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Research Article

Robust Stabilization of Extended Nonholonomic Chained-Form Systems with Dynamic Nonlinear Uncertain Terms by Using Active Disturbance Rejection Control

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In this paper, the stabilization problem of nonholonomic chained-form systems is addressed with uncertain constants. In this paper, the active disturbance rejection control (ADRC) is designed to solve this problem. The proposed control strategy combines extended state observer (ESO) and adaptive sliding mode controller. The control of nonholonomic chained-form systems with dynamic nonlinear uncertain terms and uncertain constants is first discussed in this paper. In comparison with existing methods, the proposed method in this paper has better performance. It is proved that, with the application of the proposed control strategy, semiglobal finite-time stabilization of the systems is achieved. An example is given to illustrate the effectiveness of the proposed method.

1. Introduction

The nonholonomic chained-form system was first proposed by Murray and Sastry in [1]. In recent years, more attention has been paid to the finite-time stabilization of the nonholonomic chained-form systems [2–6]. According to Brockett's necessary conditions [7], there is no smooth-time-invariant static state feedback control law that can stabilize a nonholonomic system. A number of approaches have been proposed to solve the stabilization problem including continuous time-varying feedback control laws, discontinuous time invariant control, and hybrid stabilization [8–14].

However, the complexity of understanding complex systems, the inevitable changes in system architecture, and the difficulty of predicting changes in the environment are three key points, leading to the dilemma that uncertainties always exist in the modeling of actual power systems [15]. Plenty of control methods have been developed [16–20] such as adaptive control [21, 22] and robust control [23]. In recent years, the active disturbance rejection control [2] technique has been widely recognized for its abilities to handle with uncertainties and its simplicity in the control structure.

Nevertheless, those control algorithms rely significantly on a priori known amplitude of interference. In addition, the finite-time control algorithm has the advantages of fast convergence in the aspect of control performance compared with other algorithms, such as continuous time-varying feedback control laws [24–26].

In recent years, more and more studies have been done on the stability of nonholonomic systems [27–34]. Yasir Awais Butt [3] proposed a robust switching controller based on discrete switching logic and ISM. This approach can guarantee the desired performance and robustness properties of the feedback control system. But this method does not take dynamic nonlinear uncertain terms into consideration. Qing Wang [2] designed the active disturbance rejection control (ADRC), which proves that it is an effective method to achieve finite-time stabilization of nonholonomic chained-form systems when the magnitude of the interference is unknown, but it can only be applied to relatively simple chained-form systems. Wang [4] constructed an adaptive output feedback controller by utilizing an adaptive control method and a parameter separation technique to stabilize the whole systems with unknown nonlinear parameters. To the best of the authors' knowledge, there is no research on stabilizing the dynamic feedback systems with bounded unknown uncertain positive parameters of nonholonomic robots about this issue.

In comparison with existing methods, our main contributions can be summarized as the following three respects:

(1) There are no a priori assumptions and it can deal with robust stability effectively in contrast with the existing methods. The reason why a disturbance of lower magnitude has an impact on the overall closed-loop system is that a priori can be estimated and the disturbance can be well compensated.

(2) The proposed controller can be applicable in the nonholonomic systems in chained-form with bounded unknown uncertain positive parameters.

(3) The proposed controllers can achieve the stabilization of extended nonholonomic chained-form systems with dynamic nonlinear uncertain terms. Compared with existing methods, the proposed controllers considered the dynamic nonlinear uncertain terms, resulting in the fact that the proposed controllers become more practical.

In this paper, a finite-time switching controller integrates ESO and adaptive sliding mode controller, which is set up to realize stabilization of a class of nonholonomic chained-form systems. Numerical simulation demonstrates the effectiveness of the proposed control method.

This paper's fundamental framework is as follows. Section 2 gives a formalization of the problem considered and introduces some preliminaries. In Section 3, we present the proposed switching controller and its stability analysis results. Section 2 gives a formalization of the problem considered and some preliminaries in this paper. In Section 3, we present the proposed switching controller and its stability analysis results. Section 4 states an illustrative example and the validity of the proposed methodology. Section 5 will summarize the full content. Section 6, as the last part of this paper, will introduce the future research direction.

2. Problem Statement and Preliminaries

Nonholonomic system in extended chained-form [35] can be described by

$$
\begin{aligned}
\dot{x}_1 &= k_1 u_1 \\
\dot{x}_2 &= k_2 x_3 u_1 \\
&\vdots \\
\dot{x}_{n-1} &= k_{n-1} x_n u_1 \\
\dot{x}_n &= k_n u_2 \\
\dot{u}_1 &= \zeta_1 (x,t)\, \psi_1 + f_1 (x, z_1, \omega_1) \\
\dot{u}_2 &= \zeta_2 (x,t)\, \psi_2 + f_2 (x, z_2, \omega_2) \\
\dot{z}_1 &= f_{01} (x, z_1, \omega_1) \\
\dot{z}_2 &= f_{02} (x, z_2, \omega_2) \\
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned}
\tag{1}
$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is a representation of the system state vector. $[u_1, u_2]^T \in \mathbb{R}^2$ can be treated equally as the velocity input for the kinematics model. $f_i(.)$, $f_{0i}(.)$, and $\zeta_i(.)$ are some unknown continuously dynamic nonlinear terms, $f_i(0, z_i, \omega_i) = 0$, $(i = 1, 2)$. $f_i(x, t) \in \mathbb{R}$ and $\zeta_i(x, t) \neq 0 (\in \mathbb{R})$ for all $(x, t) \in \mathbb{R}^n \times \mathbb{R} (i = 1, 2)$ are system dynamics and smooth nonlinear control directions, respectively. $z_i \in \mathbb{R}^m (i = 1, 2)$ represents nonlinear dynamic auxiliary variable, $y_i \in \mathbb{R}$, $(i = 1, 2)$ is the measured output, and $\omega_i \in \mathbb{R}^p, (i = 1, 2)$ are external time-varying uncertain disturbances, assuming that ω_i and its derivative $\dot{\omega}_i$ are continuous and bounded. We donate the practical control input $[\psi_1, \psi_2]^T \in \mathbb{R}^2$ as the formal inputs of force or torque for the extended dynamic model, and $k_i \in \mathbb{R}^+ (i = 1, 2, \ldots, n)$ are uncertain normal number parameter with bounded unknowns.

Remark 1. System 2 can describe the motion state of multiple (2,0) wheeled mobile robots. The pose of the robot in the inertial coordinate system can be represented by a vector $q = [x, y, \theta]^T$. u means the forward speed and steering velocity of the robot. $[\psi_1, \psi_2]^T \in \mathbb{R}^2$ can be donated as the formal inputs of force or torque for the extended dynamic model. As a result, we can control the pose of the robot by means of devising $[\psi_1, \psi_2]^T \in \mathbb{R}^2$. $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ represents the state vectors of i robots. In this case, x_1 and x_2 are targets, and their motion state can be measured. $x_3 \sim x_i$ $(i = 3, 4, \ldots)$ robots follow x_1 and x_2. In addition, there are dynamic nonlinear uncertainties in the process of motion. According to the constraints of the robot motion and the motion state, it is practicable to establish a model of the nonholonomic motion system. After proper coordinate transformation and input transformation, the model can be converted into a nonholonomic chain system of system 2.

System 2 can be rewritten as

$$
\begin{aligned}
\dot{x}_1 &= k_1 u_1 \\
\dot{u}_1 &= \zeta_1 (x,t)\, \psi_1 + f_1 (x, z_1, \omega_1) \\
\dot{z}_1 &= f_{01} (x, z_1, \omega_1) \\
y_1 &= x_1
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\dot{x}_2 &= k_2 x_3 u_1 \\
&\vdots \\
\dot{x}_{n-1} &= k_{n-1} x_n u_1 \\
\dot{x}_n &= k_n u_2 \\
\dot{u}_2 &= \zeta_2 (x,t)\, \psi_2 + f_2 (x, z_2, \omega_2) \\
\dot{z}_2 &= f_{02} (x, z_2, \omega_2) \\
y_2 &= x_2
\end{aligned}
\tag{3}
$$

To begin with, consider system (2)

$$\dot{u}_1 = \zeta_1(x,t)\psi_1 + f_1(x, z_1, \omega_1) \tag{4}$$

Lemma 2 (see [36, 37]). *Consider a first-order disturbed system:*

$$\dot{x} = u + f(x,t), \tag{5}$$

where $x, u \in R^1$ are state variable and control input, respectively, and $f(x,t)$ represents an external disturbance with a known bound ($0 < c < +\infty$), satisfying

$$\begin{gathered} |f(x,t)| \le c < +\infty, \\ f(0,t) = 0, \\ \forall t \ge 0. \end{gathered} \tag{6}$$

Taking a continuous sliding mode control law,

$$u = -c \cdot \begin{cases} \operatorname{sgn}(x), & |x| \ge \varpi(t) \\ \dfrac{x}{\varpi(t)}, & |x| < \varpi(t) \end{cases} \tag{7}$$

where $\varpi(t)$ denotes a continuous, time-variable boundary layer and satisfies that

$$\begin{gathered} \varpi(t) > 0, \\ \int_0^{+\infty} \varpi(t)\, dt < \sigma, \end{gathered} \tag{8}$$

where σ is a nonnegative constant. Then, system 2 can be asymptotically stabilized to the zero equilibrium point by (6).

Proof. For the proof, see Yang and Wang (2011).

Let $x = u_1 - 1$, then system (4) can be rewritten as

$$\begin{aligned} \dot{x} &= \dot{u}_1 \\ \dot{x} &= \zeta_1(x,t)\psi_1 + f_1(x, z_1, \omega_1) \end{aligned} \tag{9}$$

Our goal is to stabilize system dynamics (9) regardless of external disturbances and uncertainties. Just in this case, system (9) could be straightforwardly stabilized to the zero equilibrium point in finite time by using the first-order continuous sliding mode control law. As a result, u_1 could be stabilized to the constant 1. The control signal is designed as

$$\psi_1 = -\frac{c}{\zeta_1(x,t)} \cdot \begin{cases} \operatorname{sgn}(x), & |x| \ge \varpi(t) \\ \dfrac{x}{\varpi(t)}, & |x| < \varpi(t) \end{cases} \tag{10}$$

□

Assumption 3 (see [2]). The time derivative of the $f_1(x, z_1, \omega_1)$ is bounded

$$|f_1(x, z_1, \omega_1)| \le M \tag{11}$$

There exists some $M > 0$ such that

$$Mq + |\varepsilon(0)| < E. \tag{12}$$

$\varepsilon(0)$ is the initial value of the estimation error ε. Then the resulting closed-loop system is stabilized in finite time. Assumption 3 implies that the first control component ψ_{11} is bounded and the second control component ψ_{12} is certainly uniformly bounded. Thus, the composed signal ψ_1 is uniformly bounded and velocity input $u_1 \longrightarrow 1$ in finite time.

Let $d_i = x_{i+1}$, $(i = 1, 2, \ldots n-1)$, $d_n = u_2$, then system (3) can be rewritten as

$$\begin{aligned} \dot{d}_1 &= k_2 d_2 \\ &\vdots \\ \dot{d}_{n-2} &= k_{n-1} d_n \\ \dot{d}_{n-1} &= k_n d_n \\ \dot{d}_n &= \zeta_2(d,t)\psi_2 + f_2(d, z_2, \omega_2) \\ \dot{z}_2 &= f_{02}(d, z_2, \omega_2) \\ y_2 &= d_1 \end{aligned} \tag{13}$$

which are extended nonholonomic chained-form systems.

The following assumptions are made for the nonlinear system.

Assumption 4. There exists a unbounded positive definite function $L_0(z_2)$ such that, $\forall (d, z_2, \omega_2) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^b$,

$$\frac{\partial L_0}{\partial z_2}(z_2) f_{02}(d, z_2, \omega_2) \le 0, \quad \forall z_2 : \|z_2\| \ge \delta(d, \omega_2) \tag{14}$$

where $\delta(d, \omega_2)$ is a nonnegative continuous function.

Assumption 5. $\forall (d, z_2, \omega_2) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^b$, $\zeta_2(d,t) \ne 0$. The sign of $\zeta_2(d,t)$ is known.

Remark 6. Assumption 3 implies that $\dot{z}_2 = f_{02}(d, z_2, \omega_2)$. Assumption 5 ensures that the control signal always has an effect on the system (13). Since the sign of $\zeta_2(d,t)$ is fixed, we assume $\zeta_2(d,t) > 0$ and let $\zeta_0(d) > 0$ be a nominal model of $\zeta_2(d,t)$.

Remark 7. Set continuous and saturated control law $\dot{z}_2 = -k\operatorname{sgn}(z_2)|z_2|^{\alpha}$ where k and α are design parameters. For instance, $L_0(z_2) = z_2^2/2$ can satisfy Assumption 4.

Let $d_{n+1} = f_2(d, z_2, \omega_2) + (\zeta_2(d,t) - \zeta_0(d))\psi_2$ where the unknown system dynamics $f_2(d, z_2, \omega_2)$ and the parameter mismatch of control $(\zeta_2(d,t) - \zeta_0(d))u_2$ are viewed as an extended state of the system. Assume d_{n+1} is differentiable with $m = \dot{d}_{n+1}$, then system (13) can be rewritten as

$$\begin{aligned} \dot{d}_i &= k_{i+1} d_{i+1} \\ \dot{d}_n &= d_{n+1} + \zeta_0 (d) \psi_2 \\ \dot{d}_{n+1} &= m \\ y_2 &= d_1 \end{aligned} \tag{15}$$

The extended state observer was first proposed by Jingqing Han in [38]. The extended state observer (ESO) is designed as [2, 39, 40]

$$\begin{aligned} \dot{\hat{d}}_i &= k_{i+1}\hat{d}_{i+1} + \phi^{n-i} g_i\left(\frac{d_1 - \hat{d}_1}{\phi^n}\right) \\ & \qquad\qquad i = 1, 2, \ldots, n-1 \\ \dot{\hat{d}}_n &= \hat{d}_{n+1} + g_n\left(\frac{d_1 - \hat{d}_1}{\phi^n}\right) + \zeta_0\left(\hat{d}\right)\psi_2 \\ \dot{\hat{d}}_{n+1} &= \phi^{-1} g_{n+1}\left(\frac{d_1 - \hat{d}_1}{\phi^n}\right) \end{aligned} \tag{16}$$

which is a nonlinear generalization of LESO for gain ϕ and pertinent chosen functions $g_i(.)$, $i = 1, 2, \ldots, n+1$. $[\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_n, \hat{d}_{n+1}]^T \in \mathbb{R}^{n+1}$ is the nonlinear extended state observer state and depends on a small positive constant parameter ϕ.

Remark 8. In theory, the value of ϕ is chosen to be arbitrarily small to make the trajectory tracking error as small as possible. However, the existence of noise and sampling constraints in practice are responsible for the restrictions on the values of ϕ.

Now with the state estimates $[\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_n, \hat{d}_{n+1}]^T \in \mathbb{R}^{n+1}$, the active disturbance rejection control (ADRC) law, which is based on the output of the ESO (16), can be designed as

$$\psi_{2nom} = -\frac{1}{\zeta_2 (d, t)} \hat{d}_{n+1} + \psi_{20}\left(\hat{d}\right) \tag{17}$$

where $-(1/\zeta_2(x, t))\hat{d}_{n+1}$ is to compensate the total uncertainties and $\psi_{20}(\hat{d})$ is to guarantee the stability and performance requirements of the closed-loop system.

In order to protect the system from the peaking in the observer's transient response caused by the nonzero initial error $\|[d_1(t_0) - \hat{d}_1(t_0), \ldots, d_n(t_0) - \hat{d}_n(t_0)]\|$, we design the system that uses a special controller as [39]. The control is modified as

$$\psi_2 = W sat_\phi \left(\frac{\psi_{2nom}}{W}\right) \tag{18}$$

where the function $sat_\phi(.)$ is shown by [41]

$$sat_\phi (e) \begin{cases} e & \text{for } 0 \le e \le 1 \\ e + \dfrac{e-1}{\phi} - \dfrac{e^2 - 1}{2\phi} & \text{for } 1 < e \le 1 + \phi \\ 1 + \dfrac{\phi}{2} & \text{for } e > 1 + \phi \end{cases} \tag{19}$$

The function $sat_\phi(.)$ is nondecreasing, continuously differentiable. The saturation bound W ensures that the saturation is not invoked in the steady state of the ESO (16).

Set the scaled ESO estimation error as

$$\begin{aligned} \Delta_i (t) &= d_i - \hat{d}_i, \quad i = 1, 2, \ldots, n \\ \Delta_{n+1} (t) &= f_2 (d, z_2, \omega_2) \\ &\quad + (\zeta_2 (d, t) - \zeta_0 (d)) W sat_\phi \left(\frac{\psi_{2nom}}{W}\right) \\ &\quad - \hat{d}_{n+1}. \end{aligned} \tag{20}$$

For the purpose of getting a compact form of the closed-loop equation for the state estimation error, we design these scaled variables

$$\eta_i = \frac{d_i - \hat{d}_i}{\phi^{n+1-i}} = \frac{\Delta_i (t)}{\phi^{n+1-i}}, \quad i = 1, 2, \ldots, n+1. \tag{21}$$

Then substituting (15) and (16) into (22), the estimation error state dynamics can be written as

$$\begin{aligned} \phi\dot{\eta}_i &= k_{i+1}\eta_{i+1} - g_i (\eta_1), \quad i = 1, 2, \ldots, n-1. \\ \phi\dot{\eta}_n &= \eta_{n+1} - g_n (\eta_1) \\ \phi\dot{\eta}_{n+1} &= \phi m - g_{n+1} (\eta_1) \end{aligned} \tag{22}$$

Assumption 9 (see [2, 39, 40]). $\forall \eta = [\eta_1, \eta_2, \ldots, \eta_{n+1}]^T \in \mathbb{R}^{n+1}$, there exist constants $\theta_i, (i = 1, 2, 3, 4), \gamma$ and positive definite, continuous differentiable functions $L_1, Q_1 : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}$ such that

$$\begin{aligned} (i) \quad & \theta_1 \|\eta\|^2 \le L_1 (\eta) \le \theta_2 \|\eta\|^2, \\ & \theta_3 \|\eta\|^2 \le Q_1 (\eta) \le \theta_4 \|\eta\|^2 \\ (ii) \quad & \sum_{i=1}^{n-1} (k_{i+1}\eta_{i+1} - g_i (\eta_1)) \frac{\partial L_1}{\partial \eta_i} (\eta) \\ & \quad + (\eta_{n+1} - g_n (\eta_1)) \frac{\partial L_1}{\partial \eta_n} (\eta) \\ & \quad - g_{n+1} (\eta_1) \frac{\partial L_1}{\partial \eta_{n+1}} (\eta) \le -Q_1 (\eta) \\ (iii) \quad & \left| \frac{\partial L_1}{\partial \eta_{n+1}} (\eta) \right| \le \gamma \|\eta\| \end{aligned} \tag{23}$$

where $\eta = (\eta_1, \eta_2, \ldots, \eta_{n+1})$. $\|.\|$ denotes the Euclid norm of $\mathbb{R}^{n+1}$.

Assumption 10 (see [2, 41]). The functions $|g_i(\eta_1)| \leq \varphi_i \eta_i$ for some positive constants φ_i for all $i = 1, 2, \ldots, n+1$. For any (d, z_2, ω_2) belonging to the domain of interest and $\forall \hat{d} \in \mathbb{R}^n$, the following inequality holds:

$$\rho_a \triangleq \left| \frac{\zeta(d,t) - \zeta_0(d)}{\zeta_0(\hat{d})} \right| < \frac{\theta_3}{\varphi_{n+1}\gamma} \tag{24}$$

Remark 11. Assumption 10 implies that the nominal model $\zeta_0(.)$ is close to $\zeta(.)$. The functions $g_i(.)$, $i = 1, 2, \ldots, n+1$, should be chosen appropriately to make the zero balance of the subsequent system asymptotically stable [40]:

$$\begin{aligned} \phi\dot{\eta}_i &= k_{i+1}\eta_{i+1} - g_i(\eta_1), \quad i = 1, 2, \ldots, n-1. \\ \phi\dot{\eta}_n &= \eta_{n+1} - g_n(\eta_1) \\ \phi\dot{\eta}_{n+1} &= -g_{n+1}(\eta_1) \end{aligned} \tag{25}$$

Two useful lemmas will be presented in the following section.

Lemma 12 (see [2, 42]). *Consider the system*

$$\begin{aligned} \dot{d}_i &= k_{i+1} d_{i+1}, \quad i = 1, 2, \ldots, n-1 \\ \dot{d}_n &= \omega_{2nom}(d) \end{aligned} \tag{26}$$

Let $h_1, h_2, \ldots, h_n > 0$ so that the polynomial $\theta^n + h_n\theta^{n-1} + \cdots + h_2\theta + h_1$ is Hurwitz stable. Then there exists $\phi \in (0,1)$ such that, for any $\nu \in (1-\phi, 1)$, the origin of (26) is a globally finite-time stable equilibrium under the feedback

$$\begin{aligned} \omega_{2nom}(d) = -h_1 \operatorname{sgn}(d_1)|d_1|^{\nu_1} - \cdots \\ - h_n \operatorname{sgn}(d_n)|d_n|^{\nu_n}, \end{aligned} \tag{27}$$

where $\nu_{n+1} = 1$, $\nu_n = \nu$, $\nu_{i-1} = \nu_i\nu_{i+1}/(2\nu_{i+1} - \nu_i)$, $i = 2, 3, \ldots, n$.

Lemma 13 (see [2, 43]). *If the continuously differentiable, nonnegative function $L(j)$ satisfies*

$$\dot{L}(j) + \nu L(j) + bL^{w}(j) \leq 0, \tag{28}$$

where $\nu, b > 0$, $0 < w < 1$, then j will converge to $j = 0$ in finite time.

3. Control Design and Stability Analysis

We design the active disturbance rejection controllers to achieve finite stabilization for a class of systems (3) by integrating extended state observer and adaptive sliding mode controller. The analysis is as follows.

Step 1. According to Lemma 13, we can design a controller to achieve the finite-time sliding mode stabilization of system (2). The control signal is designed as (10).

Step 2. Design the active disturbance rejection control to achieve finite-time stabilization for a class of systems (3) by combining extended state observer with adaptive sliding mode controller.

The sliding surface is selected as [2, 44]

$$s = d_n(t) - d_n(0) - \int_0^t \omega_{2nom}(d(\beta))\, d\beta. \tag{29}$$

Once the ideal sliding mode $s = 0$ is established, (29) can be rewritten as

$$d_n(t) = d_n(0) - \int_0^t \omega_{2nom}(d(\beta))\, d\beta. \tag{30}$$

Differentiating (30) yields (26), and this implies that system (13) will converge to the origin from any initial condition along the sliding surface $s = 0$ in finite time.

Define an odd continuous and differentiable function

$$\pi(\beta, \mu, \tau) = \begin{cases} \operatorname{sgn}(\beta)|\beta|^{\mu}, & |\beta| \geq \tau \\ (\mu - 1)\tau^{\mu-2}\operatorname{sgn}(\beta)o^2 + (2-\mu)\tau^{\mu-1}\beta, & |\beta| < \tau \end{cases} \tag{31}$$

where $0 < \mu < 1$, τ is a sufficiently small positive constant. The ADRC law is designed for system (13):

$$\begin{aligned} \psi_{2nom} = -\frac{1}{\zeta(d,t)} \Big(\hat{d}_{n+1} - \hat{\omega}_{2nom}(\hat{d}) + m_1\hat{s} + n_0\hat{\varsigma}_{\max}\hat{s} \\ + m_2\pi(\hat{s}, l, \tau_{n+1}) \Big) \end{aligned} \tag{32}$$

where

$$\begin{aligned} \hat{\omega}_{2nom}(\hat{d}) &= -h_1\pi(\hat{d}_1, c_1, \tau_1) - \cdots - h_n\pi(\hat{d}_n, c_n, \tau_n), \\ \hat{s} &= \hat{d}_n(t) - \hat{d}_n(0) - \int_0^t \hat{\omega}_{2nom}(\hat{d}(\beta))\, d\beta, \end{aligned} \tag{33}$$

where τ_i, $1 \leq i \leq n+1$ are sufficiently small positive constants, and $0 < l < 1$, $m_1, m_2, n_0 > 0$. Define the estimation of the upper bound of $\sqrt{\kappa_1^2 + \kappa_2^2}$ as $\hat{\varsigma}_{\max}$. κ_1 and κ_2 will be specified latter. The updating law of $\hat{\varsigma}_{\max}$ is

$$\dot{\hat{\varsigma}}_{\max} = m_3 n_0 \left(\hat{s}^2 - \hat{\varsigma}_{\max}\right), \tag{34}$$

where $m_3 > 0$.

Theorem 14. *Consider the closed-loop system (13) formed of the nonlinear extended observer (16) and active disturbance rejection control law (18) and (32). Suppose Assumptions 3–10 are satisfied, for any $d(0) \in \mathbb{R}^n$, $\eta(0) \in \mathbb{R}^n$ [2, 45]*

(i) *$\|\eta\| \longrightarrow 0$ and $|d_i(t) - \hat{d}_i(t)| \longrightarrow 0$ as $\phi \longrightarrow 0$, uniformly in $t \in (0, \infty)$;*

(ii) *there exists $\phi_0 > 0$ such that, for any $\phi \in (0, \phi_0)$,*

there exists ϕ-dependent T_ϕ such that $d = 0$, $\forall t \in [T_\phi, \infty)$.

Proof. Associating (13), (16), and (15), we can compute the derivative of the extended state d_{n+1} with respect to t in the time interval $[0, t_1]$. The derivative of the extended state d_{n+1} is shown as

$$\begin{aligned} m &= \sum_{i=1}^{n-1} k_{i+1} d_{i+1} \left(\frac{\partial f_2}{\partial d_i} + \psi_2 \frac{\partial \zeta_2}{\partial d_i} - \psi_2 \frac{\partial \zeta_0}{\partial d_i} \right) \\ &\quad + \left(f_2 \left(d, z_2, \omega_2 \right) + \zeta \left(d, t \right) \psi_2 \right) \\ &\quad \cdot \left(\frac{\partial f_2}{\partial d_n} + \psi_2 \frac{\partial \zeta_2}{\partial d_n} - \psi_2 \frac{\partial \zeta_0}{\partial d_n} \right) \\ &\quad + f_{20} \left(d, z_2, \omega_2 \right) \frac{\partial f_2}{\partial z_2} + f_{20} \left(d, z_2, \omega_2 \right) \psi_2 \frac{\partial \zeta_2}{\partial z_2} \\ &\quad + \dot{\omega}_2 \frac{\partial f_2}{\partial \omega_2} + \dot{\omega}_2 \psi_2 \frac{\partial \zeta_2}{\partial \omega_2} + \left(\zeta_2 \left(d, t \right) - \zeta_0 \left(d \right) \right) \dot{\psi}_2, \end{aligned} \tag{35}$$

where

$$\begin{aligned} \dot{\psi}_2 &= -sat_\phi \left(\frac{\psi_{2nom}}{W} \right)' \left(\frac{1}{\zeta \left(d, t \right)} \left(\dot{\hat{d}}_{n+1} \right. \right. \\ &\quad + \sum_{i=1}^{n} h_i \dot{\hat{d}}_i \frac{d\pi \left(d_i, c_i, \tau_i \right)}{d\hat{d}_i} + m_1 \dot{\hat{s}} + m_2 \dot{s} \frac{d\pi \left(\hat{s}, c_n, \tau_n \right)}{d\hat{s}} \\ &\quad \left. + n_0 \dot{\hat{\varsigma}}_{\max} \hat{s} + n_0 \hat{\varsigma}_{\max} \dot{s} \right) - \frac{\sum_{i=1}^{n} \dot{\hat{d}}_i \left(\partial \zeta_0 / \partial \hat{d}_i \right)}{\zeta_0^2 \left(\hat{d} \right)} \left(d_{n+1} \right. \\ &\quad \left. \left. - \hat{\omega}_{2nom} \left(\hat{d} \right) + m_1 \hat{s} + m_2 \pi \left(\hat{s}, l, \tau_{n+1} \right) + n_0 \hat{\varsigma}_{\max} \hat{s} \right) \right) \end{aligned} \tag{36}$$

$$\dot{\hat{s}} = \dot{\hat{d}}_n - \hat{\omega}_{2nom} \left(\hat{d} \right) \tag{37}$$

We can know that $\hat{d}_i = d_i - \eta_i \phi^{n+1-i}$, $i = 1, 2, \ldots, n+1$, from (21). In addition, $d(t)$ and $\hat{d}(t)$ are continuous in t, and $d(0)$ and $\hat{d}(0)$ and ψ_2 are bounded in the time interval $[0, t_1]$. Then considering Assumptions 3–10 and substituting (36) and (37) into (35), we can infer that the derivative of the extended state d_{n+1} with respect to t in the time interval $[0, t_1]$ is bounded

$$|m| \leq W_0 + W_1 \|\eta\| + \frac{\rho_a \varphi_{n+1}}{\phi} \|\eta\| \tag{38}$$

where W_0 and W_1 are independent positive constants.

Let $L_1(\eta)$ be a positive definite, continuous differentiable function satisfying Assumption 9. The derivative of $L_1(\eta)$ with respect to t in the time interval $[0, t_1]$ satisfies

$$\begin{aligned} \frac{dL_1(\eta)}{dt} &= \frac{1}{\phi} \left(\sum_{i=1}^{n-1} \left(k_{i+1} \eta_{i+1} - g_i \left(\eta_1 \right) \right) \frac{\partial L_1}{\partial \eta_i} \left(\eta \right) \right. \\ &\quad + \left(\eta_{n+1} - g_n \left(\eta_1 \right) \right) \frac{\partial L_1}{\partial \eta_n} \left(\eta \right) \\ &\quad \left. + \left(\phi m - g_{n+1} \left(\eta_1 \right) \right) \frac{\partial L_1}{\partial \eta_{n+1}} \left(\eta \right) \right) \leq -\frac{1}{\phi} Q_1 \left(\eta \right) + |m| \\ &\quad \cdot \left| \frac{\partial L_1}{\partial \eta_{n+1}} \left(\eta \right) \right| \leq -\frac{\theta_3}{\phi} \|\eta\|^2 + \left(W_0 + W_1 \|\eta\| \right. \\ &\quad \left. + \frac{\rho_a \varphi_{n+1}}{\phi} \|\eta\| \right) \gamma \|\eta\| \leq -\frac{1}{\theta_2 \phi} \left(\theta_3 \right. \\ &\quad \left. - \frac{\theta_2 \left(\rho_a \varphi_{n+1} \gamma + \phi W_1 \right) \gamma}{\theta_1} \right) L_1 \left(\eta \right) + W_0 \gamma \frac{\sqrt{L_1 \left(\eta \right)}}{\sqrt{\theta_1}} \end{aligned} \tag{39}$$

Considering $dL_1(\eta)/dt = 2\sqrt{L_1(\eta)}(d\sqrt{L_1(\eta)}/dt)$, we can get

$$\begin{aligned} \frac{d\sqrt{L_1(\eta)}}{dt} &\leq -\frac{1}{2\theta_2 \phi} \left(\theta_3 - \frac{\theta_2 \left(\rho_a \varphi_{n+1} \gamma + \phi W_1 \right) \gamma}{\theta_1} \right) \sqrt{L_1(\eta)} \\ &\quad + \frac{W_0 \gamma}{2\sqrt{\theta_1}} \end{aligned} \tag{40}$$

Considering (41) and Assumption 9 $\forall \phi \in (0, (\theta_1 \theta_3 - \theta_2 \rho_a \varphi_{n+1} \gamma)/\theta_2 W_1 \gamma)$, the following inequality holds:

$$\|\eta\| \leq \frac{\sqrt{L_1(\eta)}}{\sqrt{\theta_1}} \tag{41}$$

This together with (20) yields

$$\begin{aligned} \left| \Delta_i(t) \right| &\leq \phi^{n+1-i} \left\| \eta \left(\frac{t}{\phi} \right) \right\| \leq \phi^{n+1-i} \times \left(\frac{\sqrt{L_1(\eta(0))}}{\sqrt{\theta_1}} \right. \\ &\quad \left. - \frac{W_0 \gamma \theta_2 \phi}{\theta_1 \theta_3 - \theta_2 \rho_a \varphi_{n+1} \gamma - \theta_2 \phi W_1 \gamma} \right) \\ &\quad \times e^{-(1/2\theta_2 \phi)[\theta_3 - (\theta_2 (\rho_a \varphi_{n+1} \gamma - \phi W_1 \gamma)/\theta_1)](t/\phi)} + \phi^{n+1-i} \\ &\quad \times \frac{W_0 \gamma \theta_2 \phi}{\theta_1 \theta_3 - \theta_2 \rho_a \varphi_{n+1} \gamma - \theta_2 \phi W_1 \gamma} \end{aligned} \tag{42}$$

The right hand side of the inequality (41) converges to 0 in the time interval $(0, t_1]$ as $\phi \longrightarrow 0$, and there exists an $\phi^* > 0$ such that, for any $\phi \in (0, \phi^*)$, there exists an ϕ-independent

$t_0 \in (0, t_1)$ such that $|\Delta_i| = |d_i(\phi, t) - \hat{d}_i(\phi, t)| \longrightarrow 0$, $i = 1, 2, \ldots, n+1$, $t \in [t_0, t_1]$ as $\phi \longrightarrow 0$. What is more, the control will be out of saturation after the transient period of the nonlinear extended observer by appropriately selecting the bound W.

Let κ_1 and κ_2 be defined as

$$\kappa_1 = \begin{cases} \dfrac{1}{n_0 s}\left(\zeta_2\psi_2 + f_2 - \omega_{2nom} + m_1 s + m_2 \operatorname{sgn}(s)|s|^l + n_0\hat{\varsigma}_{\max} s\right), & |s| \geq 1 \\ \dfrac{1}{n_0}\left(\zeta_2\psi_2 + f_2 - \omega_{2nom} + m_1 s + m_2 \operatorname{sgn}(s)|s|^l + n_0\hat{\varsigma}_{\max}\operatorname{sgn}(s) + m_1 \operatorname{sgn}(s)\right), & |s| < 1 \end{cases}$$

$$\kappa_2 = \hat{s}^2 - s^2 \tag{43}$$

It can be concluded that both κ_1 and κ_2 are bounded in the time interval $[0, t_1]$. Thus, $\sqrt{\kappa_1^2 + \kappa_2^2}$ is bounded in the time interval $[0, t_1]$.

In the reaching phase ($s \neq 0$), we consider the time derivative of the sliding variable in the time interval $[t_0, t_1]$. In the case $|s| \geq 1$, associating with (13) and (43), we can deduce that

$$\begin{aligned} \dot{s} &= \dot{d}_n - \omega_{2nom} = f_2 + \zeta_2\psi_2 - \omega_{2nom} \\ &= n_0 s\kappa_1 - m_1 s - m_2 \operatorname{sgn}(s)|s| - n_0\hat{\varsigma}_{\max} s \end{aligned} \tag{44}$$

In the case $|s| < 1$, associating (13) with (43), we can get

$$\begin{aligned} \dot{s} &= f_2 + \zeta_2\psi_2 - \omega_{2nom} \\ &= n_0 s\kappa_1 - m_1 s - m_2 \operatorname{sgn}(s)|s| - n_0\hat{\varsigma}_{\max}\operatorname{sgn}(s) \\ &\quad - m_1 \operatorname{sgn}(s) \end{aligned} \tag{45}$$

Let $\tilde{\varsigma}_{\max} = \varsigma_{\max} - \hat{\varsigma}_{\max}$. Consider the following Lyapunov function:

$$L_2 = \frac{1}{2}s^2 + 2\frac{1}{2m_3}\tilde{\varsigma}_{\max}^2, \tag{46}$$

and differentiate L_2 with respect to t in the time interval $[t_0, t_1]$. In the case $|s| \geq 1$, associating with (34), (43), and (44), we can get

$$\begin{aligned} \frac{dL_2}{dt} &= s\dot{s} + \frac{1}{m_3}\dot{\hat{\varsigma}}_{\max} \\ &= -m_1 s^2 - m_2|s|^{l+1} + n_0(\kappa_1 - \varsigma_{\max})s^2 \\ &\quad + n_0\tilde{\varsigma}_{\max}(\varsigma_{\max} - \kappa_2) - n_0\tilde{\varsigma}_{\max}^2. \end{aligned} \tag{47}$$

Since $\kappa_1, \kappa_2 \leq \varsigma_{\max}$ and

$$\begin{aligned} (\kappa_{\max} - \kappa_2)\tilde{\varsigma}_{\max} &\leq \frac{1}{2}\tilde{\varsigma}_{\max}^2 + \frac{1}{2}(\varsigma_{\max} - \kappa_2)^2 \\ &\leq \frac{1}{2}\tilde{\varsigma}_{\max}^2 + 2{\varsigma_{\max}}^2, \end{aligned} \tag{48}$$

inequality (46) can be simplified as

$$\frac{dL_2}{dt} \leq -m_1 s^2 - \frac{n_0}{2}\tilde{\varsigma}_{\max}^2 + 2n_0{\varsigma_{\max}}^2. \tag{49}$$

In the case $|s| < 1$, associating with (34), (43), and (45), we can get

$$\begin{aligned} \frac{dL_2}{dt} &= s\dot{s} - \frac{1}{m_3}\tilde{\varsigma}_{\max}\dot{\hat{\varsigma}}_{\max} \\ &= n_0\kappa_1 s - m_1 s^2 - m_2|s|^{l+1} - n_0\hat{\varsigma}_{\max}|s| - m_1|s| \\ &\quad - n_0\tilde{\varsigma}_{\max}s^2 - n_0\hat{\varsigma}_{\max}\kappa_2 + n_0\tilde{\varsigma}_{\max}\hat{\varsigma}_{\max}. \end{aligned} \tag{50}$$

Considering (47) and

$$\begin{aligned} \kappa_1 s - \hat{\varsigma}_{\max}|s| - \tilde{\varsigma}_{\max}s^2 &\leq \varsigma_{\max}|s| - \hat{\varsigma}_{\max}|s| - \tilde{\varsigma}_{\max}s^2 \\ &\leq \frac{1}{4}\tilde{\varsigma}_{\max}^2 + \left(|s| - s^2\right)^2 \leq \frac{1}{4}\tilde{\varsigma}_{\max}^2 + \frac{1}{16}, \end{aligned} \tag{51}$$

inequality (50) can be simplified as

$$\frac{dL_2}{dt} \leq -m_1 s^2 - \frac{n_0}{4}\tilde{\varsigma}_{\max}^2 + 2n_0{\varsigma_{\max}}^2 + \frac{1}{16}n_0. \tag{52}$$

According to the boundedness theorem, both s and $\tilde{\varsigma}_{\max}$ are bounded in the time interval $[t_0, t_1]$. Assume $|\tilde{\varsigma}_{\max}| \leq a$.

In order to show the finite-time stability, we consider the Lyapunov function

$$L_3 = \frac{1}{2}s^2 \tag{53}$$

and differentiate L_3 with respect to t in the time interval $[t_0, t_1]$.

In the case $|s| \geq 1$, associating with (44) and (45), we can get

$$\begin{aligned} \frac{dL_3}{dt} &= s\dot{s} = n_0\kappa_1 s^2 - m_1 s^2 - m_2|s|^{l+1} - n_0\hat{\varsigma}_{\max}s^2 \\ &\leq -2(m_1 - n_0\tilde{\varsigma}_{\max})L_3 - 2^{(l+1)/2}m_2{L_3}^{(l+1)/2}. \end{aligned} \tag{54}$$

In the case $|s| < 1$, associating with (42) and (46), we can get

$$\begin{aligned} \frac{dL_3}{dt} &= n_0\kappa_1 s^2 - m_1 s^2 - m_2|s|^{l+1} - n_0\hat{\varsigma}_{\max}|s| - m_1|s| \\ &\leq -2(m_1 - n_0\tilde{\varsigma}_{\max})L_3 - 2^{(l+1)/2}m_2{L_3}^{(l+1)/2} \\ &\quad + |s|\left(n_0\tilde{\varsigma}_{\max}(1 - |s|) - m_1\right). \end{aligned} \tag{55}$$

Choose m_1 and n_0 satisfying $m_1 > n_0 a$, and then together with (54) and (55), we can get

$$\frac{dL_3}{dt} \leq -2\left(m_1 - n_0 \widetilde{\varsigma}_{\max}\right) L_3 - 2^{(l+1)/2} m_2 {L_3}^{(l+1)/2}. \quad (56)$$

According to Lemma 13, the sliding surface s will converge to zero in finite time t_{l1}. Besides, on the basis of Lemma 12, $d_i = 0$, $(i = 1, 2, \ldots n)$ will be arrived in finite t_{l2}(here one can select $t_1 > t_{l2}$).

Next, it can be illustrated that system (13) will stay at the origin for all $t > t_{l2}$. We can get that $d = 0$ in the time interval $[t_{l2}, t_1]$ in the first step. Considering d is continuous in t, d is bounded in the time interval $[0, 2t_1]$. Then running the analysis above, we can get $d = 0$ in the time interval $[t_{l2}, 2t_1]$, and then d is bounded in the time interval $[0, 3t_1]$. We can get $d = 0$ in the time interval $[t_{l2}, 3t_1]$ similarly.

Finally, it can be summarized that there exists $\phi_0 > 0$ such that, for any $\phi \in (0, \phi_0)$, there exists ϕ-dependent T_ϕ such that $d = 0$, $\forall t \in [T_\phi, \infty)$. As a result, inequality (41) holds in the time interval $[0, \infty)$, and consequently $\|\eta\| \longrightarrow 0$ as $\phi \longrightarrow 0$ uniformly in $t \in (0, \infty)$. □

Remark 15. We can get that the closed-loop can converge to 0 only when $\phi \longrightarrow 0$ according to the analysis above [36–44]. However, the condition $\phi = 0$ cannot be met in practice. What is more, reducing the value of will increase the high-frequency oscillations. In this paper, the proposed control law can guarantee the closed-loop converge to 0 asymptotically and in finite time, without relying on the condition $\phi = 0$.

Step 3. Rethink system 2 and design the ψ_1 so that the sliding surface $s = 0$ will be reached in finite time and the nonholonomic system in extended chained-form (2) will converge to the origin in finite time as system (3).

Let $n = 2$, $x_1 = x_{11}, u_1 = x_{21}$; system (2) can be rewritten as

$$\begin{aligned} \dot{x}_{11} &= k_1 x_{21} \\ \dot{x}_{21} &= \zeta_1(x,t)\psi_1 + f_1(x, z_1, \omega_1) \\ \dot{z}_1 &= f_{01}(x, z_1, \omega_1) \\ y_1 &= x_{11} \end{aligned} \quad (57)$$

In accordance with the state estimations of the nonlinear extended state observer (15), the output of the ESO (16), and control input (17), the ADRC control law of the system (57) can be designed as (17) and the control injected into the system (57) is modified as (18).

On the basis of Lemmas 12 and 13, the sliding surface $s = 0$ will be reached in finite time, and system (59) will converge to the origin in finite time.

Associating with system (11) and (57), we can get a conclusion that, under the condition (15), (16), and (17), the nonholonomic system in extended chained-form (2) and (3) will converge to the origin in finite time. Finally, we can get a conclusion that there exists $\phi_2 > 0$ such that, for any $\phi \in (0, \phi_2)$, there exists ϕ-dependent T_ϕ such that $x = [x_1, x_2, \ldots, x_n]^T = 0$ $\forall t \in [T_\phi, \infty)$, and consequently $\|\eta\| \longrightarrow 0$ as $\phi \longrightarrow 0$ uniformly in $t \in (0, \infty)$.

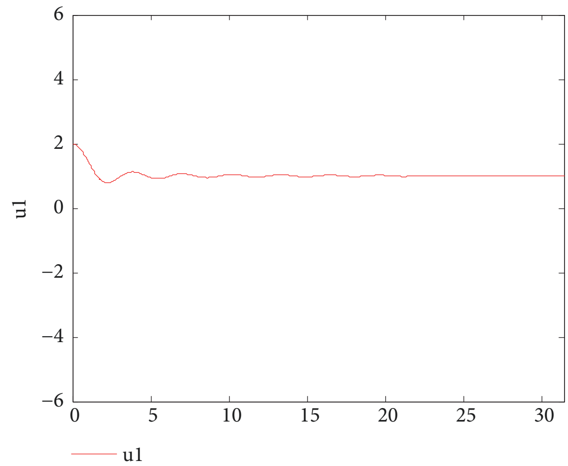

Figure 1: The velocity input u_1 for the kinematics model.

4. Simulations

In this section, we demonstrate the effectiveness of the proposed control strategy for the following nonholonomic systems in extended chained-form 2 through a series of simulations, $n = 3$ for 2. The control signal ψ_1 is designed as (10).

The ESO is designed as

$$\begin{aligned} \dot{\hat{x}}_2 &= 2\hat{x}_3 + \frac{2}{\phi}\left(x_1 - \hat{x}_1\right), \\ \dot{\hat{x}}_3 &= 4\hat{x}_4 + \frac{6}{\phi^2}\left(x_3 - \hat{x}_3\right) + \zeta_0 \psi_2, \\ \dot{\hat{x}}_4 &= \frac{4}{\phi^3}\left(x_4 - \hat{x}_4\right), \end{aligned} \quad (58)$$

In this example, we assume $x(0) = [0.2, -0.6, 1]^T$, $\hat{x}(0) = [0.1, -1, 1.2]^T$, $q = 0.01$, $r(0) = 0$, $E = 2$, $\zeta_{20}(0) = 15$, and $\widehat{\varsigma}_{\max}(0) = 0$. The control parameters are selected as $m_1 = 300$, $m_2 = 1$, $m_3 = 1$, $n_0 = 0.01$, $\nu_1 = 1/2$, $\nu_2 = 1/4$, $h_1 = 9$, $h_2 = 6$, $h_3 = 3$, and $W = 30$.

Figures 1~4 are the simulation results of the three steps. Figure 1 shows that the velocity input u_1 for the kinematics model can converge to 1 in a finite time $t \leq 6s$ in Step 1 and keep it in Step 2 until it is driven to zero in the last step. Figure 2 shows that the sliding surface $s = 0$ is reached in a finite time $t \leq 6s$. Figures 3–7 show the time histories of $x, \hat{x}$ and η_i. These figures suggest that the system state vector $x = [x_1, x_2, \ldots, x_n]^T$ is well estimated by the ESO and finally the state x scaled ESO estimation error η_i converge to zero in finite time. In addition, Figure 8 suggests that the control signal ψ_2 converges to zero in a finite time $t \leq 8s$.

What is more, in [46], a finite-time tracking control law is designed for the nonholonomic mobile robot. The control law in [46] also used the switching control method and the simulation results are depicted in Figures 9 and 10. We can see that the state x converges to zero in finite time $t \leq 10s$ and the tracking distance is stabilized to a constant in finite time $t \leq 15s$. From Figure 10 we can get that the controller

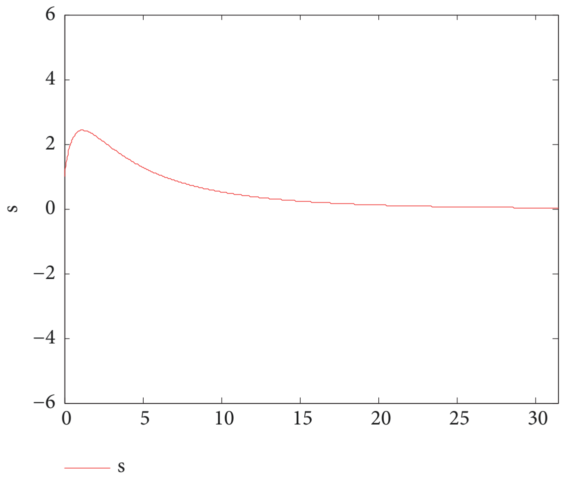

Figure 2: The sliding surface s.

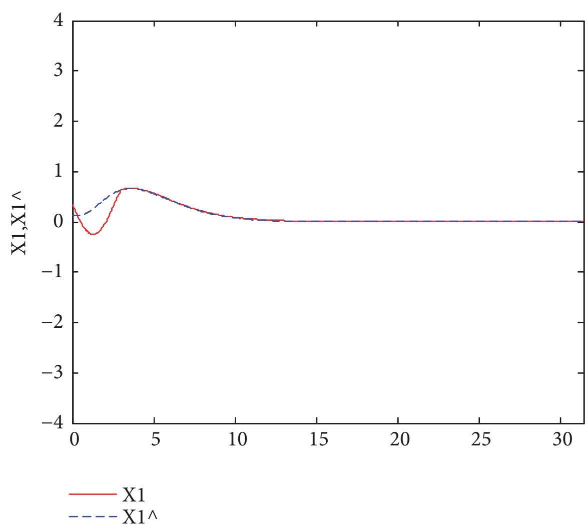

Figure 3: System states and ESO outputs of x_1.

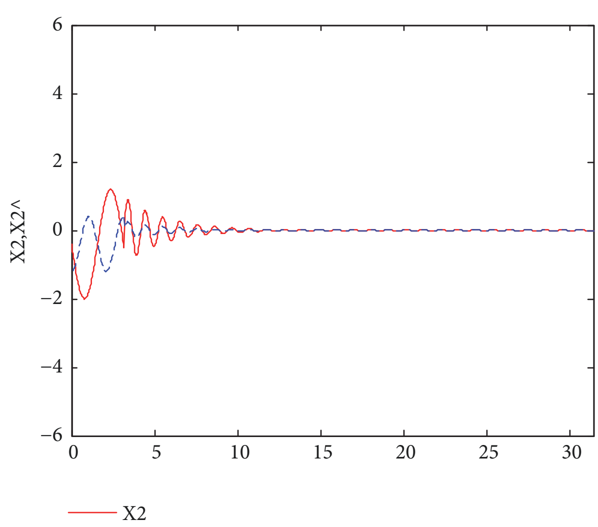

Figure 4: System states and ESO outputs of x_2.

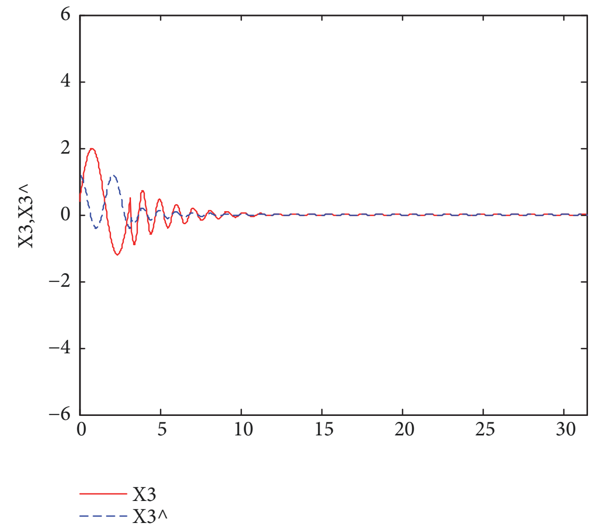

Figure 5: System states and ESO outputs of x_3.

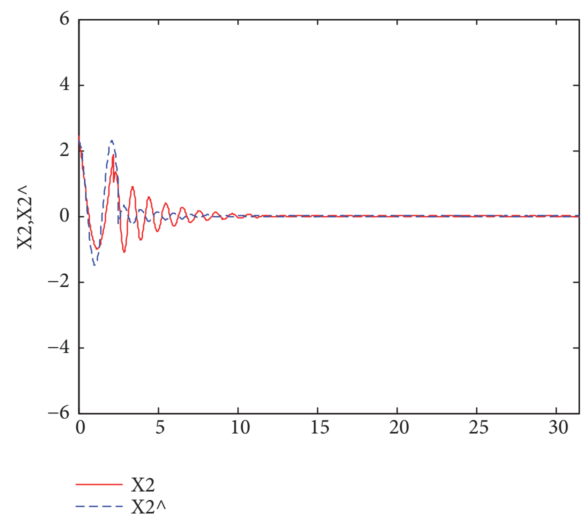

Figure 6: System states and ESO outputs of x_4.

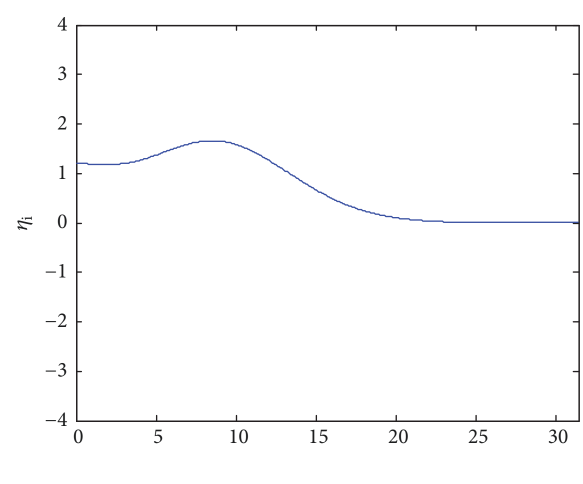

Figure 7: The convergence of the scaled ESO estimation error η_i.

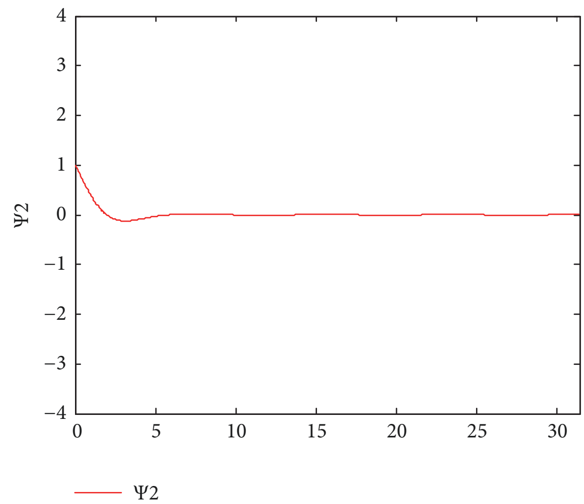

FIGURE 8: The control signal ψ_2.

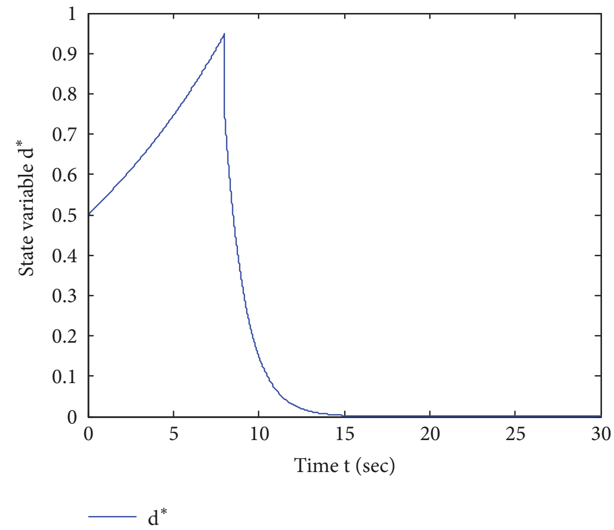

FIGURE 10: The stability of distance error d^* with respect of time.

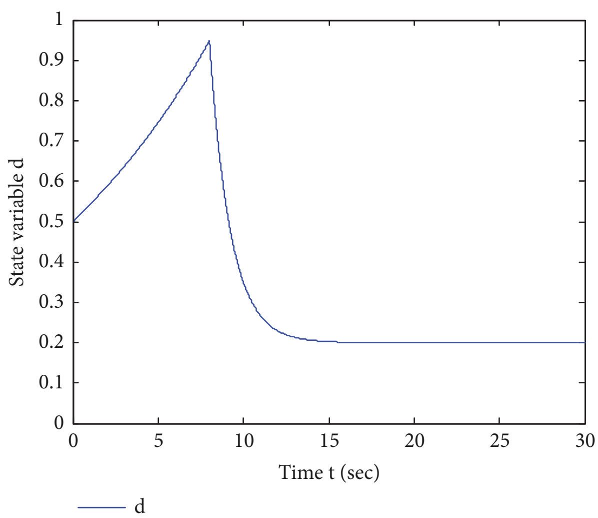

FIGURE 9: The convergence of tracking distance d with respect of time.

proposed in this paper is more smooth than the controller in [46] in switching control. We can see that the controller proposed in this paper has faster convergence speed and more stable performance than that obtained in [46].

Remark 16. For your convenience review, we make Table 1 to explain how to choose the design parameters.

Lemma 12 shows that $h_1, h_2, \ldots, h_n > 0$ ensure that the polynomial $\theta^n + h_n\theta^{n-1} + \cdots + h_2\theta + h_1$ is Hurwitz stable. Then there exists $\phi \in (0,1)$ such that, for any $\nu \in (1-\phi, 1)$, the origin of (22) is a globally finite-time stable equilibrium under the feedback (27), where $\nu_{n+1} = 1$, $\nu_n = \nu$, $\nu_{i-1} = \nu_i\nu_{i+1}/(2\nu_{i+1} - \nu_i)$, $i = 2, 3, \ldots, n$. In addition, $m_1, m_2, m_3, n_0 > 0$ allow four terms $-\widehat{\omega}_{2nom}(\widehat{d}) + m_1\widehat{s} + n_0\widehat{\varsigma}_{\max}\widehat{s} + m_2\pi(\widehat{s}, l, \tau_{n+1})$ to guarantee the stability and performance requirements of the closed-loop system. The saturation bound W is chosen so that the saturation is not invoked in the steady state of the ESO. Practically, we can choose a group of available parameters $m_1 = 300$, $m_2 = 1$, $m_3 = 1$, $n_0 = 0.01$, $\nu_1 = 1/2$, $\nu_2 = 1/4$, $h_1 = 9$, $h_2 = 6$, $h_3 = 3$, and $W = 30$ in the simulation section.

Remark 17. By comparing the performance of the controller proposed in this paper with the performance of the controller proposed in [47, 48], we can know that the fixed and predefined-time controllers have better performance for nonholonomic systems. The fixed and predefined-time controllers predetermine the time, so the operation of the controller is independent of the initial value of the nonholonomic systems. However, for the nonholonomic chained-form systems with dynamic nonlinear uncertain terms considered in this paper, it is difficult to estimate the time in advance due to the existence of dynamic nonlinear uncertain terms. Achieving fixed-time control of nonholonomic chained-form systems with dynamic nonlinear uncertain terms is one of our future research directions.

5. Conclusion

In this paper, finite-time switching controllers are put forward in order to address the stabilization problem of nonholonomic chained-form systems with uncertain parameters and external perturbations. The proposed control strategy is able to guarantee the semiglobal finite-time stabilization of the extended nonholonomic chained-form systems. The simulation results of the numerical example show that the method is effective.

6. Future Research Directions and Prospects

We consider the application of the finite-time switching controllers proposed in the theory to the anti-interference of the robot in the source seeking work as our future research direction. It is very practical for realistic engineering. We

Table 1

Parameters	Source of each parameter
$m_1, m_2, m_3, n_0 > 0$	From (28) and (29)
$\nu_{n+1} = 1,\ \nu_n = \nu,\ \nu_{i-1} = \dfrac{\nu_i \nu_{i+1}}{2\nu_{i+1} - \nu_i},\ i = 2, 3, \ldots, n$	From (26) and (27)
$h_1, h_2, \ldots h_n > 0$	From Lemma 2
$W > 0$	From (18) and (19)

will conduct more research and experiments in practical application.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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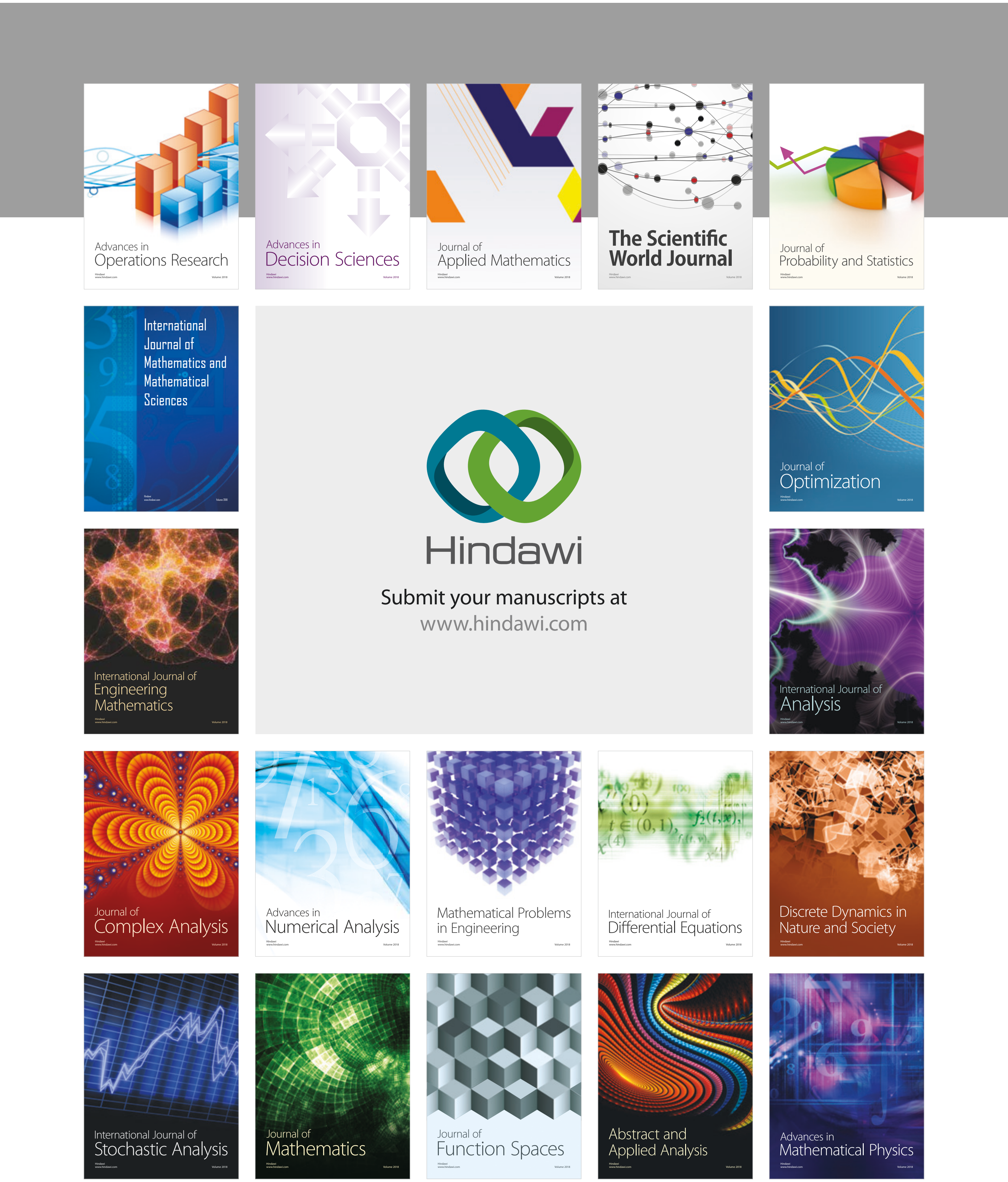

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