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Research Article

Consensus Tracking of Fractional-Order Multiagent Systems via Fractional-Order Iterative Learning Control

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In this work, the consensus problem of fractional-order multiagent systems with the general linear model of fixed topology is studied. Both distributed PD^{α}-type and D^{α}-type fractional-order iterative learning control (FOILC) algorithms are proposed. Here, a virtual leader is introduced to generate the desired trajectory, fixed communication topology is considered, and only a subset of followers can access the desired trajectory. The convergence conditions are proved using graph theory, fractional calculus, and λ norm theory. The theoretical analysis shows that the output of each agent completely tracks the expected trajectory in a limited time as the iteration number increases for both PD^{α}-type and D^{α}-type FOILC algorithms. Extensive numerical simulations are given to demonstrate the feasibility and effectiveness.

1. Introduction

In recent years, the research on the coordinating control of multiagent systems (MASs) has become a major task in the field of control because of its application in various areas, such as formation control of unmanned aerial vehicles [1], air traffic control [2], and formation of multiple robots [3]. The consensus is an important and fundamental problem for the coordination control of MASs. One object of the consensus problem is that all agents are expected to reach an agreement in some amounts (such as speed, position, phases, and attitudes) by communicating with their local neighbors [4–6].

The consensus problem, as an interesting topic for MASs, has achieved many results [5–10]. However, most studies have focused on the consensus of integer-order MASs (IOMASs). In fact, many systems cannot be well described for some complex problems by integer-order models, such as viscoelastic systems, underwater vehicle, and electrical machines [11–13]. Recently, fractional-order calculus is found to be suitable for describing some complex systems, such as macromolecule fluids, porous media, and viscoelastic systems, and have excellent memory and hereditary [14]. Therefore, it is meaningful to study the consensus problem for fractional-order MASs (FOMASs). In particular, integer-order systems are special cases of fractional-order systems, and many researchers have studied the convergence problem and focused on the coordination control of FOMASs. The necessary and sufficient conditions of FOMASs for containment control were presented in [15]. For FOMASs, several consensus problems of FOMASs were studied in [16–19]. Du et al. studied the problem of robust consensus for second-order MASs and proposed a class of novel continuous nonsmooth consensus algorithms [20]. Zhu et al. studied the leader-following consensus problem of FOMASs with general linear and nonlinear models with input time delay and designed a control gain matrix and sufficient condition [21]. In [22], group multiple-lag consensus of nonlinear FOMAS with leader-following was investigated, and two kinds of lag consensus were considered.

Notably, the aforementioned studies consider the asymptotic convergence problem of FOMAS, that is, the tracking errors of each agent decreases gradually as time increases. For MAS with repetitive tasks, such as multi-mechanical arms on industrial production lines, the asymptotic convergence evidently cannot meet the requirements. The complete convergence within a limited time is required [23–25]. Among the control methods, iterative learning control (ILC) can achieve full tracking in a limited time for the repetitive system [26–28]. Therefore, the ILC algorithm is applied for IOMAS to solve the consensus problem recently, especially for complex MASs [29], MASs with switching topologies and communication time delays [30], and nonlinear MASs [31]. The ILC algorithm can achieve high-precision tracking performance and achieve exact tracking within finite time as the iteration number increases. However, most existing works using ILC are suitable only for IOMAS.

The consensus problem for the fractional MASs with ILC is seldom discussed. In this study, we consider the leader-following consensus problem of FOMAS with general linear leader and design PD^{α}-type and D^{α}-type fractional-order ILC (FOILC) controllers. The convergence of the proposed controller is analyzed, and the convergence conditions are derived. The convergence conditions can ensure that the tracking errors of all the agents are gradually decreased to a sufficiently small value as the number of iterations increases. Furthermore, the tracking errors can also tend to zero if the initial states of the agents tend to zero.

The rest of the paper is organized as follows. In Section 2, graph theory, fractional calculus, and problem formulation are given. The FOILC and the analysis of convergence for FOMAS are discussed in Section 3. In Section 4, the effectiveness of the proposed algorithm is verified by simulation. Section 5 elaborates the conclusions.

2. Preliminary Knowledge

In this section, some basic definitions and lemmas are introduced, which will be used in the following sections.

2.1. Mathematical Knowledge. The set of real numbers is denoted by $\mathbb{R}$, and the set of complex numbers is denoted by $\mathbb{C}$. The set of natural numbers is denoted by $\mathbb{N}$, and $i \in \mathbb{N}$ is the iteration number. For a given vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, $|\mathbf{x}|$ denotes any l_p vector norm, where $1 \leq p \leq \infty$. In particular, $|\mathbf{x}|_1 = \sum_{k=1}^{n} |x_k|$, $|\mathbf{x}|_2 = \sqrt{\mathbf{x}^T\mathbf{x}}$, and $|\mathbf{x}|_{\infty} = \max_{k=1,\ldots,n} |x_k|$. For any matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, $|\mathbf{A}|$ is the induced matrix norm. $\rho(\mathbf{A})$ is the spectral radius. Moreover, $\otimes$ denotes the Kronecker product, and $\mathbf{I}_m$ is the $m \times m$ identity matrix.

Remark 1. $C^m[0,T]$ denotes a set consisting of all functions with mth derivatives that are continuous on the finite time interval $[0,T]$.

Definition 1. Continuous vector function $\mathbf{f}(\cdot) \in C[0,T]$, where $\mathbf{f}(t) = [f_1(t), f_2(t), \ldots, f_n(t)]^T$, is given. The λ norm of the function $\mathbf{f}$ is defined as

$$\|\mathbf{f}(\cdot)\|_{\lambda} = \sup_{0\leq t\leq T} e^{-\lambda t}\left(\max_{1\leq i\leq n}\left|f_i(t)\right|\right), \tag{1}$$

where λ is a positive real number.

Definition 2 (see [32]). The α-order fractional integral of the function $\mathbf{f}(t)$ over the interval $[t_0, t]$ is defined as

$${}_{t_0}D_t^{-\alpha}\mathbf{f}(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}\mathbf{f}(\tau)\mathrm{d}\tau, \tag{2}$$

where $\alpha > 0$ and $\Gamma(\cdot)$ is the Gamma function defined as $\Gamma(\alpha) = \int_0^{\infty}\exp(-t)t^{\alpha-1}\mathrm{d}t$.

For the function $\mathbf{f}(t)$, the α-order Caputo derivative over the interval $[t_0, t]$ is defined as

$${}_{t_0}^{C}D_t^{\alpha}\mathbf{f}(t) = {}_{t_0}D_t^{-([\alpha]-\alpha+1)}\left[\frac{\mathrm{d}^{[\alpha]+1}}{\mathrm{d}t^{[\alpha]+1}}\mathbf{f}(t)\right], \tag{3}$$

where $\alpha > 0$ and $[\alpha]$ represents the integer part of α.

Lemma 1 (see [33, 34]). *The function $\mathbf{f}(\mathbf{x}(t), t)$ is assumed continuous. Then, the initial value problem is described as*

$$\begin{cases} {}_{t_0}^{C}D_t^{\alpha}\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), t), \\ \mathbf{x}(t_0) = \mathbf{x}_0, \\ 0 < \alpha < 1. \end{cases} \tag{4}$$

The function in equation (4) is equivalent to the following nonlinear Volterra integral equation:

$$\mathbf{x}(t) = \mathbf{x}_0 + \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}\mathbf{f}(\mathbf{x}(\tau), \tau)\mathrm{d}\tau. \tag{5}$$

2.2. Graph Theory. In this study, N FOMAS and a virtual leader are considered, and v_i indicates the ith agent, $i = 1, 2, \ldots, N$. The communications among the agents can be described by a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{M})$, where $\mathbf{V} = \{v_1, v_2, \ldots, v_n\}$ represents the node set, $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ is the set of edges, and $\mathbf{M} = [a_{lm}]_{n\times n}$ denotes the weighted adjacency matrix, that is, $a_{mm} = 0$, $a_{lm} > 0$ if $(l, m) \in \mathbf{E}$ and $a_{lm} = 0$ otherwise [35]. $N_l = \{m \mid (m, l) \in \mathbf{E}\}$ is the set of neighborhood of the lth node. The Laplacian matrix $\mathbf{L} \in \mathbb{R}^{n\times n}$ of the graph is defined as

$$\mathbf{L} = \mathbf{D}_{\text{in}} - \mathbf{M}, \tag{6}$$

where $\mathbf{D}_{\text{in}} = \operatorname{diag}[d_1^{\text{in}}, d_2^{\text{in}}, \ldots, d_n^{\text{in}}]$, $d_i^{\text{in}} = \sum_{m\in N_i} a_{lm}$.

When joining a virtual leader, the relationship between the followers and the leader is expressed by a diagonal matrix $\mathbf{D}$, that is, $\mathbf{D} = \operatorname{diag}[d_1, d_2, \ldots, d_n]$, where $d_l > 0$ if the lth agent can receive information from the virtual leader. The trajectory of the leader is only accessible to a small portion of the followers due to communication or sensor limitations. The communication among followers is described by the graph $\mathbf{G}\{\mathbf{V}, \mathbf{E}\}$. If the leader is labeled by vertex 0, then the complete information flow among all the agents can be characterized by a new graph $\overline{\mathbf{G}} = \{0 \cup \mathbf{V}, \overline{\mathbf{E}}\}$, where $\overline{\mathbf{E}}$ is the new edge set. If the graph is connected or a spanning tree, it can be described as follows.

Definition 3 (see [36]). Among all agents, any agent can receive indirectly or directly the information of the virtual leader in a path. In other words, we do not consider the isolated agent, which means the agent cannot accept any information of the leader, and self-loop agents, which means the agent accepts their own information only.

Lemma 2. *If $\overline{\mathbf{G}}$ is a connected graph, then the matrix $\mathbf{L} + \mathbf{D}$ associated with the graph $\overline{\mathbf{G}}$ is a positive definite matrix, where $\mathbf{L}$ is the Laplacian matrix and $\mathbf{D} = diag[d_1, d_2, \ldots, d_n]$.*

The proof of Lemma 2 is described in [36].

3. Problem Description

A FOMAS consists of N agents indexed by $1, 2, \ldots, N$ and a leader. The dynamic of each follower can be described as follows:

$$\begin{cases} D^{\alpha}\mathbf{x}_{i,j}(t) = \mathbf{A}\mathbf{x}_{i,j}(t) + \mathbf{B}\mathbf{u}_{i,j}(t), \\ \mathbf{y}_{i,j}(t) = \mathbf{C}\mathbf{x}_{i,j}(t), \end{cases} \tag{7}$$

where $t \in [0, T]$ and i are the time and number of iterations; $\mathbf{x}_{i,j}(t) \in \mathbb{R}^m$ is the state vector of the jth agent; $\mathbf{u}_{i,j}(t) \in \mathbb{R}^{m1}$ and $\mathbf{y}_{i,j}(t) \in \mathbb{R}^{m2}$ are the input and output vectors, respectively; $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are constant matrices with compatible dimensions; and $D^{\alpha}\mathbf{x}_{i,j}(t)$ is the α-order Caputo derivative of $\mathbf{x}_{i,j}(t)$.

The dynamic equations described by equation (7) are rewritten in the following compact form:

$$\begin{cases} D^{\alpha}\mathbf{x}_i(t) = (I_N \otimes \mathbf{A})\mathbf{x}_i(t) + (I_N \otimes \mathbf{B})\mathbf{u}_i(t), \\ \mathbf{y}_i(t) = (I_N \otimes \mathbf{C})\mathbf{x}_i(t), \end{cases} \tag{8}$$

where $\mathbf{x}_i(t) = [\mathbf{x}_{i,1}(t)^T, \mathbf{x}_{i,2}(t)^T, \ldots, \mathbf{x}_{i,N}(t)^T]^T$, $\mathbf{u}_i(t) = [\mathbf{u}_{i,1}^T(t), \mathbf{u}_{i,2}^T(t), \ldots, \mathbf{u}_{i,N}^T(t)]^T$, $\mathbf{y}_i(t) = [\mathbf{y}_{i,1}^T(t), \mathbf{y}_{i,2}^T(t), \ldots, \mathbf{y}_{i,N}^T(t)]^T$, $\xi_i(t) = [\xi_{i,1}(t)^T, \xi_{i,2}(t)^T, \ldots, \xi_{i,N}(t)^T]^T$, and $\otimes$ is the Kronecker product.

The expected trajectory $\mathbf{y}_d(t)$ is defined on a finite time interval $[0, T]$ and is generated by the leader. The dynamics of the leader agent are as follows:

$$\begin{cases} D_t^{\alpha}\mathbf{x}_d(t) = \mathbf{A}\mathbf{x}_d(t) + \mathbf{B}\mathbf{u}_d(t), \\ \mathbf{y}_d(t) = \mathbf{C}\mathbf{x}_d(t), \end{cases} \tag{9}$$

where $\mathbf{u}_d(t)$ is the continuous and unique desired control input.

We define that $\mathbf{u}_d(t)$ is the continuous and unique desired control input for all followers, that is, $\mathbf{u}_d(t) = [\mathbf{u}_{1,d}, \mathbf{u}_{2,d}, \ldots, \mathbf{u}_{N,d}]^T$.

This study aims to find the appropriate control input of each agent with ILC to ensure that the trajectories of each agent are exactly the same as $\mathbf{y}_d(t)$ in a finite time $[0, T]$, that is, finding appropriate $\mathbf{u}_{i,k}(t)$ satisfies

$$\lim_{i \longrightarrow \infty} \left\| \mathbf{y}_d(t) - \mathbf{y}_{i,j}(t) \right\| = 0, \quad j = 1, 2, \ldots, N,\ t \in [0, T]. \tag{10}$$

In other words, the goal of the function in equation (10) is that each agent in the FOMAS converges to the desired reference trajectory in a finite time with FOILC as the number of iterations increases. The major task is to design a set of distributed FOILC rules for guaranteeing that each individual agent in the network can track the trajectory of the leader under the sparse communication graph $\overline{\mathbf{G}}$.

$\xi_{i,j}(t)$ denotes the distributed information measured or received by the jth agent at the ith iteration. Specifically,

$$\xi_{i,j}(t) = \sum_{k \in N_j} a_{j,k}\left(\mathbf{y}_{i,k}(t) - \mathbf{y}_{i,j}(t)\right) + d_j\left(\mathbf{y}_d(t) - \mathbf{y}_{i,j}(t)\right), \tag{11}$$

where $a_{j,k}$ is the element of the weighted adjacency matrix $\mathbf{M}$, that is, $a_{j,k} \in \mathbf{M}$ and d_j is the element of the diagonal matrix $\mathbf{D}$ and describes the relationship between the followers and the leader, that is, $d_j \in \mathbf{D}$. We define the tracking error between the j-th and the leader fractional-order agents as follows:

$$\mathbf{e}_{i,j}(t) = \mathbf{y}_d(t) - \mathbf{y}_{i,j}(t). \tag{12}$$

Consequently, equation (15) can be rewritten in a compact form:

$$\xi_i(t) = ((\mathbf{L} + \mathbf{D}) \otimes \mathbf{I}_m)\mathbf{e}_i(t), \tag{13}$$

where $\mathbf{e}_i(t) = [\mathbf{e}_{i,1}(t)^T, \mathbf{e}_{i,2}(t)^T, \ldots, \mathbf{e}_{i,N}(t)^T]^T$, $\xi_i(t) = [\xi_{i,1}(t)^T, \xi_{i,2}(t)^T, \ldots, \xi_{i,N}(t)^T]^T$, $\mathbf{L}$ is the Laplacian matrix of graph $\mathbf{G}$, and $\mathbf{D} = \text{diag}\{d_i\}$, $i = 1, \ldots, N$.

3.1. Open-Loop PD^{α}-Type Iterative Learning Control. To solve the consensus tracking problem of FOMASs, the following distributed PD^{α}-type ILC for the jth agent is designed as

$$\mathbf{u}_{i+1,j}(t) = \mathbf{u}_{i,j}(t) + \mathbf{\Gamma}_P\xi_{i,j}(t) + \mathbf{\Gamma}_D{\xi_{i,j}}^{(\alpha)}(t). \tag{14}$$

Based on (12), the distributed measurement in equation (11) can be rewritten as

$$\begin{aligned} \xi_{i,j}(t) &= \sum_{k \in N_j} a_{j,k}\left(\mathbf{y}_d(t) - \mathbf{y}_{i,j}(t) - \mathbf{y}_d(t) + \mathbf{y}_{i,k}(t)\right) \\ &\quad + d_j\left(\mathbf{y}_d(t) - \mathbf{y}_{i,j}(t)\right) \\ &= \sum_{k \in N_j} a_{j,k}\left(\mathbf{e}_{i,j}(t) - \mathbf{e}_{i,k}(t)\right) + d_j\mathbf{e}_{i,j}(t). \end{aligned} \tag{15}$$

By using equation (13), the updating law in equation (14) can be rewritten in terms of the tracking errors for all the followers as

$$\mathbf{u}_{i+1}(t) = \mathbf{u}_i(t) + ((\mathbf{L} + \mathbf{D}) \otimes \mathbf{\Gamma}_P)\mathbf{e}_i(t) + ((\mathbf{L} + \mathbf{D}) \otimes \mathbf{\Gamma}_D){\mathbf{e}_i}^{(\alpha)}(t). \tag{16}$$

For convenience, we define γ_j and $j = 1, 2, \ldots, N$ as the jth eigenvalue of $\mathbf{L} + \mathbf{D}$.

To simplify the controller design and convergence analysis, the following assumptions are imposed.

Assumption 1. When the system described by (7) is repeatedly run over the interval $[0, T]$, the initial state of the each subsystem in equation (7) is equal to the expected initial state. In particular, $\mathbf{x}_{i,j}(0) = \mathbf{x}_d(0)$ is satisfied for all j and i.

Remark 2. Assumption 1 is the standard condition for ILC to ensure perfect tracking performance. If the assumption is removed, then the tracking performance will be scarified and the extra system information or additional control mechanisms are required. For instance, the initial state learning control is discussed in [35–37]. Notably, perfect tracking can never be achieved without perfect initial condition.

Assumption 2. The communication graph $\overline{\mathbf{G}}$ contains a spanning tree with the leader and the followers being the root.

Remark 3. Assumption 3 is the necessary condition of the consensus tracking problem for the FOMAS. If the graph $\overline{\mathbf{G}}$ does not contain a spanning tree, then an isolated agent exists and cannot follow the trajectory of the leader because it does not know the control objective. However, the graph $\mathbf{G}$ does not necessarily contain a spanning tree because the isolated agent can communicate with the selected leader.

Theorem 1. *We consider the FOMAS in equation (7) under Assumptions 1 and 2, the communication graph* $\mathbf{G}$, *and* PD^{α}*-type updating rule in equation (14). If the learning gains* $\mathbf{\Gamma}_P$ *and* $\mathbf{\Gamma}_D$ *are chosen, then*

$$\left\| \mathbf{I} - (\mathbf{L} + \mathbf{D}) \otimes \mathbf{\Gamma}_D \mathbf{C}\mathbf{B} \right\| + \beta < 1, \tag{17}$$

where $\mathbf{I}$ *is the identity matrix with the subscript denoting its dimension,* $\mathbf{L}$ *is the Laplacian matrix of* $\mathbf{G}$, *and* $\mathbf{D} = diag\{d_j, j = 1, \ldots, N\}$. *The control input* $\mathbf{u}_{i,j}(t)$ *and output* $\mathbf{y}_{i,j}(t)$ *converge to* $\mathbf{u}_d(t)$ *and* $\mathbf{y}_d(t)$, *respectively, as the iteration number tends to infinity.*

Proof. The convergence analysis is presented as follows. We assume that

$$\begin{cases} \Delta \mathbf{x}_i(t) = \mathbf{1}_N \otimes \mathbf{x}_d(t) - \mathbf{x}_i(t), \\ \Delta \mathbf{u}_i(t) = \mathbf{1}_N \otimes \mathbf{u}_d(t) - \mathbf{u}_i(t), \end{cases} \tag{18}$$

where $\beta = ((\|\mathbf{B}\| \|\mathbf{L} + \mathbf{D}\| \|\mathbf{\Gamma}_P \mathbf{C} + \mathbf{\Gamma}_D \mathbf{C}\mathbf{A})\|)/\lambda^{\alpha} - \|\mathbf{A}\|)$ and $\mathbf{1}_N$ is a vector in which all entries are 1. From (1), we can obtain

$$\begin{aligned} \mathbf{e}_i^{(\alpha)}(t) &= \mathbf{1}_N \otimes \mathbf{y}_d^{(\alpha)}(t) - \mathbf{y}_i^{(\alpha)}(t) \\ &= (\mathbf{I}_n \otimes \mathbf{C})\left(\mathbf{x}_d^{(\alpha)}(t) - \mathbf{x}_i^{(\alpha)}(t)\right) \\ &= (\mathbf{I}_n \otimes \mathbf{C})\Delta \mathbf{x}_i^{(\alpha)}(t) \\ &= (\mathbf{I}_n \otimes \mathbf{C})(\mathbf{I}_n \otimes \mathbf{A})\Delta \mathbf{x}_i(t) + (\mathbf{I}_n \otimes \mathbf{C})(\mathbf{I}_n \otimes \mathbf{B})\Delta \mathbf{u}_i(t). \end{aligned} \tag{19}$$

Lemma 1 yields

$$\begin{aligned} \mathbf{x}_i(t) = \mathbf{x}_i(0) + \frac{1}{\mathbf{\Gamma}(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} &\left((\mathbf{I}_n \otimes \mathbf{A})\mathbf{x}_i(\tau)\right. \\ &\left.+ \left(\mathbf{I}_n \otimes \left(\mathbf{I}_n \otimes \mathbf{B}\right)\right)\mathbf{u}_i(\tau)\right)\mathrm{d}\tau. \end{aligned} \tag{20}$$

Accordingly, we can obtain

$$\begin{aligned} \Delta \mathbf{x}_i(t) &= \mathbf{1}_N \otimes \mathbf{x}_d(t) - \mathbf{x}_i(t) \\ &= \frac{1}{\mathbf{\Gamma}(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} \left((\mathbf{I}_n \otimes \mathbf{A})\Delta \mathbf{x}_i(\tau) + (\mathbf{I}_n \otimes \mathbf{B})\Delta \mathbf{u}_i(\tau)\right)\mathrm{d}\tau. \end{aligned} \tag{21}$$

By taking the λ norm on both sides of (11) and noting Definition 1, we have

$$\begin{aligned} &\left\|\Delta \mathbf{x}_i(t)\right\|_{\lambda} \\ &\leq \sup_{0 \leq t \leq T} \mathbf{e}^{-\lambda t} \left\|\Delta \mathbf{x}_i(t)\right\| \\ &\leq \sup_{0 \leq t \leq T} \frac{1}{\mathbf{\Gamma}(\alpha)} \mathbf{e}^{-\lambda t} \int_0^t (t - \tau)^{\alpha - 1} \left(\left\|\mathbf{I}_n \otimes \mathbf{A}\right\| \left\|\Delta \mathbf{x}_i(\tau)\right\|\right. \\ &\quad \left.+ \left\|\mathbf{I}_n \otimes \mathbf{B}\right\| \left\|\Delta \mathbf{u}_i(\tau)\right\|\right)\mathrm{d}\tau \\ &\leq \sup_{0 \leq t \leq T} \frac{1}{\mathbf{\Gamma}(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} \mathbf{e}^{-\lambda (t - \tau)} \mathbf{e}^{-\lambda \tau} \left(\left\|\mathbf{I}_n \otimes \mathbf{A}\right\| \left\|\Delta \mathbf{x}_i(\tau)\right\|\right. \\ &\quad \left.+ \left\|\mathbf{I}_n \otimes \mathbf{B}\right\| \left\|\Delta \mathbf{u}_i(\tau)\right\|\right)\mathrm{d}\tau \\ &\leq \sup_{0 \leq t \leq T} \frac{1}{\mathbf{\Gamma}(\alpha)} \int_0^t (t - \tau)^{\alpha - 1} \mathbf{e}^{-\lambda (t - \tau)} \left(\left\|\mathbf{I}_n \otimes \mathbf{A}\right\| \left\|\Delta \mathbf{x}_i(\tau)\right\|_{\lambda}\right. \\ &\quad \left.+ \left\|\mathbf{I}_n \otimes \mathbf{B}\right\| \left\|\Delta \mathbf{u}_i(\tau)\right\|_{\lambda}\right)\mathrm{d}\tau \\ &\leq \frac{\left(\|\mathbf{A}\| \left\|\Delta \mathbf{x}_i(t)\right\|_{\lambda} + \|\mathbf{B}\| \left\|\Delta \mathbf{u}_i(\tau)\right\|_{\lambda}\right)}{\mathbf{\Gamma}(\alpha)} \sup_{0 \leq t \leq T} \int_0^t (t - \tau)^{\alpha - 1} \mathbf{e}^{-\lambda (t - \tau)} \mathrm{d}\tau. \end{aligned} \tag{22}$$

By contrast, we have

$$\begin{aligned} \int_0^t (t - \tau)^{\alpha - 1} \mathbf{e}^{-\lambda (t - \tau)} \mathrm{d}\tau &= e^{-\lambda t} \int_0^t (t - \tau)^{\alpha - 1} \mathbf{e}^{\lambda \tau} \mathrm{d}\tau \\ &\overset{t - \tau = \omega}{=} \mathbf{e}^{-\lambda t} \int_0^t \omega^{\alpha - 1} \mathbf{e}^{\lambda (t - \omega)} \mathrm{d}\omega \\ &\overset{\lambda \omega = s}{=} \frac{\mathbf{e}^{-\lambda t}}{\lambda^{\alpha}} \int_0^t s^{\alpha - 1} \mathbf{e}^{\lambda (t - s)} \mathrm{d}s \\ &\leq \frac{1}{\lambda^{\alpha}} \int_0^{+\infty} s^{\alpha - 1} \mathbf{e}^{-s} \mathrm{d}s = \frac{\mathbf{\Gamma}(\alpha)}{\lambda^{\alpha}}. \end{aligned} \tag{23}$$

Substituting equation (23) into equation (22) yields

$$\left\|\Delta \mathbf{x}_i(t)\right\|_{\lambda} \leq \frac{\|\mathbf{A}\| \left\|\Delta \mathbf{x}_i(t)\right\|_{\lambda} + \|\mathbf{B}\| \left\|\Delta \mathbf{u}_i(t)\right\|_{\lambda}}{\lambda^{\alpha}}. \tag{24}$$

Thus, we can find sufficiently large λ to make

$$\lambda^{\alpha} - \|\mathbf{A}\| > 0. \tag{25}$$

Therefore, equation (24) is reformulated as

$$\left\|\Delta\mathbf{x}_i(t)\right\|_\lambda \le \frac{\|\mathbf{B}\|}{\lambda^\alpha - \|\mathbf{A}\|}\left\|\Delta\mathbf{u}_i(t)\right\|_\lambda. \tag{26}$$

Equations (7) and (18) yield

$$\mathbf{e}_i(t) = \mathbf{y}_d(t) - \mathbf{y}_i(t) = (\mathbf{I}_n \otimes \mathbf{C})\Delta\mathbf{x}_i(t), \tag{27}$$

$$\begin{aligned}\Delta\mathbf{u}_{i+1}(t) = \Delta\mathbf{u}_i(t) &- ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_P)\mathbf{e}_i(t)\\ &- ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)\mathbf{e}_i{}^{(\alpha)}(t).\end{aligned} \tag{28}$$

By substituting (19) and (27) into (28), we can obtain

$$\begin{aligned}\Delta\mathbf{u}_{i+1}(t) &= \Delta\mathbf{u}_i(t) - ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_P)(\mathbf{I}_n\otimes\mathbf{C})\Delta\mathbf{x}_i(t)\\ &\quad - ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)(\mathbf{I}_n\otimes\mathbf{C})(\mathbf{I}_n\otimes\mathbf{A})\Delta\mathbf{x}_i(t)\\ &\quad - ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)(\mathbf{I}_n\otimes\mathbf{C})(\mathbf{I}_n\otimes\mathbf{B})\Delta\mathbf{u}_i(t)\\ &= (\mathbf{I} - ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)(\mathbf{I}_n\otimes\mathbf{C})(\mathbf{I}_n\otimes\mathbf{B}))\Delta\mathbf{u}_i(t)\\ &\quad - (((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_P)(\mathbf{I}_n\otimes\mathbf{C}) + ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)\\ &\quad \cdot (\mathbf{I}_n\otimes\mathbf{C})(\mathbf{I}_n\otimes\mathbf{A}))\Delta\mathbf{x}_i(t).\end{aligned} \tag{29}$$

By taking the λ norm on both sides of equation (29), we have

$$\begin{aligned}&\left\|\Delta\mathbf{u}_{i+1}(t)\right\|_\lambda\\ &\le \left\|\mathbf{I} - ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)(\mathbf{I}_n\otimes\mathbf{C})(\mathbf{I}_n\otimes\mathbf{B})\right\|\left\|\Delta\mathbf{u}_i(t)\right\|_\lambda\\ &\quad + \left\|((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_P)(\mathbf{I}_n\otimes\mathbf{C}) + ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)(\mathbf{I}_n\otimes\mathbf{C})\right.\\ &\quad \left.\cdot(\mathbf{I}_n\otimes\mathbf{A})\right\|\left\|\Delta\mathbf{x}_i(t)\right\|_\lambda\\ &= \left\|\mathbf{I} - (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CB}\right\|\left\|\Delta\mathbf{u}_i(t)\right\|_\lambda\\ &\quad + \left\|(\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_P\mathbf{C} + (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CA}\right\|\left\|\Delta\mathbf{x}_i(t)\right\|_\lambda.\end{aligned} \tag{30}$$

By substituting equation (26) into equation (30), we can obtain

$$\left\|\Delta\mathbf{u}_{i+1}(t)\right\|_\lambda \le \left(\left\|\mathbf{I} - (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CB}\right\| + \beta\right)\left\|\Delta\mathbf{u}_i(t)\right\|_\lambda, \tag{31}$$

where

$$\beta = \frac{\|\mathbf{B}\|\|\mathbf{L}+\mathbf{D}\|\left\|\mathbf{\Gamma}_P\mathbf{C} + \mathbf{\Gamma}_D\mathbf{CA}\right\|}{\lambda^\alpha - \|\mathbf{A}\|}. \tag{32}$$

Given that $\|\mathbf{I} - (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CB}\| < 1$, sufficiently large λ is obtained, thereby satisfying $\|\mathbf{I} - (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CB}\| + \beta \le \widetilde{\rho} < 1$. Thus, from equation (31), we have

$$\left\|\Delta\mathbf{u}_{i+1}(t)\right\|_\lambda \le \widetilde{\rho}\left\|\Delta\mathbf{u}_i(t)\right\|_\lambda \le \widetilde{\rho}^k\left\|\Delta\mathbf{u}_1(t)\right\|_\lambda. \tag{33}$$

When the number of iterations increases, that is, $i \longrightarrow \infty$, we have

$$\begin{aligned}\lim_{i\longrightarrow\infty}\left\|\Delta\mathbf{u}_{i+1}(t)\right\|_\lambda &= 0,\\ \lim_{i\longrightarrow\infty}\left\|\Delta\mathbf{x}_{i+1}(t)\right\|_\lambda &= 0.\end{aligned} \tag{34}$$

Taking the λ norm on both sides yields

$$\left\|\mathbf{e}_{i+1}(t)\right\|_\lambda \le \|\mathbf{C}\|\left\|\Delta\mathbf{x}_{i+1}(t)\right\|_\lambda. \tag{35}$$

Accordingly, we can obtain

$$\lim_{i\longrightarrow\infty}\left\|\mathbf{e}_{i+1}(t)\right\|_\lambda = 0. \tag{36}$$

The tracking errors of the agents tend to zero with $i \longrightarrow \infty$. Thus, $\mathbf{y}_{i,k}(t)$ converges to $\mathbf{y}_d(t)$. The proof is completed. □

3.2. Open-Loop D^α-Type Iterative Learning Control. For the FOMASs described in (7), if we let $\mathbf{\Gamma}_P = 0$ in (14), then the distributed open-loop PD^α-type FOILC becomes a distributed open-loop D^α-type FOILC as follows:

$$\mathbf{u}_{i+1,j}(t) = \mathbf{u}_{i,j}(t) + \mathbf{\Gamma}_D\xi_{i,j}{}^{(\alpha)}(t). \tag{37}$$

Similar to the PD^α-type FOILC, the updating law (37) can be rewritten in the vector compact form

$$\mathbf{u}_{i+1}(t) = \mathbf{u}_i(t) + ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)\mathbf{e}_i{}^{(\alpha)}(t). \tag{38}$$

Then, a corollary can be obtained as follows:

Theorem 2. *Consider the FOMAS (7) under the directed graph $\overline{\mathbf{G}}$, suppose Assumption 1 and 2 are satisfied. Let the distributed open-loop D^α-type ILC scheme (38) be applied for the system with learning gain $\mathbf{\Gamma}_{D1}$ satisfying*

$$\left\|\mathbf{I}_N - (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CB}\right\| + \beta_1 = \rho_1 < 1, \tag{39}$$

where β_1 is an expression about the system parameters and the control parameter as follows: $\beta_1 = ((\|\mathbf{B}\|\|\mathbf{L}+\mathbf{D}\|\|\mathbf{\Gamma}_D\mathbf{CA}\|)/(\lambda^\alpha - \|\mathbf{A}\|))$, and then $\lim_{i\longrightarrow\infty}\|\mathbf{e}_i(t)\|_\lambda = 0$. Namely, for the achievable desired trajectory $\mathbf{y}_d(t), t \in [0,T]$, the outputs $\mathbf{y}_i(t)$ of the FOMASs converge to the desired trajectory $\mathbf{y}_d(t)$ when $i \longrightarrow \infty$, i.e., $\lim_{i\longrightarrow\infty}\mathbf{y}_i(t) = \mathbf{y}_d(t)$, $t \in [0,T]$.

Proof. The proof process of Theorem 2 is similar to the proof process of Theorem 1. □

Remark 4. In the proof, the constant λ is only a tool for analysis and can be very large, that is, $\lambda \longrightarrow \infty$. The reason is that λ is not used in the implementation and does not affect the actual control performance.

Remark 5. The convergence of FOILC for the FOMASs is uninfluenced by the initial states. We define $M \triangleq \|\mathbf{I} - (\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D\mathbf{CB}\| + \beta$. Suitable $\mathbf{\Gamma}_P$ and $\mathbf{\Gamma}_D$ can be designed such that $\rho(M) < \widetilde{\rho}$ for all $t \in [0,T]$. Thus, an appropriate matrix norm can be designed such that $M < \widetilde{\rho}$.

Remark 6. According to Lemma 2, the matrix $\mathbf{L}+\mathbf{D}$ is a positive definite matrix when the directed graph $\overline{\mathbf{G}}$ is a connected graph. Then, the matrix $-(\mathbf{L}+\mathbf{D})$ is the Hurwitz stable matrix. Accordingly, the gain matrix $\mathbf{\Gamma}_D$ can be found to satisfy the condition in equation (17).

Remark 7. Compared with D^α-type FOILC, PD^α-type FOILC can reduce the deviation of the limit trajectory from the expected trajectory. So, the increased P-type component reduces the adverse effect of the D^α-type component on the disturbance sensitivity and improves the control performance.

4. Simulations

A network consisting of four follower fractional-order agents and a virtual leader fractional-order agent is used to verify the effectiveness of the proposed method. The dynamic function of the *ith* agent is a MIMO system. In equation (7), the matrix **A**, **B**, and **C** and α are given as follows:

$$\mathbf{A} = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & -0.1 \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} 0.3 & 0.02 \\ 0.01 & 0.2 \end{bmatrix},$$
$$\mathbf{C} = \begin{bmatrix} 0.7 & 0.01 \\ 0.01 & 0.9 \end{bmatrix}, \tag{40}$$
$$\alpha = 0.9,$$

and the desired reference trajectory is

$$\mathbf{y}_d = \begin{bmatrix} t + \sin(t * 2\pi) \\ 25t^2 + \sin(t * 2\pi) \end{bmatrix}, \quad t \in [0, 1]. \tag{41}$$

The virtual leader and the four followers are labeled as 0, 1, 2, 3, and 4. The information topology-directed graph is shown in Figure 1.

From Figure 1, the weighted adjacency matrix **M** and the in-degree $\mathbf{D}_{\text{in}}$ can be obtained as follows on the basis of graph theory:

$$\mathbf{M} = \begin{bmatrix} 0 & 0 & 0 & 0.7 \\ 0.9 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.6 & 0 \end{bmatrix}, \tag{42}$$
$$\mathbf{D}_{\text{in}} = \text{diag}\begin{bmatrix} 0.7 & 0.9 & 0.8 & 0.6 \end{bmatrix}.$$

The Laplacian matrix and the relationship between the followers and the leader are shown as follows:

$$\mathbf{L} = \mathbf{D}_{\text{in}} - \mathbf{M} = \begin{bmatrix} 0.7 & 0 & 0 & -0.7 \\ -0.9 & 0.9 & 0 & 0 \\ 0 & -0.8 & 0.8 & 0 \\ 0 & 0 & -0.6 & 0.6 \end{bmatrix}, \tag{43}$$
$$\mathbf{D} = \text{diag}\begin{bmatrix} 1.8 & 0 & 0 & 2.0 \end{bmatrix}.$$

4.1. The Results of the Open-Loop PD^α-Type FOILC. For the open-loop PD^α-type FOILC described in (16), $\mathbf{\Gamma}_P$ and $\mathbf{\Gamma}_D$ are designed to satisfy the convergence conditions, i.e., $\mathbf{\Gamma}_P = \text{diag}\{1.1 \;\; 1.6\}$ and $\mathbf{\Gamma}_D = \text{diag}\{1.5 \;\; 0.8\}$.

By checking the convergence condition and defining $\lambda = 100$, we obtain that

$$\begin{gathered} \left\| \mathbf{I} - (\mathbf{L} + \mathbf{D}) \otimes \mathbf{\Gamma}_D \mathbf{CB} \right\| = 0.9312 < 1, \\ \left\| \mathbf{I} - (\mathbf{L} + \mathbf{D}) \otimes \mathbf{\Gamma}_D \mathbf{CB} \right\| + \beta = 0.9621 < 1. \end{gathered} \tag{44}$$

Therefore, the convergence condition is satisfied.

The initial states of all the agents, the initial errors, and the initial inputs are 0, that is, $\mathbf{e}_i(0) = 0$. The input signals at the zeroth iteration are set to zero, that is, $\mathbf{u}_{0,j}(t) = 0$, for all agents. Figures 2 and 3 show the tracking performance by the open-loop PD^α-type FOILC at 5th, 20th, and 70th iterations. At the 5th iteration, the deviations are very large between the trajectories of the follower and the desired one. As the iteration number increases, the outputs $y_1(t)$ and $y_2(t)$ of the followers nearly completely track the reference trajectory over the time [0, 1]. By 70 iterations, the maximum tracking errors of $y_1(t)$ of the four agents are 0.000326, 0.000843, 0.000916, and 0.000276, and the maximum tracking errors of $y_2(t)$ of the four agents are 0.0016, 0.0041, 0.0045, and 0.0014, respectively. And moreover, compared with Agents 2 and 4, both Agents 1 and 3 can get information from the leader directly, so they can converge to the expected trajectory more quickly. We define the maximum errors at the ith iteration as $\max_{j=1,2,3,4}\|y_{d,1} - y_{i,j,1}\|$ and $\max_{j=1,2,3,4}\|y_{d,2} - y_{i,j,2}\|$. Compared with Figure 2(d) and Figure 3(d), the outputs $y_2(t)$ converge to the expected trajectory more quickly than $y_1(t)$. That is because the trajectory $y_{1d}(t)$ is more complicated than $y_{2d}(t)$ (Figure 4).

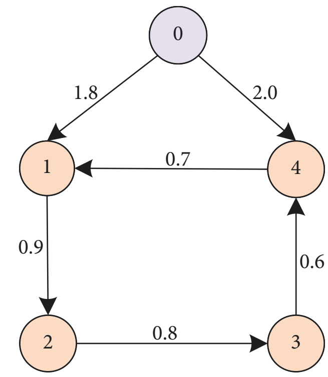


Figure 1: Communication topology among agents in the network.

4.2. The Results of the Open-Loop D^α-Type FOILC. In order to compare with the performance of the open-loop PD^α-type FOILC, the model and the structure of the FOMASs with open-loop D^α-type FOILC are the same as example 1. The control parameters of the open-loop D^α-type FOILC described in (38) are designed as $\mathbf{\Gamma}_D = \text{diag}\{1.5, 0.8\}$ which are the same as $\mathbf{\Gamma}_D$ of the open-loop PD^α-type FOILC. Thus, if $\lambda = 100$ is selected, the convergence condition (39) can be calculated as follows:

$$\left\| \mathbf{I} - (\mathbf{L} + \mathbf{D}) \otimes \mathbf{\Gamma}_D \mathbf{CB} \right\| + \beta_1 = 0.9382 < 1. \tag{45}$$

Thus, the convergence condition in Theorem 1 is satisfied.

The simulation results are plotted in (Figure 4 and 5). Similar to results of the open-loop PD^α-type FOILC, as iteration number increases, the outputs $y_1(t)$ and $y_2(t)$ of the followers nearly completely track the reference trajectory over the time [0, 1]. But, the maximum tracking errors of $y_1(t)$ of the four agents are 0.013, 0.034, 0.037, and 0.011, and the maximum tracking errors of $y_2(t)$ of the four agents are 0.1, 0.26, 0.28, and 0.085 by 70 iterations, respectively. Obviously, compared with the open-loop PD^α-type FOILC, the convergence speed is slower than the open-loop D^α-type FOILC.

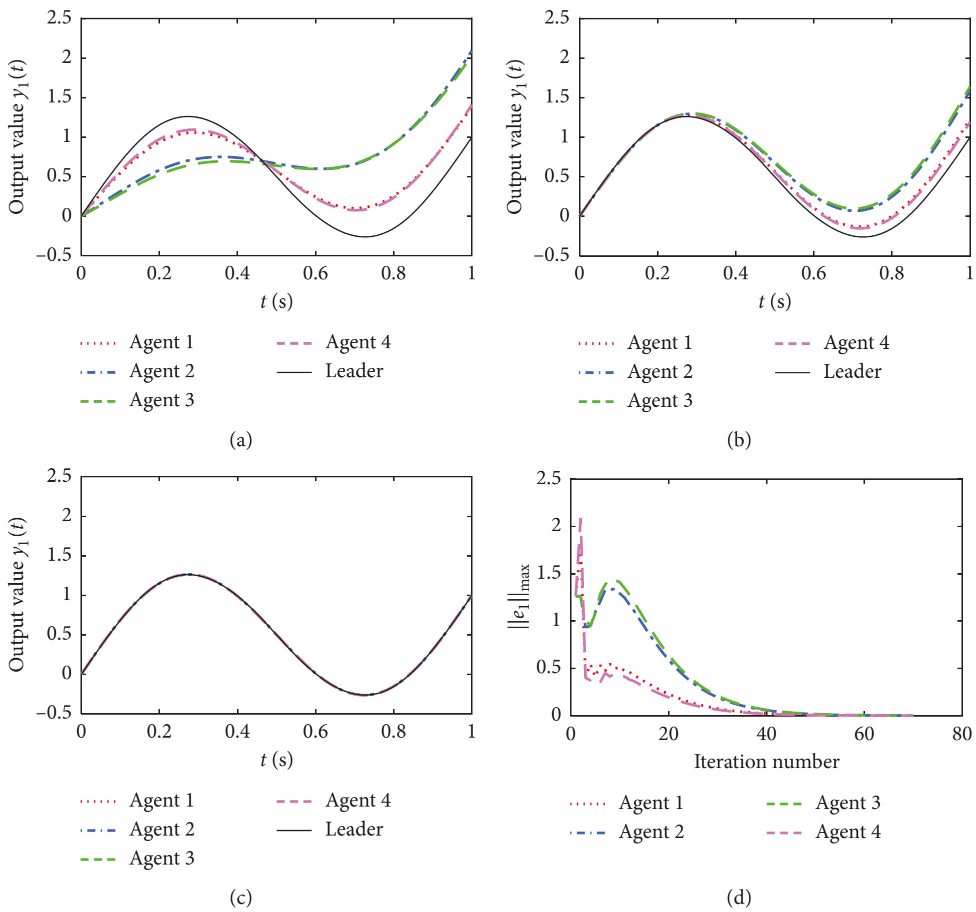


FIGURE 2: Results of y_1 with open-loop PD^{α}-type FOILC. (a) Trajectories $y_1(t)$ at the 5th iteration. (b) Trajectories $y_1(t)$ at the 20th iteration. (c) Trajectories $y_1(t)$ at the 70th iteration. (d) Tracking error e_1 vs. iteration number.

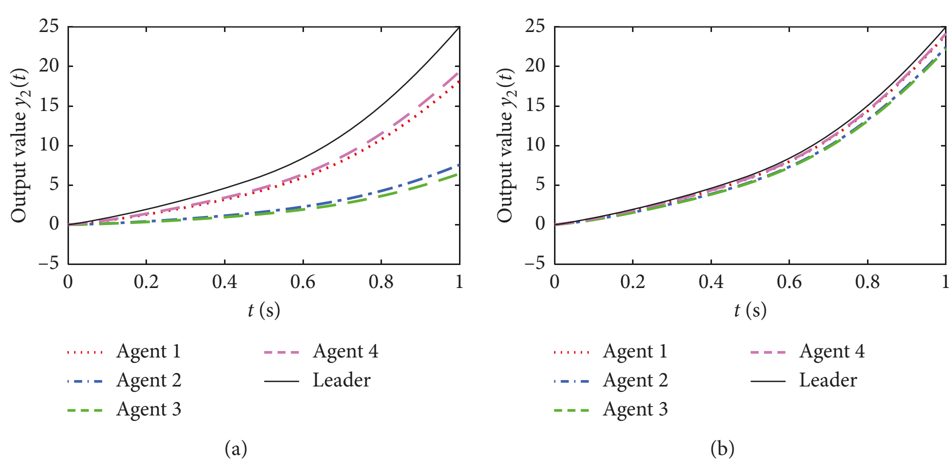


FIGURE 3: Continued.

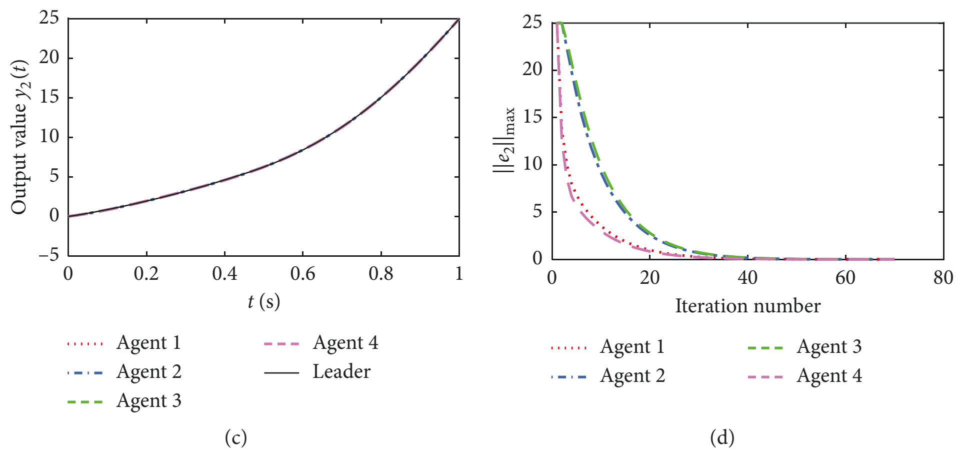


FIGURE 3: Results of y_2 with open-loop PD^{α}-type FOILC. (a) Trajectories $y_2(t)$ at the 5th iteration. (b) Trajectories $y_2(t)$ at the 20th iteration. (c) Trajectories $y_2(t)$ at the 70th iteration. (d) Tracking error e_2 vs. iteration number.

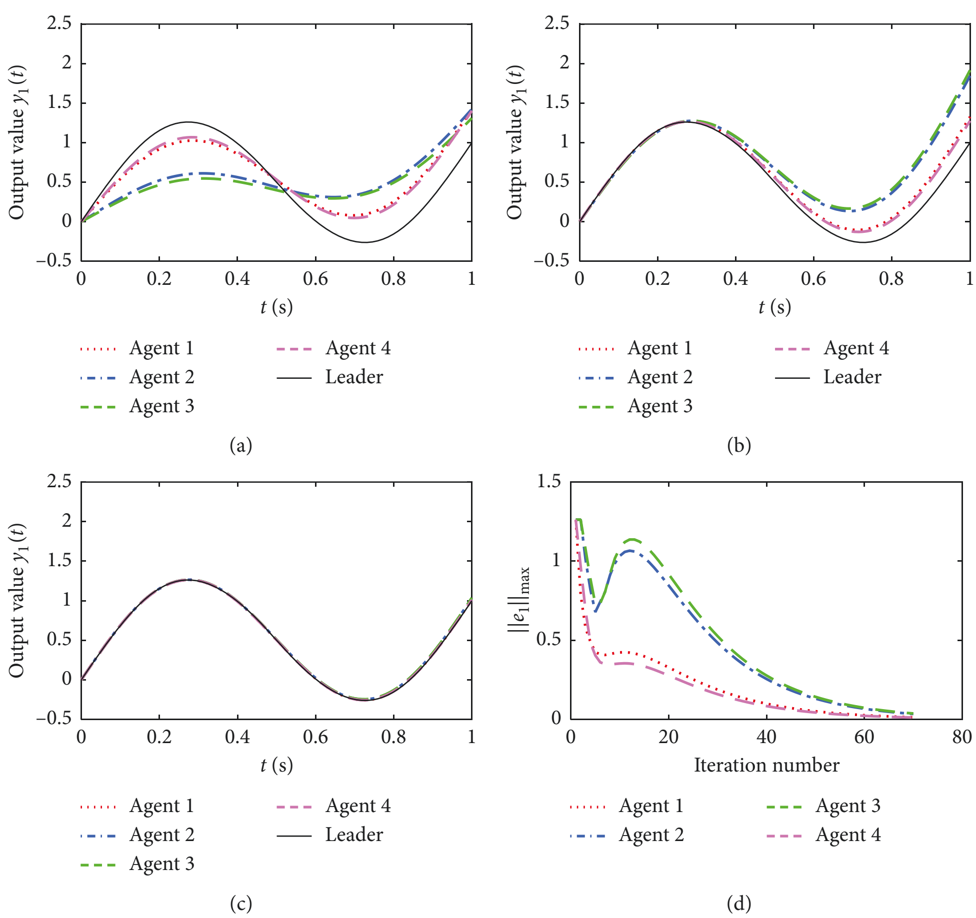


FIGURE 4: Results of y_1 with open-loop D^{α}-type FOILC. (a) Trajectories $y_1(t)$ at the 5th iteration. (b) Trajectories $y_1(t)$ at the 20th iteration. (c) Trajectories $y_1(t)$ at the 70th iteration. (d) Tracking error e_1 vs. iteration number.

4.3. The Results of the Open-Loop PD-Type and D-Type IOILC. In order to reflect the superiority of the fractional controller, we designed an integer-order iterative learning control (IOILC) to apply the FOMASs. The *PD*-type and *D*-type IOILC are as follows:

$$\mathbf{u}_{i+1}(t) = \mathbf{u}_i(t) + ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_P)\mathbf{e}_i(t) + ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)\frac{\mathrm{d}\mathbf{e}_i(t)}{\mathrm{d}t},$$
$$\mathbf{u}_{i+1}(t) = \mathbf{u}_i(t) + ((\mathbf{L}+\mathbf{D})\otimes\mathbf{\Gamma}_D)\frac{\mathrm{d}\mathbf{e}_i(t)}{\mathrm{d}t}. \quad (46)$$

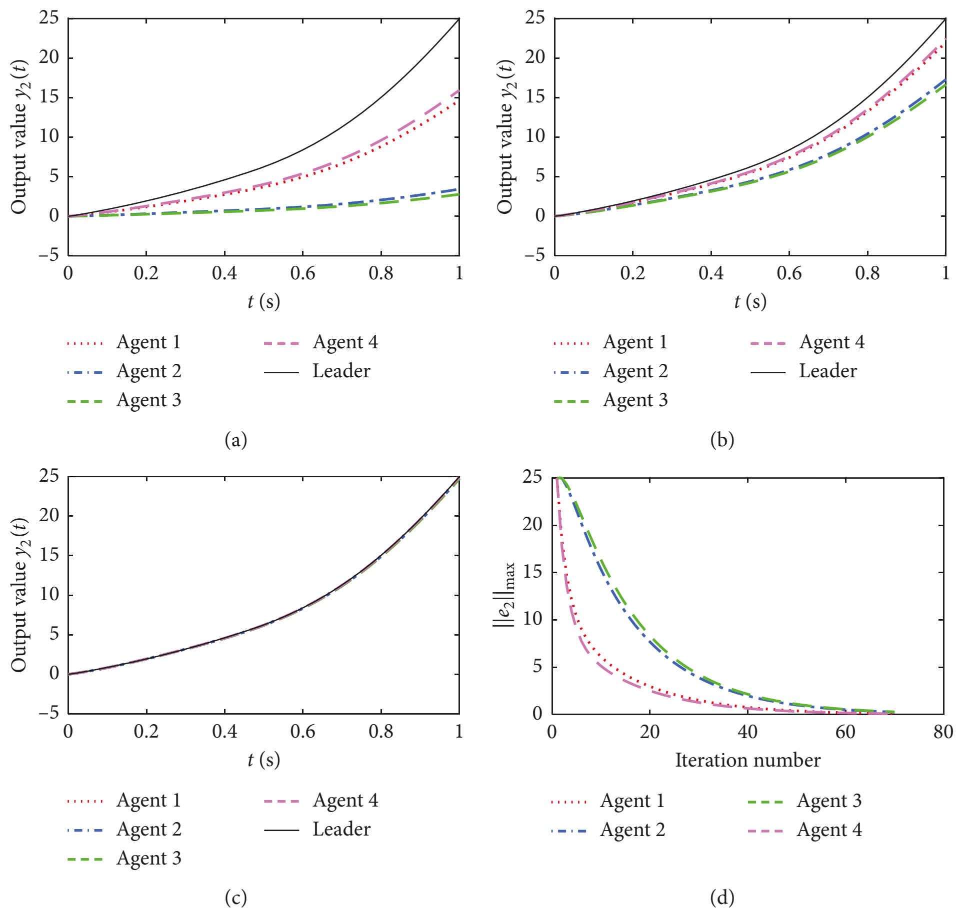


FIGURE 5: Results of y_2 with open-loop D^{α}-type FOILC. (a) Trajectories $y_2(t)$ at the 5th iteration. (b) Trajectories $y_2(t)$ at the 20th iteration. (c) Trajectories $y_2(t)$ at the 70th iteration. (d) Tracking error e_2 vs. iteration number.

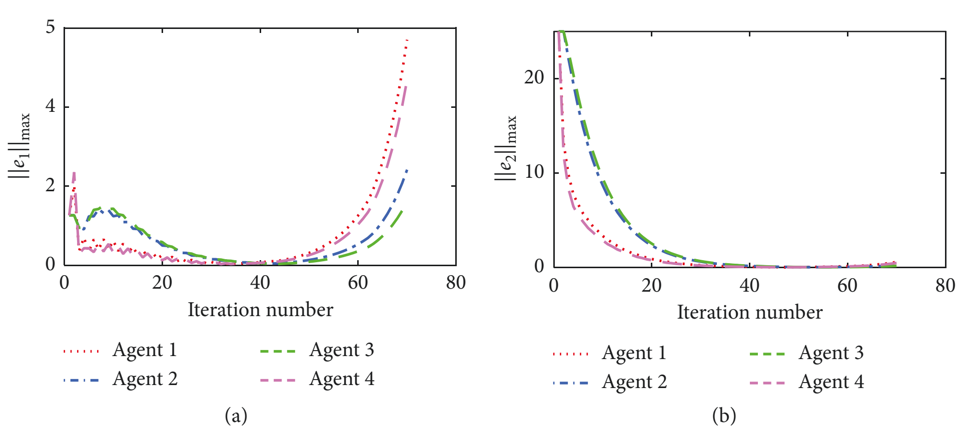
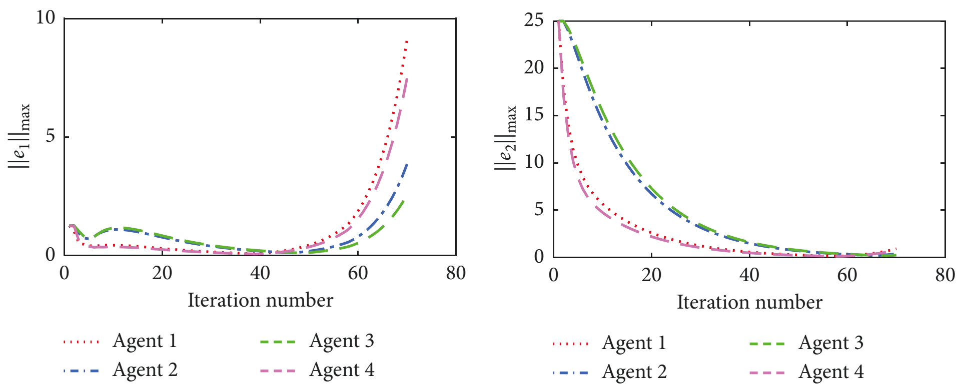


FIGURE 6: Results with open-loop PD-type IOILC. (a) Tracking error e_1 vs. iteration number. (b) Tracking error e_2 vs. iteration number.

The parameters of the FOMASs are the same as Figure 1 and equations (40) and (41). The learning gains of PD-type are $\mathbf{\Gamma}_P = \operatorname{diag}\{1.1 \quad 1.6\}$, $\mathbf{\Gamma}_D = \operatorname{diag}\{1.5 \quad 0.8\}$, and the learning gain of D-type is $\mathbf{\Gamma}_D = \operatorname{diag}\{1.5 \quad 0.8\}$, respectively, which are the same as the parameters of the FOILC proposed in this paper. The results are shown in Figures 6 and 7. It can be seen that the errors of $y_1(t)$ and $y_2(t)$ decrease first as the iteration number increases when employing the PD-type and D-type IOILC. But, when the iteration number is greater than 60, the error gradually increases. Compared with IOILC, we can see that the FOILC has better performance.



(a)

(b)

Figure 7: Results with open-loop D-type FOILC. (a) Tracking error e_1 vs. iteration number. (b) Tracking error e_2 vs. iteration number.

5. Conclusion

In this study, a consensus tracking problem of fractional-order multiagent systems with general linear models is formulated. Distributed PD^{α}-type and D^{α}-type fractional-order ILC control algorithms are proposed to solve the consensus tracking problem of the FOMAS. The convergence is proved, and the convergence condition is presented based on graph theory, fractional calculus, and norm theory. It is shown theoretically, together with numerical simulation, that the proposed method can make the agents reach agreement exactly on a common output trajectory over a finite time interval. Through comparison, we find that the PD^{α}-type FOILC can obtain much better performance in terms of convergence speed. Further research may include the closed-loop PD^{α}-type iterative learning control schemes for fractional-order nonlinear multiagent systems, as well as for experimental verifications.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

All authors declare that there are no conflicts of interest regarding the publication of this paper.

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