

## Research Article

# Bertrand Game with Nash Bargaining Fairness Concern

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The classical Bertrand game is assumed that players are perfectly rational. However, many empirical researches indicate that people have bounded rational behavior with fairness concern, which is important in the two-person game and has attracted much attention. In this paper, fairness concern is incorporated into the Bertrand game with two homogeneous products and the effect of fairness concern on this extended Bertrand game is explored. Nash bargaining solution of player is applied to be his own fairness reference point. Then, a Bertrand game model with fairness concern is established, and its equilibrium price is also derived and analyzed. It is shown from some numerical examples that fairness concern and bargaining power of players have a significant influence on their equilibrium price, expected profits, and utilities. As a player gets more fair-minded, if the other player has a less focus on fairness, the price competition between them will be intensified and both of them suffer loss. Thus, fairness concern may be advantageous or disadvantageous for players. In most situations, the fairness concern behavior is not beneficial for players. Additionally, the effect of bargaining power is relative to fairness concern. A player who manufactures a low-cost product should have a weak bargaining power if he terribly focuses on fairness and should have a strong bargaining power if he pays little attention to fairness. However, a player who manufactures a high-cost product should have a weak bargaining power if he is rarely concerned about fairness. Anyway, the same bargaining power is the best for two players.

## 1. Introduction

The Bertrand game is a model about price competition between players, which presents the interactions among players that make price decisions and their consumers that choose quantities at a set prices [1]. There are a lot of researches about the Bertrand game, such as equilibrium of game models [2, 3], strategic choice of timing [4, 5], asymmetric costs [6–8], and some applications to the economic problems [9]. Those above achievements are based on the assumption that players are fully rational and only desire to maximize their own profits. However, people often tend to show characteristic behavior of bounded rationality [10–12], such as risk aversion, loss aversion, and so on, in the real word [13–19].

In a market, only two firms sell homogeneous products at their own competitive price to fight for market share. This Bertrand game is just like a two-person bargaining through pricing. Obviously, price is a display signal for the profit distribution of these two firms, and thus the nature of the

Bertrand game is profit distribution. Once a firm reduces his price to obtain more market share, the other firm who has the homogenous products may also take price-off promotions, which may lead to a more intense price competition. Both firms will suffer loss. For example, a new supermarket always carries out a series of promotional activities. Meanwhile, in another supermarket who has been around for a long time, there are more favorable promotions for stopping the new supermarket from seizing the market. Another famous example, Galanz Group, has occupied 60% share of the Chinses market for microwave oven through a series of price wars from 1996 to now. During the process of price competition, some firms won and benefited from it, while others failed and lost a lot. This kind of event naturally occurs from time to time. In China, “Double-Eleven” Day is a shopping festival and has been the carnival day of merchants, consumers, and couriers. Most merchants offer a lot of marketing promotions and some of them earn a lot of money while others are in heavy debt.

Why does this phenomenon appear? There are some explanations for intense price competition. One of them is entry deterrence. Wang et al. [20] point that when a new company with a huge amount of products enters the market, the existing enterprise will reduce his price if the capacity of the entrant is unlimited. Then, price competition between the two sides becomes more intense, which leads to lower profits. However, with the growth of the two firms, the competition gets more serious than before. For example, China Mobile and China Unicom are the two powerful competitors who roughly enjoy 80 percent of the Chinese mobile communications market. They are always engaged in the Bertrand competition game, but China Mobile has a higher bargaining power than China Unicom. From 1999 to now, the price competition between China Mobile and China Unicom is increasingly severe. Each of them considers that there is a behavior of unfair competition of the other side, which is the reason why they keep on competing. Another common explanation is the breakdown of cooperation. Generally, the unfair distribution of profits is the main reason to cause the breakdown of cooperation. After the partnership broke down, the unfair competition incidents occur frequently, even if the law of the PRC against unfair competition was already published in 1993. So, fairness concern exists in price competition.

Fairness concern is that individuals not only care about their own income but also concern the fairness of their income distribution [21, 22]. Some scholars also prove the existence of fairness concern behavior in competition. For example, Iris and Santos-Pinto [23] explore the existence of fairness concern behavior in experimental Cournot game, which is a model about quality competition between players. They also find that fairness concern is the reason the experiment results are inconsistent with the Cournot-Nash production level. Thus, fairness concern may be a more plausible explanation for intense price competition than other known mechanisms, such as entry deterrence and the breakdown of cooperation. In this paper, we attempt to incorporate fairness concern into the Bertrand game to account for this phenomenon and investigate how the equilibrium price changes as players have fairness concern.

However, many scholars incorporate the difference between players' monetary payoff to model the fairness concern utility. For example, a fairness model (namely, F-S model) proposed by Fehr and Schmidt is widely applied in economy, management, and so on, where the fairness reference point of player is opponent's income [21]. However, this kind of fairness reference point is not completely applicable in practice. A player would not like to get the same benefit with his opponent, if his competitive power or contribution is comparatively larger, and vice versa. After that, Cui et al. [24] use  $u$  and  $r$  times of opponent's profit as player's fairness reference point on the basis of F-S model. Loch and Wu [25] propose a simple model to depict fairness concern as considering the influence of social preference. However, these models do not take the endogenous power and contribution into account. Thus, in order to consider their significant impact, Du et al. [26] put forward a fairness concern model with Nash bargaining reference, in which

Nash bargaining solution is employed as fairness reference point. Individual psychological perception is the focus of this model. Actually, Nash bargaining solution comes from a Nash bargaining process, which is just a psychological game for fairness perception. Moreover, the psychological game tends to happen in individuals' mind rather than reality. Additionally, it is more appropriate to adopt Nash bargaining solution as fairness reference point to depict the individuals' fairness perception. The reason is that it emphasizes the relative fairness through self-enforcingly integrating the power and contribution of players, rather than the absolute fairness expressed by a couple of parameters as the instinct or default properties of individuals. Thus, in this paper, the fairness model of Du et al. [26] is applied to formulate the fairness concern in the Bertrand game.

Why use the Nash bargaining solution as the fairness reference point to depict perceptively fair compromise in the Bertrand game with fairness concern? A set of axioms appealing in defining fairness characterize the Nash bargaining solution, which can be regarded as representing all anticipations that the two bargainers might agree upon as fair bargains [26]. Nash bargaining solution also can be seen as a natural extension of the proportional fairness criterion that is probably the most popular fairness notion. In long-term interactions, both sides will gradually come into consensus about the fairness reference of Nash bargaining solution, even though there is no real bargaining process. In fact, abundant evidence can illustrate it. Firstly, in 2018, China Mobile had 928 million mobile customers and the net profit reached 117.8 billion RMB, while China Unicom had 315 million mobile customers and the net profit reached 4.1 billion RMB. Secondly, in the ultimatum game experiment where a proposer and a responder bargain about the distribution of a surplus of fixed size, 60%-80% of the proposers provide an offer, which is in the range of 40%-50% of the surplus. No one offers above 50% of the surplus. There are almost no offers below 20% of the surplus. Similarly, in the dictator game experiment where the receiver has no choice and must accept the dictator's offer, about 80% of the players offer a positive amount while practically nobody offers more than 50% of an available monetary surplus. So, all these show that the individuals or firms are fair-minded and may consider competitive power and contribution when they compete for a surplus.

In this paper, the role of fairness concern on the Bertrand game is explored. In the Bertrand game with fairness concern, players not only concern about his own monetary payoff but also are willing to reduce the difference between his own and rival's monetary payoff by sacrificing his income. Because of its characteristics (Pareto efficiency, symmetric, invariant to affine transformations, and independence of irrelevant alternatives), Nash bargaining solution is always appropriate to be regarded as fairness concern point of players. Thus, we try to investigate how the equilibrium prices of players change with that fairness concern point in this paper.

The remainder of this paper is composed of the following four sections. In Section 2, literature review is provided. In Section 3, the basic concepts of Bertrand games fairness concern with Nash bargaining reference are introduced,

and then a Bertrand game model with fairness concern is established. In Section 4, the analysis of the equilibrium price for the Bertrand game with fairness concern is conducted. In Section 5, some numerical examples are used to explore the effect of fairness concern and bargaining power on the equilibrium price and expected profits and utilities in the extended Bertrand game. In Section 6, some conclusions and management insight are summarized.

## 2. Literature Review

Our research focuses on the Bertrand game with fairness concern, with the aim to understand how the players behave when they are fair-minded. Therefore, in this section, a review of the existing literatures related to our research is provided and our contributions are highlighted.

Fairness concern is very popular in the researches of game theory. A broad range of experimental results for many different games have also been explained by fairness concern, such as ultimatum games, gift exchange games, trust games, and bargaining games [23, 27–30]. For instance, a player has a preference for a much greater offer than one predicted by subgame perfect equilibrium and rejects a relative lower offer. According to the fairness concern, these offers are consistent with the equilibrium in a situation, where players are aware of the fact that the opponents may toss out the inequitable distribution. Nishimura et al. [31] define two kinds of fairness in a three-person ultimatum game, namely, universal fairness and in-coalition fairness, and study the evolution of fairness. Ho and Su [32] propose that individuals show peer-induced fairness concerns as they regard their peers to be a reference when evaluating their endowments. In order to prove its validity, two independent ultimatum games with a leader and two followers are studied. According to the experimental results, they find that 50 percent of subjects are fairness-minded. With peer-induced fairness, the second follower rejects receiving less than the first follower. Peer-induced fairness between followers is two times stronger than distributional fairness between leader and follower and can limit the degree of price discrimination. Moreover, Iris and Santos-Pinto [23] analyze the fairness concern in experimental Cournot oligopolies. They find that fairness concern will affect the equilibrium strategy and can explain the observed behavior. They also indicate that, under the influence of fairness concern, the outcomes of players may be more than or less than or equal to the Cournot-Nash outcomes. Therefore, according to the Iris and Santos-Pinto [23], it is meaningful to use fairness concern to explain intense price competition.

It is well known that the Bertrand game is a model about price competition between players. Recently, the researches about the Bertrand game with bounded rationality have already been investigated. Ahmed et al. [33] investigate a dynamic Bertrand game with differentiated duopoly where players with bounded rationality update their prices in each period by a gradient adjustment mechanism. Andaluz and Jarne [34] compare the complexity of Cournot and Bertrand duopolies with vertical product differentiation and bounded

rationality. Xin and Chen [35] investigate a master-slave Bertrand game model, which was proposed for upstream and downstream monopolies owned by different parties with bounded rationality. Tu and Wang [36] explore the complex dynamics of an R&D two-stage input competition triopoly game model with bounded rational expectations. Elsadany [37] analyzes the complex dynamics on a triopoly game with bounded rationality and radical form inverse demand function. Wambach [38] and Cheng [39] study a Bertrand game with cost uncertainty and risk averse firms and find that a risk-averse firm will charge a price higher than the competitive price. Anderson et al. [40] explore the relationship between subject-specific risk tolerance and tacit collusion in Bertrand duopoly experiments and find that less risk-averse subjects charge price higher than do their more risk-averse counterparts, but this relationship is only significant when actions are strategic substitutes. All of these researches focused on the Bertrand game with bounded rationality or risk aversion.

However, few researchers have investigated the case of Bertrand game with fair-minded players. In the Bertrand game, players prefer to reduce price to obtain more profits, while the opponents only get a small benefit or even a loss. This makes Bertrand game be in a prisoner's dilemma. If players care about fairness, the consequence of the Bertrand competition may be different. So, this paper takes the fairness concern behavior of players into account, adopts the Nash bargaining solution as the fairness reference point, then constructs a model of the Bertrand game with Nash bargaining fairness concern, and finally analyzes the effect of fairness concern on the equilibrium price, and the expected profits and utilities. The contribution of this paper is concluded as follows: (1) the equilibrium price for the Bertrand game with Nash bargaining fairness concern is presented; (2) the effects of fairness concern and bargaining power on the Bertrand game with Nash bargaining fairness concern are analyzed; (3) another important explanation for the price competition is explored, namely, fairness concern.

## 3. Basic Concepts of Bertrand Games and Fairness Concern

The basic concept of the classical Bertrand game is reviewed and fairness concern with Nash bargaining reference is modeled in this section. After that, we can obtain fairness reference points of fair-minded players in the duopoly Bertrand game. Then, a Bertrand game model with fairness concern is established.

*3.1. The Classical Bertrand Games.* Consider a Bertrand game where players make their price decisions. There are two players with two homogenous products. Product 1 is manufactured by player 1 and product 2 is manufactured by player 2. Both players choose their prices independently and come to an agreement, which is providing any oncoming customer demand at a set price.

For player  $i$  ( $i = 1, 2$ ) with product  $i$ ,  $Q_i$  represents the customer demand,  $p_i$  represents the selling price, and  $c_i$

represents the marginal production cost. Let  $j (j = 1, 2)$  denote the rival of player  $i$ . Then, the customer demand function of player  $i$  is  $Q_i = a_i - b_i p_i + \theta p_j, i, j = 1, 2$ , where  $a_i$  denotes the market potential of player  $i$ ,  $b_i$  denotes the price flexibility coefficient,  $\theta$  is the cross-price flexibility coefficients, and  $b_i > \theta$ . In order to guarantee that the market demand conforms to reality, we assume that  $a_i > b_i p_i$ , i.e.,  $p_i < a_i/b_i$ .

According to the above description, the expected profit of player  $i$  is given as follows:

$$\pi_i = (p_i - c_i) Q_i = (p_i - c_i) (a_i - b_i p_i + \theta p_j), \quad (1)$$

$$i, j = 1, 2.$$

And let  $\pi = \pi_1 + \pi_2$  denote the total expected profit of two players.

Notice that all information, including price, cost, and so on, is common knowledge for two players.

**3.2. Fairness Concern with Nash Bargaining Reference.** Since a fair-minded player not only cares about his own income but also concerns the fairness of income distribution, his utility is composed of the realized monetary payoff as well as the difference between the monetary payoff and the fairness reference point. Thus, according to the fairness concern model proposed by Du et al. [26], when player  $i$  cares about fairness, his utility function can be written as follows:

$$u_i(\pi_i) = [\rho_i - \phi_i(\bar{\rho}_i - \rho_i)] \pi, \quad i = 1, 2, \quad (2)$$

where  $\rho_i$  is the proportion of the expected profit  $\pi_i$  of player  $i$  to the total expected profit  $\pi$  of two players,  $\bar{\rho}_i$  represents player  $i$ 's relative reference point for fairness perception, and  $\phi_i > 0$  denotes player  $i$ 's fairness concern coefficient and is common knowledge for players. (Some methods can be used to achieve information sharing, such as paying the cost of revealing information and bargaining which involves side payments to reveal cost structure. Additionally, during the long-term competition, players can know the real degree of their fairness concern through price signal) It is well known that  $\rho_1 + \rho_2 = 1$ . Since the relative fairness reference point is from the psychological Nash bargaining for the fairness allocation of the total expected profit between two players, they must satisfy the Pareto efficiency axiom, which means that  $\bar{\rho}_1 + \bar{\rho}_2 = 1$ . This utility function contains two parts. The first part is the monetary payoff of player  $i$ . The second part denotes the fairness concern utility, which is the difference between the material payoff and the fairness reference point.  $\phi_i$  reflects the intrinsic characteristic of player  $i$ . And  $\phi_i$  does not hinge on player  $i$ 's relative power and contribution. The higher  $\phi_i$  is, the more important the fairness concern utility of player  $i$  becomes. Notice that if  $\phi_i$  is zero, the player  $i$ 's utility is identical to his monetary payoff.

**Lemma 1.** *The fairness references depend on the fairness-concerned degree of both sides. The more one (or the less his counterpart) concerns fairness, the higher his reference for fairness perception is, and vice versa. The relative fairness reference points for player 1 and player 2 are as follows:*

$$\bar{\rho}_1 = \frac{(1 + \phi_1) \alpha}{[1 + \phi_2 (1 - \alpha) + \phi_1 \alpha]}, \quad (3)$$

$$\bar{\rho}_2 = \frac{(1 + \phi_2) (1 - \alpha)}{[1 + \phi_2 (1 - \alpha) + \phi_1 \alpha]}, \quad (4)$$

where  $0 < \alpha < 1$  denotes the bargaining power of player 1 and  $1 - \alpha$  is the bargaining power of player 2.

*Proof.* See Appendix A.  $\square$

Note that bargaining power refers to the bargainer's capability to change the bargaining relationship [41], to win accommodations from the other [42], and to influence the outcomes of a negotiation [43]. If one's bargaining power is stronger than his rival, he can occupy the advantage and master the initiative, to achieve his bargaining goal. In the firm competition, bargaining power refers to the status of firm in his industry or the firm's ability to price assets. For example, in a community, a store will rise his price if his only opponent is temporarily facing closure, because he has a bigger bargaining power. But, this behavior is regarded as unfair. In fact, in Chinese market, Huawei Company who has a strong bargaining power does not charge a high price because of the existence of fairness concern. Moreover, studies on the legitimization of occupational income inequality indicate that individuals prefer outcomes related to individuals' status [44].

Lemma 1 implies that fairness references point is affected by fairness concern and bargaining power of two players. Then, the utility functions of player 1 and player 2 are obtained as follows, respectively,

$$u_1(\pi_1) = (1 + \phi_1) \pi_1 - \frac{\phi_1 (1 + \phi_1) \alpha \pi}{[1 + \phi_2 (1 - \alpha) + \phi_1 \alpha]}, \quad (5)$$

$$u_2(\pi_2) = (1 + \phi_2) \pi_2 - \frac{\phi_2 (1 + \phi_2) (1 - \alpha) \pi}{[1 + \phi_2 (1 - \alpha) + \phi_1 \alpha]}. \quad (6)$$

#### 4. Equilibrium Analysis of the Bertrand Game with Nash Bargaining Fairness Concern

In this section, we investigate equilibrium strategies of the Bertrand game with Nash bargaining fairness concern where both players are fair-minded. As players 1 and 2 are fair-minded, they not only concern about their own material interests but also pay attention to the fairness of payoff distribution. Substituting (1) into (5) and (6), the expected utility functions of player 1 and player 2 are

$$u_1(\pi_1) = \frac{(1 + \phi_1) \{ [1 + \phi_2 (1 - \alpha)] (p_1 - c_1) (a_1 - b_1 p_1 + \theta p_2) - \phi_1 \alpha (p_2 - c_2) (a_2 - b_2 p_2 + \theta p_1) \}}{[1 + \phi_2 (1 - \alpha) + \phi_1 \alpha]}, \quad (7)$$

$$u_2(\pi_2) = \frac{(1 + \phi_2) [(1 + \phi_1\alpha)(p_2 - c_2)(a_2 - b_2 p_2 + \theta p_1) - \phi_2(1 - \alpha)(p_1 - c_1)(a_1 - b_1 p_1 + \theta p_2)]}{[1 + \phi_2(1 - \alpha) + \phi_1\alpha]}, \quad (8)$$

respectively.

Owing to  $\partial^2 u_1(\pi_1)/\partial p_1^2 = -2(1 + \phi_1)[1 + \phi_2(1 - \alpha)]b_1/(1 + \phi_2(1 - \alpha) + \phi_1\alpha) < 0$  and  $\partial^2 u_2(\pi_2)/\partial p_2^2 = -2(1 + \phi_2)(1 + \phi_1\alpha)b_2/(1 + \phi_2(1 - \alpha) + \phi_1\alpha) < 0$ ,  $u_1(\pi_1)$  is strictly concave in

$p_1$  and  $u_2(\pi_2)$  is strictly concave in  $p_2$ . It means that there is a unique equilibrium price in the Bertrand game with Nash bargaining fairness concern.

Then, taking the first-order deviation of  $u_1(\pi_1)$  with respect to  $p_1$ , we have

$$\frac{\partial u_1(\pi_1)}{\partial p_1} = \frac{(1 + \phi_1)\{[1 + \phi_2(1 - \alpha)][a_1 - b_1 p_1 + \theta p_2 - b_1(p_1 - c_1)] - \theta\phi_1\alpha(p_2 - c_2)\}}{[1 + \phi_2(1 - \alpha) + \phi_1\alpha]}. \quad (9)$$

And, taking the first-order derivative of  $u_2(\pi_2)$  with respect to  $p_2$ , we have

$$\frac{\partial u_2(\pi_2)}{\partial p_2} = \frac{(1 + \phi_2)\{(1 + \phi_1\alpha)[a_2 - b_2 p_2 + \theta p_1 - b_2(p_2 - c_2)] - \theta\phi_2(1 - \alpha)(p_1 - c_1)\}}{[1 + \phi_2(1 - \alpha) + \phi_1\alpha]}. \quad (10)$$

Letting (9) and (10) equal to 0, we get

$$\begin{aligned} & [1 + \phi_2(1 - \alpha)][a_1 - 2b_1 p_1 + \theta p_2 + b_1 c_1] \\ & - \theta\phi_1\alpha(p_2 - c_2) = 0 \\ & (1 + \phi_1\alpha)[a_2 - 2b_2 p_2 + \theta p_1 + b_2 c_2] \\ & - \theta\phi_2(1 - \alpha)(p_1 - c_1) = 0 \end{aligned} \quad (11)$$

By solving above these equations, we can obtain the following conclusion.

**Proposition 2.** The equilibrium price  $(p_1^F, p_2^F)$  of the Bertrand game with Nash bargaining fairness concern is given as follows, respectively,

$$p_1^F = \frac{\theta[1 + \phi_2(1 - \alpha) - \phi_1\alpha][(1 + \phi_1\alpha)(a_2 + b_2 c_2) + \theta\phi_2(1 - \alpha)c_1] + 2(1 + \phi_1\alpha)b_2\{[1 + \phi_2(1 - \alpha)](a_1 + b_1 c_1) + \theta\phi_1\alpha c_2\}}{4(1 + \phi_1\alpha)[1 + \phi_2(1 - \alpha)]b_1 b_2 - [1 + \phi_2(1 - \alpha) - \phi_1\alpha][1 + \phi_1\alpha - \phi_2(1 - \alpha)]\theta^2}, \quad (12)$$

$$\begin{aligned} p_2^F &= \frac{2[1 + \phi_2(1 - \alpha)]b_1[(1 + \phi_1\alpha)(a_2 + b_2 c_2) + \theta\phi_2(1 - \alpha)c_1] + \theta[1 + \phi_1\alpha - \phi_2(1 - \alpha)][[1 + \phi_2(1 - \alpha)](a_1 + b_1 c_1) + \theta\phi_1\alpha c_2]}{4(1 + \phi_1\alpha)[1 + \phi_2(1 - \alpha)]b_1 b_2 - [1 + \phi_2(1 - \alpha) - \phi_1\alpha][1 + \phi_1\alpha - \phi_2(1 - \alpha)]\theta^2}. \end{aligned} \quad (13)$$

Proposition 2 indicates that the equilibrium prices of two players are influenced by fairness concern and bargaining power, which will further affect the profit allocation and competition of Bertrand game. If  $\phi_1 = \phi_2 = 0$ , then we have

$$p_1^F = \frac{\theta(a_2 + b_2 c_2) + 2b_2(a_1 + b_1 c_1)}{4b_1 b_2 - \theta^2} = p_1^*, \quad (14)$$

$$\text{and } p_2^F = \frac{2b_1(a_2 + b_2 c_2) + \theta(a_1 + b_1 c_1)}{4b_1 b_2 - \theta^2} = p_2^*,$$

which means that both players only concern about maximizing their own expected profits.

Let  $G = (1 + \phi_1\alpha)(a_2 + b_2 c_2) + \theta\phi_2(1 - \alpha)c_1$ ,  $H = [1 + \phi_2(1 - \alpha)][a_1 + b_1 c_1] + \theta\phi_1\alpha c_2$ ,  $M_1 = \theta[1 + \phi_2(1 - \alpha) - \phi_1\alpha]$ ,  $N_1 = 2(1 + \phi_1\alpha)b_2$ ,  $M_2 = \theta[1 + \phi_1\alpha - \phi_2(1 - \alpha)]$ ,  $N_2 = 2[1 + \phi_2(1 - \alpha)]b_1$ , then we have  $p_1^F = (M_1 G + N_1 H)/(N_1 N_2 - M_1 M_2)$ ,  $p_2^F = (N_2 G + M_2 H)/(N_1 N_2 - M_1 M_2)$ . By taking the first-order deviation of  $p_1^F$  and  $p_2^F$  with respect to  $\phi_1$ ,  $\phi_2$ , and  $\alpha$ , we can obtain the following conclusion.

**Proposition 3.** In the Bertrand game with Nash bargaining fairness concern, the equilibrium price has a relationship with the fairness concern coefficient:

- (i)  $\partial p_1^F / \partial \phi_1 < 0$ ;  
 if  $(N_1 N_2 - M_1 M_2)[\theta M_1 c_1 + \theta G + (a_1 + b_1 c_1) N_1] > [2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H)$ , then  $\partial p_1^F / \partial \phi_2 > 0$ ;  
 if  $(N_1 N_2 - M_1 M_2)[\theta M_1 c_1 + \theta G + (a_1 + b_1 c_1) N_1] < [2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H)$ , then  $\partial p_1^F / \partial \phi_2 < 0$ ;  
 and if  $(N_1 N_2 - M_1 M_2)[\theta M_1 c_1 + \theta G + (a_1 + b_1 c_1) N_1] = [2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H)$ , then  $\partial p_1^F / \partial \phi_2 = 0$ .
- (ii)  $\partial p_2^F / \partial \phi_2 < 0$ ;  
 if  $(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2) N_2 + \theta H + \theta M_2 c_2] > [2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)$ , then  $\partial p_2^F / \partial \phi_1 > 0$ ;  
 if  $(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2) N_2 + \theta H + \theta M_2 c_2] < [2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)$ , then  $\partial p_2^F / \partial \phi_1 < 0$ ;  
 and if  $(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2) N_2 + \theta H + \theta M_2 c_2] = [2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)$ , then  $\partial p_2^F / \partial \phi_1 = 0$ .

*Proof.* See Appendix B.  $\square$

Proposition 3 (i) shows that player 1's price  $p_1$  decreases with his own fairness concern coefficient  $\phi_1$ . It suggests that the more player 1 concerns fairness, the lower price he will charge. This is because when player 1 pays more attention to fairness, the fairness-concerned utility accounts more in his utility and player 2 has more difficulty in capturing more market shares than player 1 does. Then, player 1 will decline his price to attract more consumers. In addition, there is a threshold of player 1's fairness concern coefficient  $\phi_1^*$ , which must satisfy

$$\begin{aligned} & [N_2 N_1(\phi_1^*) - M_1(\phi_1^*) M_2(\phi_1^*)] \\ & \cdot [\theta c_1 M_1(\phi_1^*) + \theta G(\phi_1^*) + (a_1 + b_1 c_1) N_1(\phi_1^*)] \\ & = [2b_1 N_1(\phi_1^*) + \theta M_1(\phi_1^*) - \theta M_2(\phi_1^*)] \\ & \cdot [M_1(\phi_1^*) G(\phi_1^*) + N_1 H(\phi_1^*)]. \end{aligned} \quad (15)$$

If the fairness concern coefficient  $\phi_1$  of player 1 is less than this threshold, namely,  $0 < \phi_1 < \phi_1^*$ , the price of player 1 decreases with player 2's fairness concern coefficient  $\phi_2$ . Correspondingly, if the fairness concern coefficient  $\phi_1$  of player 1 is more than this threshold, namely,  $\phi_1 > \phi_1^*$ , the price  $p_1$  of player 1 increases with player 2's fairness concern coefficient  $\phi_2$ . Moreover, there is no relationship between the price  $p_1$  of player 1 and the fairness concern coefficient  $\phi_2$  of player 2 if  $\phi_1 = \phi_1^*$ . This happens because when player 2 gets more sensitive to fairness, player 2 will cut down his price. If the degree of fairness concern of player 1 is low, the fairness-concerned utility covers small proportion in player 1's utility so that player 1 reduces his price to boost sales and grab the market. If the degree of fairness concern of player 1 is high, the fairness-concerned utility covers big proportion in player 1's

utility so that player 1 rises his price to bring down his fairness reference point and increases his marginal utility.

Proposition 3 (ii) indicates that the more player 2 cares about fairness, the lower price  $p_2$  he will set. Meanwhile, there is a threshold of player 2's fairness concern coefficient  $\phi_2^*$ , which must satisfy

$$\begin{aligned} & [N_1 N_2(\phi_2^*) - M_1(\phi_2^*) M_2(\phi_2^*)] \\ & \cdot [\theta H(\phi_2^*) + (a_2 + b_2 c_2) N_2(\phi_2^*) + \theta M_2(\phi_2^*) c_2] \\ & = [2b_2 N_2(\phi_2^*) + \theta M_2(\phi_2^*) - \theta M_1(\phi_2^*)] \\ & \cdot [M_2(\phi_2^*) H(\phi_2^*) + N_2(\phi_2^*) G(\phi_2^*)]. \end{aligned} \quad (16)$$

The price  $p_2$  of player 2 decreases with the increase of player 1's fairness concern coefficient  $\phi_1$  when  $0 < \phi_2 < \phi_2^*$ , and increases with the increase of player 1's fairness concern coefficient  $\phi_1$  when  $\phi_2 > \phi_2^*$ , and have nothing to do with player 1's fairness concern  $\phi_1$  when  $\phi_2 = \phi_2^*$ . In the Bertrand game with complete information, players 1 and 2 only compete through pricing. The influence of player 2's fairness concern on his own price is similar to that of player 1. Thus, we do not explain why player 2 makes his pricing decision as he gets more sensitive to fairness concern.

According to Proposition 3 (i) with (ii), the fairness concern behaviors of two players have a significant influence on the prices of players. To be specific, the price of player  $i$  is decreasing in his own fairness concern, but may be increasing, decreasing, or invariable as the other player pays more attention to fairness. Therefore, the equilibrium price is biased by fairness concerns of two players.

**Proposition 4.** *In the Bertrand game with Nash bargaining fairness concern, the equilibrium price has the following relationship with bargaining power:*

- (i) if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) > 0$ , then  $\partial p_1^F / \partial \alpha > 0$ ;  
 if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) < 0$ , then  $\partial p_1^F / \partial \alpha < 0$ ;  
 and if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) = 0$ , then  $\partial p_1^F / \partial \alpha = 0$ .
- (ii) if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) > 0$ , then  $\partial p_2^F / \partial \alpha > 0$ ;  
 if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) < 0$ , then  $\partial p_2^F / \partial \alpha < 0$ ;  
 and if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) = 0$ , then  $\partial p_2^F / \partial \alpha = 0$ ,

where

$$\begin{aligned} E_1 &= \frac{\{[\phi_1(a_2 + b_2 c_2) - \theta \phi_2 c_1] M_1 - \theta(\phi_1 + \phi_2) G - [\phi_2(a_1 + b_1 c_1) - \theta \phi_1 c_2] N_1 + 2\phi_1 b_2 H\}}{(M_1 G + N_1 H)}, \\ E_2 &= \frac{\{[\phi_1(a_2 + b_2 c_2) - \theta \phi_2 c_1] N_2 - 2\phi_2 b_1 G + [\theta \phi_1 c_2 - \phi_2(a_1 + b_1 c_1)] M_2 + \theta(\phi_1 + \phi_2) H\}}{(N_2 G + M_2 H)}. \end{aligned} \quad (17)$$

*Proof.* See Appendix C  $\square$

Proposition 4 (i) shows that if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) = 0$ , then the bargaining power  $\alpha$  of player 1 has no influence on the price  $p_1$  of player 1. Moreover, if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) > 0$ , then player 1 will raise his price as his own bargaining power increases. Finally, if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) < 0$ , then player 1 will decline his price as his own bargaining power increases. Proposition 4 (ii) is similar to Proposition 4 (i). It indicates that if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) > 0$ , then the weaker bargaining power player 2 has, the higher price he will set; if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) < 0$ , then a stronger bargaining power of player 2 will lead to a lower price of player 2; and if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) = 0$ , then player 2's bargaining power does not affect his own price. According to Proposition 4 (i) and (ii), when the bargaining power of player 1 increases, the price of player  $i$  may increase or decrease or not change. For two players, the impact of bargaining power on the price is also related to the fairness concern coefficients. Because of the complexity of these equations, numerical studies are used to analyze it in detail in Section 5.1.

**Corollary 5.** *The equilibrium price of players in the Bertrand game with fairness concern is less than that in the Bertrand game without fairness concern, namely,  $p_1^F < p_1^*$  and  $p_2^F < p_2^*$ .*

*Proof.* See Appendix D.  $\square$

Corollary 5 states that, under the influence of fairness concern and bargaining power, players set a smaller price than the price in the Bertrand game without fairness concern. Therefore, fairness concern leads to a more intense price competition. This may be the reason why some companies in a price war continue to offer discount or sell at a low price.

According to Proposition 2, we substitute (12) and (13) into (1) and (2), then we can obtain that the equilibrium expected profits of players 1 and 2 are

$$\begin{aligned} \pi_1^F &= \left( \frac{M_1 G + N_1 H}{N_1 N_2 - M_1 M_2} - c_1 \right) \\ &\quad \cdot \left( a_1 + \frac{(\theta N_2 - b_1 M_1) G + (\theta M_2 - b_1 N_1) H}{N_1 N_2 - M_1 M_2} \right), \\ \pi_2^F &= \left( \frac{N_2 G + M_2 H}{N_1 N_2 - M_1 M_2} - c_2 \right) \\ &\quad \cdot \left( a_2 + \frac{(\theta M_1 - b_2 N_2) G + (\theta N_1 - b_2 M_2) H}{N_1 N_2 - M_1 M_2} \right), \end{aligned} \quad (18)$$

and the total expected profit of two players can be written as  $\pi^F = \pi_1^F + \pi_2^F$ . Meanwhile, the expected utilities of player 1 and player 2 are  $u_1(\pi_1^F) = (1 + \phi_1)\pi_1^F - \phi_1(1 + \phi_1)\alpha\pi_1^F/(1 + \phi_2(1 - \alpha) + \phi_1\alpha)$ , and  $u_2(\pi_2^F) = (1 + \phi_2)\pi_2^F - \phi_2(1 + \phi_2)(1 - \alpha)\pi_2^F/(1 + \phi_2(1 - \alpha) + \phi_1\alpha)$ , respectively, and the total expected utility of

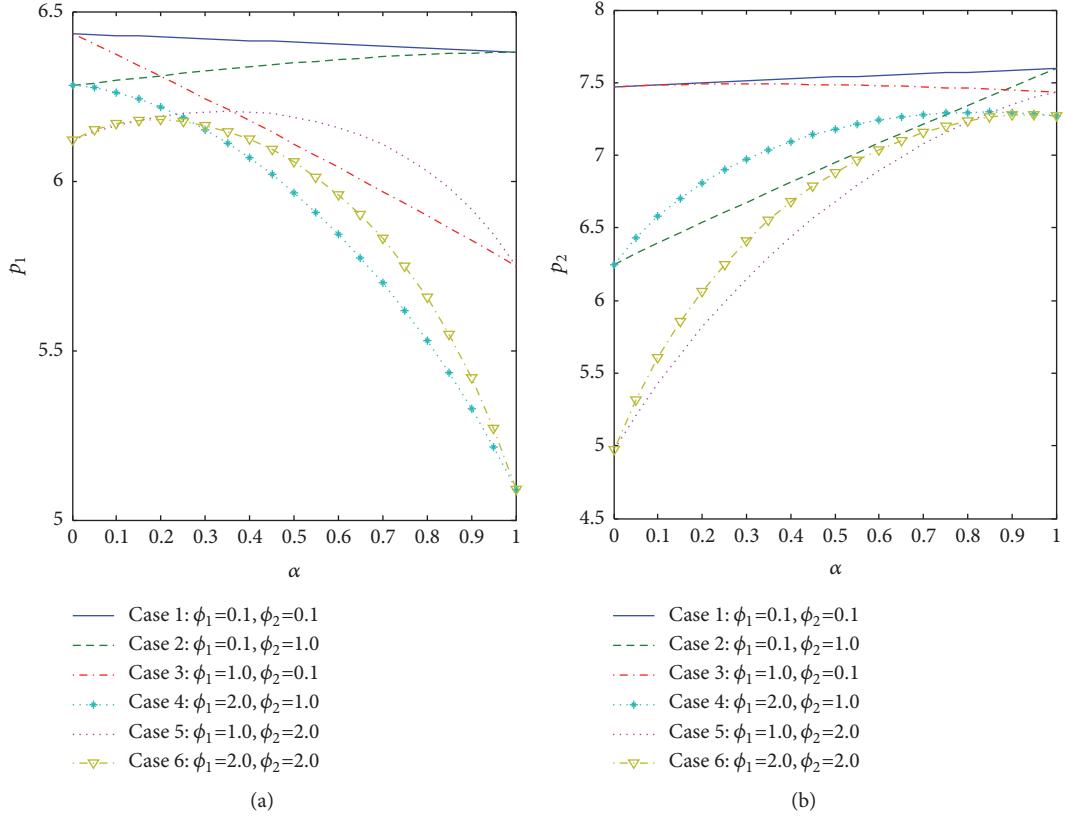
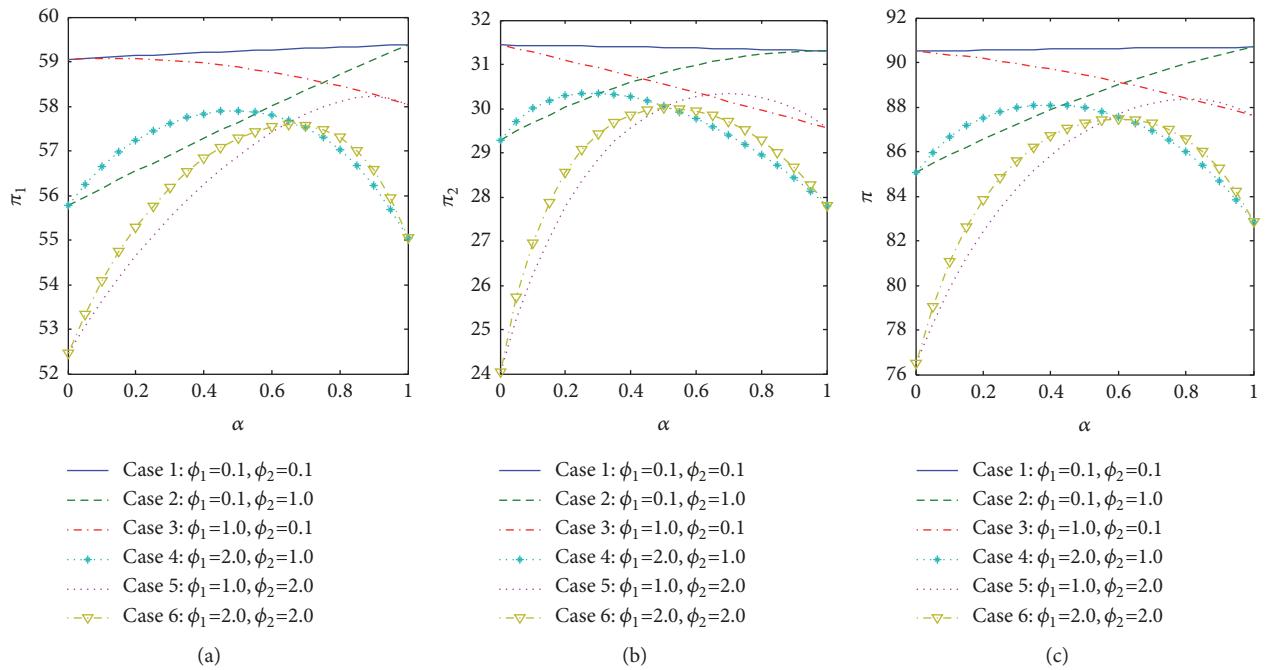
two players can be written as  $u(\pi^F) = u_1(\pi_1^F) + u_2(\pi_2^F)$ . Since the equations of players' expected profits and utilities are very complicated, numerical studies are used to analyze the impact of fairness concern and bargaining power on the equilibrium price, expected profits, and utilities of players 1 and 2.

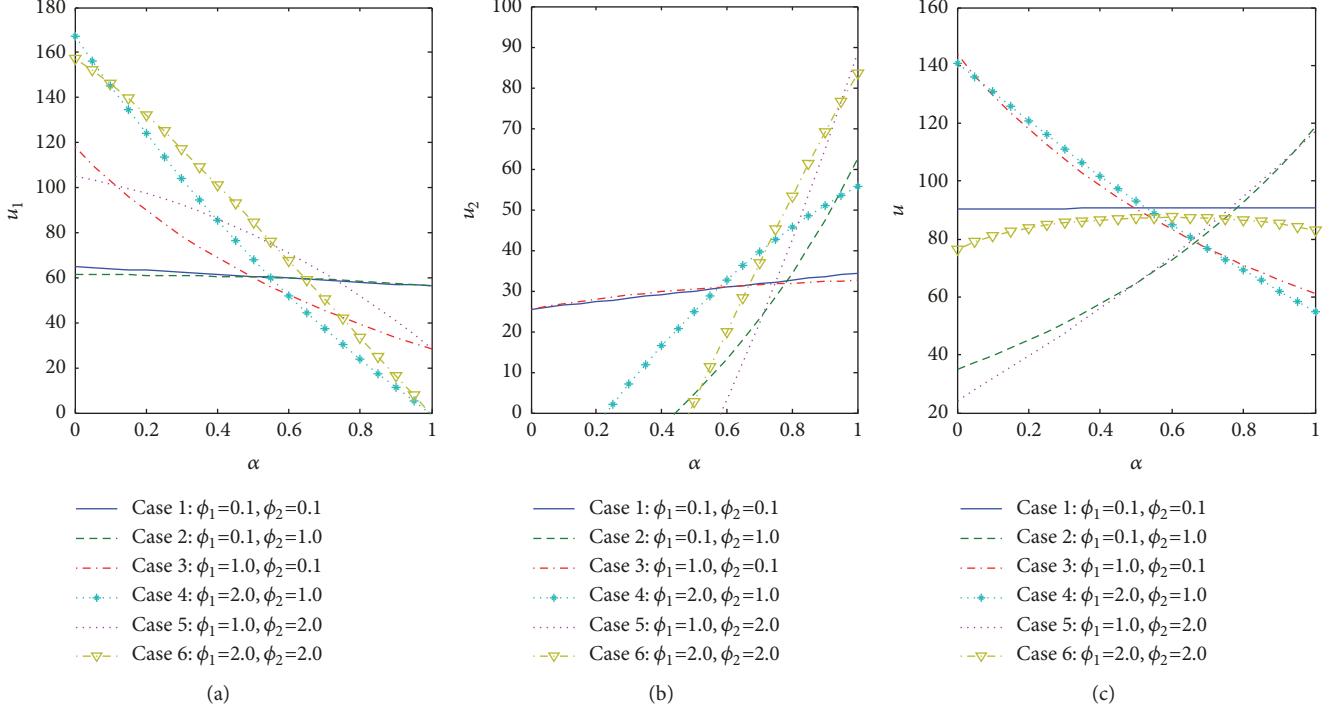
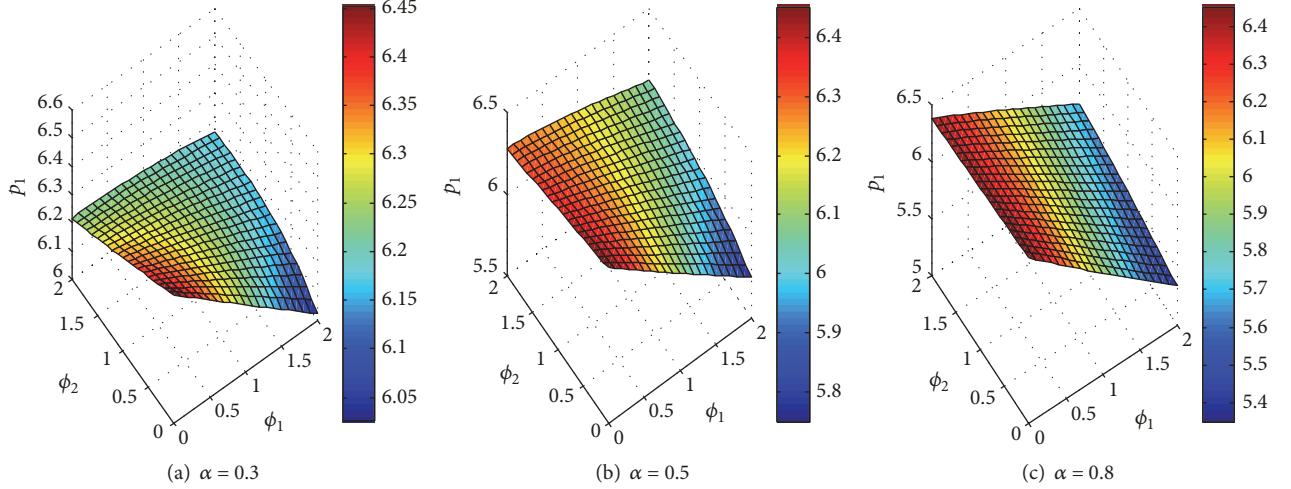
## 5. Numerical Analysis

To understand more about the influence of bargaining power and fairness concern on the equilibrium price, expected profits, and utilities of players in the Bertrand game with Nash bargaining fairness concern, some numerical examples by changing the parametric values of bargaining power and fairness concern coefficient are provided to analyze the theoretical results and further explain players' behavior regarding fairness concern in this subsection. The results of these numerical examples are summarized in Figures 1–11, where  $a_1 = 20$ ,  $a_2 = 10$ ,  $b_1 = 2$ ,  $b_2 = 1$ ,  $c_1 = 1$ ,  $c_2 = 2$ ,  $\theta = 0.5$ . Notice that player 1 manufactures a low-cost product while player 2 manufactures a high-cost product.

**5.1. The Impact of Bargaining Power on the Bertrand Game with Nash Bargaining Fairness Concern.** To focus on the effect of bargaining power on the equilibrium price, expected profits, and utilities, some numerical examples are conducted, where the bargaining power  $\alpha$  of player 1 varies from 0 to 1, and the fairness concern coefficients  $\phi_1$  and  $\phi_2$  are constant. In these numerical examples, six cases are used to demonstrate the change of the equilibrium price, expected profits, and utilities with respect to bargaining power under different degrees of fairness concern. Namely, Case 1 is that  $\phi_1=0.1$ ,  $\phi_2=0.1$ ; Case 2 is that  $\phi_1=0.1$ ,  $\phi_2=1.0$ ; Case 3 is that  $\phi_1=1.0$ ,  $\phi_2=0.1$ ; Case 4 is that  $\phi_1=2.0$ ,  $\phi_2=1.0$ ; Case 5 is that  $\phi_1=1.0$ ,  $\phi_2=2.0$ ; Case 6 is that  $\phi_1=2.0$ ,  $\phi_2=2.0$ . Then, some interesting insights can be found from Figures 1–3.

Figure 1 presents the changes of two players' prices with the bargaining power  $\alpha$  in the six cases. Figure 1 illustrates the results in Proposition 4 more clearly. See from Figure 1(a), in Cases 1, 3, and 4, where the fairness concern coefficient of player 1 is no less than that of player 2, the price of player 1 decreases as bargaining power  $\alpha$  gets stronger. In Case 2, where the fairness concern coefficient of player 1 is much less than that of player 2, the price of player 1 increases along with bargaining power  $\alpha$  increasing. In Cases 5 and 6, where the fairness concern coefficient of player 1 is a little less than that of player 2, the price of player 1 first rises and then falls with an increasing bargaining power  $\alpha$ . Obviously, the player with a stronger bargaining power desires a higher expected profit, which leads to a higher fairness reference point he would prefer, or vice versa [45, 46]. Interestingly, when player 1 is more sensitive to fairness than player 2, he wants to obtain more fairness-concerned utility. Thus, as player 1's bargaining power increases, player 1 can only reduce his price to expend his customer demand. When player 1 is much less sensitive to fairness than player 2, player 1 is more eager for obtaining the material profit than player 2. So, as the bargaining power of player 1 increases, player 1 will raise his price to increase marginal profit because of the increase of player 2's price.

FIGURE 1: The impact of bargaining power  $\alpha$  on the price.FIGURE 2: The impact of bargaining power  $\alpha$  on the expected profits.

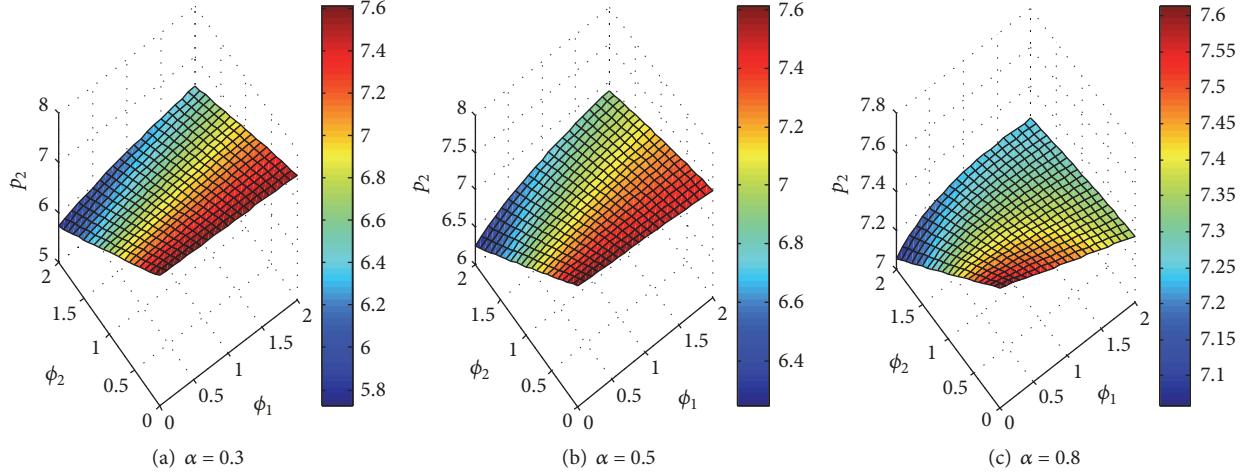
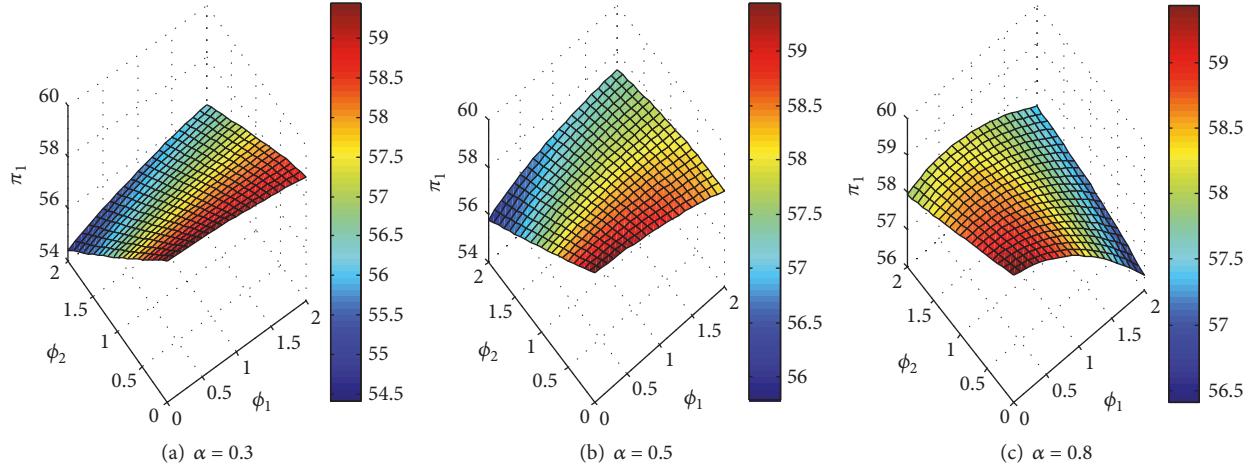
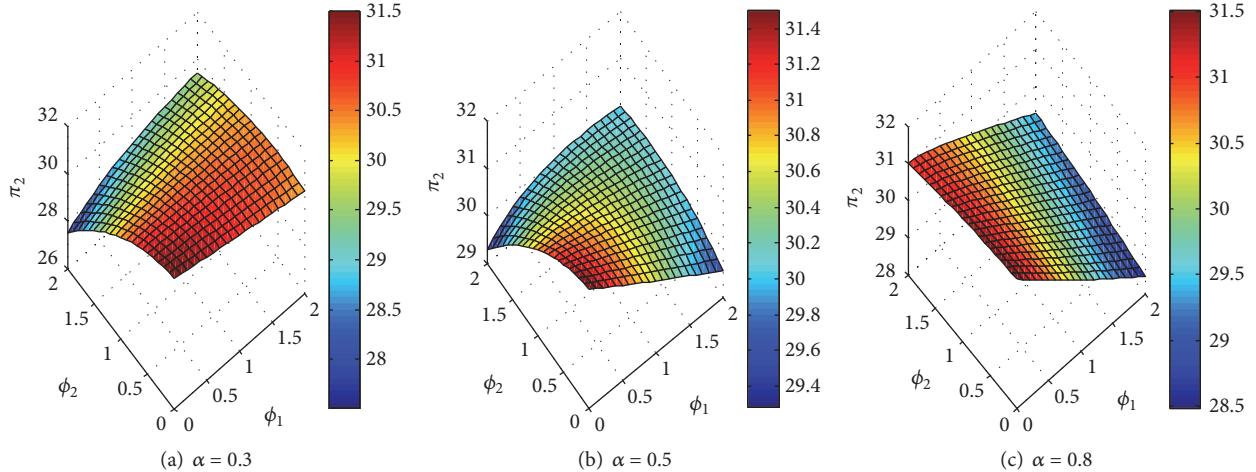
FIGURE 3: The impact of bargaining power  $\alpha$  on the expected utilities.FIGURE 4: The impact of fairness concern on player 1's price  $p_1$ .

When player 1 is little less sensitive to fairness than player 2, the prices of players 1 and 2 are low. Therefore, as the bargaining power of player 1 increases, player 1 will first improve his price to increase marginal profit and then cut it down to expand the customer demand because of the increase of player 2's price. So Proposition 4 (i) is proved.

See from Figure 1(b), in Case 3, where the fairness concern coefficient of player 1 is far more than that of player 2, the price of player 2 decreases when the bargaining power  $\alpha$  of player 1 increases. In Cases 1-2 and 4-6, where the fairness concern coefficient of player 1 is not much more than that of player 2, player 2's price is increasing as the bargaining power

$\alpha$  is increasing. So it verifies Proposition 4 (ii). When player 1 is far more sensitive to fairness than player 2, he desires to obtain the fairness-concerned utility than player 2; thus he should increase his expected profit. As the bargaining power of player 1 increases, player 2 can only decrease his price to expand his customer demand on account of the reduction of player 1's price. When player 1 is not far more sensitive to fairness than player 2, as player 1's bargaining power gets stronger, he should increases his expected profit by improving his price because player 1's price is declining.

Additionally, as bargaining power  $\alpha$  increases, the rate of change of player 1's price is smaller than that of player 2's price.

FIGURE 5: The impact of fairness concern on player 2's price  $p_2$ .FIGURE 6: The impact of fairness concern on player 1's expected profit  $\pi_1$ .FIGURE 7: The impact of fairness concern on player 2's expected profit  $\pi_2$ .

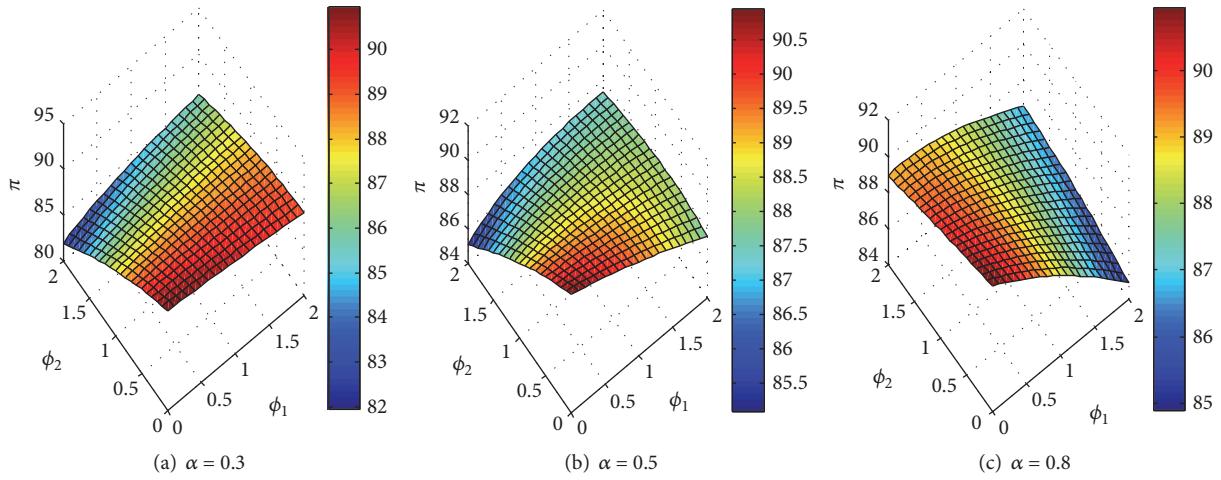


FIGURE 8: The impact of fairness concern on the total expected profit  $\pi$ .

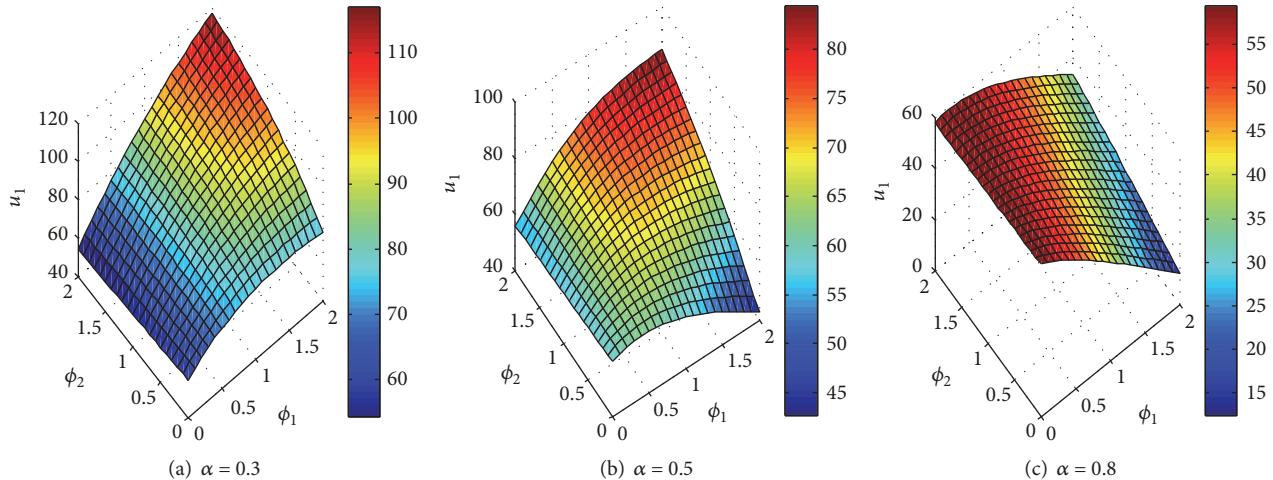


FIGURE 9: The impact of fairness concern on player 1's expected utility  $u_1$ .

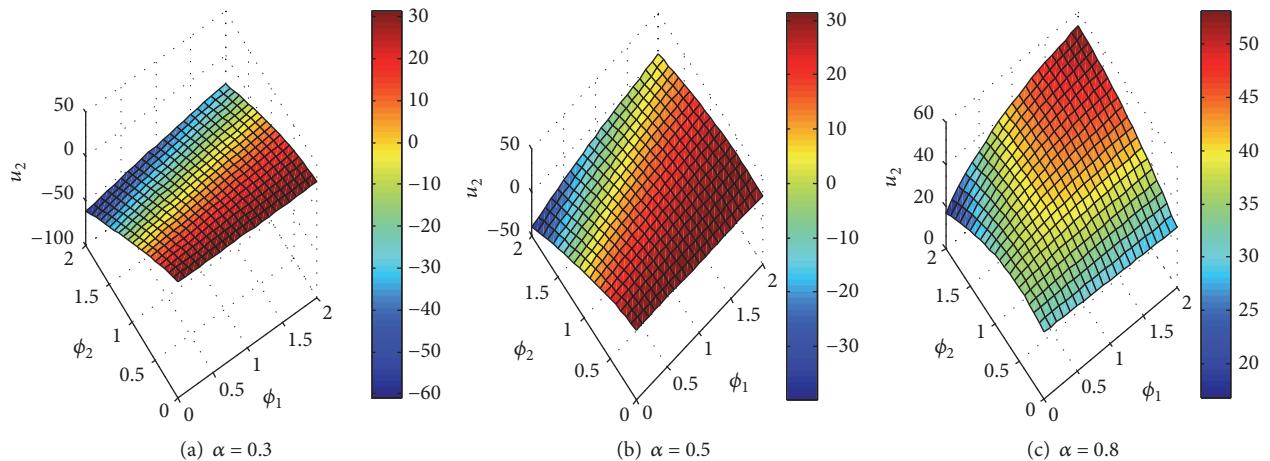


FIGURE 10: The impact of fairness concern on player 2's expected utility  $u_2$ .

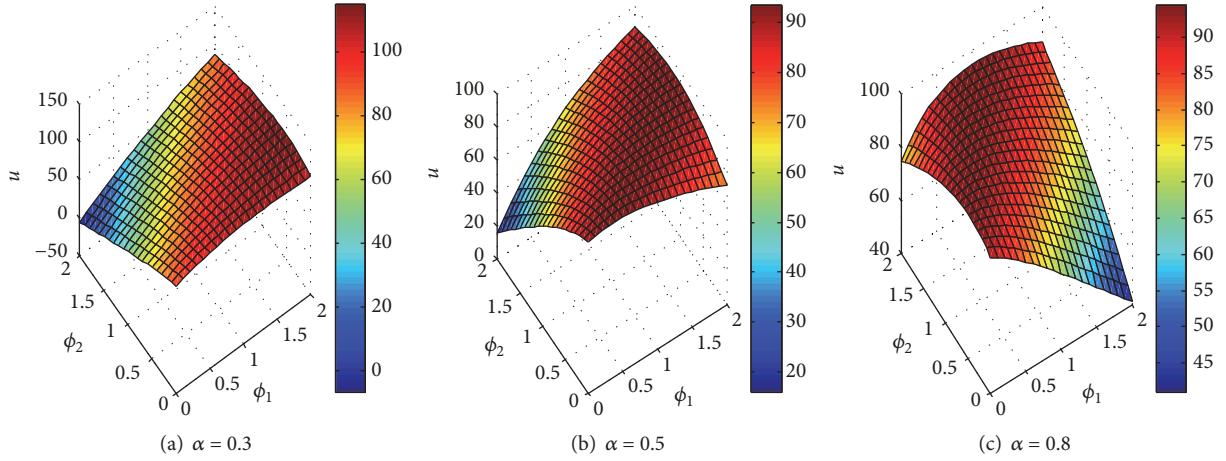


FIGURE 11: The impact of fairness concern on the total expected utility  $u$ .

It means that bargaining power's effect on the price of player 2 with high cost is more than that on the price of player 1 with low cost. And, if player 1 is much more fair-minded than player 2, the bargaining power may intensify the competition between two players. Moreover, both player 1's and 2's prices are the highest in Case 1. Therefore, the player who has a strong bargaining power may offer a product at a low price.

Figure 2 shows the changes of the expected profits with the bargaining power  $\alpha$  in the six cases. See from Figure 2(a) that the expected profit of player 1 in six cases has been influenced by the bargaining power  $\alpha$ . More specifically, in Cases 1 and 2, the expected profit of player 1 is increasing in the bargaining power  $\alpha$ . In Case 3, the expected profit of player 1 is declining in the bargaining power  $\alpha$ . In Cases 4-6, the expected profit of player 1 is first increasing and then decreasing with an increasing bargaining power  $\alpha$ . That is, in Case 1, when player 1 gets stronger bargaining power, the increase of customer demand is more than the reduction of marginal profit; thereby he will earn a higher expected profit. In Case 2, as player 1 has a stronger bargaining power, the increase of both customer demand and marginal profit leads to a bigger expected profit of his own. On the contrary, in Case 3, as player 1's bargaining power gets stronger, the reduction of both customer demand and marginal profit leads to a smaller expected profit of his own. In Cases 4-6, when player 1 gets stronger bargaining power, the increase of customer demand cannot make up the loss caused by the reduction of marginal profit all the time; thus the expected profit of player 1 increases at the beginning and then decreases. Additionally, among the six cases, player 1's expected profit in Case 1 is the highest.

Figure 2(b) presents the influence of bargaining power  $\alpha$  on the expected profit of player 2 in the six cases. See from Figure 2(b) that player 2's expected profit in six cases has been affected by the bargaining power  $\alpha$ . To be specific, in Cases 1 and 3, the expected profit of player 2 decreases with the bargaining power  $\alpha$ . In Case 2, the expected profit of player 2 increases with the bargaining power  $\alpha$ . However, in Cases 4-6, when the bargaining power  $\alpha$  is increasing, the expected profit of player 2 is increasing at first and then decreasing. In

Case 1, as player 1 gets stronger bargaining power, although the marginal profit is increasing, the reduction of customer demand is more than the augment of marginal profit; thus the expected profit of player 2 is declining. In Case 2, as player 1's bargaining power becomes stronger, the increase of both customer demand and marginal profit leads to a larger expected profit of player 2. On the contrary, in Case 3, as player 1 has a stronger bargaining power, the reduction of both customer demand and marginal profit leads to a smaller expected profit of player 2. In Cases 4-6, when player 1 gets stronger bargaining power, the increase of marginal profit cannot make up the loss that is caused by the reduction of customer demand all the time; thus the expected profit of player 1 increases at the beginning and then decreases. In addition, among the six cases, player 2's expected profit in Case 1 is the highest.

Figure 2(c) reveals the effect of bargaining power  $\alpha$  on the total expected profit of two players in the six cases. See from Figure 2(c) that, in Cases 1 and 2, the total expected profit of two players increases with the bargaining power  $\alpha$  increasing. In Case 3, the total expected profit of two players decreases with the bargaining power  $\alpha$  increasing. In Cases 4-6, when the bargaining power  $\alpha$  is increasing, the total expected profit of two players increases at first and then decreases. In other words, if both the degrees of two players' fairness concerns are low and the degree of fairness concern of player 1 is less than that of player 2, the total expected profit is increasing with the increase of player 1's bargaining power. Moreover, if both the degrees of two players' fairness concerns are low and the degree of fairness concern of player 1 is more than that of player 2, the total expected profit is decreasing with the increase of player 1's bargaining power. If both the degrees of two players' fairness concerns are low, the total expected profit first increases and then decreases when player 1 gets a stronger bargaining power. Besides, among the six cases, the total expected profit in Case 1 is the highest.

Figure 3 depicts the impact of the bargaining power  $\alpha$  on the expected utilities and the total expected utility of two players. In Figures 3(a) and 3(b), when the bargaining

TABLE 1: The impact of fairness concern on player 1's price  $p_1$  under three cases.

$p_1$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	6.4198	6.3242	6.2168	6.2039	6.1636	6.1524	6.0496	6.1939	6.1999
Case (b): $\alpha = 0.5$	6.4093	6.3482	6.2756	6.1890	6.0551	5.9659	5.7822	6.1091	6.1761
Case (c): $\alpha = 0.8$	6.3920	6.3735	6.3501	6.0301	5.6565	5.5294	5.3708	5.8972	5.9731

TABLE 2: The impact of fairness concern on player 2's price  $p_2$  under three cases.

$p_2$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	7.5129	6.6765	5.7815	6.1499	6.4114	6.9745	7.4572	7.4904	6.8558
Case (b): $\alpha = 0.5$	7.5379	6.9504	6.3128	6.6824	6.8819	7.1811	7.4157	7.4847	7.1127
Case (c): $\alpha = 0.8$	7.5730	7.3446	7.0922	7.2281	7.2350	7.3658	7.3843	7.4607	7.3551

power  $\alpha$  of player 1 increases, the expected utility of player 1 decreases, while the expected utility of player 2 increases. The reason is that player 1 has a lower expected profit and a higher fairness reference point while player 2 has a higher expected profit and a lower fairness reference point if the bargaining power of player 1 is stronger. Thus, an increase of bargaining power of player 1 leads to a smaller expected utility of player 1 and a bigger expected utility of player 2. Figure 3(c) suggests that the effect of the bargaining power  $\alpha$  on the total expected utility of two players is different according to the different fairness concern coefficient. Specifically, if the degree of player 1's fairness concern is less than that of player 2's fairness concern, the total expected utility of two players increases with the bargaining power  $\alpha$ . If the degree of player 1's fairness concern is more than that of player 2's fairness concern, the total expected utility of two players decreases with the bargaining power  $\alpha$ .

Comparing Figures 2 and 3, player 1 may get hurt by his own strong bargaining power while player 2 may benefit from his own weak bargaining power when both players have fairness concern behavior. However, for both players, a middle level of bargaining power is always beneficial. Thus, in a market, a firm who has a high-cost product should show his weakness to the firm who has a low-cost product. Moreover, a firm who has a low-cost product should not be too strong when he is more sensitive to fairness, while a firm who has a low-cost product should be strong when he is less sensitive to fairness.

**5.2. The Impact of Fairness Concern on the Bertrand Game with Nash Bargaining Fairness Concern.** We use some numerical examples to explore the impact of fairness concern on the equilibrium price, expected profits, and utilities, where player 1's fairness concern coefficient  $\phi_1$  and player 2's fairness concern coefficient  $\phi_2$  vary from 0 to 2, and the bargaining power  $\alpha$  is constant. In these numerical examples, six cases are used to illustrate the change of the equilibrium price, expected profits, and utilities with respect to fairness concern under different levels of the bargaining power  $\alpha$ . Namely, the first Case is labeled by (a) where  $\alpha = 0.3$ ; the second Case is labeled by (b) where  $\alpha = 0.5$ ; the third Case is labeled by (c)

where  $\alpha = 0.8$ . Then, some interesting insights can be found from Figures 4–11 and Tables 1–8.

In Figure 4 and Table 1, we consider the effect of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on player 1's price  $p_1$ . Figure 4 shows that the price of player 1 decreases as his own fairness concern coefficient  $\phi_1$  increases. Meanwhile, if player 1's fairness concern coefficient  $\phi_1$  is less than a threshold, the price of player 1 decreases as player 2's fairness concern coefficient  $\phi_2$  increases. On the contrary, if player 1's fairness concern coefficient  $\phi_1$  is more than a threshold, the price of player 1 increases as player 2's fairness concern coefficient  $\phi_2$  increases. These conclusions also can be seen from Table 1. Specifically, no matter how big  $\phi_2$  and  $\alpha$  are, the price of player 1 decreases in  $\phi_1$ . Moreover, if  $\phi_1 = 0.1$ , then the price of player 1 decreases in  $\phi_2$ ; if  $\phi_1 = 1.0$  or  $2.0$ , then the price of player 1 increases in  $\phi_2$ . Thus, it proves Proposition 3 (i). Moreover, in Figures 4(a), 4(b), and 4(c), the price of player 1 in the Bertrand game with fairness concern is less than that in the Bertrand game without fairness concern. So, Corollary 5 is demonstrated.

In Figure 5 and Table 2, the effect of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on player 2's price  $p_2$  is analyzed. As we can see from Figure 5, the price of player 2 decreases with his own fairness concern coefficient  $\phi_2$ . Meanwhile, if player 2's fairness concern coefficient  $\phi_2$  is less than a threshold, the price of player 2 decreases with player 1's fairness concern coefficient  $\phi_1$ . On the contrary, if player 2's fairness concern coefficient  $\phi_2$  is more than a threshold, the price of player 2 increases with player 1's fairness concern coefficient  $\phi_1$ . Table 2 also reveals these conclusions. More specifically, no matter how big  $\phi_1$  and  $\alpha$  are, the price of player 2 decreases in  $\phi_2$ . Moreover, if  $\phi_2 = 0.1$ , then the price of player 2 decreases in  $\phi_1$ ; if  $\phi_2 = 1.0$  or  $2.0$ , then the price of player 2 increases in  $\phi_1$ . Therefore, Proposition 3 (ii) is also verified. Additionally, Figures 5(a), 5(b), and 5(c) also show that the price of player 2 in the Bertrand game with fairness concern is less than that in the Bertrand game without fairness concern. So, Corollary 5 is furtherly proved.

Figure 6 and Table 3 present the impact of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on player 1's expected profit  $\pi_1$ . See from Figures 6(a) and 6(b) that, with an increasing

TABLE 3: The impact of fairness concern on player 1's expected profit  $\pi_1$  under three cases.

$\pi_1$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	59.1670	56.9148	54.5529	55.5108	56.1721	57.6165	58.7238	59.0326	57.3713
Case (b): $\alpha = 0.5$	59.2338	57.6471	55.9488	56.8881	57.2781	57.8962	58.0724	58.8781	57.9937
Case (c): $\alpha = 0.8$	59.3256	58.7071	58.0265	58.1169	57.2960	57.0197	56.4952	58.4528	58.3413

TABLE 4: The impact of fairness concern on player 2's expected profit  $\pi_2$  under three cases.

$\pi_2$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	31.4069	30.3299	27.7066	28.8504	29.4256	30.3528	30.3836	30.9194	30.4416
Case (b): $\alpha = 0.5$	31.3818	30.8097	29.4349	30.0240	30.1025	30.0599	29.6532	30.5490	30.5300
Case (c): $\alpha = 0.8$	31.3370	31.2241	30.9752	30.2547	29.2807	28.9620	28.5445	29.9678	30.1571

player 1's fairness concern coefficient  $\phi_1$ , the expected profit of player 1 is decreasing if the player 2's fairness concern coefficient  $\phi_2$  is less than a threshold, or vice versa. These also can be seen from Table 3. In Table 3, under Cases (a) and (b), if  $\phi_2 = 0.1$ , the expected profit of player 1 decreases with  $\phi_1$ ; if  $\phi_2 = 1.0$  or  $2.0$ , the expected profit of player 1 increases with  $\phi_1$ . As player 1 pays more attention to fairness, his marginal profit is decreased. For player 1, if player 2 with a strong bargaining power is less fair-minded, the declined customer demand and marginal profit lead to a reduction in his expected profit. However, if player 2 with a strong bargaining power is more fair-minded, though the marginal profit is dropping gradually, his expected profit is increasing because the growth rate of customer demand is higher than the droop rate of marginal profit. Meanwhile, as player 2's fairness concern coefficient  $\phi_2$  is increasing, the expected profit of player 1 is declining. Table 3 also shows that the expected profit of player 1 decreases with  $\phi_2$  in Cases (a) and (b). This happens because as player 2 with a strong bargaining power gets more fair-minded, the customer demand declines so sharply that player 1 obtains a lower expected profit.

See from Figure 6(c) that, as player 1's fairness concern coefficient  $\phi_1$  is increasing, the expected profit of player 1 is decreasing, which is also shown from Table 3. That is, when player 1 with a strong bargaining power pays more attention to fairness, the marginal profit goes down so sharply that player 1 only obtains a lower expected profit. However, with an increasing player 2's fairness concern coefficient  $\phi_2$ , if player 1's fairness concern coefficient  $\phi_1$  is less than a threshold then the expected profit of player 1 decreases, or vice versa. In Table 3, under Case (c), if  $\phi_1 = 0.1$  or  $1.0$ , the expected profit of player 1 decreases with  $\phi_2$ ; if  $\phi_2 = 2.0$ , the expected profit of player 1 increases with  $\phi_2$ . As player 2 pays more attention to fairness, the customer demand of player 1 decreases. If player 1 with a strong bargaining power is less fair-minded, the declined customer demand and marginal profit leads to a reduction in his expected profit. However, if player 1 with a strong bargaining power becomes more fair-minded, though the customer demand is dropping gradually, his expected profit is increasing because the growth rate of

marginal profit is higher than the droop rate of customer demand.

Additionally, for player 1, when his bargaining power is weak, his expected profit will be high if player 2's fairness concern degree is low; when his bargaining power is strong, his expected profit will be high if his own fairness concern degree is low. Thus, a player with low cost may get hurt or benefited by the fairness concern behavior. Moreover, when both players pay more attention to fairness, the expected profit of player 1 is declining. In most situation, fairness concern is not good for a player with low cost.

Figure 7 and Table 4 illustrate the influence of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on player 2's expected profit  $\pi_2$ . See from Figure 7(a) that, as player 2's fairness concern coefficient  $\phi_2$  increases, the expected profit of player 2 decreases. Moreover, if the player 2's fairness concern coefficient  $\phi_2$  is less than a threshold, then the expected profit of player 2 is increasing with the increase of player 1's fairness concern coefficient  $\phi_1$ . If the player 2's fairness concern coefficient  $\phi_2$  is more than a threshold, then the expected profit of player 2 is decreasing with the increase of player 1's fairness concern coefficient  $\phi_1$ . Table 4 also shows the same results. For example, under Case (a), the expected profit of player 2 decreases in  $\phi_2$ . But, if  $\phi_2 = 0.1$ , the expected profit of player 2 decreases in  $\phi_1$ ; if  $\phi_2 = 2.0$ , the expected profit of player 2 increases in  $\phi_1$ . As player 2 pays more attention to fairness, his marginal profit is decreased. Nevertheless, player 2 has a strong bargaining power. If player 2 cares more about fairness, then the reduction of his marginal profit is more than the augment of his customer demand, which leads to player 2's expected profit decline. Moreover, as player 1 pays more attention to fairness, the customer demand of player 2 decreases. If player 2 is less fair-minded, the declined customer demand and marginal profit lead to a reduction in his expected profit. However, if player 2 is more fair-minded, though the customer demand is dropping gradually, his expected profit is increasing because the growth rate of marginal profit is higher than the droop rate of marginal profit.

See from Figure 7(b) that, with an increasing fairness concern coefficient  $\phi_1$  of player 1, if player 2's fairness concern

TABLE 5: The impact of fairness concern on the total expected profit  $\pi$  under three cases.

$\pi$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	90.5740	87.2447	82.2595	84.3612	85.5977	87.9694	89.1074	89.9520	87.8129
Case (b): $\alpha = 0.5$	90.6156	88.4567	85.3837	86.9121	87.2806	87.9561	87.7256	89.4271	88.5437
Case (c): $\alpha = 0.8$	90.6625	89.9312	89.0017	88.3716	86.5768	85.9817	85.0397	88.4206	88.4984

TABLE 6: The impact of fairness concern on player 1's expected utility  $u_1$  under three cases.

$u_1$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	62.3665	60.9421	58.8910	92.2748	117.1577	104.0040	80.1275	78.6702	88.3987
Case (b): $\alpha = 0.5$	60.6264	60.2730	59.2529	79.0113	84.5537	68.1412	45.8383	60.0612	71.7155
Case (c): $\alpha = 0.8$	58.0051	58.3951	58.5372	51.9636	33.3653	23.6618	13.6877	39.1732	45.8839

coefficient  $\phi_2$  is less than a threshold, then the expected profit of player 2 decreases, while if player 2's fairness concern coefficient  $\phi_2$  is more than a threshold, then the expected profit of player 2 increases. As the fairness concern coefficient  $\phi_2$  of player 2 is increasing, if player 1's fairness concern coefficient  $\phi_1$  is less than a threshold, then the expected profit of player 2 decreases, while if player 2's fairness concern coefficient  $\phi_2$  is more than a threshold, then the expected profit of player 2 increases. As we can see from Table 4, in Case (b), if  $\phi_2 = 0.1$  or  $1.0$ , the expected profit of player 2 decreases with  $\phi_1$ ; if  $\phi_2 = 2.0$ , the expected profit of player 2 increases with  $\phi_1$ . And, if  $\phi_1 = 0.1$  or  $1.0$ , the expected profit of player 2 decreases with  $\phi_2$ ; if  $\phi_1 = 2.0$ , the expected profit of player 2 increases with  $\phi_2$ . This happens because, under the situation where player 1 and player 2 have a similar bargaining power, if player 2 is less fair-minded, the reduction of marginal profit and customer demand result in a lower expected profit of player 2 as player 1 concerns more about fairness; if player 2 is more fair-minded, the augment of marginal profit is more than the reduction of customer demand, so that player 2 can obtain a higher expected profit as player 1 concerns more about fairness. However, if player 1 is less fair-minded, the reduction of marginal profit and customer demand results in a lower expected profit of player 2 as player 2 concerns more about fairness; if player 1 is more fair-minded, the augment of customer demand is more than the reduction of marginal profit, so that player 2 can obtain a higher expected profit as he concerns more about fairness.

See from Figure 7(c) that, as player 1's fairness concern coefficient  $\phi_1$  increases, the expected profit of player 2 decreases. As the fairness concern coefficient  $\phi_2$  of player 2 is increasing, if player 1's fairness concern coefficient  $\phi_1$  is less (more) than a threshold, then the expected profit of player 2 decreases (increases). Likewise, in Table 4, under Case (c), the expected profit of player 2 decreases in  $\phi_2$ . However, if  $\phi_1 = 0.1$ , the expected profit of player 2 decreases with  $\phi_2$ ; if  $\phi_2 = 1.0$  or  $2.0$ , the expected profit of player 2 increases with  $\phi_1$ . In other words, under the situation where player 2 has a weak bargaining power, the customer demand cuts down so sharply that player 2 has to obtain a lower expected profit when player 1 gets more fair-minded. At the same time, as

player 2 pays more attention to fairness, if player 1 is less fair-minded, then the reduction of marginal profit and customer demand results in a lower expected profit of player 2; if player 1 is more fair-minded, then the augment of customer demand is more than the reduction of marginal profit, so that player 2 can obtain a higher expected profit.

Therefore, the effect of fairness concern on player 2's expected profit is different with the different bargaining power  $\alpha$ . For player 2, when his bargaining power is strong, his expected profit will be high if his own fairness concern coefficient is low; when his bargaining power is weak, his expected profit will be high if player 1's fairness concern coefficient is low. All these manifest that a higher degree of fairness concern hurts player 2.

Figure 8 and Table 5 present the impact of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on the total expected profit  $\pi$  of two players. See from Figures 8(a) and 8(b) that, as player 2's fairness concern coefficient  $\phi_2$  is increasing, the expected profit of two players is decreasing. Moreover, as player 1's fairness concern coefficient  $\phi_1$  is increasing, the expected profit of player 2 is increasing if player 2 is more fair-minded, but the expected profit of player 2 is decreasing if player 2 is less fair-minded. See from Figure 8(c) that the change of the total expected profit with respect to fairness concern is opposite to Figures 8(a) and 8(b). It also means that the effect of fairness concern on the total expected profit is relative to the different bargaining power. At the same time, as player 2 has a strong bargaining power, the total expected profit is high if his fairness concern coefficient is low; but, as player 1 has a strong bargaining power, the total expected profit is high if his fairness concern coefficient is low. These also can be seen from Table 5. Thus, one with strong bargaining power should be less fair-minded.

Figure 9 and Table 6 show the impact of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on player 1's expected utility  $u_1$ . See from Figure 9 and Table 6 that we find that the change of player 1's expected utility with respect to fairness concern is different when the bargaining power of players alters. As the degree of player 1's fairness concern increases, the expected utility of player 1 increases if his bargaining power is weak, but decreases if his bargaining power is strong. If players have

TABLE 7: The impact of fairness concern on player 2's expected utility  $u_2$  under three cases.

$u_2$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	28.2075	-9.9428	-59.0570	-44.6774	-31.5599	7.1591	29.3134	28.9556	-0.5859
Case (b): $\alpha = 0.5$	29.9892	4.5505	-36.6470	-14.2225	2.7269	24.9374	30.2649	30.4307	16.8282
Case (c): $\alpha = 0.8$	32.6574	34.3446	20.7619	42.5613	53.2115	45.6409	30.6849	31.8958	42.6145

TABLE 8: The impact of fairness concern on the total expected utility  $u$  under three cases.

$u$	$\phi_1=0.1,$ $\phi_2=0.1$	$\phi_1=0.1,$ $\phi_2=1.0$	$\phi_1=0.1,$ $\phi_2=2.0$	$\phi_1=1.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=2.0$	$\phi_1=2.0,$ $\phi_2=1.0$	$\phi_1=2.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=0.1$	$\phi_1=1.0,$ $\phi_2=1.0$
Case (a): $\alpha = 0.3$	90.5740	50.9993	-0.1660	47.5973	85.5977	111.1631	109.4409	107.6258	87.8129
Case (b): $\alpha = 0.5$	90.6156	64.8235	22.6059	64.7888	87.2806	93.0786	76.1032	90.4919	88.5437
Case (c): $\alpha = 0.8$	90.6625	92.7397	79.2991	94.5249	86.5768	69.3028	44.3726	71.0690	88.4984

the same bargaining power, with the increase of player 1's fairness concern coefficient, the expected utility of player 1 is decreasing as player 2 is less fair-minded, but increasing as player 2 is more fair-minded. However, when player 2 pays more attention to fairness, the expected utility of player 1 is decreasing if player 1 is less fair-minded, but increasing if player 1 is more fair-minded. Therefore, if player 1 has a weak bargaining power, the more two players care about fairness, the higher expected utility he will obtain; if player 1 has a strong bargaining power, the less he cares about fairness, the higher expected utility he will get.

Figure 10 and Table 7 present the impact of the fairness concern coefficients  $\phi_1$  and  $\phi_2$  on player 2's expected utility  $u_2$ . See from Figure 10 and Table 7 that the change of player 2's expected utility with respect to fairness concern is also different when the bargaining power of players alters. As the degree of player 1's fairness concern increases, the expected utility of player 2 is decreasing if player 2 is less fair-minded, but increasing if player 2 is more fair-minded. However, when player 2 pays more attention to fairness, the expected utility of player 2 increases if his bargaining power is weak. And, if player 2 has a strong bargaining power, with the increase of his own fairness concern coefficient, the expected utility of player 2 is decreasing as player 1 is less fair-minded, but increasing as player 1 is more fair-minded. Thus, if player 2 has a strong bargaining power, the less he concerns fairness, the higher expected utility he will get; if player 2 has a weak bargaining power, the more two players concerns fairness, the higher expected utility he will obtain.

The change of the total expected utility  $u$  with respect to the fairness concern coefficients  $\phi_1$  and  $\phi_2$  is shown in Figure 11 and Table 8. See from Figure 11 and Table 8 that if the bargaining power of player 1 is weak, the total expected utility is increasing with an increase of the degree of player 1's fairness concern, but decreasing with an increase of the degree of player 2's fairness concern. If the bargaining power of player 1 is strong, with an increasing fairness concern coefficient  $\phi_1$  of player 1, if player 2's fairness concern coefficient  $\phi_2$  is low, the total expected utility decreases, while if player 2's fairness concern coefficient  $\phi_2$  is high, the total expected utility increases. As the fairness concern coefficient

$\phi_2$  of player 2 is increasing, if player 1's fairness concern coefficient  $\phi_1$  is low, the total expected utility decreases, while if player 2's fairness concern coefficient  $\phi_2$  is high, the total expected utility increases. Therefore, under the influence of bargaining power, the total expected utility of two players changes differently with the different degree of fairness concern. If the bargaining power of player 1 is weak, the total expected utility will be high as player 2 is less fair-minded; if the bargaining power of player 1 is strong, the total expected utility will be high as player 1 is less fair-minded.

## 6. Conclusion

*6.1. Managerial Insights.* One possible field of this study is duopoly service market where two firms compete vigorously for consumers on the basis of price, such as mobile communication and fixed broadband, and so on. To some extent, price competition promotes the development of industry. However, unfair price competition is not good for the market, which inevitably results in a bad quality. This research is the first to use fairness concern as an explanation for this phenomenon, which is more plausible and reasonable. And, the model developed in this research helps fair-minded individuals or firms to make the price decisions in the service market. More specially, as a firm pays more attention to fairness, he prefers to make a lower price, while the opponent would like a lower (higher) price if the opponent's fairness concern degree is low (high). Furthermore, if the fairness concern degree of a firm with low cost is much more than that of the opponent with high cost, the firm provides the product at a lower price as his bargaining power increases. If the fairness concern degree of a firm with low cost is much less than that of the opponent with high cost, the firm will set a higher price as his bargaining power increases. If the fairness concern degree of the opponent with high cost is far less than that of the firm with low cost, he prefers a lower price as his bargaining power increases. While if the fairness concern degree of the opponent with high cost is not far less than that of the firm with low cost, he prefers a higher price as his bargaining power increases. Therefore, a firm with low cost and great focus on fairness should show that he has a weak bargaining

power. A firm with low cost and a little attention on fairness should show that he has a strong bargaining power. At the same time, a firm with high-cost product and a little bit of focus on fairness should show that he has a weak bargaining power. Anyway, the equal bargaining power is the best for two players. This research not only promotes further theoretical study of the Bertrand game with fairness concern, but also provides some management insights for helping firms make their price decisions. Thus, our research is of important theoretical and practical significance.

**6.2. Concluding Remarks.** In this paper, the fairness concern behavior of players in a Bertrand game is considered. Based on the existing researches, the Nash bargaining solution of player is regarded as his fairness reference point. Then, a Bertrand game model with Nash bargaining fairness concern is established. The equilibrium price of the extended Bertrand game is derived, and its characteristic is analyzed. Some numerical examples are conducted to explore how bargaining power and fairness concern influence the equilibrium price, expected profits, and utilities. The main results of this paper are summarized as follows.

(i) The role of fairness concern is of great importance in the Bertrand game. The more fair-minded a player is, the lower price he will charge. Besides, if a player is less fair-minded, the more the opponent is concerned with fairness, the lower price he will set. If a player is more fair-minded, the more the opponent cares about fairness, the higher price he will make. These indicate that as a player gets more fair-minded, if the other player has a less focus on fairness, the price competition between them will be intensified and both of them suffer loss. Thus, the expected profits and utilities of two players may go up or down as they pay more attention to fairness. When they simultaneously pay more attention to fairness, the equilibrium prices and expected profits of players are declining. Fairness concern may be advantageous or disadvantageous for players. In most situation, it is not beneficial for players.

(ii) Bargaining power also has a great effect on players' price decisions and their expected profits and utilities. For a player with a low cost, his price declines with his own bargaining power if his fairness concern degree is much more than that of the opponent, but increases with his own bargaining power if his fairness concern degree is much less than that of the opponent. And, if his fairness concern degree is little less than that of the opponent, his price increases first and then decreases with his own bargaining power. For a player with a high cost, if his fairness concern degree is far less than that of the opponent, his price decreases with his own bargaining power, otherwise his price increases with his own bargaining power. A player who manufactures a low-cost product should have a weak bargaining power if he terribly focuses on fairness and should have a strong bargaining power if he pays little attention to fairness. However, a player

who manufactures a high-cost product should have a weak bargaining power if he is rarely concerned about fairness. Anyway, the same bargaining power is the best for two players.

- (iii) The effect of fairness concern on the expected profit and utility is relative to bargaining power, and vice versa. Thus, one with strong bargaining power should be less fair-minded.

However, our analysis may have some limitations. Firstly, we assume that there are only two players in the Bertrand game. But, actually, more than two companies are in the price war. For example, in the Chinese express industry, YTO Express, SF Express, and China Post Group are the big three. During the students' graduation, they compete fiercely. Therefore, the two-person Bertrand game should be extended to the  $n$ -person Bertrand game. Secondly, some of the information may be asymmetric, such as fairness concern. But, the fairness concern characteristics of players in this paper are assumed to be common knowledge. The future research should be in a condition where the player has no ideal of the degree of the other player's Nash bargaining fairness concern degree. Finally, the customer demand may be an exponential function of the selling price because the customer demand following an exponential distribution is more conformable to the reality. Therefore, the Bertrand game with an exponential customer demand function should also be discussed.

## Appendix

### A. Proof of Lemma 1

*Proof.* Based on Nash's axiomatic definition [47–49], Nash bargaining solution  $(\bar{\rho}_1, \bar{\rho}_2)$  is the partition  $(\rho_1, \rho_2)$  that maximizes the Nash product  $\varphi = [\rho_1 - \phi_1(\bar{\rho}_1 - \rho_1)]^\alpha [\rho_2 - \phi_1(\bar{\rho}_2 - \rho_2)]^{1-\alpha}$  as follows.

$$\begin{aligned} & \max \quad \varphi \\ &= [\rho_1 - \phi_1(\bar{\rho}_1 - \rho_1)]^\alpha [\rho_2 - \phi_1(\bar{\rho}_2 - \rho_2)]^{1-\alpha} \\ & s.t. \quad \rho_1 + \rho_2 = 1 \\ & \quad \rho_1, \rho_2 > 0 \end{aligned} \tag{A.1}$$

Through mathematical conversion, the Nash product  $\varphi$  can be written product  $\varphi$  can be written as

$$\begin{aligned} \varphi &= [\rho_1 - \phi_1(\bar{\rho}_1 - \rho_1)]^\alpha [\rho_2 - \phi_2(\bar{\rho}_2 - \rho_2)]^{1-\alpha} \\ &\stackrel{\rho_1=1-\rho_2}{=} \stackrel{\bar{\rho}_2=1-\bar{\rho}_1}{=} [(1 + \phi_1)\rho_1 - \phi_1\bar{\rho}_1]^\alpha \\ &\quad \cdot [(1 + \phi_2)(1 - \rho_1) - \phi_2(1 - \bar{\rho}_1)]^{1-\alpha} \\ &\stackrel{\rho_1=1-\rho_2}{=} \stackrel{\bar{\rho}_1=1-\bar{\rho}_2}{=} [(1 + \phi_1)(1 - \rho_2) - \phi_1(1 - \bar{\rho}_2)]^\alpha \\ &\quad \cdot [(1 + \phi_2)\rho_2 - \phi_2\bar{\rho}_2]^{1-\alpha}. \end{aligned} \tag{A.2}$$

Then, we take the second-order derivative of (A.2) with respect to  $\rho_1$ , and we have  $d^2\varphi/d\rho_1^2 < 0$ . Hence,  $\varphi$  is strictly concave in  $\rho_1$ . There exists a unique equilibrium solution, i.e.,  $\bar{\rho}_1$ , which satisfies the first-order condition (A.3)

$$\begin{aligned} \frac{(1+\phi_1)\alpha}{[(1+\phi_1)\rho_1-\phi_1\bar{\rho}_1]} \\ - \frac{(1+\phi_2)(1-\alpha)}{[(1+\phi_2)(1-\rho_1)-\phi_2(1-\bar{\rho}_1)]} = 0. \end{aligned} \quad (\text{A.3})$$

Similarly, there also exists a unique equilibrium solution, i.e.,  $\bar{\rho}_2$ , which satisfies the first-order condition (A.4)

$$\begin{aligned} \frac{(1+\phi_2)(1-\alpha)}{[(1+\phi_2)\rho_2-\phi_2\bar{\rho}_2]} \\ - \frac{(1+\phi_1)\alpha}{[(1+\phi_1)(1-\rho_2)-\phi_1(\rho-\bar{\rho}_2)]} = 0. \end{aligned} \quad (\text{A.4})$$

Then, let  $\rho_1 = \bar{\rho}_1$  and  $\rho_2 = \bar{\rho}_2$ , and we have  $\bar{\rho}_1 = (1+\phi_1)\alpha/[1+\phi_2(1-\alpha)+\phi_1\alpha]$ ,  $\bar{\rho}_2 = (1+\phi_2)(1-\alpha)/[1+\phi_2(1-\alpha)+\phi_1\alpha]$ .

These complete the proof of Lemma 1.  $\square$

## B. Proof of Proposition 3

After taking the first derivative of (12) with respect to  $\phi_1$ , we have

$$\begin{aligned} \frac{\partial p_1^F}{\partial \phi_1} &= \frac{N_1 N_2 - M_1 M_2}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (M_1 G + N_1 H)}{\partial \phi_1} \\ &- \frac{M_1 G + N_1 H}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \phi_1}. \end{aligned} \quad (\text{B.1})$$

Since  $\partial(M_1 G + N_1 H)/\partial \phi_1 = \alpha(a_2 + b_2 c_2)M_1 - \theta \alpha G + 2b_2 \alpha H + \theta \alpha c_2 N_1$  and  $\partial(N_1 N_2 - M_1 M_2)/\partial \phi_1 = 2b_2 \alpha N_2 - \theta \alpha M_1 + \theta \alpha M_2$ , we can get  $\partial p_1^F/\partial \phi_1 = (\alpha(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2)M_1 - \theta G + 2b_2 H + \theta c_2 N_1] - \alpha[2b_2 N_2 + \theta(M_2 - M_1)](M_1 G + N_1 H))/(N_1 N_2 - M_1 M_2)^2$ .

Due to  $\alpha(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2)M_1 - \theta G] - \alpha[2b_2 N_2 + \theta(M_2 - M_1)]M_1 G = -\theta \phi_2(1-\alpha)\alpha M_1(N_1 N_2 - M_1 M_2)c_1/(1+\phi_1\alpha) - \alpha[2b_2 N_2 M_1 + \theta(N_1 N_2 - M_1 M_1) - M_1(N_1 N_2 - M_1 M_2)/(1+\phi_1\alpha)]G < 0$  and  $\alpha(N_1 N_2 - M_1 M_2)(2b_2 H + \theta c_2 N_1) - \alpha[2b_2 N_2 + \theta(M_2 - M_1)]N_1 H = \theta \alpha(N_1 N_2 - M_1 M_2)N_1 c_2 - \alpha[2b_2 M_1 M_2 + \theta N_1(M_2 - M_1)]H < 0$ , we have  $\partial p_1^F/\partial \phi_1 < 0$ .

Taking the first derivative of (12) with respect to  $\phi_2$ , we have

$$\begin{aligned} \frac{\partial p_1^F}{\partial \phi_2} &= \frac{N_1 N_2 - M_1 M_2}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (M_1 G + N_1 H)}{\partial \phi_2} \\ &- \frac{M_1 G + N_1 H}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \phi_2}. \end{aligned} \quad (\text{B.2})$$

Since  $\partial(M_1 G + N_1 H)/\partial \phi_2 = (1-\alpha)[\theta c_1 M_1 + \theta G + (a_1 + b_1 c_1)N_1]$ ,  $\partial(N_1 N_2 - M_1 M_2)/\partial \phi_2 = (1-\alpha)[2b_1 N_1 + \theta(M_1 - M_2)]$ , we have  $\partial p_1^F/\partial \phi_2 = ((1-\alpha)(N_1 N_2 - M_1 M_2)[\theta c_1 M_1 + \theta G + (a_1 + b_1 c_1)N_1] - (1-\alpha)[2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H))/(N_1 N_2 - M_1 M_2)^2$ .

Because of  $\theta(N_1 N_2 - M_1 M_2)[c_1 M_1 + G] - [2b_1 N_1 + \theta(M_1 - M_2)]M_1 G = (\theta N_1 N_2 - \theta M_1 M_1 - 2b_1 N_1 M_1)G + \theta c_1(N_1 N_2 - M_1 M_2)M_1 > 0$  and  $(N_1 N_2 - M_1 M_2)(a_1 + b_1 c_1)N_1 - [2b_1 N_1 + \theta(M_1 - M_2)]N_1 H = -N_1(N_1 N_2 - M_1 M_2)\theta \phi_1 \alpha c_2/(1+\phi_2(1-\alpha)) - (2b_1 M_1 M_2 + \theta N_2(M_1 - M_2))/N_2 N_1 H < 0$ , thus we can find that  $\partial p_1^F/\partial \phi_2 > 0$  as  $[2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H) < (N_1 N_2 - M_1 M_2)[\theta c_1 M_1 + \theta G + (a_1 + b_1 c_1)N_1]$ ,  $\partial p_1^F/\partial \phi_2 < 0$  as  $(N_1 N_2 - M_1 M_2)[\theta c_1 M_1 + \theta G + (a_1 + b_1 c_1)N_1] < [2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H)$ , and  $\partial p_1^F/\partial \phi_2 = 0$  as  $(N_1 N_2 - M_1 M_2)[\theta c_1 M_1 + \theta G + (a_1 + b_1 c_1)N_1] = [2b_1 N_1 + \theta(M_1 - M_2)](M_1 G + N_1 H)$ .

(ii) After taking the first derivative of (13) with respect to  $\phi_1$ , we have

$$\begin{aligned} \frac{\partial p_2^F}{\partial \phi_1} &= \frac{N_1 N_2 - M_1 M_2}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_2 G + M_2 H)}{\partial \phi_1} \\ &- \frac{N_2 G + M_2 H}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \phi_1}. \end{aligned} \quad (\text{B.3})$$

Since  $\partial(N_2 G + M_2 H)/\partial \phi_1 = \alpha[(a_2 + b_2 c_2)N_2 + \theta H + \theta c_2 M_2]$ , we have

$$\frac{\partial p_2^F}{\partial \phi_1} = \frac{\alpha(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2)N_2 + \theta H + \theta c_2 M_2] - \alpha[2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)}{(N_1 N_2 - M_1 M_2)^2}. \quad (\text{B.4})$$

Because of  $(N_1 N_2 - M_1 M_2)(a_2 + b_2 c_2)N_2 - [2b_2 N_2 + \theta(M_2 - M_1)]N_2 G = -\theta \phi_2(1-\alpha)N_2(N_1 N_2 - M_1 M_2)c_1/(1+\phi_1\alpha) - [\theta M_2 - (\theta - M_2)/(1+\phi_1\alpha)]M_1 N_2 G < 0$  and  $\theta(N_1 N_2 - M_1 M_2)(H + c_2 M_2) - [2b_2 N_2 + \theta(M_2 - M_1)]M_2 H = \theta M_2(N_1 N_2 - M_1 M_2)c_2 + [(\theta - M_2)/(1+\phi_1\alpha))N_1 N_2 - \theta M_2^2]H > 0$ , we can find that  $\partial p_2^F/\partial \phi_1 > 0$  as  $(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2)N_2 + \theta H + \theta c_2 M_2] >$

$[2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)$ ,  $\partial p_2^F/\partial \phi_1 < 0$  as  $(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2)N_2 + \theta H + \theta c_2 M_2] < [2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)$ , and  $\partial p_2^F/\partial \phi_1 = 0$  as  $(N_1 N_2 - M_1 M_2)[(a_2 + b_2 c_2)N_2 + \theta H + \theta c_2 M_2] = [2b_2 N_2 + \theta(M_2 - M_1)](N_2 G + M_2 H)$ .

Taking the first derivative of (13) with respect to  $\phi_2$ , we obtain

$$\begin{aligned} \frac{\partial p_2^F}{\partial \phi_2} &= \frac{N_1 N_2 - M_1 M_2}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_2 G + M_2 H)}{\partial \phi_2} \\ &\quad - \frac{N_2 G + M_2 H}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \phi_2}. \end{aligned} \quad (\text{B.5})$$

Since  $\partial(N_2 G + M_2 H)/\partial \phi_2 = (1 - \alpha)[\theta c_1 N_2 + 2b_1 G + (a_1 + b_1 c_1)M_2 - \theta H]$ , we have

$$\begin{aligned} \frac{\partial p_1^F}{\partial \phi_2} &= \frac{(1 - \alpha)(N_1 N_2 - M_1 M_2)[\theta c_1 N_2 + 2b_1 G + (a_1 + b_1 c_1)M_2 - \theta H] - (1 - \alpha)[2b_1 N_1 + \theta(M_1 - M_2)](N_2 G + M_2 H)}{(N_1 N_2 - M_1 M_2)^2}. \end{aligned} \quad (\text{B.6})$$

On account of  $(N_1 N_2 - M_1 M_2)(\theta c_1 N_2 + 2b_1 G) - [2b_1 N_1 + \theta(M_1 - M_2)]N_2 G = \theta N_2(N_1 N_2 - M_1 M_2)c_1 - 2b_1 M_1 M_2 G - \theta N_2(M_1 - M_2)G < 0$  and  $(N_1 N_2 - M_1 M_2)[(a_1 + b_1 c_1)M_2 - \theta H] - [2b_1 N_1 + \theta(M_1 - M_2)]M_2 H = (a_1 + b_1 c_1)M_2(N_1 N_2 - M_1 M_2) - [2b_1 N_1 M_2 + \theta(N_1 N_2 - M_1 M_2)]H < 0$ , then  $\partial p_2^F/\partial \phi_2 < 0$ .

These complete the proof of Proposition 3.

### C. Proof of Proposition 4

*Proof.* (i) After taking the first derivative of (12) with respect to  $\alpha$ , we obtain

$$\frac{\partial p_1^F}{\partial \alpha} = \frac{N_1 N_2 - M_1 M_2}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (M_1 G + N_1 H)}{\partial \alpha}$$

$$- \frac{M_1 G + N_1 H}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \alpha}. \quad (\text{C.1})$$

Since  $\partial(M_1 G + N_1 H)/\partial \alpha = [\phi_1(a_2 + b_2 c_2) - \theta \phi_2 c_1]M_1 - \theta(\phi_1 + \phi_2)G - [\phi_2(a_1 + b_1 c_1) - \theta \phi_1 c_2]N_1 + 2\phi_1 b_2 H$ ,

$$\begin{aligned} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \alpha} &= 2\phi_1 b_2 N_2 - 2\phi_2 b_1 N_1 \\ &\quad + \theta(\phi_1 + \phi_2)(M_1 - M_2), \end{aligned} \quad (\text{C.2})$$

then we can have

$$\begin{aligned} \frac{\partial p_1^F}{\partial \alpha} &= \frac{\{[\phi_1(a_2 + b_2 c_2) - \theta \phi_2 c_1]M_1 - \theta(\phi_1 + \phi_2)G - [\phi_2(a_1 + b_1 c_1) - \theta \phi_1 c_2]N_1 + 2\phi_1 b_2 H\}}{(N_1 N_2 - M_1 M_2)} \\ &\quad - \frac{[2\phi_1 b_2 N_2 - 2\phi_2 b_1 N_1 + \theta(\phi_1 + \phi_2)(M_1 - M_2)](M_1 G + N_1 H)}{(N_1 N_2 - M_1 M_2)^2}. \end{aligned} \quad (\text{C.3})$$

Due to  $(N_1 N_2 - M_1 M_2)\{[\phi_1(a_2 + b_2 c_2) - \theta \phi_2 c_1]M_1 - \theta(\phi_1 + \phi_2)G - [2\phi_1 b_2 N_2 - 2\phi_2 b_1 N_1 + \theta(\phi_1 + \phi_2)(M_1 - M_2)]M_1 G = -\theta(1 + \phi_1)\phi_2 M_1(N_1 N_2 - M_1 M_2)c_1/(1 + \phi_1 \alpha) - [2M_1(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)M_1(M_1 - M_2) + (\theta(\phi_1 + \phi_2) - \phi_1 M_1)/(1 + \phi_1 \alpha))(N_1 N_2 - M_1 M_2)\}G$  and  $(N_1 N_2 - M_1 M_2)\{-[\phi_2(a_1 + b_1 c_1) - \theta \phi_1 c_2]N_1 + 2\phi_1 b_2 H\} - [2\phi_1 b_2 N_2 - 2\phi_2 b_1 N_1 + \theta(\phi_1 + \phi_2)(M_1 - M_2)]N_1 H = \theta\phi_1(1 + \phi_2)c_2 N_1(N_1 N_2 - M_1 M_2)/(1 + \phi_2(1 - \alpha)) - [2N_1(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)N_1(M_1 - M_2) + (2\phi_1 b_2 - \phi_2 N_1)/(1 + \phi_2(1 - \alpha))](N_1 N_2 - M_1 M_2)H$ , it is impossible to directly determine whether the value of  $\partial p_1^F/\partial \alpha$  is positive or negative.

Therefore, if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) > 0$ , then  $\partial p_1^F/\partial \alpha > 0$ ; if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) <$

0, then  $\partial p_1^F/\partial \alpha < 0$ ; and if  $E_1 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)]/(N_1 N_2 - M_1 M_2) = 0$ , then  $\partial p_1^F/\partial \alpha = 0$ .

(ii) After taking the first derivative of (13) with respect to  $\alpha$ , we obtain

$$\begin{aligned} \frac{\partial p_2^F}{\partial \alpha} &= \frac{N_1 N_2 - M_1 M_2}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_2 G + M_2 H)}{\partial \alpha} \\ &\quad - \frac{N_2 G + M_2 H}{(N_1 N_2 - M_1 M_2)^2} \frac{\partial (N_1 N_2 - M_1 M_2)}{\partial \alpha}. \end{aligned} \quad (\text{C.4})$$

Since  $\partial(N_2 G + M_2 H)/\partial \alpha = [\phi_1(a_2 + b_2 c_2) - \theta \phi_2 c_1]N_2 - 2\phi_2 b_1 G + [\theta \phi_1 c_2 - \phi_2(a_1 + b_1 c_1)]M_2 + \theta(\phi_1 + \phi_2)H$ , we have

$$\begin{aligned} \frac{\partial p_2^F}{\partial \alpha} &= \frac{(N_1 N_2 - M_1 M_2) \{[\phi_1 (a_2 + b_2 c_2) - \theta \phi_2 c_1] N_2 - 2 \phi_2 b_1 G + [\theta \phi_1 c_2 - \phi_2 (a_1 + b_1 c_1)] M_2 + \theta (\phi_1 + \phi_2) H\}}{(N_1 N_2 - M_1 M_2)^2} \\ &\quad - \frac{[2 \phi_1 b_2 N_2 - 2 \phi_2 b_1 N_1 + \theta (\phi_1 + \phi_2) (M_1 - M_2)] (N_2 G + M_2 H)}{(N_1 N_2 - M_1 M_2)^2}. \end{aligned} \quad (\text{C.5})$$

Owing to  $(N_1 N_2 - M_1 M_2) \{[\phi_1 (a_2 + b_2 c_2) - \theta \phi_2 c_1] N_2 - 2 \phi_2 b_1 G\} = [2 \phi_1 b_2 N_2 - 2 \phi_2 b_1 N_1 + \theta (\phi_1 + \phi_2) (M_1 - M_2)] N_2 G = [(2 \phi_2 b_1 - \phi_1 N_2 / (1 + \phi_1 \alpha)) M_1 M_2 + \theta (\phi_1 + \phi_2) N_2 (M_2 - M_1)] G - \theta (1 + \phi_1 \alpha) \phi_2 N_2 (N_1 N_2 - M_1 M_2) c_1 / (1 + \phi_1 \alpha)$  and  $(N_1 N_2 - M_1 M_2) \{[\theta \phi_1 c_2 - \phi_2 (a_1 + b_1 c_1)] M_2 + \theta (\phi_1 + \phi_2) H\} = [2 \phi_1 b_2 N_2 - 2 \phi_2 b_1 N_1 + \theta (\phi_1 + \phi_2) (M_1 - M_2)] M_2 H = \theta \phi_1 (1 + \phi_2) M_2 (N_1 N_2 - M_1 M_2) c_2 / (1 + \phi_2 (1 - \alpha)) + [\theta (\phi_1 + \phi_2) N_1 N_2 - 2 \phi_1 b_2 N_2 M_2 + \phi_2 M_2 M_1 M_2 / (1 + \phi_2 (1 - \alpha)) - \theta (\phi_1 + \phi_2) M_2 (2 M_1 - M_2)] H$ , it is impossible to directly determine whether the value of  $\partial p_1^F / \partial \alpha$  is positive or negative.

Therefore, if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) > 0$ , then  $\partial p_2^F / \partial \alpha > 0$ ; if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) < 0$ , then  $\partial p_2^F / \partial \alpha < 0$ ; and if  $E_2 - [2(\phi_1 b_2 N_2 - \phi_2 b_1 N_1) + \theta(\phi_1 + \phi_2)(M_1 - M_2)] / (N_1 N_2 - M_1 M_2) = 0$ , then  $\partial p_2^F / \partial \alpha = 0$ .

These complete the proof of Proposition 4.  $\square$

## D. Proof of Corollary 5

According to Propositions 3 and 4, we divide (12) and (13) by  $(1 + \phi_1 \alpha)[1 + \phi_2(1 - \alpha)]$ , and we can obtain

$$\begin{aligned} p_1^F &= \frac{([1 + \phi_2 (1 - \alpha) - \phi_1 \alpha] / (1 + \phi_2 (1 - \alpha))) \theta (a_2 + b_2 c_2) + 2 b_2 (a_1 + b_1 c_1) + \phi_2 (1 - \alpha) [1 + \phi_2 (1 - \alpha) - \phi_1 \alpha] \theta^2 c_1 / (1 + \phi_1 \alpha) [1 + \phi_2 (1 - \alpha)] + 2 \phi_1 \alpha \theta b_2 c_2 / (1 + \phi_2 (1 - \alpha))}{4 b_1 b_2 - ([1 + \phi_1 \alpha - \phi_2 (1 - \alpha)] [1 + \phi_2 (1 - \alpha) - \phi_1 \alpha] / (1 + \phi_1 \alpha) [1 + \phi_2 (1 - \alpha)]) \theta^2} \\ &< \frac{\theta (a_2 + b_2 c_2) + 2 b_2 (a_1 + b_1 c_1)}{4 b_1 b_2 - \theta^2} = p_1^*, \\ p_2^F &= \frac{2 b_1 (a_2 + b_2 c_2) + ([1 + \phi_1 \alpha - \phi_2 (1 - \alpha)] / 1 + \phi_1 \alpha) \theta (a_1 + b_1 c_1) + 2 \phi_2 (1 - \alpha) \theta b_1 c_1 / (1 + \phi_1 \alpha) + (\phi_1 \alpha [1 + \phi_1 \alpha - \phi_2 (1 - \alpha)] / (1 + \phi_1 \alpha) [1 + \phi_2 (1 - \alpha)]) \theta^2 c_2}{4 b_1 b_2 - ([1 + \phi_1 \alpha - \phi_2 (1 - \alpha)] [1 + \phi_2 (1 - \alpha) - \phi_1 \alpha] / (1 + \phi_1 \alpha) [1 + \phi_2 (1 - \alpha)]) \theta^2} \\ &< \frac{2 b_1 (a_2 + b_2 c_2) + \theta (a_1 + b_1 c_1)}{4 b_1 b_2 - \theta^2} = p_2^*. \end{aligned} \quad (\text{D.1})$$

These complete the proof of Corollary 5.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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