Research Article

Numerical Simulation of a Class of Hyperchaotic System Using Barycentric Lagrange Interpolation Collocation Method

Xiaofei Zhou,¹ Junmei Li,² Yulan Wang,² and Wei Zhang¹

¹Institute of Economics and Management, Jining Normal University, Jining 012000, Inner Mongolia, China
²Department of Mathematics, Inner Mongolia University of Technology, Hohhot 010051, China

Correspondence should be addressed to Yulan Wang; wylnei@163.com and Wei Zhang; jnsfxyzw@163.com

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Hyperchaotic system, as an important topic, has become an active research subject in nonlinear science. Over the past two decades, hyperchaotic system between nonlinear systems has been extensively studied. Although many kinds of numerical methods of the system have been announced, simple and efficient methods have always been the direction that scholars strive to pursue. Based on this problem, this paper introduces another novel numerical method to solve a class of hyperchaotic system. Barycentric Lagrange interpolation collocation method is given and illustrated with hyperchaotic system (\( \dot{x} = ax + dz - yz, \dot{y} = xz - by, 0 \leq t \leq T, \dot{z} = c(x - z) + xy, \dot{w} = c(y - w) + xz, \)) as examples. Numerical simulations are used to verify the effectiveness of the present method.

1. Introduction

Many chaotic systems have been developed such as Lorenz system [1], Rossler system [2], and Chen system [3]. As chaos theory progresses, many new chaotic systems [4–8] have been proposed, specially hyperchaotic systems [9–15]. A hyperchaotic system is usually characterized as a chaotic system with more than one positive Lyapunov exponent, implying that the dynamics expand in more than one direction, giving rise to more complex chaotic dynamics. Barycentric interpolation collocation method [16, 17] is a high precision method. Some authors have used barycentric interpolation collocation method to solve various kinds of problems [16–23]. This paper suggests the barycentric interpolation collocation method to solve a class of hyperchaotic system, and a hyperchaotic system (1) is adopted as an example to elucidate the solution process.

We consider the following 4D butterfly hyperchaotic system with butterfly phenomenon [24]:

\[
\begin{align*}
\dot{x} &= ax + dz - yz, \\
\dot{y} &= xz - by, & 0 \leq t \leq T, \\
\dot{z} &= c(x - z) + xy, \\
\dot{w} &= c(y - w) + xz,
\end{align*}
\]

where \( x, y, z, w \) are the state variables and \( a, b, c, d \) are the positive constant parameters of the system which satisfy the following initial conditions:

\[
\begin{align*}
x(0) &= c_1, \\
y(0) &= c_2, \\
z(0) &= c_3, \\
w(0) &= c_4.
\end{align*}
\]

2. The Numerical Solution of System (1)

First of all, we give initial function \( x_0(t), y_0(t) \) and construct the following linear iterative format of system (1):

\[
\begin{align*}
x'_n &= ax_n + dz_n - y_{n-1} z_n, \\
y'_n &= x_{n-1} z_n - by_n, & n = 1, 2, \ldots, \\
z'_n &= c(x_n - z_n) + x_{n-1} y_n, \\
w'_n &= c(y_n - w_n) + x_{n-1} z_n.
\end{align*}
\]

Next, we use the barycentric Lagrange interpolation collocation method to solve (3).
In the interval $[0, T]$ takes $M$ different nodes, $0 \leq t_1 < t_2 < \cdots < t_M \leq T$. The barycentric interpolation of $x_n(t), y_n(t), z_n(t), w_n(t)(n = 1, 2, 3, \cdots)$ can be written as [16, 17]

\[
\begin{align*}
\xi_j(t) &= \omega_j/(t - t_j)/\sum_{k=1}^{M} \omega_k/(t - t_k)) \text{ is, respectively, barycentric Lagrange interpolation primary function and } \\
\omega_j &= 1/\prod_{k=1, k \neq j}^{M} (t_i - t_j) \text{ is center of gravity interpolation weight.}
\end{align*}
\]

Use formula (4), the functions $x'_n(t), y'_n(t), z'_n(t), w'_n(t)$ can be expressed as

\[
\begin{align*}
x'_n(t) &= \sum_{j=1}^{M} \xi'_j(t) x_n(t_j), \\
y'_n(t) &= \sum_{j=1}^{M} \xi'_j(t) y_n(t_j), \\
z'_n(t) &= \sum_{j=1}^{M} \xi'_j(t) z_n(t_j), \\
w'_n(t) &= \sum_{j=1}^{M} \xi'_j(t) w_n(t_j).
\end{align*}
\]

So, linear iterative format (3) can be written in following partitioned matrix form:

\[
\begin{bmatrix}
D - al & 0 & \diag(y_{n-1}) - dl & 0 \\
0 & D + bl & -\diag(x_{n-1}) & 0 \\
-cl & \diag(x_{n-1}) & D + cl & 0 \\
0 & -cl & -\diag(y_{n-1}) & D + cl
\end{bmatrix}
\begin{bmatrix}
x_n \\
y_n \\
z_n \\
w_n
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

The matrix $D = (\xi'_j(t_j))_{j=1,2,\cdots,M}$ is $M$ order matrix. $I$ is $M$ order unit matrix, diagonal matrix $\diag(x_{n-1}) = \diag(x_{n-1}(t_1), x_{n-1}(t_2), \ldots, x_{n-1}(t_M))$, and diagonal matrix $\diag(y_{n-1}) = \diag(y_{n-1}(t_1), y_{n-1}(t_2), \ldots, y_{n-1}(t_M))$. The vector $\omega_j$ can be expressed as

\[
\begin{align*}
M &= \sum_{j=1}^{M} \xi_j(t_j) x_n(t_j), \\
M &= \sum_{j=1}^{M} \xi_j(t_j) y_n(t_j), \\
M &= \sum_{j=1}^{M} \xi_j(t_j) z_n(t_j), \\
M &= \sum_{j=1}^{M} \xi_j(t_j) w_n(t_j).
\end{align*}
\]

At last, we use initial conditions (2).

Take formula (4) into initial conditions (2); we can get the following discrete equations of initial conditions:

\[
\begin{align*}
\sum_{j=1}^{M} \xi_j(t_j) x_n(t_j) &= c_1, \\
\sum_{j=1}^{M} \xi_j(t_j) y_n(t_j) &= c_2, \\
\sum_{j=1}^{M} \xi_j(t_j) z_n(t_j) &= c_3, \\
\sum_{j=1}^{M} \xi_j(t_j) w_n(t_j) &= c_4.
\end{align*}
\]

In this paper, we use displacement method to impose the initial conditions. The detailed procedure is as follows.

The first 1 of (6) are replaced separately by the equation of initial conditions (8) in turn.

So, we can get that $x_n(t_j), y_n(t_j), z_n(t_j), w_n(t_j), (j = 1, 2, \cdots, M)$ are approximate solution of (1) and (2).

3. Numerical Experiment

In this section, six numerical experiments are studied to demonstrate the effectiveness of the present method. All experiments are computed using MatlabR2017a. In Experiments 1–6, we choose Chebyshev nodes, the accuracy of iteration control is $\varepsilon = 10^{-18}$, and the initial iteration value $x_0 = y_0 = z_0 = 0; x_1 = y_1 = z_1 = T$. Parameters of the numerical Experiments 1–5 are listed in Table I.

**Table I: Parameters used in Experiments 1–5.**

<table>
<thead>
<tr>
<th>Figures</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>k</th>
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<tbody>
<tr>
<td>Figure 1</td>
<td>1.378</td>
<td>0.5</td>
<td>0.6</td>
<td>0.097</td>
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<tr>
<td>Figure 2</td>
<td>1.378</td>
<td>0.5</td>
<td>0.6</td>
<td>0.097</td>
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<tr>
<td>Figure 3</td>
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<td>0.8</td>
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<td>Figure 4</td>
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<td>Figure 5</td>
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<td>Figure 6</td>
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<td>Figure 7</td>
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<td>Figure 8</td>
<td>8</td>
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<td>Figure 9</td>
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<td>2.5</td>
<td>0.75</td>
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<td>Figure 10</td>
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<td>0.75</td>
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<tr>
<td>Figure 11</td>
<td>15</td>
<td>2.5</td>
<td>0.75</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

$[x_n, y_n, z_n, w_n] = [x_n(t_1), x_n(t_2), \ldots, x_n(t_M), y_n(t_1), \ldots, y_n(t_M), z_n(t_1), z_n(t_2), \ldots, z_n(t_M), w_n(t_1), \ldots, w_n(t_M)].$

**Experiment 1.** We consider the following hyperchaotic system [25]:

\[
\begin{align*}
\dot{x} &= -y - z - aw, \\
\dot{y} &= x,
\end{align*}
\]

Complexity
where \( x, y, z, w \) are the state variables and \( a, b, c, d \) are the positive parameters of the system, which satisfy the following initial conditions:

\[
\begin{align*}
  x(0) &= 0, \\
  y(0) &= 0, \\
  z(0) &= 0, \\
  w(0) &= 0.1.
\end{align*}
\]

We choose Chebyshev nodes; the number of nodes \( M = 40 \). Numerical results of Experiment 1 are given in Figures 1 and 2.

Figure 1 is states of the hyperchaotic system for Experiment 1 with \( a = 1.378, b = 0.5, c = 0.6, d = 0.097 \), which is obtained by using the current method. Among them, \((c)\) is the states of \( x \) and \( y \) and \((b)\) is the states of \( z \) and \( w \). Figure 2 is hyperchaotic attractors of the system for Experiment 1 with \( a = 1.378, b = 0.5, c = 0.6, d = 0.097 \), which is obtained by using the current method. Among them, \((c)\) is the graph projected on \((x, z)\)-plane; \((b)\) is the graph projected on \((x, w)\)-plane; \((e)\) is the graph projected on \((z, w)\)-plane; \((f)\) is the graph in three-dimensional \((x, y, z)\)-space.

Experiment 2. We consider the following hyperchaotic system [26]:

\[
\begin{align*}
  \dot{x} &= a(y-x) + yz, \\
  \dot{y} &= cx - y - xz + w, \\
  \dot{z} &= xy - bz, \\
  \dot{w} &= -xz + dw,
\end{align*}
\]

where \( x, y, z, w \) are the state variables and \( a, b, c, d \) are the positive parameters of the system, which satisfy the following initial conditions:

\[
\begin{align*}
  x(0) &= 1, \\
  y(0) &= 0, \\
  z(0) &= 1, \\
  w(0) &= 0.
\end{align*}
\]

We choose Chebyshev nodes; the number of nodes \( M = 40 \), and the parameters \( b = 0.5, c = 0.8, d = 0.063 \). Numerical results of Experiment 2 are given in Figures 3–6. Figure 3 is phase portraits of a new hyperchaotic system for Experiment 2 with \( a = 0.2 \) by using the current method. \((a)\) is the graph projected on \((x, y)\)-plane; \((b)\) is the graph projected on \((x, z)\)-plane; \((c)\) is the graph projected on \((y, z)\)-plane; \((d)\) is the graph projected on \((y, w)\)-plane; \((e)\) is the graph projected on \((z, w)\)-plane; \((f)\) is the graph in three-dimensional \((x, y, z)\)-space. Figures 4 and 5 are phase portraits of a new hyperchaotic system for Experiment 2 with different parameter value \( a \). \((a)\) and \((d)\) represent time series when \( a = 0.2 \); \((b)\) and \((e)\) represent time series when \( a = 0.3 \); \((c)\) and \((f)\) represent time series when \( a = 0.6 \).

Experiment 3. We consider the following butterfly hyperchaotic system [24]:

\[
\begin{align*}
  \dot{x} &= ax + dz - yz, \\
  \dot{y} &= xz - by, \\
  \dot{z} &= c(x-z) + xy, \\
  \dot{w} &= c(y-w) + xz,
\end{align*}
\]

where \( x, y, z, w \) are the state variables and \( a, b, c, d \) are the positive constant parameters of the system, which satisfy the following initial conditions:

\[
\begin{align*}
  x(0) &= 1, \\
  y(0) &= 0, \\
  z(0) &= 1, \\
  w(0) &= 0.
\end{align*}
\]

We choose Chebyshev nodes; the number of nodes \( M = 30 \). Numerical results of Experiment 3 are given in Figures 7 and 8. Figure 7 is states of a novel butterfly hyperchaotic system for Experiment 3 with \( a = 1, b = 0.5, c = 2, d = 1 \), which is obtained by using the current method. Among them, \((c)\) is the graph projected on \((x, z)\)-plane; \((d)\) is the graph projected on \((x, w)\)-plane; \((e)\) is the graph projected on \((z, w)\)-plane; \((f)\) is the graph in three-dimensional \((x, y, z)\)-space.

Experiment 4. We consider the following hyperchaotic Chen system [27]:

\[
\begin{align*}
  \dot{x} &= a(y-x), \\
  \dot{y} &= (d-z)x + cy - w, \\
  \dot{z} &= xy - bz, \\
  \dot{w} &= x + k,
\end{align*}
\]

where \( x, y, z, w \) are the state variables and \( a, b, c, d, k \) are the positive constant parameters of the system, which satisfy the following initial conditions:
Figure 1: States of the hyperchaotic system for Experiment 1 with $a = 1.378, b = 0.5, c = 0.6, \text{ and } d = 0.097$: (a) $x, y$ states; (b) $z, w$ states.

Figure 2: Hyperchaotic attractors of the system for Experiment 1 with $a = 1.378, b = 0.5, c = 0.6, d = 0.097$ (c) on $x-z$ plane, (d) on $x-w$ plane, (e) on $z-w$ plane, and (f) in $x-y-z$ space.
Figure 3: Phase portraits of a new hyperchaotic system for Experiment 2 with $a = 0.2$ on $x - y$ plane, $(b_1)$ on $x - z$ plane, $(c_1)$ on $y - z$ plane, $(d_1)$ on $y - w$ plane, $(e_1)$ on $z - w$ plane, and $(f_1)$ in $x - y - z$ space.

Figure 4: Phase portraits of a new hyperchaotic system for Experiment 2 with $a = 0.3$ on $x - y$ plane, $(b_2)$ on $x - z$ plane, $(c_2)$ on $y - z$ plane, $(d_2)$ on $y - w$ plane, $(e_2)$ on $z - w$ plane, and $(f_2)$ in $x - y - z$ space.
Figure 5: Phase portraits of a new hyperchaotic system for Experiment 2 with $a = 0.6$ ($a_3$) on $x - y$ plane, ($b_3$) on $x - z$ plane, ($c_3$) on $y - z$ plane, ($d_3$) on $y - w$ plane, ($e_3$) on $z - w$ plane, and ($f_3$) in $x - y - z$ space.

Figure 6: The time series plots of a new hyperchaotic system for Experiment 2. ($a$) and ($d$) represent time series when $a = 0.2$; ($b$) and ($e$) represent time series when $a = 0.3$; ($c$) and ($f$) represent time series when $a = 0.6$. 

Figure 7: States of a novel butterfly hyperchaotic system for Experiment 3 with $a = 1, b = 0.5, c = 2, d = 1$: (a) $x, y$ states; (b) $z, w$ states.

Figure 8: Phase portraits of a novel butterfly hyperchaotic system for Experiment 3 with $a = 1, b = 0.5, c = 2, d = 1$ (c) projected on the $(x, z)$-plane, (d) projected on the $(x, w)$-plane, (e) projected on the $(y, z)$-plane, (f) projected on the $(z, w)$-plane, and (g) in the three-dimensional $(y, z, w)$ space.

We choose Chebyshev nodes; the number of nodes $M = 35$. Numerical results of Experiment 4 are given in Figures 9 and 10.

Figure 9 is time response of the hyperchaotic Chen system's variable states for Experiment 4 with $a = 8, b = 3, c = 4, d = -2, k = 0.2$, which is obtained by using the current

$x(0) = 1,$
$y(0) = 0,$
$z(0) = 1,$
$w(0) = 0.$

(16)
Experiment 4. We consider the following hyperchaotic system [28]:

\[
\begin{align*}
\dot{x} &= a(y - x) + yz, \\
\dot{y} &= cx - xz + kw, \\
\dot{z} &= -bz + xy + 0.5w, \\
\dot{w} &= -dy,
\end{align*}
\]

(17)

where \(a = 8, b = 3, c = 4, d = -2, k = 0.2\). Figure 10 shows phase portraits of the hyperchaotic Chen system for Experiment 4 with these parameters, obtained by using the current method. Among them, (c) is the graph projected on \((x, y)\)-plane; (d) is the graph projected on \((x, z)\)-plane; (e) is the graph projected on \((y, w)\)-plane; (f) is the graph projected on \((w, z)\)-plane; (g) is the graph in three-dimensional \((x, y, z)\)-space; (h) is the graph in three-dimensional \((x, z, w)\)-space.
where \( x, y, z, w \) are the state variables and \( a, b, c, d, k \) are the positive constant parameters of the system, which satisfy the following initial conditions:

\[
\begin{align*}
  x(0) &= 0, \\
  y(0) &= 1, \\
  z(0) &= 2, \\
  w(0) &= 3.
\end{align*}
\]  

(18)

We choose Chebyshev nodes; the number of nodes \( M = 40 \). Numerical results of Experiment 5 are given in Figures 11 and 12.

Figure 11 is the time series plots of a new hyperchaotic system for Experiment 5 with \( a = 15, b = 2.5, c = 0.75, d = 2, k = 0.2 \), which is obtained by using the current method, and (a) is the states of \( x \) and \( y \) and (b) is the states of \( z \) and \( w \). Figure 12 is hyperchaotic attractors of a new hyperchaotic system for Experiment 5 with \( a = 15, b = 2.5, c = 0.75, d = 2, k = 0.2 \), which is obtained by using the current method.
4. Conclusions and Remarks

In this paper, a class of hyperchaotic system has been solved by using barycentric Lagrange interpolation collocation method. The numerical simulation results are in accord with the theoretical analyses and circuit implementation.
Figure 14: Phase portraits of a 4D hyperchaotic system for Experiment 6 with $1/a^2 = 3, 1/b^2 = 3, 1/c^2 = 3, d = 3, k = 1, h = 5$ ($a_2$) projected on the $(x, y)$-plane, ($b_2$) projected on the $(x, z)$-plane, ($c_2$) projected on the $(y, z)$-plane, ($d_2$) projected on the $(y, w)$-plane, and ($e_2$) projected on the $(w, z)$-plane.

Figure 15: The time series plots of a 4D hyperchaotic system for Experiment 6: ($f_1$) and ($g_1$) $1/a^2 = 2.7, 1/b^2 = 1, 1/c^2 = 1.5, d = 3, k = 1, h = 5$; ($f_2$) and ($g_2$) $1/a^2 = 3, 1/b^2 = 3, 1/c^2 = 3, d = 3, k = 1, h = 5$. 

$\begin{array}{c}
\text{(a)}
\end{array}$
Numerical simulations are provided to verify the effectiveness and feasibility of the proposed numerical results, which are in agreement with theoretical analysis. In the further work, we will be devoted to studying fractional-order hyperchaotic system.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this article.

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