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Research Article

Stability and Bifurcation for a Single-Species Model with Delay Weak Kernel and Constant Rate Harvesting

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In this paper, we consider the effect of constant rate harvesting on the dynamics of a single-species model with a delay weak kernel. By a simple transformation, the single-species model is transformed into a two-dimensional system. The existence and the stability of possible equilibria under different conditions are carried out by analysing the two-dimensional system. We show that there exists a critical harvesting value such that the population goes extinct in finite time if the constant rate harvesting u is greater than the critical value, and there exists a degenerate critical point or a saddle-node bifurcation when the constant rate harvesting u equals the critical value. When the constant rate harvesting u is less than the critical value, sufficient conditions about the existence of the Hopf bifurcation are derived by topological normal form for the Hopf bifurcation and calculating the first Lyapunov coefficient. The key results obtained in the present paper are illustrated using numerical simulations. These results indicate that it is important to select the appropriate constant rate harvesting u.

1. Introduction

Ecological population dynamics is an important research field of mathematical biology. The single-species model is the cornerstone of research for mathematical biology. As early as 1798, Malthus [1] put forward the famous Malthus population model, that is,

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t), \tag{1}$$

where $N(t)$ represents the unit density of population at time t, and $r > 0$ is the intrinsic rate of growth for population. Considering the carrying capacity of the environment, Verhulst [2] improved the Malthus population model to logistic equation in 1838:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - \frac{N(t)}{K}\right), \tag{2}$$

where K is the maximum carrying capacity of the environment. Logistic equation successfully predicted the total population of the world and was widely used to study the dynamical behavior of single species. Especially because of its simple form, clear biological meaning of model parameters, and clear dynamic behavior, it has very important applications in many fields such as ecology [3], biological resource management [4], life science [5], cell and molecular biology [6], biostatistics [7, 8], stock market [9], and medicine [10–12]. Other continuous single-species models include Gompertz model, Food-limit model, Allee model, Rosenzweig model, and Beverton–Holt model (see [13] and references therein).

Time-delay phenomenon is an important factor affecting the stability of systems. Many scholars in the field of mathematical biology solved biological problems by studying this phenomenon, for example, discrete delay [14], delay dependent parameters [15], distributed delays [16], variable delays [17], and stochastic model with delays [18–22]. The logistic growth model with discrete delay τ (Hutchinson [23]) is governed by

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - \frac{N(t-\tau)}{K}\right). \tag{3}$$

In 1934, in order to solve the problem that Verhulst's logistic equation could not be applied due to population disappearance, Volterra [24] established a more accurate model than the former model:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(t)\left(1 - \frac{1}{K}\int_{-\infty}^{t} G(t-s)N(s)\mathrm{d}s\right), \tag{4}$$

where $G(t)$, called the delay kernel, is a weighting factor which says how much emphasis should be given to the size of the population at earlier times to determine the present effect on resource availability. The delay kernel is usually normalized so that $\int_0^{\infty} G(u)\mathrm{d}u = 1$. Two special cases,

$$\begin{aligned} G(u) &= \alpha e^{-\alpha u}, \\ G(u) &= \alpha^2 u e^{-\alpha u}, \end{aligned} \tag{5}$$

are called weak delay kernel and strong delay kernel, respectively. Let $G(u) = \alpha e^{-\alpha u}$, Volterra's model [25] follows the form

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - \frac{1}{K}\int_{-\infty}^{t} \alpha e^{-\alpha(t-s)}N(s)\mathrm{d}s\right). \tag{6}$$

In 1977, Cushing [25] proposed the following single-species model with weak delay kernel:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = rx(t)\left(1 - ax(t) - \omega\int_{-\infty}^{t} \alpha e^{-\alpha(t-s)}x(s)\mathrm{d}s\right), \tag{7}$$

and detailed qualitative results of this model were obtained.

The study concerning population dynamics with harvesting is a subject of mathematical bioeconomics. The study on mathematical models of bioeconomics for resource management can be found in [4, 26–34]. Renewable resources, which are vital to human survival, are increasingly being overexploited. In the light of the reduction of resource stocks and the deterioration of the environment, the exploitation and management of renewable resources has become a major issue concerned nowadays. In order to protect the natural environment which human beings depend on, the development and utilization of renewable resources must be moderate. The maximum sustained yield (MSY) with the minimum effort is what we want. The MSY is the largest harvest rate that can be sustained indefinitely. If the harvest rate of a population is more than its MSY, the population will go extinct. So the determination of the MSY is very important. Generally, harvesting strategies include constant harvesting, linear harvesting, time-dependent harvesting, impulsive harvesting, and seasonal harvesting. Which harvesting strategy can be used to achieve the best economic benefits under the premise of ensuring the sustainable development of species and ecological environment? It is a question we need to answer. Harvesting models are introduced by Kot [3] and Murray [35]. A model with a constant rate harvesting Y_0 was studied by Brauer and Sanchez [36]. The model is

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = rN(t)\left(1 - \frac{N(t)}{K}\right) - Y_0. \tag{8}$$

Ludwig et al. [37] considered the following equation with harvesting function $p(N)$ on the budworm population dynamics:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = r_B N(t)\left(1 - \frac{N(t)}{K_B}\right) - p(N(t)). \tag{9}$$

Population models with different harvesting strategies have been studied by scholars, for example, impulsive harvesting [38–44], constant rate harvesting [45–48], optimal harvesting strategy [49], linear harvesting [50, 51], and nonlinear harvesting [52, 53].

Volterra's model [24] and Cushing's model [25] can sufficiently describe the deterministic dynamical behaviors. However, it is inevitable for us to encounter some questions for resource management. Hence, the models do not effectively predict population behaviors in reality. Due to the size of the population at earlier times to determine the present effect on resource availability, motivated by the above work, introducing a constant rate harvesting u into the Volterra's model [24], we consider a single-species model with a delay weak kernel and a constant rate harvesting:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = rx(t)\left(1 - \frac{1}{K}\int_{-\infty}^{t} \alpha e^{-\alpha(t-s)}x(s)\mathrm{d}s\right) - u, \tag{10}$$

where $x(t)$ represents the unit density of the species at time t, $r > 0$ represents the intrinsic growth rate of the species (reflecting the characteristics of the species itself), $K > 0$ is carrying capacity, $\alpha e^{-\alpha(t-s)}$ is a common weak kernel function, and u stands for constant rate harvesting, here α and u are positive constants.

Let $y(t) = \int_{-\infty}^{t} \alpha e^{-\alpha(t-s)}x(s)\mathrm{d}s$, we have the following system:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = rx(t)\left(1 - \dfrac{1}{K}y(t)\right) - u, \\[2ex] \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = \alpha x(t) - \alpha y(t). \end{cases} \tag{11}$$

The purpose of this paper is to study the effect of constant rate harvesting u in system (11). The present paper is built up as follows. In Section 2, we give a simple analysis of system (11) and some lemmas and a definition are given. In Section 3, the existence of degenerate critical point and saddle-node bifurcation of system (11) is analysed. The stability of the equilibria and the existence of Hopf bifurcation of system (11) are discussed in Section 4. In order to illustrate the theoretical analysis, Section 5 gives some examples and their simulations. Finally, a brief conclusion is given in Section 6.

2. A Simple Analysis of System and Preliminaries

In this section, a simple analysis of system (11) is performed, and some lemmas and a definition are given.

Let the right-hand side of system (11) equal to zero, that is,

$$\begin{cases} \dfrac{dx(t)}{dt} = rx(t)\left(1 - \dfrac{1}{K}y(t)\right) - u := P(x(t), y(t)) = 0, \\ \dfrac{dy(t)}{dt} = \alpha x(t) - \alpha y(t) := Q(x(t), y(t)) = 0. \end{cases} \tag{12}$$

By the second formula of system (12) $\alpha x(t) - \alpha y(t) = 0$, i.e., $x(t) = y(t)$, and substituting it into the first formula, we obtain the quadratic equation $rx(t)[1 - (1/K)x(t)] - u = 0$. In this system, denote Δ as $\Delta = r^2 - (4ur/K)$, which is a discriminant of quadratic equation $rx(t)[1 - (1/K)x(t)] - u = 0$. Obviously, the equation $rx(t)[1 - (1/K)x(t)] - u = 0$ has two different real roots if $\Delta > 0$, i.e., $u < (1/4)Kr$; the equation $rx(t)[1 - (1/K)x(t)] - u = 0$ has two identical real roots if $\Delta = 0$, i.e., $u = (1/4)Kr$; the equation $rx(t)[1 - (1/K)x(t)] - u = 0$ has no real roots for $\Delta < 0\,(u > (1/4)Kr)$. From the above analysis, we can find $dx(t)/dt < 0$ as $u > (1/4)Kr$, and there is no equilibrium in system (11) in this case.

Furthermore, we get the Jacobian matrix of system (11):

$$A = \begin{pmatrix} \dfrac{\partial P}{\partial x} & \dfrac{\partial P}{\partial y} \\ \dfrac{\partial Q}{\partial x} & \dfrac{\partial Q}{\partial y} \end{pmatrix} = \begin{pmatrix} r\left(1 - \dfrac{1}{K}y(t)\right) & -\dfrac{r}{K}x(t) \\ \alpha & -\alpha \end{pmatrix}. \tag{13}$$

Naturally, the determinant D of the Jacobian matrix A is written by the following calculation:

$$D = \det(A) = \begin{vmatrix} r\left(1 - \dfrac{1}{K}y(t)\right) & -\dfrac{r}{K}x(t) \\ \alpha & -\alpha \end{vmatrix}. \tag{14}$$

The trace T of the matrix A also can be obtained as follows:

$$T = tr(A) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = r\left(1 - \frac{1}{K}y(t)\right) - \alpha. \tag{15}$$

Because system (11) has no equilibrium and the population $x(t)$ is reduced to extinction in finite time if $u > (1/4)Kr$, we will not discuss this case. Therefore, we analyse the stability of the equilibria and the existence of bifurcations of system (11) under two different cases: $\Delta = 0$ and $\Delta > 0$.

Before the main results are given, some lemmas and a definition are introduced.

Lemma 1 (see [54]). *Given the rank of the matrix A is equal to 1 (i.e., $rank(A) = 1$) and the trace of the matrix A is equal to 0 (that is $T = tr(A) = 0$) in system as follows:*

$$\frac{dx}{dt} = f(t, x). \tag{16}$$

System (16) can be transformed into the following system by a one-to-one transformation in a neighborhood of the origin O:

$$\begin{cases} \dfrac{dx}{dt} = y, \\ \dfrac{dy}{dt} = Q_2(x, y), \end{cases} \tag{17}$$

where $Q_2(x, y)$ is analytic, with terms of order no less than 2. Suppose

$$Q_2(x, y) = a_l x^l (1 + h(x)) + b_n x^n y(1 + g(x)) + y^2 f(x, y), \tag{18}$$

where $h(x)$, $g(x)$, and $f(x, y)$ are analytic, $h(0) = g(0) = 0$, $l \geq 2$, $a_l \neq 0$, n is a natural number, and O is the lth critical point.

If $l = 2m + 1$ is an odd number, let $\Delta_l = b_n^2 + 4(m + 1)a_l$, then the following holds:

(1) If $a_l > 0$, then the critical point O is a saddle, its index $I(O) = -1$
(2) If (i) $a_l < 0$, $b_n \neq 0$, n is an even number, $n < m$, or (ii) $a_l < 0$, $b_n \neq 0$, n is an even number, $n = m$, $\Delta_l \geq 0$, then the critical point O is a node, its index $I(O) = +1$
(3) If (i) $a_l < 0$, $b_n \neq 0$, n is an odd number, $n < m$, or (ii) $a_l < 0$, $b_n \neq 0$, n is an odd number, $n = m$, $\Delta_l \geq 0$, then O is a critical point with an elliptic domain, its index $I(O) = +1$
(4) If (i) $a_l < 0$, $b_n = 0$, or $a_l < 0$, $b_n \neq 0$, and $n > m$, or (ii) $a_l < 0$, $b_n \neq 0$, $n = m$, $\Delta_l < 0$, then the critical point O is a center or focal, its index $I(O) = +1$

If $l = 2m$ is an even number, then the following holds:

(1) If $b_n = 0$ or $b_n \neq 0$ and $n \geq m$, then O is a degenerate critical point, its index $I(O) = 0$
(2) If $b_n \neq 0$, $1 \leq n < m$, then the critical point O is a saddle-node, its index $I(O) = 0$

Lemma 2 (see [54]). *Assume that the rank of matrix A is equal to 1, i.e., $rank(A) = 1$ and the trace of matrix A satisfies $tr(A) = T \neq 0$ in system (16), system (16) can be transformed into*

$$\begin{cases} \dfrac{dx}{dt} = P_2(x, y) = P(x, y), \\ \dfrac{dy}{dt} = y + Q_2(x, y) = Q(x, y), \end{cases} \tag{19}$$

where P_2 and Q_2 are analytic functions of order no less than 2. So $Q = 0$ ensures the existence of $G(x)$ (or the existence of $G(y)$). If $G(x) = a_l x^l + \cdots$, $l \geq 2$ $(a_l \neq 0)$, then the following properties are satisfied:

(1) If l is an odd number and $a_l > 0$, then O is a node, the index $I(O) = +1$
(2) If l is an odd number and $a_l < 0$, then O is a saddle point, its index $I(O) = -1$
(3) If l is an even number, then O is a saddle-node, the index $I(O) = 0$

Lemma 3 (see [55]). *Assuming that $\gamma = 0$, we stand a function $\varphi(x, 0)$ for*

$$\varphi(x,0) = \frac{1}{2}B(x,x) + \frac{1}{6}C(x,x,x) + O\left(\|x\|^4\right). \tag{20}$$

In this formula, $B(\delta, \eta)$ and $C(\delta, \eta, \zeta)$ are symmetric multilinear vector functions. We have

$$B_i(\delta,\eta) = \sum_{j,k=1}^{2} \left.\frac{\partial^2 \varphi_i(\xi,0)}{\partial\xi_j \partial\xi_k}\right|_{\xi=0} \delta_j \eta_k, \quad i = 1,2, \tag{21}$$

$$C_i(\delta,\eta,\zeta) = \sum_{j,k,l=1}^{2} \left.\frac{\partial^3 \varphi_i(\xi,0)}{\partial\xi_j \partial\xi_k \partial\xi_l}\right|_{\xi=0} \delta_j \eta_k \zeta_l, \quad i = 1,2. \tag{22}$$

Moreover, the Taylor coefficients g_{kl} of the equation

$$B(zq + \overline{zq}, zq + \overline{zq}) = z^2 B(q,q) + 2z\overline{z}B(q,\overline{q}) + \overline{z}^2 B(\overline{q},\overline{q}), \tag{23}$$

can be denoted by the following equations:

$$\begin{aligned} g_{20} &= \langle p, B(q,q)\rangle, \\ g_{11} &= \langle p, B(q,\overline{q})\rangle, \\ g_{02} &= \langle p, B(\overline{q},\overline{q})\rangle, \\ g_{21} &= \langle p, C(q,q,\overline{q})\rangle, \end{aligned} \tag{24}$$

where the vector $\overline{q}$ is the conjugate vector of q and the vector $\overline{z}$ is the conjugate vector of z.

Definition 1 (see [55]). The first Lyapunov coefficient denotes as $l_1(\beta)$, and when $\beta = 0$, the following statement is completely true:

$$l_1(0) = \frac{1}{2\omega_0^2} Re\left(i g_{20} g_{11} + \omega g_{21}\right), \tag{25}$$

where the value of $l_1(0)$ depends on the normalization of the eigenvectors p and q. The most important thing in bifurcation analysis is that the sign of $l_1(0)$ does not change under $\langle p, q\rangle = 1$.

Lemma 4 (see [55]). *Suppose a two-dimensional system,*

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x,\gamma), \tag{26}$$

with smooth f, has for all sufficiently small $|\gamma|$ the equilibrium $x = 0$ with eigenvalues,

$$\lambda_{1,2}(\gamma) = \mu(\gamma) \pm i\omega(\gamma), \tag{27}$$

where $\mu(0) = 0$, $\omega(0) = \omega_0 > 0$. Let the following conditions be satisfied:

(1) $l_1(0) \neq 0$, *where l_1 is the first Lyapunov coefficient*

(2) $\mu'(0) \neq 0$

Then, there are invertible coordinate and parameter changes and a time reparameterization transforming into

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \beta & -1 \\ 1 & \beta \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \pm \left(y_1^2 + y_2^2\right)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + O\left(\|y\|^4\right). \tag{28}$$

Lemma 5 (see [55]). *Any generic two-dimensional, one-parameter system (topological normal form for the Hopf bifurcation):*

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x,\gamma), \tag{29}$$

having at $\gamma = 0$ the equilibrium $x = 0$ with eigenvalues,

$$\lambda_{1,2}(0) = \pm i\omega_0, \quad \omega_0 > 0, \tag{30}$$

is locally topologically equivalent near the origin to one of the following normal forms:

$$\begin{pmatrix} \dfrac{\mathrm{d}y_1}{\mathrm{d}t} \\ \dfrac{\mathrm{d}y_2}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} \beta & -1 \\ 1 & \beta \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \pm \left(y_1^2 + y_2^2\right)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}. \tag{31}$$

3. Degenerate Critical Point and Saddle-Node Bifurcation

In this section, we investigate the properties of the equilibrium under $\Delta = 0$, i.e., $u = (1/4)Kr$. System (11) has two different cases for the discriminant $\Delta = 0$. One case is linear equations of system (11) with one zero eigenvalue and one nonzero eigenvalue, and the other case is linear equations of system (11) with two zero eigenvalues.

If $u = (1/4)Kr$, the solution of the equation (12) is

$$x_1^* = y_1^* = \frac{K}{2}. \tag{32}$$

So system (11) has a unique equilibrium $E_1(x_1^*, y_1^*) = (K/2, K/2)$. Substituting $x_1^* = y_1^* = K/2$ into (13) and (14), A_1^* and D_1^* are as follows:

$$\begin{aligned} A_1^* &= \begin{pmatrix} \dfrac{r}{2} & -\dfrac{r}{2} \\ \alpha & -\alpha \end{pmatrix}, \\ D_1^* &= \begin{vmatrix} \dfrac{r}{2} & -\dfrac{r}{2} \\ \alpha & -\alpha \end{vmatrix} = 0. \end{aligned} \tag{33}$$

Consequently,

$$\det\left(\lambda I - A_1^*\right) = \left|\lambda I - A_1^*\right| = \begin{vmatrix} \lambda - \dfrac{r}{2} & \dfrac{r}{2} \\ -\alpha & \lambda + \alpha \end{vmatrix} = \lambda^2 + \left(\alpha - \frac{r}{2}\right)\lambda, \tag{34}$$

where I denotes the identity matrix. Calculating $|\lambda I - A_1^*| = 0$, one can obtain the eigenvalues of matrix A_1^*: $\lambda_1 = 0$ and $\lambda_2 = (r/2) - \alpha$.

Denote X_1 and Y_1 as $X_1 = x(t) - (K/2)$ and $Y_1 = y(t) - (K/2)$, system (11) can be transformed into

$$\begin{cases} \dfrac{dX_1}{dt} = \dfrac{r}{2}(X_1 - Y_1) - \dfrac{r}{K}X_1Y_1, \\ \\ \dfrac{dY_1}{dt} = \alpha(X_1 - Y_1). \end{cases} \quad (35)$$

Theorem 1. *If $u = (1/4)Kr$ and $\alpha = (r/2)$, then the unique equilibrium $E_1(K/2, K/2)$ of system (11) is a degenerate critical point and system (11) will not admit any closed orbits. If $u = (1/4)Kr$ and $\alpha \neq r/2$, then the unique equilibrium $E_1(K/2, K/2)$ of system (11) is a saddle-node and system (11) will not also admit any closed orbits.*

Proof. It is easy to obtain that $\lambda_1 = \lambda_2 = 0$ if $u = (1/4)Kr$ and $\alpha = r/2$; in the other word, the linear equations of system (11) has two zero eigenvalues.

In this case, substituting $\alpha = r/2$ into system (35), we get

$$\begin{cases} \dfrac{dX_1}{dt} = \dfrac{r}{2}(X_1 - Y_1) - \dfrac{r}{K}X_1Y_1, \\ \\ \dfrac{dY_1}{dt} = \dfrac{r}{2}(X_1 - Y_1). \end{cases} \quad (36)$$

Let

$$\begin{pmatrix} s \\ v \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{2}{r} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \quad (37)$$

that is, $s = (2/r)Y_1$ and $v = X_1 - Y_1$.

Take the derivatives of s and v, respectively, and the following system can be obtained:

$$\begin{cases} \dfrac{ds}{dt} = v, \\ \\ \dfrac{dv}{dt} = -\dfrac{r^3}{4K}s^2 - \dfrac{r^2}{2K}sv. \end{cases} \quad (38)$$

Since O is an isolated equilibrium of system (38), it is assumed

$$\begin{aligned} \frac{dv}{dt} = -\frac{r^3}{4K}s^2 - \frac{r^2}{2K}sv = a_l s^l (1 + h(s)) + b_n s^n v (1 + g(s)) \\ + v^2 f(s, v), \end{aligned} \quad (39)$$

where $h(s)$, $g(s)$, and $f(s, v)$ are analytic, $h(0) = g(0) = 0$. By Lemma 1, $Q_2(s, v) = a_l s^l + b_n s^n v$. On account of $l = 2 = 2m$ is an even number, $b_1 = -r^2/2K \neq 0$ and $1 = n = m$, we can obtain that the equilibrium $E_1(K/2, K/2)$ is a degenerate critical point by Lemma 1. It is a degenerate critical point whose index is not one, so it has no closed orbits if system (11) has a unique equilibrium.

If $u = (1/4)Kr$ and $\alpha \neq r/2$ in system (11), then the linear equations of system (11) has one zero eigenvalue and one nonzero eigenvalue.

Let

$$\begin{pmatrix} w \\ v \end{pmatrix} = \begin{pmatrix} \dfrac{4\alpha}{r(2\alpha - r)} & -\dfrac{2}{2\alpha - r} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \end{pmatrix}, \quad (40)$$

and take the derivatives of w and v, system (35) is replaced by

$$\begin{cases} \dfrac{dw}{dt} = -\dfrac{\alpha r^2}{K(2\alpha - r)}w^2 + \dfrac{2\alpha r(2\alpha + r)}{K(2\alpha - r)^2}wv - \dfrac{8\alpha^2 r}{K(2\alpha - r)^3}v^2, \\ \\ \dfrac{dv}{dt} = -\dfrac{r^3}{4K}w^2 + \left(\dfrac{r}{2} - \alpha\right)v + \dfrac{r^2(2\alpha + r)}{2K(2\alpha - r)}wv - \dfrac{2\alpha r^2}{K(2\alpha - r)^2}v^2. \end{cases} \quad (41)$$

We know that $dv/dt = 0$ can ensure the existence of $G(w)$ by Lemma 2. Let $dv/dt = 0$, we have

$$v = \frac{2}{r - 2\alpha}\left[\frac{r^3}{4K}w^2 - \frac{r^2(2\alpha + r)}{2K(2\alpha - r)}wv - \frac{2\alpha r^2}{K(2\alpha - r)^2}v^2\right]. \quad (42)$$

Substituting equation (42) to

$$\frac{dw}{dt} = -\frac{\alpha r^2}{K(2\alpha - r)}w^2 + \frac{2\alpha r(2\alpha + r)}{K(2\alpha - r)^2}wv - \frac{8\alpha^2 r}{K(2\alpha - r)^3}v^2, \quad (43)$$

we have

$$\frac{dw}{dt} = -\frac{\alpha r^2}{K(2\alpha - r)}w^2 + O(3). \quad (44)$$

Because $l = 2$ is an even number, we can obtain that the equilibrium $E_1(K/2, K/2)$ is a saddle-node by Lemma 2. Because the unique equilibrium point $E_1(K/2, K/2)$ is a saddle-node whose index is not one, system (11) does not admit any closed orbits. □

4. Hopf Bifurcation

In this section, the case $\Delta > 0$, that is $u < (1/4)Kr$, is discussed. If $u < (1/4)Kr$, system (11) has two equilibria:

$$\begin{aligned} E_2(x_2^*, y_2^*) &= \left(\frac{K}{2} - \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}, \frac{K}{2} - \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}\right), \\ E_3(x_3^*, y_3^*) &= \left(\frac{K}{2} + \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}, \frac{K}{2} + \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}\right). \end{aligned} \quad (45)$$

We analyse and discuss qualitative characteristics of these two equilibria, respectively.

Theorem 2. *The equilibrium $E_2(x_2^*, y_2^*)$ is a saddle point if $u < (1/4)Kr$ in system (11).*

Proof. The determinant D_2^* of the linear matrix corresponding to $E_2(x_2^*, y_2^*)$ of system (11) is as follows:

$$D_2^* = \begin{vmatrix} \frac{r}{2}\left(1+\sqrt{1-\frac{4u}{Kr}}\right) & -\frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right) \\ \alpha & -\alpha \end{vmatrix} = -\alpha r\sqrt{1-\frac{4u}{Kr}}, \tag{46}$$

where $r > 0$ and α is a positive constant, so $D_2^* < 0$. Obviously, $E_2(x_2^*, y_2^*)$ is a saddle point. □

Theorem 3. *If $u < (1/4)Kr$ in system (11), then the following holds:*

(1) If $2\alpha > r(1 - \sqrt{1-(4u/Kr)})$, then $E_3(x_3^*, y_3^*)$ is asymptotically stable

(2) If $2\alpha < r(1 - \sqrt{1-(4u/Kr)})$, then $E_3(x_3^*, y_3^*)$ is unstable

(3) If $2\alpha = r(1 - \sqrt{1-(4u/Kr)})$, a unique and stable limit cycle bifurcation emerges via the Hopf bifurcation from the equilibrium $E_3(x_3^*, y_3^*)$ for small enough:

$$T = \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right) - \alpha > 0. \tag{47}$$

Proof. The linear matrix A_3^* corresponding to the equilibrium $E_3(x_3^*, y_3^*)$ of system (11) is governed by

$$A_3^* = \begin{pmatrix} \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right) & -\frac{r}{2}\left(1+\sqrt{1-\frac{4u}{Kr}}\right) \\ \alpha & -\alpha \end{pmatrix}, \tag{48}$$

$D_3^* = \alpha r\sqrt{1-(4u/Kr)}$ is the determinant of Jacobian matrix A_3^*, where $r > 0$ and α is a positive constant, so $D_3^* > 0$.

It is easy to derive the trace T of the matrix A_3^* from the following formula:

$$T = \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right) - \alpha. \tag{49}$$

Obviously, if $T < 0$, the real part $\mathrm{Re}\lambda$ of the eigenvalues of A_3^* satisfies $\mathrm{Re}\lambda_i < 0\ (i = 1, 2)$, so $E_3(x_3^*, y_3^*)$ is asymptotically stable; if $T > 0$, the real part $\mathrm{Re}\lambda$ of the eigenvalues of A_3^* satisfies $\mathrm{Re}\lambda_i > 0\ (i = 1, 2)$, so $E_3(x_3^*, y_3^*)$ is unstable. This completes the proofs of (1) and (2).

Denote

$$\mu(\alpha) = \frac{T}{2} = \frac{r}{4}\left(1-\sqrt{1-\frac{4u}{Kr}}\right) - \frac{\alpha}{2}. \tag{50}$$

Let

$$\alpha_0 = \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right), \tag{51}$$

we have $\mu(\alpha_0) = 0$. The condition (2) of Lemma 4 is easy to verify:

$$\mu'(\alpha_0) = -\frac{1}{2} < 0. \tag{52}$$

Moreover,

$$\omega^2(\alpha_0) = \frac{r^2}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right)\sqrt{1-\frac{4u}{Kr}} > 0. \tag{53}$$

To compute the first Lyapunov coefficient, denote X_2 and Y_2 as

$$\begin{aligned} X_2 &= x(t) - \frac{K}{2}\left(1+\sqrt{1-\frac{4u}{Kr}}\right), \\ Y_2 &= y(t) - \frac{K}{2}\left(1+\sqrt{1-\frac{4u}{Kr}}\right). \end{aligned} \tag{54}$$

System (11) is converted into

$$\begin{cases} \frac{\mathrm{d}X_2}{\mathrm{d}t} = \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right)X_2 - \frac{r}{2}\left(1+\sqrt{1-\frac{4u}{Kr}}\right)Y_2 \\ \quad -\frac{r}{K}X_2Y_2, \\ \frac{\mathrm{d}Y_2}{\mathrm{d}t} = \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right)X_2 - \frac{r}{2}\left(1-\sqrt{1-\frac{4u}{Kr}}\right)Y_2. \end{cases} \tag{55}$$

Perform the transformation

$$\begin{aligned} M: \xi_1 &= X_2 - Y_2, \\ \xi_2 &= Y_2. \end{aligned} \tag{56}$$

The transformation M transforms the system into

$$\begin{cases} \frac{\mathrm{d}\xi_1}{\mathrm{d}t} = -ar\xi_2 - \frac{r}{K}\xi_1\xi_2 - \frac{r}{K}\xi_2^2 \equiv F_1(\xi_1, \xi_2), \\ \frac{\mathrm{d}\xi_2}{\mathrm{d}t} = \frac{r}{2}(1-a)\xi_1 \equiv F_2(\xi_1, \xi_2), \end{cases} \tag{57}$$

where $a = \sqrt{1-(4u/Kr)}, 0 < a < 1$. The Jacobian matrix A_4^* of system (57) at $O(0, 0)$ is

$$A_4^* = \begin{pmatrix} 0 & -ar \\ \frac{r}{2}(1-a) & 0 \end{pmatrix}. \tag{58}$$

The determinant D_4^* of the Jacobian matrix A_4^* evaluates

$$D_4^* = \begin{vmatrix} 0 & -ar \\ \frac{r}{2}(1-a) & 0 \end{vmatrix}. \tag{59}$$

By

$$\det\left(\lambda I - D_4^*\right) = \begin{vmatrix} \lambda & ar \\ -\frac{r}{2}(1-a) & \lambda \end{vmatrix} = \lambda^2 + \frac{r^2}{2}\left(1 - \sqrt{1 - \frac{4u}{Kr}}\right)\sqrt{1 - \frac{4u}{Kr}} = 0, \tag{60}$$

we can calculate eigenvalues

$$\lambda_{1,2} = \pm\sqrt{\frac{a(1-a)}{2}}ri. \tag{61}$$

Thus, it is easy to find complex vectors

$$q \sim \begin{pmatrix} \frac{a(1-a)}{2} \\ -\frac{1-a}{2}\sqrt{\frac{a(1-a)}{2}}i \end{pmatrix}, \quad p \sim \begin{pmatrix} \frac{1-a}{2}\sqrt{\frac{a(1-a)}{2}} \\ -\frac{a(1-a)}{2}i \end{pmatrix}. \tag{62}$$

In order to accomplish the normalization $\langle p, q\rangle = 1$ from the expressions (62), we can take, for example,

$$q = \begin{pmatrix} \frac{a(1-a)}{2} \\ -\frac{1-a}{2}\sqrt{\frac{a(1-a)}{2}}i \end{pmatrix}, \quad p = \frac{1}{\left(\left(a(1-a)^2\right)/2\right)\sqrt{(a(1-a))/2}} \begin{pmatrix} \frac{1-a}{2}\sqrt{\frac{a(1-a)}{2}} \\ -\frac{a(1-a)}{2}i \end{pmatrix}. \tag{63}$$

To compute $B(\delta, \eta)$ and $C(\delta, \eta, \zeta)$ in Lemma 3, let $\delta = (\delta_1, \delta_2)^T$, $\eta = (\eta_1, \eta_2)^T$, and $\zeta = (\zeta_1, \zeta_2)^T$. On the basis of the (21) and (22), we find

$$B(\delta, \eta) = \begin{pmatrix} -\frac{2r}{K}\delta_2\eta_2 - \frac{r}{K}(\delta_1\eta_2 + \delta_2\eta_1) \\ 0 \end{pmatrix}, \tag{64}$$

$$C(\delta, \eta, \zeta) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{65}$$

Substituting the value of the vector q and the conjugate vector $\overline{q}$ of q into equations (64) and (65), we obtain

$$B(q, q) = \begin{pmatrix} \frac{ar(1-a)^3}{4K} + \frac{ar(1-a)^2}{2K}\sqrt{\frac{a(1-a)}{2}}i \\ 0 \end{pmatrix}, \quad B(q, \overline{q}) = \begin{pmatrix} -\frac{ar(1-a)^3}{4K} \\ 0 \end{pmatrix}, \quad C(q, q, \overline{q}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{66}$$

Basing on Lemma 3, we can calculate

$$g_{20} = \langle p, B(q, q)\rangle = \frac{r(1-a)^2 + 2(1-a)\sqrt{(a(1-a))/2}ri}{4K}, \tag{67}$$

$$g_{11} = \langle p, B(q, \overline{q})\rangle = -\frac{r(1-a)^2}{4K}, \tag{68}$$

$$g_{21} = \langle p, C(q, q, \overline{q})\rangle = 0. \tag{69}$$

Finally, the first Lyapunov coefficient is calculated; as the following equation shows, substitute the formulas (67)–(69) into formula (25):

$$l_1(\alpha_0) = \frac{1}{2\omega^2}\operatorname{Re}\left(ig_{20}g_{11} + \omega g_{21}\right) = \frac{(1-a)^2}{8aK^2}\sqrt{\frac{a(1-a)}{2}} > 0. \tag{70}$$

It is clear that the condition (1) of Lemma 4 holds as $l_1(\alpha_0) > 0$. Therefore, by Lemmas 4 and 5, there exists a unique and stable limit cycle bifurcation via the Hopf bifurcation from the equilibrium $E_3(x_3^*, y_3^*)$ for small enough $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$,

For simplicity, we use α as bifurcation parameter in this section. If u is chosen as bifurcation parameter, the conditions obtained can be converted. So we have the following remark: □

Remark 1. If $u < (1/4)Kr$ in system (11), then $E_2(x_2^*, y_2^*)$ is a saddle and $u = \alpha K(1 - (\alpha/r))$ is a bifurcation point. The following holds:

(1) $E_3(x_3^*, y_3^*)$ is stable if $u < \min\{\alpha K(1 - (\alpha/r)), (1/4)Kr\}$

(2) $E_3(x_3^*, y_3^*)$ is unstable if $(1/4)Kr > u > \alpha K(1 - (\alpha/r))$, a unique and stable limit cycle bifurcation emerges near $E_3(x_3^*, y_3^*)$ for small enough $T(u_0) = u_0 - \alpha K(1 - (\alpha/r)) > 0$.

5. Some Examples and Their Simulations

The main results obtained are listed in Table 1 by the previous analysis.

Table 1: The results under different conditions.

Conditions	Subconditions	Equilibria	Stability
$u = Kr/4$	$\alpha = r/2$	$E_1(K/2, K/2)$	$E_1(K/2, K/2)$ is a degenerate critical point, system (11) has no limit circle
	$\alpha \neq r/2$	$E_1(K/2, K/2)$	$E_1(K/2, K/2)$ is a saddle-node, system (11) has no limit circle
$u < Kr/4$	$2\alpha > r(1 - \sqrt{1-(4u/Kr)})$	$E_2(x_2^*, y_2^*)$, $E_3(x_3^*, y_3^*)$	$E_2(x_2^*, y_2^*)$ is a saddle, $E_3(x_3^*, y_3^*)$ is asymptotically stable
	$2\alpha < r(1 - \sqrt{1-(4u/Kr)})$	$E_2(x_2^*, y_2^*)$, $E_3(x_3^*, y_3^*)$	$E_2(x_2^*, y_2^*)$ is a saddle, $E_3(x_3^*, y_3^*)$ is unstable
	$2\alpha = r(1 - \sqrt{1-(4u/Kr)})$	$E_2(x_2^*, y_2^*)$, $E_3(x_3^*, y_3^*)$	$E_2(x_2^*, y_2^*)$ is a saddle, a Hopf bifurcation occurs near $E_3(x_3^*, y_3^*)$ for small enough $T = (r/2)(1 - \sqrt{1-(4u/Kr)}) - \alpha > 0$
$u > Kr/4$			System (11) has no equilibrium point and the population $x(t)$ is reduced to extinction in finite time

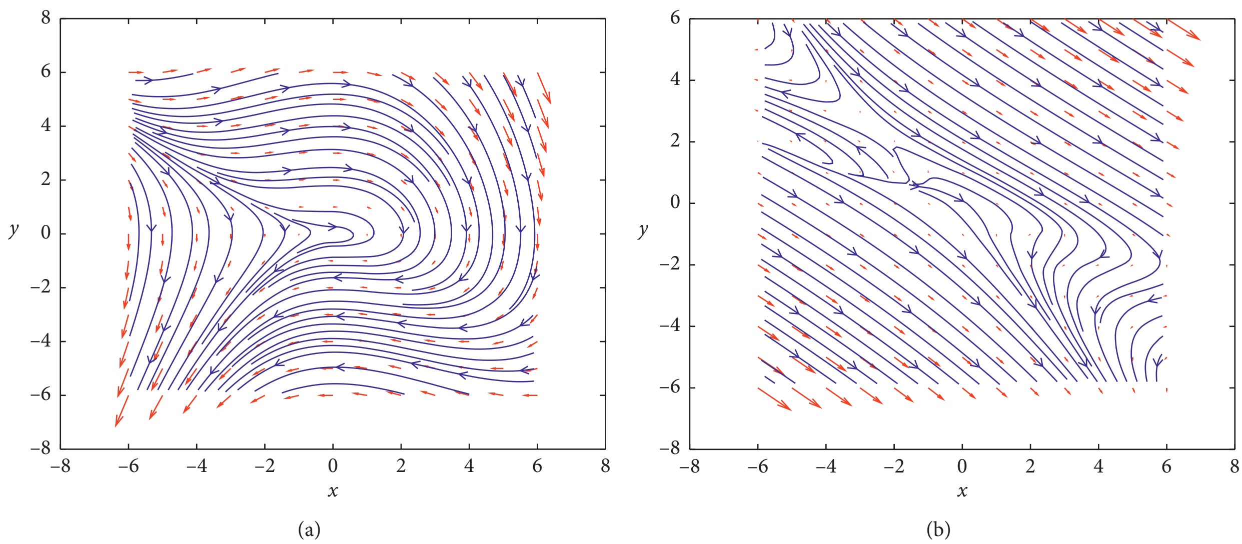


Figure 1: The vector fields for the transformation system of model (71) when $u = Kr/4$; here $u = 4$, $K = 8$, and $r = 2$. (a) The vector fields for the transformation system of model (71) for $\alpha = 1 = r/2$; (b) the vector fields for the transformation system of model (71) for $\alpha = 0.6 \neq r/2$.

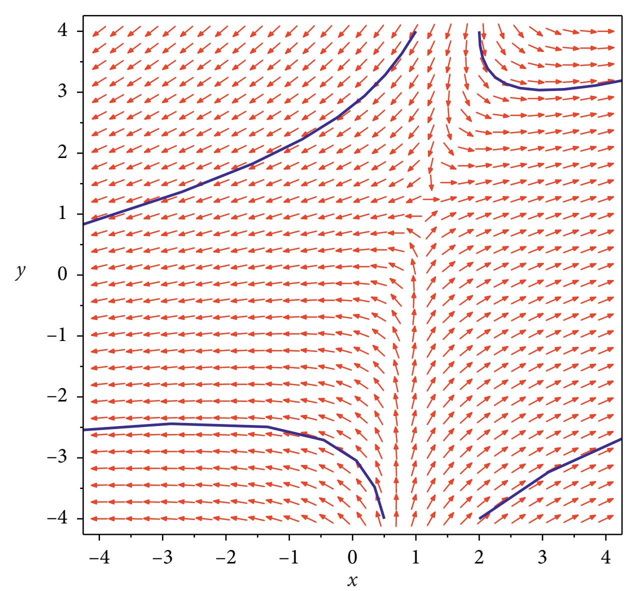


Figure 2: The vector fields of model (71) when $u < Kr/4$, here $u = 2, K = 8$, and $r = 2$.

For further verification of the results obtained, we offer an example as follows: let $r = 2$ and $K = 8$, system (11) becomes

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = 2x(t)\left(1 - \dfrac{1}{8}y(t)\right) - u, \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = \alpha x(t) - \alpha y(t). \end{cases} \tag{71}$$

System (71) has a unique equilibrium point $x_1^* = y_1^* = K/2 = 4$ if $u = 4$. Let $\alpha = 1 = r/2$, $E_1(x_1^*, y_1^*)$ is a degenerate critical point as Theorem 1 shows. By Section 3, system (11) becomes system (38) by coordinate translation and nonsingular linear transformation. By the numerical simulation of the transformation system (38), we confirm this case, the vector fields can be found by Figure 1(a). Furthermore, let $\alpha = 0.6 \neq r/2$ and $E_1(x_1^*, y_1^*)$ is saddle-node as Theorem 1 shows. Similarly, by the numerical simulation of the transformation system, Figure 1(b) verifies this result.

If $u < (1/4)Kr$, there are two equilibria in system (11):

$$\begin{aligned} E_2(x_2^*, y_2^*) &= \left(\frac{K}{2} - \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}, \frac{K}{2} - \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}\right), \\ E_3(x_3^*, y_3^*) &= \left(\frac{K}{2} + \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}, \frac{K}{2} + \frac{K}{2}\sqrt{1 - \frac{4u}{Kr}}\right). \end{aligned} \tag{72}$$

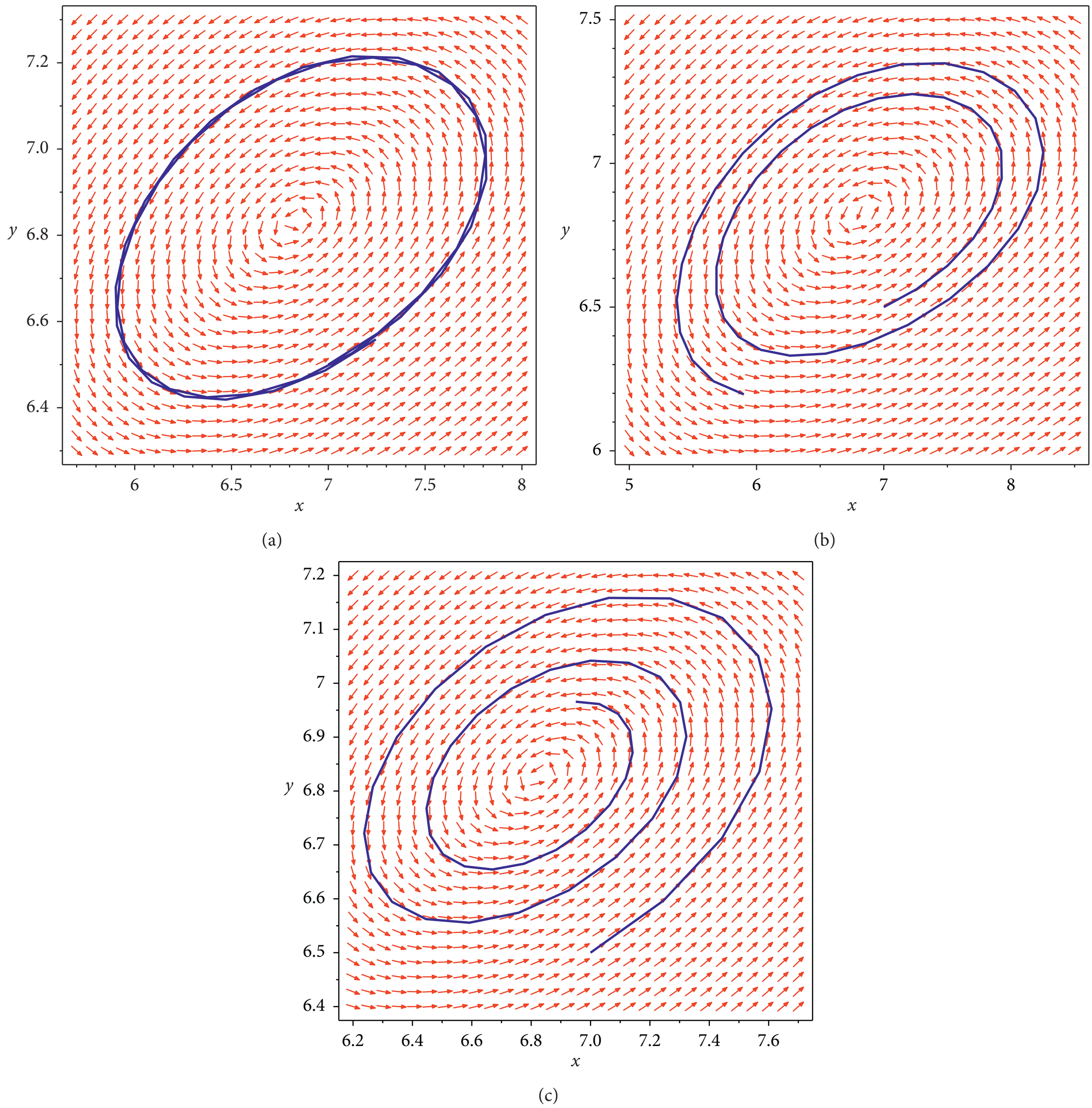


FIGURE 3: The vector fields of model (71) when $u < Kr/4$, here $u = 2$, $K = 8$, and $r = 2$; the blue part is the trajectory with initial value $(7, 6.5)$. (a) The vector fields of model (71) for $\alpha = 0.292$ which holds $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$ near $T = 0$; (b) the vector fields of model (71) for $\alpha = 0.25$ which holds $T > 0$; (c) the vector fields of model (71) for $\alpha = 0.4$ which holds $T < 0$.

Let $u = 2$, $K = 8$, and $r = 2$, we obtain $x_2^* = y_2^* \approx 1.1716$ and $x_3^* = y_3^* \approx 6.828$ by performing some simple calculations. We know that $E_2(x_2^*, y_2^*)$ is saddle point by Theorem 2, and the case is shown in Figure 2. Furthermore, by Theorem 3, Hopf bifurcation occurs near $E_3(x_3^*, y_3^*)$ in system (71) with $\alpha = 0.292$ holding small enough $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$; the blue part in Figure 3(a) is the trajectory of system (71) with initial value (7, 6.5), and it can be seen that system (71) has periodic solution by Figure 3(a); $E_3(x_3^*, y_3^*)$ is unstable for $\alpha = 0.25$ satisfying $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$, and the trajectory of system (71) with initial value (7, 6.5) goes out from the inside in anticlockwise direction as Figure 3(b) shows; $E_3(x_3^*, y_3^*)$ is stable for $\alpha = 0.4$ satisfying $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha < 0$, we can find that the trajectory of system (71) with initial value (7, 6.5) rotates from the outside to the inside in a counterclockwise direction by Figure 3(c).

The time series and portrait phase of system (71) in Figure 4 show that there occurs Hopf bifurcation periodic solution near (x_3^*, y_3^*) for small enough $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$. Figure 5 reveals the instability

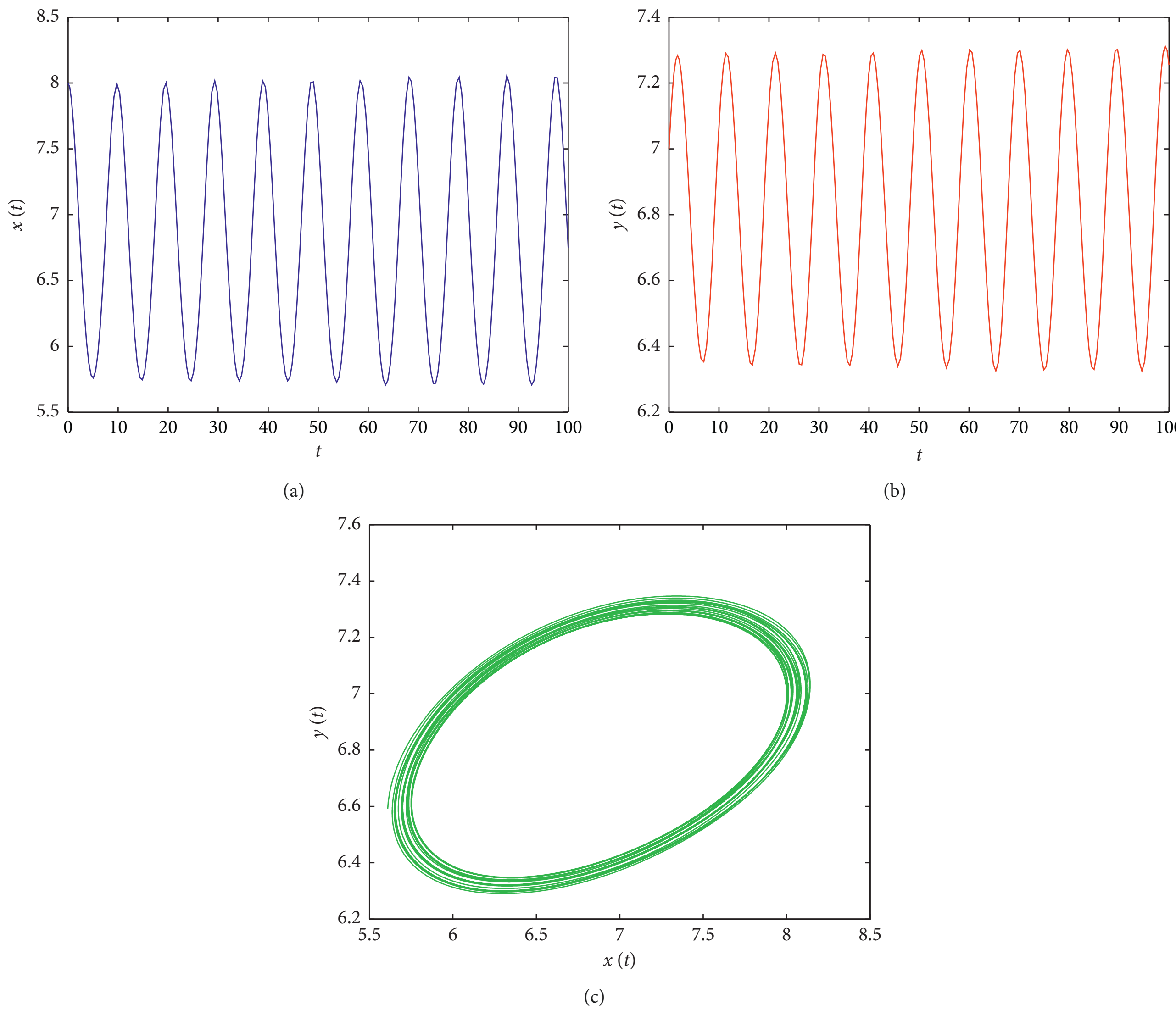


FIGURE 4: The time series and portrait phase of model (71) when $u < Kr/4$; here $u = 2, K = 8, r = 2$, and $\alpha = 0.2928$. (a) The time series of $x(t)$; (b) the time series of $y(t)$; (c) the portrait phase.

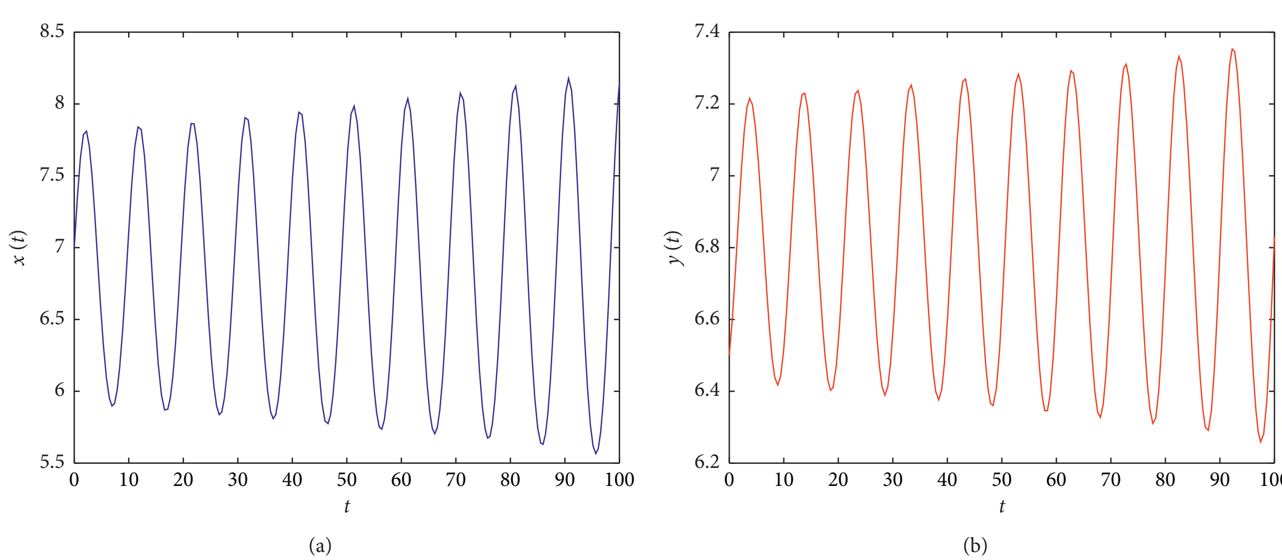
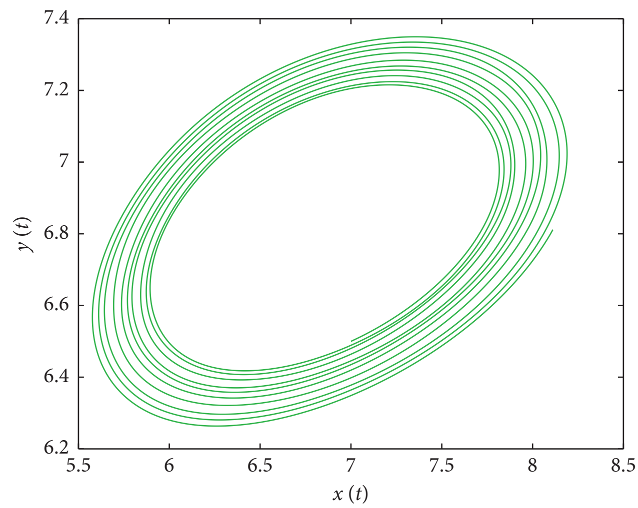
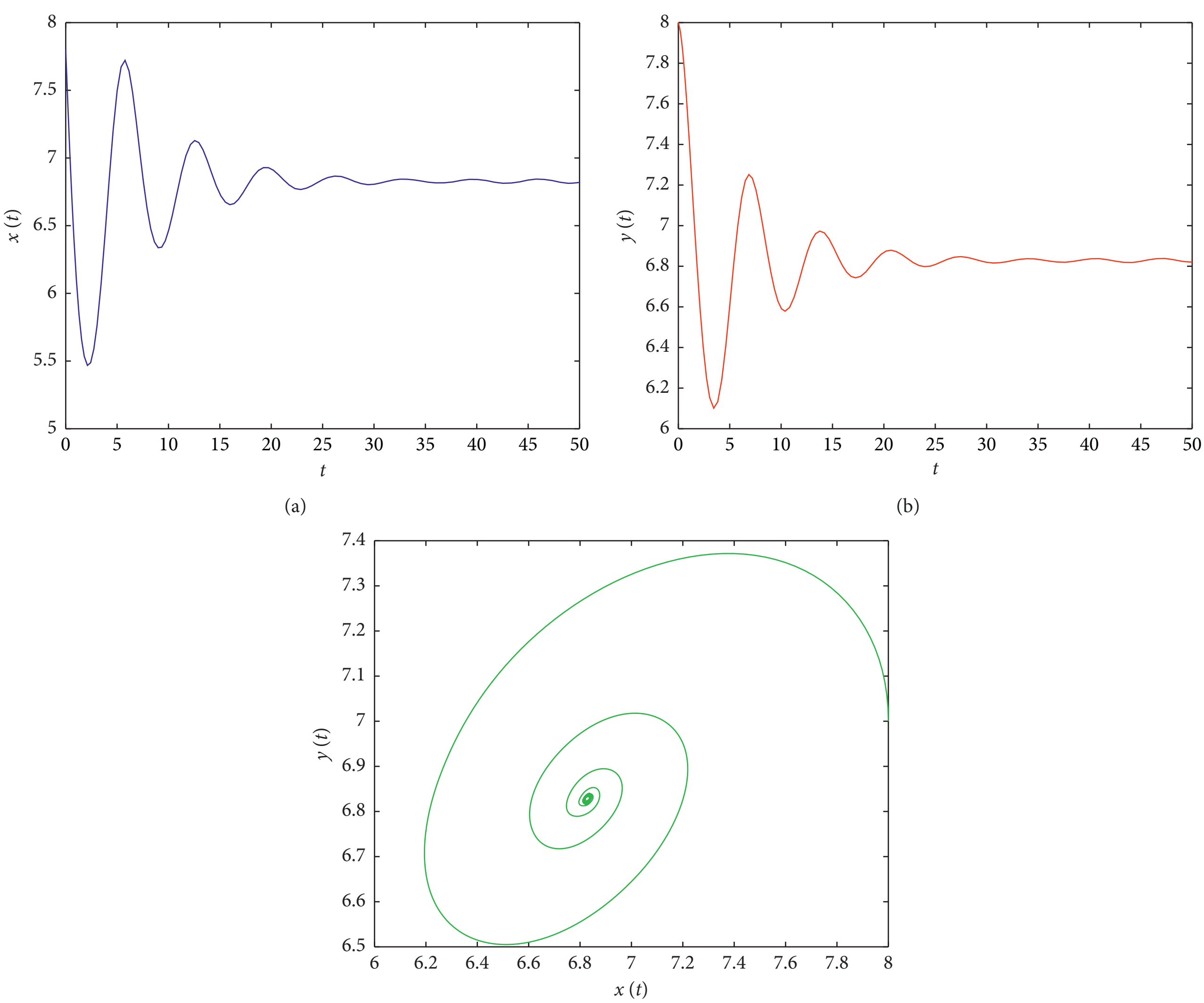


FIGURE 5: Continued.



(c)

FIGURE 5: The time series and portrait phase of model (71) when $u < Kr/4$; here $u = 2, K = 8, r = 2$, and $\alpha = 0.29$. (a) The time series of $x(t)$; (b) the time series of $y(t)$; (c) the portrait phase.



(a) (b)

(c)

FIGURE 6: The time series and portrait phase of model (71) when $u < Kr/4$; here $u = 2, K = 8, r = 2$, and $\alpha = 0.6$. (a) The time series of $x(t)$; (b) the time series of $y(t)$; (c) the portrait phase.

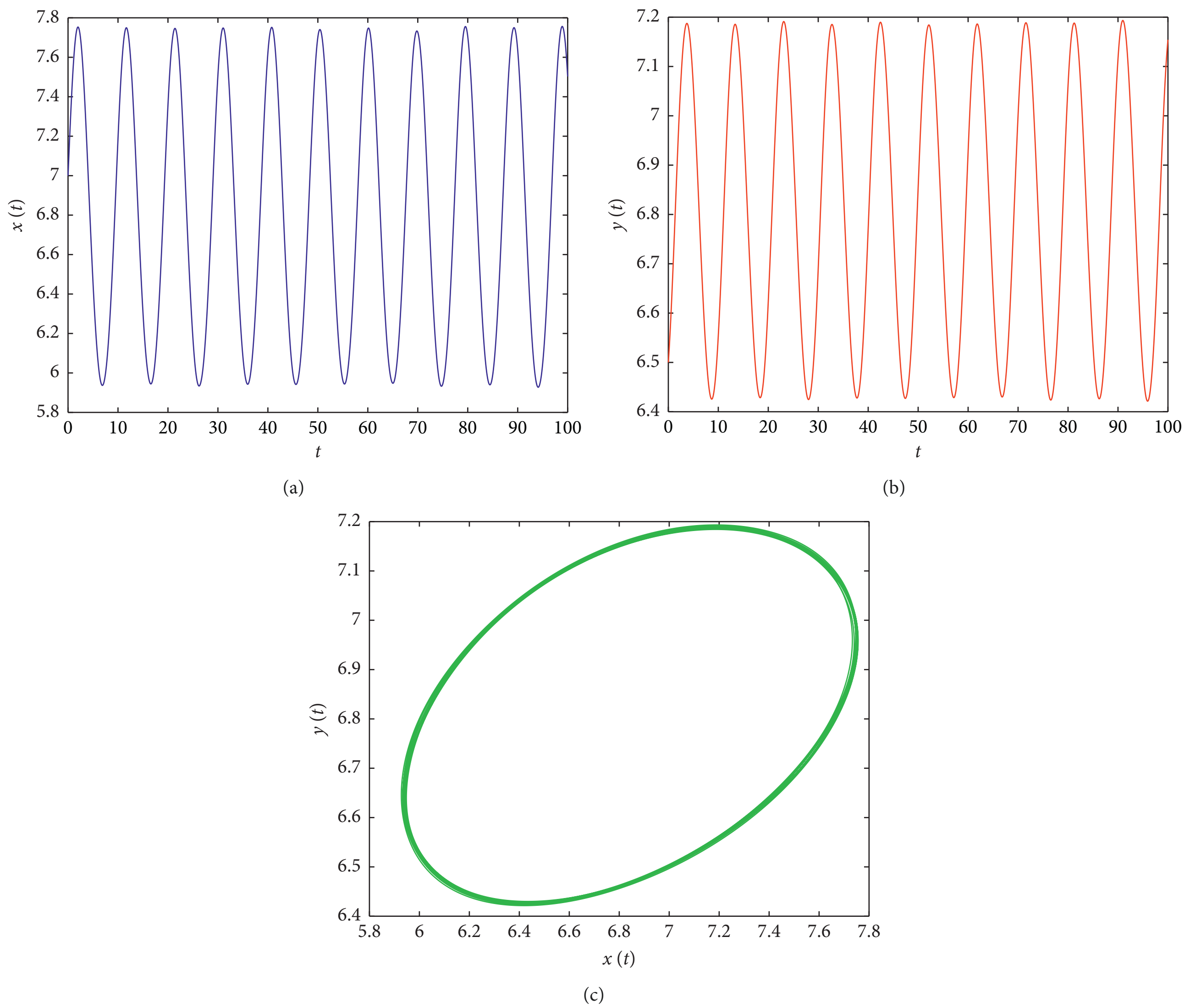


Figure 7: The time series and portrait phase of model (71) when $u < Kr/4$; here $\alpha = 0.3$, $K = 8$, $r = 2$, and (u) is close enough to 2.04. (a) The time series of $x(t)$; (b) the time series of $y(t)$; (c) the portrait phase.

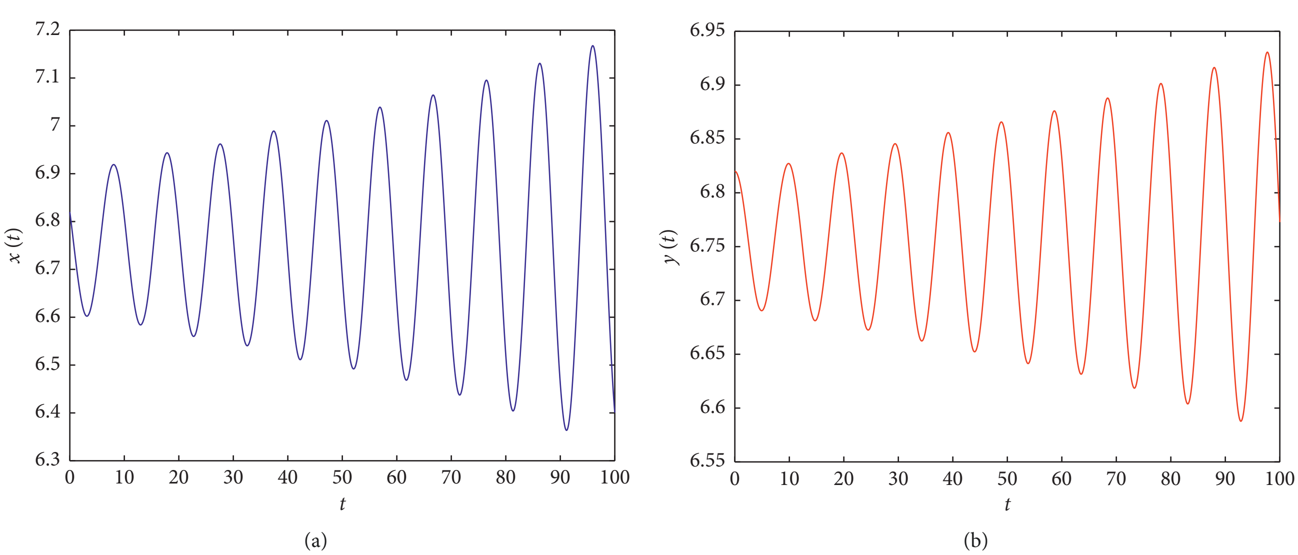
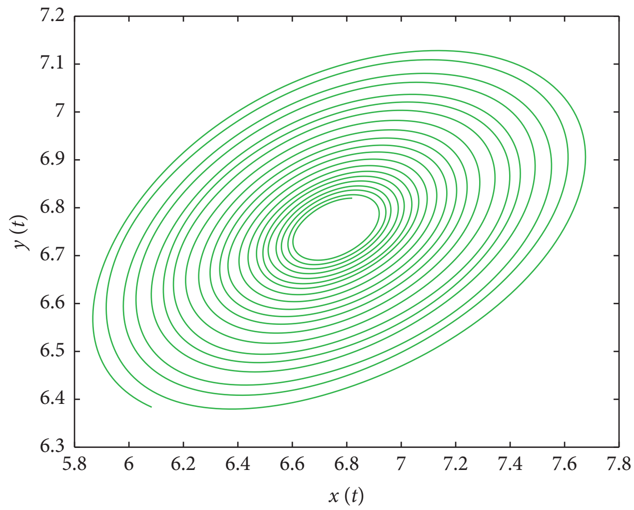
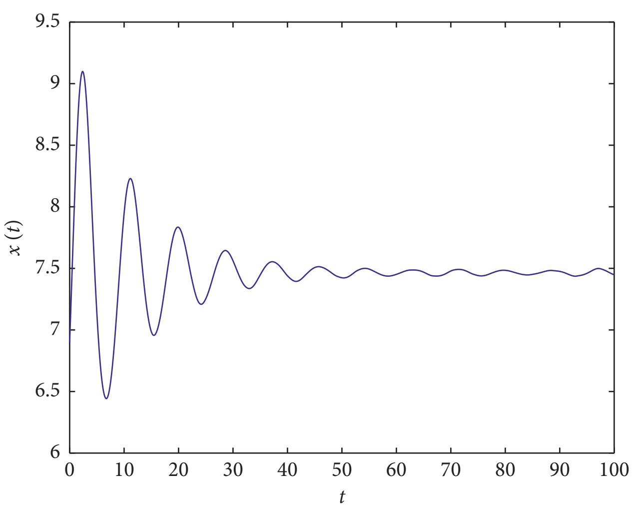


Figure 8: Continued.

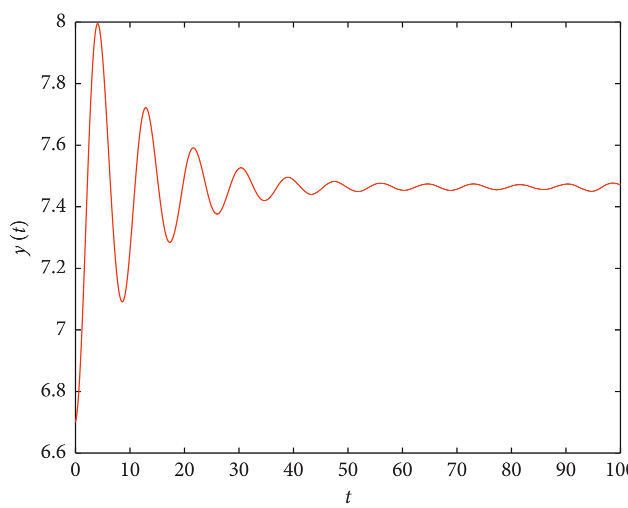


(c)

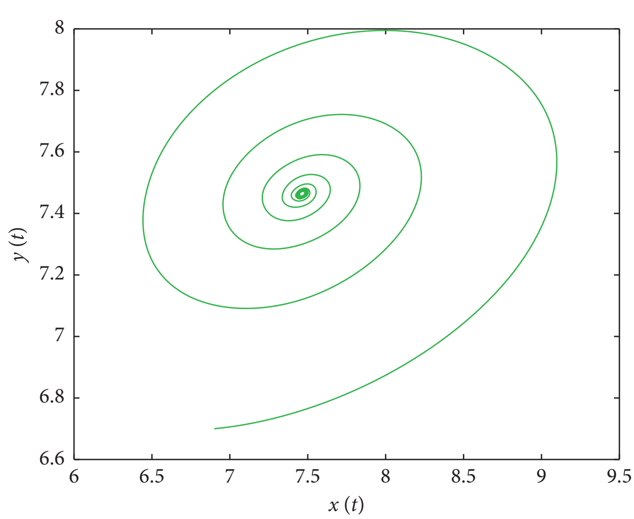
Figure 8: The time series and portrait phase of model (71) when $u < Kr/4$, here $\alpha = 0.3$, $K = 8$, $r = 2$ and $u = 2.1$. (a) The time series of $x(t)$; (b) the time series of $y(t)$; (c) the portrait phase.



(a)



(b)



(c)

Figure 9: The time series and portrait phase of model (71) when $u < Kr/4$; here $\alpha = 0.3$, $K = 8$, $r = 2$, and $u = 1$. (a) The time series of $x(t)$; (b) the time series of $y(t)$; (c) the portrait phase.

of (x_3^*, y_3^*) for $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$. Figure 6 illustrates (x_3^*, y_3^*) is stable if $T = (r/2)(1 - \sqrt{1 - (4u/Kr)}) - \alpha > 0$.

By Remark 1, if $u < (1/4)Kr$ in system (11), then $u = \alpha K(1 - (\alpha/r))$ is a bifurcation point. Let $\alpha = 0.3$, $K = 8$, and $r = 2$, then we can obtain $u = 2.04$ is bifurcation point by simple calculation. The time series and portrait phase of system (71) in Figure 7 shows that there occurs Hopf bifurcation periodic solution near (x_3^*, y_3^*) for small enough $T(u_0) = u_0 - \alpha K(1 - (\alpha/r)) > 0$. The instability of (x_3^*, y_3^*) for $T(u_0) = u_0 - \alpha K(1 - (\alpha/r)) > 0$. is shown in Figure 8. Figure 9 illustrates (x_3^*, y_3^*) is stable if $T(u_0) = u_0 - \alpha K(1 - (\alpha/r)) > 0$.

6. Conclusions

A single-species model with a delay weak kernel and a constant harvesting rate is established in this paper. Sufficient conditions of the existence and stability of equilibria in two different cases have been analysed. We know that system (11) is unstable when constant rate harvesting $u > Kr/4$, that is, harvesting rates on population exceed the MSY, which will reduce the population to extinction in finite time, since $dx/dt < 0$. If the constant rate harvesting u is equal to the critical value, system (11) has a unique equilibrium point which is a degenerate critical point or a saddle-node. When the constant rate harvesting u is less than the critical value, by topological normal form for the Hopf bifurcation and calculating the first Lyapunov coefficient, sufficient conditions about the existence of the Hopf bifurcation are obtained.

We can see that system (11) has a stable equilibrium when $u < \min\{\alpha K(1 - (\alpha/r)), (1/4)Kr\}$ by Remark 1. That is, sufficiently low harvesting rates can be sustained in perpetuity. When $(1/4)Kr > u > \alpha K(1 - \alpha/r)$, system (11) emerges a unique and stable limit cycle bifurcation near $E_3(x_3^*, y_3^*)$ for small enough $T(u_0) = u_0 - \alpha K(1- (\alpha/r)) > 0$, that is, system (11) has a stable periodic solution and population will survive. Therefore, in order to ensure the permanence of species, the control of sustained harvest is very important.

Our work on system (11) reveals that the single-species system with a constant rate harvesting is interesting and rich in dynamics. It shows that the intensity of the harvest has effect on the dynamics of system, including the existence and stability of equilibria and Hopf bifurcation. Excessive harvest will lead to extinction of species, and it has a serious impact on biodiversity. Biologically, there is still a lot of work to be done in this area. For example, the study about two-parameter bifurcations of equilibria is a very meaningful work, and it would also be interesting to study how linear rate harvesting and nonlinear rate harvesting in population impact on the dynamics of system and what is the difference between constant rate harvesting and other ways of harvesting. We will consider these issues in our future work.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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