment test for the damaged FGM structure with enough samples required for the random analysis, the actual modal data will be obtained by numerical simulation method in this study.

2. Volume Distribution of Material Constituents in FGM Beam

An FGM rectangular beam composed of ceramics (top surface/left surface) and metal (bottom surface/right surface) having the length l , width b , and thickness h is considered in this work. The effective material properties of the beam are assumed to vary continuously through its thickness direction or along its axial direction complying with power law distribution [11] and can be, respectively, expressed as

$$P(z) = \left[(P_c - P_m) \left(\frac{z}{h} + \frac{1}{2} \right)^n + P_m \right], \quad -\frac{h}{2} \leq z \leq \frac{h}{2}, \quad 0 \leq n < \infty, \quad (1)$$

$$P(x) = \left[(P_c - P_m) \left(1 - \frac{x}{l} \right)^n + P_m \right], \quad 0 \leq x \leq l, \quad 0 \leq n < \infty,$$

where z , x are the coordinates along the thickness (h) and length (l) of the FGM beam as shown in Figure 1. $P(\bullet)$ denotes the effective material properties such as Young's modulus, Poisson's ratio, and thermal expansion efficient along z coordinate for the FGM beam through its thickness direction or x coordinate for the FGM beam along its axial direction. P_c , P_m denote the corresponding material properties of ceramics and metal, respectively. n denotes the volume fraction index.

Each material property of ceramics and metal can be expressed as a function of temperature [30]:

$$P = P_0 (P_{-1} T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3), \quad (2)$$

where P_0 , P_{-1} , P_1 , P_2 , P_3 are the coefficients of temperature T and change with the constituent materials as well as the temperature.

3. Stochastic Model Updating

3.1. Updating the Means and Variances of Material Parameters. The model updating can often be posed as a minimization problem of the objective function which is a sum of square difference between the analytic and actual data,

FIGURE 1: Configuration of an FGM beam.

typically dynamic characteristics (modal frequencies and mode shapes):

$$\min_{\theta} f(\theta) = \|\mathbf{z}_{ak}(\theta) - \mathbf{z}_m\|^2, \quad \underline{\theta} \leq \theta \leq \bar{\theta}, \quad (3)$$

where the updating parameter vector θ is composed of material properties and volume fraction index of FGMs to be identified, that is, $\theta = [\mathbf{P}_c^T \ \mathbf{P}_m^T \ n]^T$. The vector $\mathbf{z}_{ak}$ is the analytic modal parameters (modal frequencies and mode shapes), that is, $\mathbf{z}_{ak} = [\mathbf{f}_{ak}^T \ \{\boldsymbol{\varphi}_{ak}^T\}]^T$, which is a nonlinear function of the updating parameters vector θ . The vector $\mathbf{z}_m = [\mathbf{f}_m^T \ \{\boldsymbol{\varphi}_m^T\}]^T$ is the actual modal parameters. The vectors $\underline{\theta}$, $\bar{\theta}$ denote the upper and lower bounds on the updating parameters.

In the stochastic model updating, the parameters can be updated by the following iterative expression [32]:

$$\begin{aligned} \mathbf{z}_{ak} &= \bar{\mathbf{z}}_{ak} + \tilde{\mathbf{z}}_{ak}, \\ \mathbf{z}_m &= \bar{\mathbf{z}}_m + \tilde{\mathbf{z}}_m, \end{aligned} \quad (4)$$

$$\boldsymbol{\theta}_k = \bar{\boldsymbol{\theta}}_k + \tilde{\boldsymbol{\theta}}_k,$$

$$\bar{\boldsymbol{\theta}}_{k+1} = \bar{\boldsymbol{\theta}}_k + \bar{\mathbf{T}}_k (\bar{\mathbf{z}}_{ak} - \bar{\mathbf{z}}_m),$$

$$\bar{\mathbf{T}}_k = [\bar{\mathbf{S}}_k^T \mathbf{W}_\varepsilon \bar{\mathbf{S}}_k + \mathbf{W}_\vartheta]^{-1} \bar{\mathbf{S}}_k^T, \quad \bar{\mathbf{S}}_k = \left[\frac{\partial \mathbf{z}_{ak}(\theta)}{\partial \theta} \right]_{\theta=\bar{\boldsymbol{\theta}}_k}, \quad (5)$$

$$\begin{aligned} \text{Cov}(\tilde{\boldsymbol{\theta}}_{k+1}, \tilde{\boldsymbol{\theta}}_{k+1}) &= \text{Cov}(\tilde{\boldsymbol{\theta}}_k, \tilde{\boldsymbol{\theta}}_k) + \text{Cov}(\tilde{\boldsymbol{\theta}}_k, \tilde{\mathbf{z}}_{ak}) \bar{\mathbf{T}}_k^T \\ &\quad + \bar{\mathbf{T}}_k \text{Cov}(\tilde{\mathbf{z}}_m, \tilde{\mathbf{z}}_m) \bar{\mathbf{T}}_k^T \end{aligned}$$

$$\begin{aligned} &- \bar{\mathbf{T}}_k \text{Cov}(\tilde{\mathbf{z}}_{ak}, \tilde{\boldsymbol{\theta}}_k) \\ &+ \bar{\mathbf{T}}_k \text{Cov}(\tilde{\mathbf{z}}_{ak}, \tilde{\mathbf{z}}_{ak}) \bar{\mathbf{T}}_k^T, \end{aligned} \quad (6)$$

where subscript k denotes the k th iteration step. The vectors $\bar{\boldsymbol{\theta}}_k$, $\bar{\mathbf{z}}_{ak}$, $\bar{\mathbf{z}}_m$ are the means of the updating parameters, the analytical modal parameters, and the actual modal parameters. The vectors $\tilde{\boldsymbol{\theta}}_k$, $\tilde{\mathbf{z}}_{ak}$, $\tilde{\mathbf{z}}_m$ are the corresponding zero-mean random parts with the same variances of $\sigma_{\boldsymbol{\theta}_k}^2$, $\sigma_{\mathbf{z}_{ak}}^2$, $\sigma_{\mathbf{z}_m}^2 \cdot \mathbf{W}_\varepsilon$, $\mathbf{W}_\vartheta$ is a diagonal weighting matrix to allow for regularization of ill-posed sensitivity equations. $\bar{\mathbf{T}}_k$ is a transformation matrix and $\bar{\mathbf{S}}_k$ is the sensitivity matrix of modal parameters with respect to updating parameters. In (6), the actual modal data and updating parameters are assumed to be uncorrelated; that is, $\text{Cov}(\tilde{\mathbf{z}}_m, \tilde{\boldsymbol{\theta}}_k) = 0$, $\text{Cov}(\tilde{\mathbf{z}}_m, \tilde{\mathbf{z}}_{ak}) = 0$, and the covariance matrix $\text{Cov}(\tilde{\mathbf{z}}_{ak}, \tilde{\boldsymbol{\theta}}_k)$ between the analytical modal parameters and the updating parameters as well as the covariance matrix $\text{Cov}(\tilde{\mathbf{z}}_{ak}, \tilde{\mathbf{z}}_{ak})$ of the analytic modal parameters can be evaluated by forward propagation of uncertainty.

3.2. Optimization Algorithm by Region Truncation. Region truncation-based method is employed in the optimization process of model updating, since the widely used trust region method is less straightforward and more expensive in computational though it is powerful and reliable in convergence [34]. In region truncation-based optimization algorithm, the constrained minimization problem (3) with the constraints is converted to the unconstrained problem and the updating parameters are limited within a controlled region in each iteration step. Thus, (5) can be replaced with the following iterative process:

$$\begin{aligned} \bar{\boldsymbol{\theta}}_{k+1} &= \begin{cases} \bar{\boldsymbol{\theta}}_k + \bar{\boldsymbol{\tau}}_k, & \Delta \boldsymbol{\theta}_k \geq \bar{\boldsymbol{\tau}}_k, \\ \bar{\boldsymbol{\theta}}_k + \left((\mathbf{S}_k^T(\boldsymbol{\theta}_k) \mathbf{W}_\varepsilon \mathbf{S}_k(\boldsymbol{\theta}_k))^{-1} + \mathbf{W}_\vartheta \right) \mathbf{S}_k^T(\boldsymbol{\theta}_k) (\bar{\mathbf{z}}_{ak} - \bar{\mathbf{z}}_m), & \underline{\boldsymbol{\tau}}_k \leq \Delta \boldsymbol{\theta}_k \leq \bar{\boldsymbol{\tau}}_k, \\ \bar{\boldsymbol{\theta}}_k + \underline{\boldsymbol{\tau}}_k, & \Delta \boldsymbol{\theta}_k \leq \underline{\boldsymbol{\tau}}_k, \end{cases} \\ \Delta \bar{\boldsymbol{\theta}}_{k+1} &= \bar{\boldsymbol{\theta}}_k - \bar{\boldsymbol{\theta}}_{k-1}, \end{aligned}$$

$$\begin{aligned}
\bar{\tau}_k &= \beta \kappa_k, \\
\underline{\tau}_k &= -\beta \kappa_k, \\
\beta &\in (0, 1), \\
\kappa_k &= \min \left\{ \left| \bar{\theta} - \bar{\theta}_k \right|, \left| \underline{\theta} - \bar{\theta}_k \right| \right\},
\end{aligned} \tag{7}$$

where $\bar{\tau}_k, \underline{\tau}_k$ are boundary vectors of the truncated region. κ_k is the distance vector of the current iterative point from the nearest constraints, and the size of truncation region depends on the parameter β . It is suggested that β should be set small for a large sensitivity of modal parameters to the updating parameters and large for a small sensitivity.

4. Finite Element Model for Damaged FGM Structures

4.1. Random Effective Material Properties. Recalling (1), the uncertainty in the effective material properties of an FGM comes from the uncertainty in both material parameters and volume fraction of constituents. Taking into account the low variability in both the physical properties (such as the Young modulus and mass density of each constituent material) and volume fraction index, and neglecting high-order terms, the effective material properties for the FGM beam can be expanded by Taylor's series:

$$P(\cdot) = \bar{P}(\cdot) + \tilde{P}(\cdot), \quad \tilde{P}(\cdot) = \sum_{i=1}^l \frac{\partial \bar{P}(\cdot)}{\partial \theta_i} \tilde{\theta}_i, \tag{8}$$

where $\bar{P}(\cdot)$ is the mean value of each effective material property, and $\tilde{P}(\cdot)$ is the corresponding zero-mean random part with the variance of $\sigma_{P(\cdot)}^2$. θ_i is the i th element of the updating parameter vector θ , and $\tilde{\theta}_i$ is the corresponding zero-mean random part.

4.2. Finite Element Model of Undamaged FGM Beam. According to the third-order shear deformation theory, the displacement field of the FGM beam can be expressed as follows [11, 12]:

$$\begin{aligned}
u(x, z, t) &= u_0(x, t) \\
&+ z \left[\phi_x(x, t) - \frac{4}{3} \left(\frac{z}{h} \right)^2 \left(\phi_x(x, t) + \frac{\partial w_0(x, t)}{\partial x} \right) \right], \tag{9} \\
w(x, t) &= w_0(x, t),
\end{aligned}$$

where u, v are the axial and transverse displacements at any point of the FGM beam in the x, z directions, respectively. u_0, w_0 are those on the mid-plane. ϕ_x is the cross-sectional rotation about x -axis.

By assuming the small deformation, the following strain field can be obtained:

$$\begin{aligned}
\varepsilon_x(x, z, t) &= \frac{\partial u_0}{\partial x} \\
&+ z \left[\frac{\partial \phi_x}{\partial x} - \frac{4}{3} \left(\frac{z}{h} \right)^2 \left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2} \right) \right], \tag{10} \\
\gamma_{xz}(x, z, t) &= \phi_x + \frac{\partial w_0}{\partial x} - 4 \left(\frac{z}{h} \right)^2 \left(\phi_x + \frac{\partial w_0}{\partial x} \right),
\end{aligned}$$

where ε_x and γ_{xz} are the normal and shear strains.

The constitutive relation in thermal environment can be expressed as

$$\begin{aligned}
&\begin{Bmatrix} \sigma_x(x, z, t) \\ \tau_{xz}(x, z, t) \end{Bmatrix} \\
&= \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \alpha(x, z, T) \Delta T(x, z) \right), \tag{11}
\end{aligned}$$

where σ_x and τ_{xz} are the normal and shear stresses. Q_{11} and Q_{55} are the elastic coefficients, that is, $Q_{11} = E(x, z, T)/(1 - \nu^2)$, $Q_{55} = E(x, z, T)/2(1 + \nu)$. $E(x, z, T) = \bar{E}(x, z, T) + \tilde{E}(x, z, T)$ are the effective Young's elastic modulus, which is the function of z for the FGM beam through its thickness direction, and the function of x for the FGM beam along its axial direction. $\bar{E}(x, z, T)$ is the mean and $\tilde{E}(x, z, T)$ is the corresponding zero-mean random part. Poisson's ratio ν is assumed to be constant. $\alpha(x, z, T)$ is the thermal expansion efficient. $\Delta T(x, z)$ is the temperature change distribution for the FGM beam.

The two-node shear deformable beam element with four degrees of freedom in each node is employed in the development of the finite element model. The displacement vector at the mid-plane of a beam element can be expressed as

$$\begin{aligned}
&\begin{Bmatrix} u_0 & w_0 & \frac{\partial w_0}{\partial x} & \phi_x \end{Bmatrix}^T = [\mathbf{N}] \{\mathbf{q}^e\}, \\
&\{\mathbf{q}^e\} = \left\{ u_0^{e_1}, w_0^{e_1}, \frac{\partial w_0^{e_1}}{\partial x}, \phi_x^{e_1}, u_0^{e_2}, w_0^{e_2}, \frac{\partial w_0^{e_2}}{\partial x}, \phi_x^{e_2} \right\}^T, \tag{12}
\end{aligned}$$

where $[\mathbf{N}]$ is the shape function matrix and $\{\mathbf{q}^e\}$ is the nodal displacement vector of beam element.

The strain and kinetic energies of the beam element are

$$U = \iiint_{V_e} \frac{1}{2} (\varepsilon_x Q_{11} \varepsilon_x + \gamma_{xz} Q_{55} \gamma_{xz} - \varepsilon_x Q_{11} \alpha \Delta T) dx dy dz, \quad (13)$$

$$T = \iiint_{V_e} \frac{1}{2} \rho(x, z, T) (\dot{u}^2 + \dot{w}^2) dx dy dz,$$

where V_e is the volume of a beam element. $\rho(x, z, T) = \bar{\rho}(x, z, T) + \tilde{\rho}(x, z, T)$ are the effective Young's modulus, which is the function of z for the FGM beam through its thickness, and the function of x for the FGM beam along its axial direction. $\bar{\rho}(x, z, T)$ is the mean and $\tilde{\rho}(x, z, T)$ is the corresponding zero-mean random part.

The following stochastic finite element equation of vibration for the beam element in thermal environment can be obtained by using Hamilton's principle:

$$[\mathbf{M}_e] \{\ddot{\mathbf{q}}_e\} + [\mathbf{K}_e] \{\mathbf{q}_e\} = \{\mathbf{F}_e\}, \quad (14)$$

where $[\mathbf{K}_e] = [\bar{\mathbf{K}}_e] + [\tilde{\mathbf{K}}_e]$ is the element stiffness matrix, $[\mathbf{M}_e] = [\bar{\mathbf{M}}_e] + [\tilde{\mathbf{M}}_e]$ is the element mass matrix, and $\{\mathbf{F}_e\} = [\bar{\mathbf{F}}_e] + [\tilde{\mathbf{F}}_e]$ is the thermal force. Each matrix is composed of the mean matrix and the zero-mean random part.

Recalling (8), the means $[\bar{\mathbf{K}}_e]$, $[\bar{\mathbf{M}}_e]$, $[\bar{\mathbf{F}}_e]$ and the zero-mean random parts $[\tilde{\mathbf{K}}_e]$, $[\tilde{\mathbf{M}}_e]$, $[\tilde{\mathbf{F}}_e]$ can be computed, respectively, by

$$\begin{aligned} [\bar{\mathbf{K}}_e] &= \iint \bar{E}(x, z, T) [\bar{\mathbf{K}}_e] dx dz, \\ [\bar{\mathbf{M}}_e] &= \iint \bar{\rho}(x, z, T) [\bar{\mathbf{M}}_e] dx dz, \\ \{\bar{\mathbf{F}}_e\} &= \iint \bar{E}(x, z, T) \{\bar{\mathbf{F}}_e\} dx dz, \\ [\tilde{\mathbf{K}}_e] &= \iint \left(\sum_{i=1}^l \frac{\partial \bar{E}(x, z, T)}{\partial \bar{\theta}_i} \bar{\theta}_i \right) [\tilde{\mathbf{K}}_e] dx dz, \\ [\tilde{\mathbf{M}}_e] &= \iint \left(\sum_{k=1}^l \frac{\partial \bar{\rho}(x, z, T)}{\partial \bar{\theta}_i} \bar{\theta}_i \right) [\tilde{\mathbf{M}}_e] dx dz, \\ \{\tilde{\mathbf{F}}_e\} &= \iint \left(\sum_{k=1}^l \frac{\partial \bar{E}(x, z, T)}{\partial \bar{\theta}_i} \bar{\theta}_i \right) \{\tilde{\mathbf{F}}_e\} dx dz, \end{aligned} \quad (15)$$

where the matrices $[\bar{\mathbf{K}}_e]$, $[\bar{\mathbf{M}}_e]$, $\{\bar{\mathbf{F}}_e\}$ can be computed by

$$\begin{aligned} [\bar{\mathbf{K}}_e] &= \frac{b}{1 - \gamma^2} ([\mathbf{B}_\varepsilon]^T [\mathbf{B}_\varepsilon] + [\mathbf{B}_\gamma]^T [\mathbf{B}_\gamma]), \\ [\bar{\mathbf{M}}_e] &= b [\mathbf{D}]^T [\mathbf{D}], \\ \{\bar{\mathbf{F}}_e\} &= \frac{b}{1 - \gamma^2} [\mathbf{B}_\varepsilon]^T \alpha(x, z, T) \Delta T(x, z), \\ [\mathbf{B}_\varepsilon] &= [1 \ 0 \ -c_1 z^3 \ z(1 - c_1 z^2)] \frac{\partial [\mathbf{N}]}{\partial x}, \end{aligned}$$

$$[\mathbf{B}_\gamma] = [0 \ 0 \ 1 - 3c_1 z^2 \ 1 - 3c_1 z^2] [\mathbf{N}],$$

$$[\mathbf{D}] = \begin{bmatrix} 1 & 0 & -c_1 z^3 & z(1 - c_1 z^2) \\ 0 & 1 & 0 & 0 \end{bmatrix} [\mathbf{N}], \quad (17)$$

where $[\mathbf{B}_\varepsilon]$, $[\mathbf{B}_\gamma]$, and $[\mathbf{D}]$ are the matrices mainly related to the shape function matrix $[\mathbf{N}]$ and $c_1 = 4/3h^2$.

4.3. Finite Element Model of Damaged FGM Beam. For the damaged FGM beam, a local damage often leads to the reduction in the local stiffness parameter. In this section, damage is introduced by reducing effectiveness elastic moduli of the corresponding elements [16]. Introducing a damage factor λ_e indicating the damage severity for the e th damaged element, effective Young's modulus of the e th damaged element can be expressed as

$$\bar{E}_d(x, z, T) = \bar{E}(x, z, T) (1 - \lambda_e), \quad (18)$$

where $\bar{E}(x, z, T)$ is the effective Young's elastic modulus for the undamaged FGM beam and $\bar{E}_d(x, z, T)$ is for the damaged FGM beam.

Recalling (15), the stiffness matrix of the damaged element can be expressed as

$$[\mathbf{K}_{e,d}] = [\mathbf{K}_e] (1 - \lambda_e), \quad (19)$$

where λ_e is the damage factor of the e th damaged element and $[\mathbf{K}_{e,d}]$ is the stiffness element for the e th damaged element.

With (18) and (19), the element matrices in (14) can be assembled to obtain the global stochastic finite element equation of vibration for the damaged FGM beam.

$$[\mathbf{M}] \{\ddot{\mathbf{q}}\} + [\mathbf{K}] \{\mathbf{q}\} = \{\mathbf{F}\},$$

$$[\mathbf{K}] = [\bar{\mathbf{K}}] + [\tilde{\mathbf{K}}],$$

$$[\mathbf{M}] = [\bar{\mathbf{M}}] + [\tilde{\mathbf{M}}],$$

$$\{\mathbf{F}\} = \{\bar{\mathbf{F}}\} + \{\tilde{\mathbf{F}}\},$$

$$[\bar{\mathbf{K}}]$$

$$\begin{aligned} &= \sum_e^{n_u} [\mathbf{T}_e]^T [\bar{\mathbf{K}}_e] [\mathbf{T}_e] \\ &+ \sum_e^{n_d} (1 - \lambda_e) [\mathbf{T}_e]^T [\bar{\mathbf{K}}_e] [\mathbf{T}_e], \end{aligned}$$

$$[\bar{\mathbf{M}}] = \sum_e^{n_u + n_d} [\mathbf{T}_e]^T [\bar{\mathbf{M}}_e] [\mathbf{T}_e],$$

$$\{\bar{\mathbf{F}}\} = \sum_e^{n_u} [\mathbf{T}_e]^T \{\bar{\mathbf{F}}_e\} + \sum_e^{n_d} (1 - \lambda_e) [\mathbf{T}_e]^T \{\bar{\mathbf{F}}_e\},$$

$$\begin{aligned}
& [\bar{\mathbf{K}}] \\
&= \sum_e^{n_u} [\mathbf{T}_e]^T [\bar{\mathbf{K}}_e] [\mathbf{T}_e] \\
&\quad + \sum_e^{n_d} (1 - \lambda_e) [\mathbf{T}_e]^T [\bar{\mathbf{K}}_e] [\mathbf{T}_e], \\
& [\bar{\mathbf{M}}] = \sum_e^{n_e} [\mathbf{T}_e]^T [\bar{\mathbf{M}}_e] [\mathbf{T}_e], \\
& \{\bar{\mathbf{F}}\} = \sum_e^{n_u} [\mathbf{T}_e]^T \{\bar{\mathbf{F}}_e\} + \sum_e^{n_d} (1 - \lambda_e) [\mathbf{T}_e]^T \{\bar{\mathbf{F}}_e\},
\end{aligned} \tag{20}$$

where $[\mathbf{K}]$ is the global random stiffness matrix and $[\mathbf{M}]$ is the global random mass matrix. $\{\mathbf{F}\}$ is the global thermal force and $[\mathbf{T}_e]$ is a transformation matrix that transforms the local coordinate of the e th element to the global coordinate. n_u is the number of undamaged elements and n_d is the number of damaged elements.

5. Forward Uncertainty Propagation

The j th modal frequency f_j and mode shape $\{\boldsymbol{\varphi}\}_j$ are governed by the following:

$$([\mathbf{K}] - (2\pi f_j)^2 [\mathbf{M}]) \{\boldsymbol{\varphi}\}_j = 0, \quad j = 1, 2, \dots, N. \tag{21}$$

The j th modal frequency and mode shape can also be represented as the sum of its mean and a zero-mean random part:

$$\begin{aligned}
f_j &= \bar{f}_j + \tilde{f}_j, \\
\{\boldsymbol{\varphi}\}_j &= \{\bar{\boldsymbol{\varphi}}\}_j + \{\tilde{\boldsymbol{\varphi}}\}_j
\end{aligned} \tag{22}$$

and further the random part of the j th normalized mode shape can be written as

$$\{\tilde{\boldsymbol{\varphi}}\}_j = \sum_{r=1}^N \varepsilon_{rj} \{\bar{\boldsymbol{\varphi}}\}_r, \tag{23}$$

where ε_{rj} , $r = 1, 2, \dots, N$ are the small coefficients to be determined. Substituting (22) into (21), we have

$$([\bar{\mathbf{K}}] - (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}]) \{\bar{\boldsymbol{\varphi}}\}_j = 0, \tag{24}$$

$$\begin{aligned}
& [\bar{\mathbf{K}}] \sum_{r=1}^N \varepsilon_{rj} \{\bar{\boldsymbol{\varphi}}\}_r + [\bar{\mathbf{K}}] \{\bar{\boldsymbol{\varphi}}\}_j \\
&= (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}] \sum_{r=1}^N \varepsilon_{rj} \{\bar{\boldsymbol{\varphi}}\}_r + (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}] \{\bar{\boldsymbol{\varphi}}\}_j \\
&\quad + 8\pi^2 \bar{f}_j \tilde{f}_j [\bar{\mathbf{M}}] \{\bar{\boldsymbol{\varphi}}\}_j.
\end{aligned} \tag{25}$$

Using the orthogonality of mode shapes, the random part of the j th modal frequency and the coefficients ε_{ij} ($i \neq j$)

can be obtained by premultiplying (25) by $\{\bar{\boldsymbol{\varphi}}\}_j^T$ and $\{\tilde{\boldsymbol{\varphi}}\}_j^T$, respectively,

$$\begin{aligned}
\tilde{f}_j &= \frac{1}{8\pi^2 \bar{f}_j} \{\bar{\boldsymbol{\varphi}}\}_j^T ([\bar{\mathbf{K}}] - (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}]) \{\bar{\boldsymbol{\varphi}}\}_j, \\
\varepsilon_{ij} &
\end{aligned} \tag{26}$$

$$= \frac{1}{4\pi^2 (\bar{f}_j^2 - \bar{f}_i^2)} \{\bar{\boldsymbol{\varphi}}\}_j^T ([\bar{\mathbf{K}}] - (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}]) \{\bar{\boldsymbol{\varphi}}\}_i.$$

Recalling the orthogonality condition $\{\boldsymbol{\varphi}\}_j^T [\mathbf{M}] \{\boldsymbol{\varphi}\}_j = 1$ and using (20) and (22), the coefficient ε_{jj} can be obtained by

$$\varepsilon_{jj} = -\frac{1}{2} \{\bar{\boldsymbol{\varphi}}\}_j^T [\bar{\mathbf{M}}] \{\bar{\boldsymbol{\varphi}}\}_j. \tag{27}$$

Neglecting the high-order terms, the random parts of modal parameters in (22) can be obtained by the Taylor's expansion:

$$\begin{aligned}
\tilde{f}_j &= \sum_{i=1}^l \frac{\partial \bar{f}_j}{\partial \theta_i} \tilde{\theta}_i, \\
\{\tilde{\boldsymbol{\varphi}}\}_j &= \sum_{k=1}^l \frac{\partial \{\bar{\boldsymbol{\varphi}}\}_j}{\partial \theta_i} \tilde{\theta}_i.
\end{aligned} \tag{28}$$

Using (20), (26)–(28), we have the following equation for the solution of the unknown $\partial \bar{f}_j / \partial \theta_i$ and $\partial \{\bar{\boldsymbol{\varphi}}\}_j^T / \partial \theta_i$ by equating the coefficients of each $\tilde{\theta}_i$:

$$\begin{aligned}
\sum_{i=1}^l \frac{\partial \bar{f}_j}{\partial \theta_i} \tilde{\theta}_i &= \frac{1}{8\pi^2 \bar{f}_j} \{\bar{\boldsymbol{\varphi}}\}_j^T ([\bar{\mathbf{K}}] - (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}]) \{\bar{\boldsymbol{\varphi}}\}_j, \\
\sum_{i=1}^l \frac{\partial \{\bar{\boldsymbol{\varphi}}\}_j^T}{\partial \theta_i} \tilde{\theta}_i &= -\frac{1}{2} \{\bar{\boldsymbol{\varphi}}\}_j^T [\bar{\mathbf{K}}] \{\bar{\boldsymbol{\varphi}}\}_j \\
&\quad + \sum_{i=1}^N \left\{ \frac{1}{4\pi^2 (\bar{f}_j^2 - \bar{f}_i^2)} \{\bar{\boldsymbol{\varphi}}\}_j^T \right. \\
&\quad \cdot ([\bar{\mathbf{K}}] - (2\pi \bar{f}_j)^2 [\bar{\mathbf{M}}]) \{\bar{\boldsymbol{\varphi}}\}_i \{\bar{\boldsymbol{\varphi}}\}_i \left. \right\},
\end{aligned} \tag{29}$$

$$\begin{aligned}
[\bar{\mathbf{K}}] &= \sum_e^{n_u} [\mathbf{T}_e]^T \iint \left(\sum_{i=1}^l \frac{\partial \bar{E}(x, z, T)}{\partial \tilde{\theta}_i} \tilde{\theta}_i \right) [\bar{\mathbf{K}}_e] dx dz \\
&\quad \cdot [\mathbf{T}_e] + \sum_e^{n_d} [\mathbf{T}_e]^T \iint (1 - \lambda_e) \left(\sum_{i=1}^l \frac{\partial \bar{E}(x, z, T)}{\partial \tilde{\theta}_i} \tilde{\theta}_i \right) \\
&\quad \cdot [\bar{\mathbf{K}}_e] dx dz [\mathbf{T}_e], \\
[\bar{\mathbf{M}}] &= \sum_{e=1}^{n_u+n_d} [\mathbf{T}_e]^T \iint \left(\sum_{k=1}^l \frac{\partial \bar{\rho}(x, z)}{\partial \tilde{\theta}_i} \tilde{\theta}_i \right) [\bar{\mathbf{M}}_e] dx dz \\
&\quad \cdot [\mathbf{T}_e].
\end{aligned}$$

TABLE 1: The following SuS304/Si3N4 type of FGM properties are used for computation [30].

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Ceramics (Si ₃ N ₄)	E (Pa)	$348.43e + 9$	0	$-3.070e - 4$	$2.160e - 7$	$-8.964e - 11$
	α (1/K)	$5.8723e - 6$	0	$9.095e - 4$	0	0
	ρ (kg/m ³)	2370	0	0	0	0
Metal (SUS304)	E (Pa)	$201.04e + 9$	0	$3.079e - 4$	$-6.534e - 7$	0
	α (1/K)	$12.330e - 6$	0	$8.086e - 4$	0	0
	ρ (kg/m ³)	8166	0	0	0	0

Further, the variances of the j th modal frequency and normalized mode shape as well as the covariance between the modal parameters and the random material parameters can be obtained:

$$\begin{aligned} \text{Var}(f_j) &= \sum_{k=1}^l \sum_{q=1}^l \frac{\partial \bar{f}_j}{\partial \theta_k} \frac{\partial \bar{f}_j}{\partial \theta_q} \rho_{kq} \sigma_{\bar{\theta}_k} \sigma_{\bar{\theta}_q}, \\ \text{Var}(\{\boldsymbol{\varphi}\}_j) &= \sum_{k=1}^l \sum_{q=1}^l \text{diag} \left(\frac{\partial \{\boldsymbol{\varphi}\}_j}{\partial \theta_k} \frac{\partial \{\boldsymbol{\varphi}\}_j^T}{\partial \theta_q} \right) \rho_{kq} \sigma_{\bar{\theta}_k} \sigma_{\bar{\theta}_q}, \end{aligned} \quad (30)$$

$$\text{Cov}(f_j, \theta_i) = \sum_{k=1}^l \frac{\partial \bar{f}_j}{\partial \theta_k} \rho_{ki} \sigma_{\bar{\theta}_k} \sigma_{\bar{\theta}_i},$$

$$\text{Cov}(\{\boldsymbol{\varphi}\}_j, \theta_i) = \sum_{k=1}^l \frac{\partial \{\boldsymbol{\varphi}\}_j}{\partial \theta_k} \rho_{ki} \sigma_{\bar{\theta}_k} \sigma_{\bar{\theta}_i},$$

where $\sigma_{\bar{\theta}_k}$, $\sigma_{\bar{\theta}_q}$, $\sigma_{\bar{\theta}_i}$ are the standard deviations of the random material parameters, that is, constituent material properties and volume fraction index, and ρ_{kq} is the correlation coefficient of random parameters $\bar{\theta}_k$, $\bar{\theta}_q$.

6. Numerical Simulation

In order to demonstrate the stochastic model updating-based parameter identification approach outlined in the previous sections for damaged FGM structures with random material properties in thermal environment, a damaged FGM beam is considered for the numerical simulation. The results obtained from the presented identification approach have been validated by comparing with the given values. The FGM is composed of ceramics (Si₃N₄) and metal (SUS304) with the mean values of material properties given in Table 1. In the FGM, material properties are assumed to vary, either through its thickness direction or along its axial direction, according to power law distribution. Therefore, the bottom surface is pure metal and the top surface is pure ceramics for the FGM beam through its thickness direction, while the right side is pure metal and the left side is pure ceramics for the FGM beam along its axial direction. The FGM beam has the geometric parameters of 20 m in length, 1 m in thickness, and 0.8 m in width. In this section, Young's elastic moduli of two materials and volume fraction index are chosen as the parameters to be identified, since the former may significantly change with the environmental temperature and the latter is hard to be determined by traditional experiments.

In the finite element, the FGM beam was discretized into eight two-node shear deformable beam elements with four degrees of freedom in each node. It is assumed that the damage is located at the 8th element with a reduction in the effective Young's elastic modulus by 10%, that is, $\lambda_8 = 0.1$. Analytically, modal analysis is first carried out to obtain the initial analytical modal parameters by using finite element model of the damaged FGM beam with initial material properties and distribution. Modal analysis is then again carried out on the damaged FGM beam with given material properties and distribution to obtain the actual modal parameters due to the difficulty of random experiment with large samples.

The actual modal data is generated in a simulated way assuming that the uncertainty of modal data is caused by 5% COV (coefficient of variation, the ratio of the square root of the variance to the mean) in elastic moduli of two constituents as well as volume fraction index and 0.1% COV in measurement noise. Figure 2 shows the scatter of the actual modal frequencies set compared to the initial modal frequencies in the temperature of 300 K for the damaged FGM beam through the thickness direction ($n = 2$) compared to the undamaged case and Figure 3 shows those for the FGM beam along the axial direction ($n = 2$), in which the clamped-free (CF) boundary condition is firstly considered.

The stochastic finite element model updating is conducted using the region truncation-based optimization algorithm, and, as a result, the means and variances of elastic moduli of two materials and volume fraction index are identified by the stochastic model updating-based method (SMUM).

The statistical identification of material parameters for a damaged FGM structure can be implemented by the following steps.

Step 1. Develop the finite element model of the damaged FGM structure and determine the parameters $\boldsymbol{\theta}$ to be identified, which are chosen as the updating parameters of stochastic model updating.

Step 2. Compute the mean vector $\bar{\mathbf{z}}_m$ and covariance matrix $\text{Cov}(\bar{\mathbf{z}}_m, \bar{\mathbf{z}}_m)$ of the actual modal data.

Step 3. Initialize the means and variances of the updating parameters $\boldsymbol{\theta}$.

Step 4. Obtain the mean vector $\bar{\mathbf{z}}_{ak}$ of the analytical modal data after carrying out the modal analysis, and determine the

FIGURE 2: First three actual modal frequencies set (3000 samples) and initial modal frequencies (thickness).

FIGURE 3: First three actual modal frequencies set (3000 samples) and initial modal frequencies (axial).

covariance matrices $\text{Cov}(\tilde{\mathbf{z}}_{ak}, \tilde{\boldsymbol{\theta}}_k)$ and $\text{Cov}(\tilde{\mathbf{z}}_{ak}, \tilde{\mathbf{z}}_{ak})$ using the forward uncertainty propagation (30).

Step 5. Update the mean values $\bar{\boldsymbol{\theta}}_{k+1}$ and variances $\text{Cov}(\tilde{\boldsymbol{\theta}}_{k+1}, \tilde{\boldsymbol{\theta}}_{k+1})$ of the updating parameters using (7) and (6), respectively.

Step 6. Go to Step 4 until $\|\mathbf{z}_{ak}(\boldsymbol{\theta}) - \mathbf{z}_m\|^2$ is small enough and the second-order statistics (means and variances) of the identified parameters can be obtained.

The convergences of the mean values and variances of elastic moduli (E_c -elastic modulus of ceramics and E_m -elastic modulus of metal of metal) and volume fraction index (n) for the damaged FGM beam with material properties varying through its thickness in the temperature of 300 K and 600 K are shown in Figure 4. The results show that, for different temperatures, the means and variances of elastic moduli of two constituents and volume fraction index can fast converge by the proposed method. It is also seen that

the means of elastic moduli of two materials converge to different values for different temperatures since the elastic modulus of each material changes with the environmental temperature. The situation does not occur to volume fraction index, since volume fraction index is not affected by the temperature environment and depends on the manufacturing and fabrication process. The temperature has no obvious effect on the identification results of parameter variances.

For the comparison of convergence between the trust region method and the proposed method, Figure 5 gives the iteration process of mean values and variances of E_c , E_m , and n for the damaged FGM beam with material properties varying through its thickness direction in the temperature of 600 K by two methods. It is seen that mean values and variances of E_c , E_m , and n can converge faster by the proposed method than those by the trust region method. In fact, the CPU run time for the proposed method is about 6.356 seconds, and the CPU run time for the trust region method is about 13.251 seconds.

Compared to the Monte-Carlo simulation (MCS) method with 3000 samples, the mean values and COVs (coefficient of variation, the ratio of the square root of the variance to the mean) of elastic moduli of two materials and volume fraction index for both the damaged FGM beam through its thickness direction and the FGM beam along its axial direction with different volume fraction indices are shown in Tables 2 and 3, respectively.

It is seen from Tables 2 and 3 that, for the mean values or the COVs, the identification results by the stochastic model updating-based method (SMUM) proposed in this work agree favorably with those from the MCS. It is also observed that, compared to the given values, the means of parameters can be identified with good accuracy in the case of different volume fraction indices, while the identification results of the COVs are obviously affected by the volume fraction index and present relatively large errors, especially in the case of relatively small volume fraction indices or relatively large volume fraction indices; that is, one of constituents plays an dominant role in the FGM. In fact, the identified COV of E_c will be far away from the given value when the volume fraction index is large enough (e.g., $n = 10$) so that the FGM is rich in metal and few ceramics are in it. In contrast, the identified COV of E_m will be far away from the given value when the volume fraction index is small enough (e.g., $n = 0.5$) so that the FGM is rich in ceramics and few metal is in it. However, the situation does not happen to the FGM beam along its axial direction for these two cases (i.e., $n = 0.5$ and $n = 10$) and the volume fraction has no significant influence on the identification results of COVs. This can be explained that the material properties vary along the axial direction from the value of E_m to the value of E_c much more gradually than those along the thickness since the length (8 m) of beam is much larger than its thickness (1 m).

Figure 6 gives the convergences of the mean values and variances of E_c , E_m , and n for the FGM beam with material properties varying through its thickness direction in different cases of boundary conditions ($T = 600$ K). Four boundary conditions of clamped-free (CF), clamped-clamped (CC), clamped-simply (CS), and simply-simply (SS) are considered.

FIGURE 4: Iterative process of means and variances of material elastic moduli and volume fraction index.

It is observed that the means and variances of elastic moduli of two constituents and volume fraction index can converge by the proposed method no matter what kind of boundary conditions, which means that the boundary condition has no influence on the convergence of iterative process. It is also seen that the boundary condition has more influences on the identification result of the variance than that of the mean, and this means that the identification of the dispersion (i.e., variance or COV) is more susceptible to the external disturbance compared to the identification of the mean.

In order to further investigate the influence of the boundary conditions on the means and COVs of identified parameters, the means and COVs of material elastic moduli and volume fraction index of the FGM beam through its thickness direction or along its axial direction ($n = 2$) for different cases of boundary conditions are shown in Table 4. From Table 4, under the same case of parameter randomness and measurement noise, boundary conditions have no significant effects on the identified results of means and COVs of parameters with the exception of small fluctuation in values.

In order to investigate the influence of the uncertainty source of actual modal data on the means and COVs of identified parameters, the means and COVs of material elastic moduli and volume fraction index of the FGM beam through its thickness direction or along its axial direction ($n = 2$) for different cases of uncertainty are shown in Tables 5 and 6. These cases include that the uncertainty of actual modal data is caused by the structural material

(i.e., material elastic moduli and volume fraction index) randomness (SR) only, the measurement noise (MN) only, and the combination of two sources (both SR and MN). With structural material randomness (SR) and measurement noise (MN), four combinations of levels of uncertainty are considered. These combinations are (1) Level 1: 1% COV for SR (elastic moduli and volume fraction) and 0.1% for MN; (2) Level 2: 1% for SR and 0.5% for MN; (3) Level 3: 5% for SR and 0.1% for MN; (4) Level 4: 5% for SR and 0.5% for MN.

The results from Tables 5 and 6 show that the uncertainty does not bring too much changes in the means of identified parameters no matter what sources the uncertainty of actual modal data comes from, and, unlikely, the uncertainty contributes to the COV results, whose errors increase with the uncertainty level. Taking the FGM beam through its thickness direction with the environmental temperature of 300 K as an example, the COVs of identified parameters range from 2.1465% to 11.5036% for metal constituent elastic modulus, 3.0398% to 15.3656% for ceramics constituent modulus, and 4.5745% to 22.8086% for constituent volume fraction index when the level of uncertainty varies from Level 1 (1% SR, 0.1% MN) to Level 4 (5% SR, 0.5% MN). The same situation occurs to the FGM beam along its axial direction. It is worth noting that the accuracy of identified COVs is much more sensitive to the measurement noise, for example, ranging from 6.1553% to 15.3656% for E_c , 6.2834% to 11.5036% for E_m , and 9.9623% to 22.8086% for n when the level of MN varies from 0.1% (Level 3) to 0.5% (Level 4) with the same SR of 5%. It is

TABLE 2: Means of material elastic moduli and volume fraction index for SMUM and MCS.

Temperature	Parameters (given values)	$n = 0.5$			$n = 2$			$n = 5$			$n = 10$		
		SMUM	MCS	SMUM	SMUM	MCS	MCS	SMUM	SMUM	MCS	SMUM	SMUM	MCS
$T = 300$ K	E_c (3.22272e11)	3.2174e112	3.2236e11	FGM beam through its thickness									
	E_m (2.07788e11)	2.07768e11	2.0778e11	3.2226e + 11	3.2233e11	3.2261e11	3.2287e11	3.2163e11	3.2233e11	3.2261e11	3.2079e11	3.2287e11	
	n (0.5/2/5/10)	0.4978	0.5027	1.9910	1.9986	5.0508	9.9449	4.9583	2.0783e11	2.0789e11	9.7925	9.9449	
	E_c (3.04610e11)	3.0388e11	3.0579e11	3.0465e11	3.0437e11	3.0608e11	3.0242e11	3.0396e11	3.0437e11	3.0608e11	3.0265e11	3.0242e11	
	E_m (1.90890e11)	1.9059e11	1.9176e11	1.9084e11	1.9061e11	1.9101e11	1.9069e11	1.9027e11	1.9061e11	1.9101e11	1.9008e11	1.9069e11	
$T = 600$ K	n (0.5/2/5/10)	0.4985	0.5086	1.9943	2.0049	5.0509	9.7003	4.9412	5.0509	9.7276	9.7003		
$T = 300$ K	E_c (3.22272e11)	3.2177e11	3.2210e11	FGM along its axial direction									
	E_m (2.07788e11)	2.0832e11	2.0752e11	3.2158e11	3.2247e11	3.2220e11	3.2231e11	2.0775e11	3.2247e11	3.2220e11	3.2125e11	3.2231e11	
	n (0.5/2/5/10)	0.4986	0.5013	1.9965	1.9973	4.9949	10.0179	5.0012	2.0734e11	2.0768e11	9.9779	10.0179	
	E_c (3.04610e11)	3.0386e11	3.0418e11	3.0396e11	3.0454e11	3.0478e11	3.0521e11	3.0383e11	3.0454e11	3.0478e11	3.0489e11	3.0521e11	
	E_m (1.90890e11)	1.9149e11	1.8969e11	1.9125e11	1.9035e11	1.9069e11	1.9072e11	1.9069e11	1.9035e11	1.9069e11	1.9105e11	1.9072e11	
$T = 600$ K	n (0.5/2/5/10)	0.4987	0.5014	1.9989	1.9998	5.0155	10.0383	4.9951	5.0155	9.9955	10.0383		

TABLE 3: COVs of material elastic moduli and volume fraction index for SMUM and MCS.

Temperature	Parameters (given values)	$n = 0.5$			$n = 2$			$n = 5$			$n = 10$		
		SMUM	MCS	SMUM	SMUM	MCS	SMUM	SMUM	MCS	SMUM	SMUM	MCS	
		FGM beam through its thickness											
$T = 300 \text{ K}$	$E_c \text{ (5\%)}$	6.9139%	6.3355%	6.1553%	6.0225%	5.8767%	5.4019%	5.8767%	8.4827%	5.4019%	8.4827%	4.1443%	
	$E_m \text{ (5\%)}$	8.4283%	7.4319%	6.2834%	5.3610%	5.7577%	5.0743%	5.7577%	5.9237%	5.0743%	5.9237%	5.1053%	
	$n \text{ (5\%)}$	11.9278%	12.2767%	9.9623%	6.8676%	9.5556%	10.4554%	9.5556%	26.9936%	10.4554%	26.9936%	18.9727%	
$T = 600 \text{ K}$	$E_c \text{ (5\%)}$	7.3813%	6.4775%	6.4379%	5.9881%	6.2478%	5.2932%	6.2478%	8.9632%	5.2932%	8.9632%	8.8812%	
	$E_m \text{ (5\%)}$	9.0423%	7.2826%	6.8224%	5.3459%	6.3822%	5.2756%	6.3822%	6.5709%	5.2756%	6.5709%	6.5534%	
	$n \text{ (5\%)}$	11.3814%	11.8691%	9.7212%	6.7731%	8.2936%	9.5587%	8.2936%	26.0417%	9.5587%	26.0417%	28.2942%	
		FGM beam along its axial direction											
$T = 300 \text{ K}$	$E_c \text{ (5\%)}$	5.6619%	5.0117%	5.6988%	5.1429%	6.2028%	5.6765%	6.2028%	6.4789%	5.6765%	6.4789%	6.0394%	
	$E_m \text{ (5\%)}$	5.9169%	5.8154%	5.9243%	5.0552%	5.5639%	5.0349%	5.5639%	5.0739%	5.0349%	5.0739%	5.1134%	
	$n \text{ (5\%)}$	6.2618%	6.2209%	5.9821%	5.3645%	6.0643%	5.8934%	6.0643%	6.6918%	5.8934%	6.6918%	6.1129%	
$T = 600 \text{ K}$	$E_c \text{ (5\%)}$	5.9368%	4.9651%	6.0989%	5.3348%	6.5508%	5.5821%	6.5508%	6.7065%	5.5821%	6.7065%	5.9315%	
	$E_m \text{ (5\%)}$	6.5070%	6.5424%	6.2054%	5.4021%	6.0851%	5.1826%	6.0851%	6.0854%	5.1826%	6.0854%	5.0826%	
	$n \text{ (5\%)}$	6.2779%	6.1971%	5.7743%	5.5224%	6.1823%	6.0234%	6.1823%	6.6069%	6.0234%	6.6069%	5.9734%

FIGURE 5: Iterative process of means and variances of E_c , E_m , and n for two methods ($T = 600$ K).

also observed that the influence of the measurement noise with a large value on the identified results is much more larger than that of structural material randomness although the identified COVs for each parameter obtained in the case of the combination of two uncertainty sources are higher than those obtained in the case of SR only or MN only; for example, the COVs is 15.3656% for E_c of the FGM through its thickness direction ($T = 300$ K) in the case of 5% SR and 0.5% MN compared to 5.5865% in the case of 5% SR only and 14.2591% in the case of 0.5% MN only, which means that the identified results of COVs when both SR and MN are included are not the simple linear superposition of the results with respect to SR only or MN only concerned, and, in fact, the measurement noise has a pronounced effect on the accuracy of the identified results. It is also seen that, for the parameters with the smaller dispersion, the higher level of measurement noise can lead to the results with much larger errors, for example, 3.0398% COVs for E_c in the case of 1% SR and 0.1% MN (level 1) compared to 6.1553% in the case of 5% SR and 0.1% MN (Level 3), which means an error of 203.98% for Level 1 compared to an error of 23.11% for Level 3. In fact, in the case of a relatively small level of SR combined with a relatively large level of MN, the identified

results of COVs could be totally annihilated by measurement noise.

The same situation occurs to the case of FGM beam along the axial direction. Compared to the given value, Figures 7, 8, and 9 show the mean values and COVs of E_c , E_m , and n for the FGM beam along its axial direction under different cases and levels of uncertainty. For each identified parameter, the result shows that the mean can be identified with good accuracy in the presence of uncertainty, while the identified result of COV is susceptible to the measurement noise. In fact, the increase of measurement noise may result in a large error in the identified result of COV compared to the given value.

Compared to the given probability density functions (PDFs), Figures 10(a), 11(a), and 12(a) show the identified PDFs of E_c , E_m , and n for the FGM beam (CF) through its thickness direction or along its axial direction with volume fraction index of $n = 2$ in the thermal environment of 300 K and 600 K for the uncertainty level of 5% SR and 0.1% MN, in which A3 and A6 denote the given PDFs for 300 K and 600 K, respectively, PT3 and PT6 denote the identified PDFs for 300 K and 600 K, respectively, when the FGM beam through its thickness direction is considered, and PA3 and PA6 denote

FIGURE 6: Iterative process of means and variances of E_c , E_m , and n for different boundary conditions ($T = 600$ k).

FIGURE 7: Means and COVs of elastic moduli of ceramics (E_c) for FGM along axial direction with different uncertainties.

TABLE 4: Means and COVs of material elastic moduli and volume fraction index for different boundary conditions.

Temperature	Parameters (given means/COVs)	Means				COVs			
		CF	CC	CS	SS	CF	CC	CS	SS
T = 300 k	E_c (3.22272e11/5%) E_m (2.07788e11/5%) n (2/5%)	3.2226e + 11	3.2244e11	3.2249e + 11	3.2276e + 11	6.1553%	6.1149%	6.0565%	6.0745%
		2.0731e + 11	2.07223e11	2.0734e + 11	2.0733e + 11	6.2834%	6.6021%	6.8732%	6.7610%
		1.9910	1.9954	1.9934	1.9899	9.9623%	9.4556%	10.1517%	9.4319%
		3.0465e11	3.0511e11	3.0465e11	3.0510e11	6.4379%	6.5151%	6.2227%	6.3908%
		1.9084e11	1.9032e11	1.9045e11	1.9026e11	6.8224%	6.9642%	7.47109%	7.2721%
T = 600 k	n (2/5%)	1.9943	1.9945	1.9934	1.9918	9.7212%	9.2149%	9.6028%	9.1508%
T = 300 k	E_c (3.22272e11/5%) E_m (2.07788e11/5%) n (2/5%)	3.2158e11	3.2064e11	3.2087e + 11	3.1971e + 11	5.6988%	5.7067%	6.0838%	5.9943%
		2.0775e11	2.0754e11	2.0724e + 11	2.0732e + 11	5.9243%	6.1247%	6.0863%	6.0629%
		1.9965	1.9870	1.9845	1.9782	5.9821%	6.1728%	6.0602%	5.9818%
		3.0396e11	3.0398e11	3.0356e + 11	3.0298e + 11	6.0989%	5.9839%	6.1146%	5.9315%
		1.9125e11	1.9076e11	1.9081e + 11	1.9054e + 11	6.2054%	6.6679%	6.4078%	6.5143%
T = 600 k	n (2/5%)	1.9989	1.9845	1.9854	1.9790	5.7743%	6.1718%	6.0829%	6.0728%

TABLE 5: Means of material elastic moduli and volume fraction index for different cases of uncertainty.

Temperature	Parameters (Given values)	SR and MN				SR only			MN only	
		Level 1 (1% SR 0.1% MN)	Level 2 (1% SR 0.5% MN)	Level 3 (5% SR 0.1% MN)	Level 4 (5% SR 0.5% MN)	1%	5%	0.1%	0.5%	
FGM beam through its thickness										
$T = 300 \text{ k}$	E_c (3.22272e11)	3.2223e11	3.2192e11	3.2226e + 11	3.2256e11	3.2231e11	3.2197e11	3.2231e11	3.2276e11	
	E_m (2.07788e11)	2.0769e11	2.0779e11	2.0731e + 11	2.0774e11	2.0780e11	2.0780e11	2.0798e11	2.0780e11	
	n (2)	1.9982	1.9964	1.9910	1.9952	1.9996	1.9927	2.0018	2.0029	
$T = 600 \text{ k}$	E_c (3.04610e11)	3.0465e11	3.0408e11	3.0465e11	3.0398e11	3.0461e11	3.0454e11	3.0455e11	3.0426e11	
	E_m (1.90890e11)	1.9087e11	1.9131	1.9084e11	1.9101e11	1.9088e11	1.9074e11	1.9079e11	1.9135e11	
	n (2)	2.0002	2.0019	1.9943	1.9965	2.0001	1.9936	1.9986	2.0028	
FGM beam along its axial direction										
$T = 300 \text{ k}$	E_c (3.22272e11)	3.2224e11	3.2261e11	3.2158e11	3.2236e11	3.2226e11	3.2161e11	3.2247e11	3.2261e11	
	E_m (2.07788e11)	2.0778e11	2.0779e11	2.0775e11	2.0732e11	2.0786e11	2.0768e11	2.0787e11	2.0786e11	
	n (2)	1.9981	2.0018	1.9965	1.9989	1.9998	1.9981	2.0019	2.0054	
$T = 600 \text{ k}$	E_c (3.04610e11)	3.0462e11	3.0421e11	3.0396e11	3.0357e11	3.0458e11	3.0394e11	3.0467e11	3.0432e11	
	E_m (1.90890e11)	1.9093e11	1.9080e11	1.9125e11	1.9061e11	1.9095e11	1.9076e11	1.9087e11	1.9079e11	
	n (2)	2.0001	1.9963	1.9989	1.9955	2.0001	1.9993	1.9999	1.9959	

TABLE 6: COVs of material elastic moduli and volume fraction index for different cases of uncertainty.

Temperature	Parameters (given values)	SR and MN				SR only		MN only	
		Level 1 (1% SR 0.1% MN)	Level 2 (1% SR 0.5% MN)	Level 3 (5% SR 0.1% MN)	Level 4 (5% SR 0.5% MN)	1%	5%	0.1%	0.5%
		FGM through its thickness							
$T = 300 \text{ K}$	E_c (1%/5%)	3.0398%	14.0489%	6.1553%	15.3656%	1.0956%	5.5865%	2.8725%	14.2591%
	E_m (1%/5%)	2.1465%	9.2254%	6.2834%	11.5036%	1.1123%	5.9712%	1.7987%	9.8175%
	n (1%/5%)	4.5745%	22.0164%	9.9623%	22.8086%	1.0342%	5.2633%	4.4465%	22.0228%
$T = 600 \text{ K}$	E_c (1%/5%)	3.2289%	15.0723%	6.4379%	16.0919%	1.1446%	5.6354%	2.9657%	15.0258%
	E_m (1%/5%)	2.2965%	9.9145%	6.8224%	11.6645%	1.1962%	6.2367%	1.9336%	9.8444%
	n (1%/5%)	4.4755%	21.9856%	9.7212%	22.7138%	1.0331%	5.3643%	4.2429%	21.9586%
FGM beam along its axial direction									
$T = 300 \text{ K}$	E_c (1%/5%)	2.0698%	8.6736%	5.6988%	10.1014%	1.1134%	5.3245%	1.7669%	8.6012%
	E_m (1%/5%)	1.2578%	3.2402%	5.9243%	6.2348%	1.0989%	5.3717%	0.6229%	3.0479%
	n (1%/5%)	2.7756%	12.6854%	5.9821%	13.5611%	1.0334%	4.8631%	2.6207%	12.6327%
$T = 600 \text{ K}$	E_c (1%/5%)	2.1761%	9.5164%	6.0989%	11.0243%	1.1456%	5.6354%	1.8449%	9.4465%
	E_m (1%/5%)	1.3856%	3.6823%	6.2054%	6.91654%	1.1934%	5.9669%	0.6939%	3.4742%
	n (1%/5%)	2.7799%	13.1743%	5.7743%	14.0605%	1.0299%	4.9781%	2.5838%	13.1392%

FIGURE 8: Means and COVs of elastic moduli of metal (E_m) for FGM along axial direction with different uncertainties.

FIGURE 9: Means and COVs of volume fraction index (n) for FGM along axial direction with different uncertainties.

the identified PDFs for 300 K and 600 K, respectively, when the FGM beam along its axial direction is considered. It can be seen that the identified results for the FGM beam along its axial direction are more close to the given PDFs than those for the FGM beam through its thickness direction. Compared to the given PDFs, Figures 10(b), 11(b), and 12(b)

show the identified PDFs of the FGM beam through its thickness direction in the thermal environment of 300k with different cases of uncertainty, in which A3 denotes the given PDF, P5SR denotes the identified PDF with 5% SR (5% COVs in material elastic moduli and volume fraction index), P01MN and P05MN denote the identified PDFs for 0.1%

FIGURE 10: Identified PDFs of ceramics material elastic modulus for different cases.

FIGURE 11: Identified PDFs of metal material elastic modulus for different cases.

MN (measurement noise) and 0.5% MN, respectively, and P5SR01MN denotes 5% SR and 0.1% MN, and P5SR05MN denotes 5% SR and 0.5% MN. It is obviously seen that the PDFs are far away from the given PDFs when MN reaches 0.5% no matter whether 5% SR is included, which also shows that the identified results of COVs are completely submerged in the measurement noise. The same situation occurs to the PDF results of all identified parameters.

7. Conclusions

Identification of the statistic numerical characteristics for constituent material properties and distribution is an important issue in mechanical behavior analysis and safety assessment of FGM structures, especially in the early stage of damage. The statistic numerical characteristics are identified for material elastic moduli and constituent volume fraction

FIGURE 12: Identified PDFs of volume fraction index for different cases.

index of a random FGM beam with initial damage by using a stochastic finite element model updating-based inverse technique in this paper. A region truncation-based optimization algorithm is utilized for simplifying the optimization process and improving computational efficiency without impairing the convergence of iterative process, and a new algorithm is developed for computing the covariance between the modal parameters and the identification parameters for damaged FGM structures. The influence of structural material randomness and measurement noise on the accuracy of identification results has been investigated.

The following conclusions can be drawn from this study:

- (1) Compared to the given values and the results from MCS method, the presented inverse technique based on stochastic finite element model updating can be utilized as an alternative to identify the statistics of the material properties and volume fraction index for random damaged FGM structures, provided that the modal parameters of the corresponding FGM structures are obtained from experimental test.
- (2) Compared to the trust region method, the faster convergence of the iterative process can be guaranteed by the region truncation-based optimization. The elastic moduli of constituent materials converge to the different values in the case of different temperatures, while volume fraction indices not affected by the environment converge to the same value in the case of different temperatures.
- (3) The volume distribution has more obvious influence on the accuracy of the dispersion (COVs) of identified parameters for the FGM with material properties varying quickly (through the thickness direction)

compared to that for the FGM with material properties varying slowly (along the axial direction).

- (4) The boundary conditions have no significant influence on the convergence of iterative process as well as the identified results of means and COVs of material elastic moduli and volume fraction index.
- (5) The measurement noise plays a dominant role in the accuracy of COV identification results while the uncertainty does not bring too much change in the means of identified parameters no matter what sources the uncertainty comes from. The dispersion of material properties and volume fraction can be identified with relatively good accuracy in the presence of small measurement noise, while the COV identification results could be totally annihilated by a relatively large measurement noise.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] D. K. Jha, T. Kant, and R. K. Singh, "A critical review of recent research on functionally graded plates," *Composite Structures*, vol. 96, pp. 833–849, 2013.

- [2] H. X. Nguyen, E. Atroshchenko, H. Nguyen-Xuan, and T. P. Vo, "Geometrically nonlinear isogeometric analysis of functionally graded microplates with the modified couple stress theory," *Computers & Structures*, vol. 193, pp. 110–127, 2017.
- [3] J. N. Reddy, J. Romanoff, and J. A. Loya, "Nonlinear finite element analysis of functionally graded circular plates with modified couple stress theory," *European Journal of Mechanics - A/Solids*, vol. 56, pp. 92–104, 2016.
- [4] G. Jin, T. Ye, X. Wang, and X. Miao, "A unified solution for the vibration analysis of FGM doubly-curved shells of revolution with arbitrary boundary conditions," *Composites Part B: Engineering*, vol. 89, pp. 230–252, 2016.
- [5] A.-Y. Tang, J.-X. Wu, X.-F. Li, and K. Y. Lee, "Exact frequency equations of free vibration of exponentially non-uniform functionally graded Timoshenko beams," *International Journal of Mechanical Sciences*, vol. 89, pp. 1–11, 2014.
- [6] G. Giunta, D. Crisafulli, S. Belouettar, and E. Carrera, "A thermo-mechanical analysis of functionally graded beams via hierarchical modelling," *Composite Structures*, vol. 95, pp. 676–690, 2013.
- [7] N. L. Shegokar and A. Lal, "Stochastic nonlinear bending response of piezoelectric functionally graded beam subjected to thermoelectromechanical loadings with random material properties," *Composite Structures*, vol. 100, pp. 17–33, 2013.
- [8] D. Wu, W. Gao, K. Gao, and F. Tin-Loi, "Robust safety assessment of functionally graded structures with interval uncertainties," *Composite Structures*, vol. 180, pp. 664–685, 2017.
- [9] M. Talha and B. N. Singh, "Stochastic perturbation-based finite element for buckling statistics of FGM plates with uncertain material properties in thermal environments," *Composite Structures*, vol. 108, no. 1, pp. 823–833, 2014.
- [10] A. Lal, H. Neeranjan Singh, and N. L. Shegokar, "FEM model for stochastic mechanical and thermal postbuckling response of functionally graded material plates applied to panels with circular and square holes having material randomness," *International Journal of Mechanical Sciences*, vol. 62, no. 1, pp. 18–33, 2012.
- [11] N. L. Shegokar and A. Lal, "Stochastic finite element nonlinear free vibration analysis of piezoelectric functionally graded materials beam subjected to thermo-piezoelectric loadings with material uncertainties," *Meccanica*, vol. 49, no. 5, pp. 1039–1068, 2014.
- [12] Y. Xu, Y. Qian, J. Chen, and G. Song, "Stochastic dynamic characteristics of FGM beams with random material properties," *Composite Structures*, vol. 133, pp. 585–594, 2015.
- [13] Z. Yu and F. Chu, "Identification of crack in functionally graded material beams using the p-version of finite element method," *Journal of Sound and Vibration*, vol. 325, no. 1-2, pp. 69–84, 2009.
- [14] M. Amirpour, R. Das, and S. Bickerton, "An elasto-plastic damage model for functionally graded plates with in-plane material properties variation: Material model and numerical implementation," *Composite Structures*, vol. 163, pp. 331–341, 2017.
- [15] I. Chiong, E. T. Ooi, C. Song, and F. Tin-Loi, "Computation of dynamic stress intensity factors in cracked functionally graded materials using scaled boundary polygons," *Engineering Fracture Mechanics*, vol. 131, pp. 210–231, 2014.
- [16] Z. R. Lu, X. X. Lin, Y. M. Chen, and M. Huang, "Hybrid sensitivity matrix for damage identification in axially functionally graded beams," *Applied Mathematical Modelling*, vol. 41, pp. 604–617, 2017.
- [17] Z.-G. Zhou, B. Wang, and Y.-G. Sun, "Investigation of the dynamic behavior of a finite crack in the functionally graded materials by use of the Schmidt method," *Wave Motion*, vol. 39, no. 3, pp. 213–225, 2004.
- [18] C.-H. Xia and L. Ma, "Dynamic behavior of a finite crack in functionally graded materials subjected to plane incident time-harmonic stress wave," *Composite Structures*, vol. 77, no. 1, pp. 10–17, 2007.
- [19] R. Bagheri, M. Ayatollahi, and E. Asadi, "Dynamic fracture analysis of multiple defects in an imperfect FGM coating-substrate layers," *International Journal of Mechanical Sciences*, vol. 75, pp. 55–65, 2013.
- [20] H. Egner, M. Juchno, M. Kula, and J. Skrzypek, "Numerical analysis of FG and TBC systems based on thermo-elasto-plastic- damage model," *Journal of Thermal Stresses*, vol. 30, no. 9-10, pp. 977–1001, 2007.
- [21] D. Gayen, D. Chakraborty, and R. Tiwari, "Finite element analysis for a functionally graded rotating shaft with multiple breathing cracks," *International Journal of Mechanical Sciences*, vol. 134, pp. 411–423, 2017.
- [22] T. Nakamura, T. Wang, and S. Sampath, "Determination of properties of graded materials by inverse analysis and instrumented indentation," *Acta Materialia*, vol. 48, no. 17, pp. 4293–4306, 2000.
- [23] G. Bolzon, G. Maier, and M. Panico, "Material model calibration by indentation, imprint mapping and inverse analysis," *International Journal of Solids and Structures*, vol. 41, no. 11-12, pp. 2957–2975, 2004.
- [24] G. R. Liu, X. Han, Y. G. Xu, and K. Y. Lam, "Material characterization of functionally graded material by means of elastic waves and a progressive-learning neural network," *Composites Science and Technology*, vol. 61, no. 10, pp. 1401–1411, 2001.
- [25] X. Han and G. R. Liu, "Computational inverse technique for material characterization of functionally graded materials," *AIAA Journal*, vol. 41, no. 2, pp. 288–295, 2003.
- [26] B. Rahmani, F. Mortazavi, I. Villemure, and M. Levesque, "A new approach to inverse identification of mechanical properties of composite materials: Regularized model updating," *Composite Structures*, vol. 105, pp. 1160–125, 2013.
- [27] K. Sun, Y. Zhao, and H. Hu, "Identification of temperature-dependent thermal—structural properties via finite element model updating and selection," *Mechanical Systems and Signal Processing*, vol. 52-53, pp. 147–161, 2015.
- [28] A. K. Mishra and S. Chakraborty, "Development of a finite element model updating technique for estimation of constituent level elastic parameters of FRP plates," *Applied Mathematics and Computation*, vol. 258, pp. 84–94, 2015.
- [29] A. K. Mishra and S. Chakraborty, "Inverse detection of constituent level elastic parameters of FRP composite panels with elastic boundaries using finite element model updating," *Ocean Engineering*, vol. 111, pp. 358–368, 2016.
- [30] J. N. Reddy and C. D. Chin, "Thermomechanical analysis of functionally graded cylinders and plates," *Journal of Thermal Stresses*, vol. 21, no. 6, pp. 593–626, 1998.
- [31] F. J. Ferrante and L. L. Graham-Brady, "Stochastic simulation of non-Gaussian/non-stationary properties in a functionally graded plate," *Computer Methods Applied Mechanics and Engineering*, vol. 194, no. 12-16, pp. 1675–1692, 2005.
- [32] N. Abu Husain, H. Haddad Khodaparast, and H. Ouyang, "Parameter selection and stochastic model updating using perturbation methods with parameter weighting matrix assignment," *Mechanical Systems and Signal Processing*, vol. 32, pp. 135–152, 2012.

- [33] P. G. Bakir, E. Reynders, and G. De Roeck, "Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm," *Journal of Sound and Vibration*, vol. 305, no. 1-2, pp. 211–225, 2007.
- [34] Y. Xu, Y. Qian, G. Song, and K. Guo, "Damage detection using finite element model updating with an improved optimization algorithm," *Steel and Composite Structures*, vol. 19, no. 1, pp. 191–208, 2015.

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