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Research Article

Vibration Control of an Axially Moving System with Restricted Input

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In this study, we consider the global stabilization of an axially moving system under the condition of input saturation nonlinearity and external perturbation. Based on Lyapunov redesign method, observer backstepping, and high-gain observers, an output feedback control strategy with an auxiliary system is constructed to eliminate the input saturation constraint effect and suppress the string system vibration, and a boundary disturbance observer is exploited to cope with the external disturbance. The stability of the controlled system is analyzed and proven based on Lyapunov stability without simplifying or discretizing the infinite dimensional dynamics. The presented simulation results show the effectiveness of the derived control.

1. Introduction

Axially moving systems are important components in mechanical system and play a significant role in actual production process. However, a few issues exist; nonsmooth input nonlinearities and external perturbations frequently occur and produce severe impacts on system performance. It is worth noting that nonsmooth input nonlinearities containing saturation, backlash, hysteresis, and dead-zone are generally found in industrial control systems, such as mechanical, hydraulic, biomedical, piezoelectric, and physical systems [1–5]. Such nonlinearities usually arise from inherent physical constraints of the dynamical system and constraints in the controller actuators, which are impossible to be eliminated. If the input nonlinearities are ignored in the system model, it is difficult to make the actual axially moving system stabilized. So far, some results associated with how to achieve the control objective for flexible structure systems with the input saturation have been attained [6–8]. In [6], the vibration control and input saturation problem for a vibrating flexible aerial refueling hose with variable lengths are addressed introducing the hyperbolic tangent function and adopting the backstepping approach. In [7], the authors develop the boundary control for a vibrating riser system with mixed input nonlinearities to suppress the deflection and compensate for the input saturation. In [8], the control schemes are constructed for a flexible beam system to suppress the vibration and eliminate the input saturation and output constraint in the presence of disturbances. However, these results do deal with the input saturation issue for stationary flexible systems, but there is little information on how to handle the input restriction for axially moving systems.

In recent decades, many achievements regarding vibration control for axially moving systems have been attained, whose dynamics can be mathematically considered to be distributed parameter systems (DPS) with infinite dimensional feature [9–15]. Effective solutions for controlling the DPS mainly include truncation model-based method, and boundary control. Different from truncation model-based method [16–20], which is employed in different ways to extract a finite dimensional subsystem to be controlled while showing robustness to neglecting the remaining infinite dimensional dynamics in the design, boundary control is the implementation of control design based on infinite dimensional system dynamics, which is generally considered to be physically more realistic due to nonintrusive actuation and sensing [21]. For the past few years, the vibration boundary control scheme design for the axially moving system has made great achievement [22–28]. In [22], the deflection of the axially moving string is regulated by the proposed adaptive vibration isolation and the practical experiment illustrates the theoretical results. In [23], an adaptive robust control strategy is constructed for controlling the vibrational offset of an axially moving system in the presence of parameter and disturbance uncertainties. In [24], an iterative learning control scheme is exploited for a stretched flexible string to damp out any string oscillation based on continuous and discrete Lyapunov functions. In [25], the vibration of a translating tensioned beam is exponentially stabilized and effectively suppressed via the choice of a proper Lyapunov function candidate. In [26], a stabilizing control law is derived for a translating tensioned strip to suppress the vibration and the closed-loop system is proven to be exponentially stable. In [27], simultaneous vibration control scheme design and velocity regulation issue are discussed and good stability is attained in the sense of Lyapunov. In [28], the high-gain observer technique and Lyapunov-based observer backstepping method are integrated into the context of boundary control design to generate a stabilizing robust control law for suppressing the deflection of an accelerative belt system. In this article, the axially moving system with the input saturation is studied under the condition of the external disturbance, which makes the control scheme design more complicated and difficult in comparison with previous research.

Moreover, in research achievements [22–28], the control schemes are implemented based on the assumption that all the system state signals consisting of the control law can be directly measured by sensors or obtained by algorithms. However, in practice, the measurement noise derived from sensors is completely unavoidable, and its effect will be further magnified in the procedure to obtain the terms of differentiation to time, which would limit the controller in [22–28] implementation. To resolve this issue, the observer backstepping [29] and high-gain observers [30] can be exploited to estimate the unmeasured system states and then an output feedback boundary control is developed to globally stabilize the considered axially moving system.

In this study, our interest lies in how to construct an output feedback control for the global stabilization of the axially moving system and simultaneously for the elimination of input saturation nonlinearity effect. Compared with the previous research, the main contributions are summed up as follows.

(i) When there are the unmeasured system states, observer backstepping and high-gain observers are employed to reconstruct the system states and then an output feedback boundary control is generated for vibration reduction of the axially moving system.

(ii) An auxiliary system is introduced to tackle the input saturation nonlinearity and a disturbance observer is exploited to track the external disturbance.

(iii) The uniformly and ultimately bounded stability of the controlled system are analyzed and demonstrated through rigorous Lyapunov analysis without any model reduction.

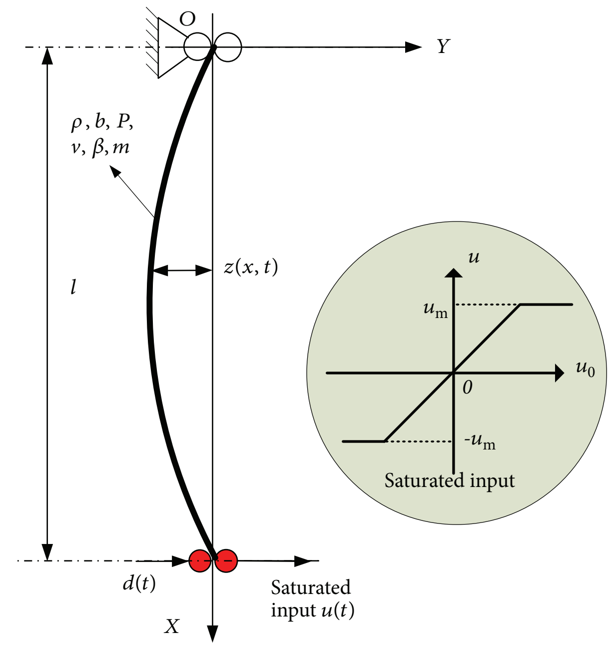

FIGURE 1: An axially moving string system with saturated input.

The arrangement of this paper is listed as below. The system dynamics and input saturation function are introduced in Section 2. Disturbance observer and boundary control law are constructed for tracking the external disturbance and suppressing the vibration in Section 3, where the stability of the controlled system is analyzed based on Lyapunov stability. Numerical simulations are presented in Section 4 and we reach a conclusion in Section 5.

2. Problem Statement

An axially moving string system with saturated input is depicted in Figure 1, where $z(x,t)$ denotes the vibration offset, l, ρ, b, P, and v denote the length, the mass per unit length, the damping coefficient, the tension, and the axial speed of the string, respectively, m and β denote the mass and the damping coefficient of the actuator, respectively, $d(t)$ denotes the external disturbance, $u(t)$ denotes the saturated control input, u_0 denotes the control command to be developed, and u_m denotes the saturation limit.

Remark 1. Notations are defined as follows: $(\star)(t) = (\star)$, $(\star)_x = \partial(\star)/\partial x$, $(\star)_t = \partial(\star)/\partial t$, $(\star)_{xt} = \partial^2(\star)/\partial x \partial t$, $(\star)_{xx} = \partial^2(\star)/\partial x^2$, and $(\star)_{tt} = \partial^2(\star)/\partial t^2$.

In this study, the dynamics of the considered string system is presented as follows:

$$\rho z_{tt}(x,t) + (\rho v_t + bv) z_x(x,t) + 2\rho v z_{xt}(x,t) + (\rho v^2 - P) z_{xx}(x,t) + b w_t(x,t) = 0, \quad 0 < x < l \tag{1}$$

$$z(0,t) = 0 \tag{2}$$

$$P z_x(l,t) + \beta z_t(l,t) = u - d - m z_{tt}(l,t) \tag{3}$$

and the input saturation function is expressed as

$$u = \begin{cases} \operatorname{sgn}(u_0)\, u_{\mathrm{m}}, & |u_0| \geq u_{\mathrm{m}}, \\ u_0, & |u_0| < u_{\mathrm{m}}, \end{cases} \tag{4}$$

Assumption 2. We assume that the axial speed v and external disturbance d are uniformly bounded and there exist two nonnegative constants α_1 and α_2 such that $0 \leq v \leq \alpha_1$ and $0 \leq |d| \leq \alpha_2$, $\forall t \in [0, +\infty)$. It is a reasonable assumption as v and d have finite energy and hence are bounded [31].

Assumption 3. We assume that the time derivative of external disturbance d_t is uniformly bounded and there exists a nonnegative constant α_3 such that $0 \leq |d_t| \leq \alpha_3$, $\forall t \in [0, +\infty)$.

3. Control Design

Before proceeding further, we first introduce the following coordinate transformation:

$$w_1 = z(l, t) \tag{5}$$

$$w_{1t} = w_2 = z_t(l, t) \tag{6}$$

$$w_{2t} = \frac{1}{m}\left[u + d - \beta w_2 - P z_x(l, t)\right] \tag{7}$$

It is evident that (6) contains the state $w_2 = z_t(l, t)$ which is available for feedback in (7) and can be computed exploiting difference algorithms to boundary measured state $z(l, t)$. Nevertheless, the impact of the measured noise arising from sensors is completely unavoidable in practice and would be further magnified in the procedure of differentiation to time, which will make the feedback irrealizable. Fortunately, observer backstepping method [29] can reconstruct the states available for feedback and guarantee both boundedness of the closed-loop states and convergence of the state errors to zero, which can be used to solve the issue aforesaid.

Then, we take the place of w_2 in (7) with $\widehat{w}_2$

$$\widehat{w}_{2t} = \frac{1}{m}\left[u + d - \beta \widehat{w}_2 - P z_x(l, t)\right] \tag{8}$$

The definition of the state estimation error $\widetilde{w}_2$ is introduced as

$$\widetilde{w}_2 = w_2 - \widehat{w}_2 \tag{9}$$

Taking the derivative of (9) and applying (7) and (8) into the consequence lead to

$$\widetilde{w}_{2t} = -\frac{\beta}{m}\widetilde{w}_2 \tag{10}$$

It can be derived from (10)

$$\widetilde{w}_2 = \widetilde{w}_2(0)\, e^{-(\beta/m)t} \tag{11}$$

Substituting (9) into (6) gives

$$w_{1t} = \widehat{w}_2 + \widetilde{w}_2 \tag{12}$$

Then, we rewrite the reconstructed states as

$$\begin{aligned} w_{1t} &= \widehat{w}_2 + \widetilde{w}_2 \\ \widehat{w}_{2t} &= \frac{1}{m}\left[u + d - \beta \widehat{w}_2 - P z_x(l, t)\right] \\ \widetilde{w}_{2t} &= -\frac{\beta}{m}\widetilde{w}_2 \end{aligned} \tag{13}$$

3.1. Disturbance Observer Dynamics. First we define $\widehat{d}$ as the estimation of d and then design $\widehat{d}_t$ as

$$\widehat{d}_t = \mu\left(d - \widehat{d}\right) \tag{14}$$

where $\mu > 0$.

Furthermore, the intermediate variable is proposed as

$$\varsigma = \widehat{d} - \mu m \widehat{w}_2 \tag{15}$$

Differentiating (15) and substituting (8) and (14) into the consequence yields

$$\varsigma_t = \mu\left[P z_x(l, t) + \beta \widehat{w}_2 - u\right] - \mu \widehat{d} \tag{16}$$

Then the boundary disturbance observer dynamics is presented as follows

$$\begin{aligned} \widehat{d} &= \varsigma + \mu m \widehat{w}_2 \\ \varsigma_t &= \mu\left[P z_x(l, t) + \beta \widehat{w}_2 - u\right] - \mu \widehat{d} \end{aligned} \tag{17}$$

The following estimation error is defined

$$\widetilde{d} = d - \widehat{d} \tag{18}$$

Differentiating (18) and substituting (14), we get

$$\widetilde{d}_t = d_t - \widehat{d}_t = d_t - \mu \widetilde{d} \tag{19}$$

3.2. Boundary Control

Step 1. In this step, we first define

$$e = \widehat{w}_2 - \omega \tag{20}$$

where ω is virtual control and e is corresponding error variable.

Then set a Lyapunov function as

$$V_m = V_a + V_b \tag{21}$$

where

$$\begin{aligned} V_a &= \frac{1}{2}\tau\rho \int_0^l \left[z_t(x, t) + v z_x(x, t)\right]^2 dx \\ &\quad + \frac{1}{2}\tau P \int_0^l z_x^2(x, t)\, dx \end{aligned} \tag{22}$$

$$V_b = \zeta\rho \int_0^l x z_x(x,t)\left[z_t(x,t) + v z_x(x,t)\right] dx \quad (23)$$

with $\tau, \zeta > 0$.

Applying $mn \leq (m^2 + n^2)/2$ to (23), we have

$$\begin{aligned}|V_b| &\leq \frac{\zeta\rho l}{2}\int_0^l z_x^2(x,t)\,dx \\ &+ \frac{\zeta\rho l}{2}\int_0^l \left[z_t(x,t) + v z_x(x,t)\right]^2 dx \leq \varrho V_a\end{aligned} \quad (24)$$

where

$$\varrho = \frac{\zeta\rho l}{\min(\tau\rho, \tau P)} \quad (25)$$

From (24), we get

$$-\varrho V_a \leq V_b \leq \varrho V_a \quad (26)$$

The proper selection of ζ and τ yields

$$\begin{aligned}\varrho_1 &= 1 - \varrho = 1 - \frac{\zeta\rho l}{\min(\tau\rho, \tau P)} > 0 \\ \varrho_2 &= 1 + \varrho = 1 + \frac{\zeta\rho l}{\min(\tau\rho, \tau P)} > 1\end{aligned} \quad (27)$$

Equation (27) shows $0 < \varrho < 1$, and then combining (25) leads to

$$\zeta < \frac{\min(\tau\rho, \tau P)}{\rho l} \quad (28)$$

Then, we further obtain

$$0 < \varrho_1 V_a \leq V_m \leq \varrho_2 V_a \quad (29)$$

Differentiating V_m, substituting (1), and then using $mn \leq m^2/\sigma + \sigma n^2$, $\sigma > 0$, we have

$$\begin{aligned}V_{mt} \leq{}& \frac{\tau P v + \zeta P l - \zeta\rho l v^2}{2} z_x^2(l,t) \\ &- \frac{\tau v\left(P - \rho v^2\right)}{2} z_x^2(0,t) + \frac{\zeta\rho l}{2} w_2^2 \\ &+ \tau P z_x(l,t)\, w_2 \\ &- \left(\frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 - \zeta b l \varpi_1\right)\int_0^l z_x^2(x,t)\,dx \\ &- \left(\tau b - \frac{\zeta b l}{\varpi_1}\right)\int_0^l \left[z_t(x,t) + v z_x(x,t)\right]^2 dx\end{aligned} \quad (30)$$

where $\varpi_1 > 0$.

Combining (9) and (20), we obtain

$$w_2 = e + \omega + \widetilde{w}_2 \quad (31)$$

Taking the substitution of (31) into (30), we derive

$$\begin{aligned}V_{mt} \leq{}& \frac{\tau P v + \zeta P l - \zeta\rho l v^2}{2} z_x^2(l,t) \\ &- \frac{\tau v\left(P - \rho v^2\right)}{2} z_x^2(0,t) \\ &+ \tau P z_x(l,t)\left(e + \omega + \widetilde{w}_2\right) \\ &- \left(\frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 - \zeta b l \varpi_1\right)\int_0^l z_x^2(x,t)\,dx \\ &- \left(\tau b - \frac{\zeta b l}{\varpi_1}\right)\int_0^l \left[z_t(x,t) + v z_x(x,t)\right]^2 dx \\ &+ \frac{\zeta\rho l}{2}(e + \omega)^2 + \frac{\zeta\rho l}{2}\widetilde{w}_2^2 + \zeta\rho l (e + \omega)\widetilde{w}_2\end{aligned} \quad (32)$$

According to (32), the virtual control ω is developed as

$$\omega = -c_1 z_x(l,t) \quad (33)$$

where $c_1 > 0$.

Applying (33) to (32) and then using $mn \leq m^2/\sigma + \sigma n^2$, $\sigma > 0$, we get

$$\begin{aligned}V_{mt} \leq{}& -\Bigg(\tau c_1 P - \zeta\rho l c_1^2 - \frac{\tau P v + \zeta P l - \zeta\rho l v^2}{2} - \frac{\tau P}{\varpi_2} \\ &- \frac{c_1\zeta\rho l}{\varpi_3}\Bigg) z_x^2(l,t) - \frac{\tau v\left(P - \rho v^2\right)}{2} z_x^2(0,t) + \big(\tau P \varpi_2 \\ &+ \zeta\rho l + c_1\zeta\rho l\varpi_3\big)\widetilde{w}_2^2 - \left(\frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 - \zeta b l\varpi_1\right) \\ &\cdot \int_0^l z_x^2(x,t)\,dx + \frac{3\zeta\rho l}{2}e^2 - \left(\tau b - \frac{\zeta b l}{\varpi_1}\right) \\ &\cdot \int_0^l \left[z_t(x,t) + v z_x(x,t)\right]^2 dx + \tau P z_x(l,t)\, e\end{aligned} \quad (34)$$

where $\varpi_2, \varpi_3 > 0$.

Step 2. In this step, the auxiliary system is developed to cope with the input saturation constraint and the control command u_0 is designed to stabilize e around zero.

Then, we first define the following auxiliary system as

$$\phi_t = \begin{cases} -c_2\phi - \dfrac{e \triangle u + 0.5(\triangle u)^2}{\phi} + \triangle u, & |\phi| \geq \phi_0, \\ 0, & |\phi| < \phi_0, \end{cases} \quad (35)$$

where $c_2, \phi_0 > 0$, $\triangle u = u - u_0$, and ϕ is the state of the auxiliary system.

Differentiate (20) and then combine (8) and (33), resulting in

$$\begin{aligned}e_t ={}& \frac{1}{m}\big[u + d - \beta e + \beta c_1 z_x(l,t) - P z_x(l,t) \\ &+ m c_1 z_{xt}(l,t)\big]\end{aligned} \quad (36)$$

Choose the Lyapunov function candidate as

$$V_n = V_m + \frac{m}{2}e^2 + \frac{\pi}{2}\widetilde{w}_2^2 + \frac{1}{2}\phi^2 + \frac{1}{2}\widetilde{d}^2 \tag{37}$$

where $\pi > 0$.

Differentiating V_n yields

$$V_{nt} = V_{mt} + mee_t + \pi\widetilde{w}_2\widetilde{w}_{2t} + \phi\phi_t + \widetilde{d}\widetilde{d}_t \tag{38}$$

Substituting (10), (19), (34)-(36) into (38), we obtain

$$\begin{aligned} V_{nt} \leq & -\left(\tau c_1 P - \zeta\rho l c_1^2 - \frac{\tau P v + \zeta P l - \zeta\rho l v^2}{2} - \frac{\tau P}{\varpi_2} \right. \\ & \left. - \frac{c_1\zeta\rho l}{\varpi_3}\right) z_x^2(l,t) - \frac{\tau v\left(P - \rho v^2\right)}{2} z_x^2(0,t) \\ & + \tau P z_x(l,t)\, e - \left(\mu - \frac{1}{2}\right)\widetilde{d}^2 + \frac{1}{2}\widetilde{d}_t^2 - \left(\frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 \right. \\ & \left. - \zeta b l \varpi_1\right) \int_0^l z_x^2(x,t)\, dx + \frac{3\zeta\rho l}{2}e^2 - \left(\tau b - \frac{\zeta b l}{\varpi_1}\right) \\ & \cdot \int_0^l \left[z_t(x,t) + v z_x(x,t)\right]^2 dx - \left(\frac{\pi\beta}{m} - \tau P \varpi_2 \right. \\ & \left. - \zeta\rho l - c_1\zeta\rho l\varpi_3\right)\widetilde{w}_2^2 - \left(c_2 - \frac{1}{2}\right)\phi^2 + e\left[u_0 + d \right. \\ & \left. - \beta e + \beta c_1 z_x(l,t) - P z_x(l,t) + m c_1 z_{xt}(l,t)\right] \end{aligned} \tag{39}$$

From (39), we design the control command u_0 as

$$\begin{aligned} u_0 = & -c_3 e - \widehat{d} - \beta c_1 z_x(l,t) + P z_x(l,t) - m c_1 z_{xt}(l,t) \\ & - \tau P z_x(l,t) + c_4\phi \end{aligned} \tag{40}$$

where $c_3, c_4 > 0$.

Substituting (40) into (39) and applying $mn \leq (m^2+n^2)/2$, we get

$$\begin{aligned} V_{nt} \leq & -\left(\tau c_1 P - \zeta\rho l c_1^2 - \frac{\tau P v + \zeta P l - \zeta\rho l v^2}{2} - \frac{\tau P}{\varpi_2} \right. \\ & \left. - \frac{c_1\zeta\rho l}{\varpi_3}\right) z_x^2(l,t) - \frac{\tau v\left(P - \rho v^2\right)}{2} z_x^2(0,t) - \left(c_3 + \beta \right. \\ & \left. - \frac{3\zeta\rho l}{2} - \frac{c_4}{2} - \frac{1}{2}\right) e^2 - \left(\frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 - \zeta b l\varpi_1\right) \\ & \cdot \int_0^l z_x^2(x,t)\, dx - \left(c_2 - \frac{c_4}{2} - \frac{1}{2}\right)\phi^2 - \left(\tau b \right. \\ & \left. - \frac{\zeta b l}{\varpi_1}\right) \int_0^l \left[z_t(x,t) + v(t) z_x(x,t)\right]^2 dx + \frac{1}{2}\widetilde{d}_t^2 \\ & - \left(\frac{\pi\beta}{m} - \tau P\varpi_2 - \zeta\rho l - c_1\zeta\rho l\varpi_3\right)\widetilde{w}_2^2 - (\mu - 1)\widetilde{d}^2 \end{aligned} \tag{41}$$

Combining (20) and (33), the designed control command u_0 in (40) is rewritten as

$$\begin{aligned} u_0(t) = & -c_3\widehat{w}_2 - c_1 c_3 z_x(l,t) - \widehat{d} - \beta c_1 z_x(l,t) \\ & + P z_x(l,t) - m c_1 z_{xt}(l,t) - \tau P z_x(l,t) + c_4\phi \end{aligned} \tag{42}$$

It is worth mentioning that the boundary state $z_{xt}(l,t)$ in the designed control command (42) can be calculated utilizing a backward difference algorithm to the measured signal $z_x(l,t)$ [22]. However, in practice, the measurement noise arising from the sensors will be further magnified in the differential process to obtain $z_{xt}(l,t)$, which will limit the controller (42) implementation. In consequence, to address this issue, high-gain observers are utilized for the estimation of the unmeasured state $z_{xt}(l,t)$.

Lemma 4. *Suppose that a system output $y(t)$ and its first m derivatives are bounded such that $|y^{(n)}| < Y_n$ with positive constants Y_n, $n = 1, \cdots, m$; then the following linear system is considered [30]*

$$\begin{aligned} \epsilon\eta_{it} &= \eta_{i+1}, \quad i = 1, \cdots, m-1, \\ \epsilon\eta_{mt} &= -\overline{\lambda}_1\eta_m - \overline{\lambda}_2\eta_{m-1} - \cdots - \overline{\lambda}_{m-1}\eta_2 - \eta_1 + x_1, \end{aligned} \tag{43}$$

where ϵ is any small positive constant and the parameters $\overline{\lambda}_i$, for $i = 1, \cdots, m-1$, are chosen such that the polynomial $s^m + \overline{\lambda}_1 s^{m-1} + \ldots + \overline{\lambda}_{m-1}s + 1$ is Hurwitz. Then, we have the following property:

$$\chi_n = \frac{\eta_n}{\epsilon^{n-1}} - x_1^{(n-1)} = -\epsilon\psi^{(n)}, \quad n = 1, \cdots, m-1 \tag{44}$$

where $\psi = \eta_m + \overline{\lambda}_1\eta_{m-1} + \cdots + \overline{\lambda}_{m-1}\eta_1$ with $\psi^{(n)}$ denoting the nth derivative of ψ. In addition, there exist positive constants T and h_n such that $\forall t > T$; we have $\|\chi_n\| \leq \epsilon h_n$, $n = 1, \cdots, m$, where $\|\cdot\|$ denotes the standard Euclidean norm.

It is evidently obtained from Lemma 4 that η_{n+1}/ϵ^n converges to $x_1^{(n)}$, which is the nth derivative of x_1; that is to say, χ_n converges to zero due to the high-gain $1/\epsilon$ provided that x_1 and its nth derivatives are bounded. As a consequence, it is proper to choose η_{n+1}/ϵ^n as an observer to estimate the output signals up to the mth order derivative.

Define $x_1 = z_x(l,t)$ and $x_2 = z_{xt}(l,t)$. We consider the observer for the system with $m = 2$ and then design the estimate of the state x_2 as

$$\widehat{x}_2 = \frac{\eta_2}{\epsilon} \tag{45}$$

where the dynamics of η_2 and the error $\widetilde{x}_2$ are defined as

$$\begin{aligned} \epsilon\eta_{1t} &= \eta_2 \\ \epsilon\eta_{2t} &= -\overline{\lambda}_1\eta_2 - \eta_1 + x_1 \\ \widetilde{x}_2 &= \widehat{x}_2 - x_2 \end{aligned} \tag{46}$$

It follows from (45) and (46) that the designed control command (40) is updated as

$$u_0 = -c_3 e - \widehat{d} - \left(\beta c_1 + \tau P - P\right) x_1 - m c_1\widehat{x}_2 + c_4\phi \tag{47}$$

Substituting (47) into (39) and applying $mn \leq (m^2+n^2)/2$, we have

$$\begin{aligned} V_{nt} \leq & -\left(\tau c_1 P - \zeta\rho l c_1^2 - \frac{\tau P v + \zeta P l - \zeta\rho l v^2}{2} - \frac{\tau P}{\varpi_2} \right. \\ & \left. - \frac{c_1\zeta\rho l}{\varpi_3}\right) z_x^2(l,t) - \frac{\tau v\left(P - \rho v^2\right)}{2} z_x^2(0,t) - \left(c_3 \right. \\ & \left. + \beta - \frac{3\zeta\rho l}{2} - \frac{c_4}{2} - \frac{1}{2} - \frac{m c_1}{\varpi_4}\right) e^2 - \left(\frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 \right. \\ & \left. - \zeta b l \varpi_1\right) \int_0^l z_x^2(x,t)\,dx - \left(c_2 - \frac{c_4}{2} - \frac{1}{2}\right)\phi^2 - \left(\tau b \right. \\ & \left. - \frac{\zeta b l}{\varpi_1}\right) \int_0^l \left[z_t(x,t) + v(t) z_x(x,t)\right]^2 dx + \frac{1}{2}\tilde{d}_t^2 \\ & + m c_1 \varpi_4 \tilde{x}_2^2 - \left(\frac{\pi\beta}{m} - \tau P \varpi_2 - \zeta\rho l - c_1\zeta\rho l \varpi_3\right)\widetilde{w}_2^2 \\ & - (\mu - 1)\tilde{d}^2 \end{aligned} \tag{48}$$

where $\varpi_4 > 0$.

The parameters $c_1, c_2, c_3, c_4, \tau, \zeta, \pi, \mu, \varpi_1, \varpi_2, \varpi_3$, and ϖ_4 are chosen such that

$$\begin{aligned} & \zeta < \frac{\min(\tau\rho, \tau T)}{\rho l} \\ & P - \rho v^2 \geq 0 \\ & \tau c_1 P - \zeta\rho l c_1^2 - \frac{\tau P v(t) + \zeta P l - \zeta\rho l v^2(t)}{2} - \frac{\tau P}{\varpi_2} \\ & \quad - \frac{c_1\zeta\rho l}{\varpi_3} \geq 0 \\ & \lambda_1 = \tau b - \frac{\zeta b l}{\varpi_1} > 0 \\ & \lambda_2 = \frac{\zeta P}{2} - \frac{\zeta\rho}{2}\alpha_1^2 - \zeta b l \varpi_1 > 0 \\ & \lambda_3 = c_3 + \beta - \frac{3\zeta\rho l}{2} - \frac{c_4}{2} - \frac{1}{2} - \frac{m c_1}{\varpi_4} > 0 \\ & \lambda_4 = \frac{\pi\beta}{m} - \tau P \varpi_2 - \zeta\rho l - c_1\zeta\rho l \varpi_3 > 0 \\ & \lambda_5 = c_2 - \frac{c_4}{2} - \frac{1}{2} > 0 \\ & \lambda_6 = \mu - 1 > 0 \end{aligned} \tag{49}$$

Substituting (49) into (48) gives

$$\begin{aligned} V_{nt} \leq & -\lambda_1 \int_0^l \left[z_t(x,t) + v z_x(x,t)\right]^2 dx \\ & - \lambda_2 \int_0^l z_x^2(x,t)\,dx - \lambda_3 e^2 - \lambda_4 \widetilde{w}_2^2 - \lambda_5 \varpi^2 \\ & - \lambda_6 \tilde{d}^2 + \gamma \end{aligned} \tag{50}$$

where $\gamma = (1/2)\alpha_3^2 + m c_1 \varpi_4 \epsilon^2 h_2^2 < +\infty$.

It can be derived from (50)

$$V_{nt} \leq -\varepsilon_1 \left(V_a + \frac{m}{2}e^2 + \frac{\pi}{2}\widetilde{w}_2^2 + \frac{1}{2}\phi^2 + \frac{1}{2}\tilde{d}^2\right) + \gamma \tag{51}$$

where $\varepsilon_1 = \min(2\lambda_1/\tau\rho, 2\lambda_2/\tau P, 2\lambda_3/m, 2\lambda_4/\pi, 2\lambda_5, 2\lambda_6)$.

Combining (21), (22), (29), and (37) yields

$$\begin{aligned} 0 & < \varepsilon_2 \left(V_a + \frac{m}{2}e^2 + \frac{\pi}{2}\widetilde{w}_2^2 + \frac{1}{2}\phi^2 + \frac{1}{2}\tilde{d}^2\right) \leq V_n \\ & \leq \varepsilon_3 \left(V_a + \frac{m}{2}e^2 + \frac{\pi}{2}\widetilde{w}_2^2 + \frac{1}{2}\phi^2 + \frac{1}{2}\tilde{d}^2\right) \end{aligned} \tag{52}$$

where $\varepsilon_2 = \min(\varrho_1, 1) > 0$ and $\varepsilon_3 = \max(\varrho_2, 1) > 0$.

Combining (51) and (52), we further obtain

$$V_{nt} \leq -\varepsilon V_n + \gamma \tag{53}$$

where $\varepsilon = (\varepsilon_1/\varepsilon_3)$.

Then, we rewrite the designed control command u_0 in (47) as

$$\begin{aligned} u_0 = & -c_3\widehat{w}_2 - \hat{d} - (c_1 c_3 + \beta c_1 + \tau P - P)x_1 - m c_1 \hat{x}_2 \\ & + c_4\phi \end{aligned} \tag{54}$$

Theorem 5. *Given the considered system described by (1), (2), and (13), under the designed disturbance observer (17), control command (54), and Assumptions 2 and 3, provided that the initial conditions are bounded and the design parameters c_1, c_2, c_3, c_4, τ, ζ, π, μ, ϖ_1, ϖ_2, ϖ_3, and ϖ_4 are chosen such that the constraints specified in (49) hold, then the close-loop system signals $z(x,t)$ and $\phi(t)$ are uniformly bounded.*

Proof. Integrating (53) from 0 to t gives

$$V_n \leq \left[V_n(0) - \frac{\gamma}{\varepsilon}\right] e^{-\varepsilon t} + \frac{\gamma}{\varepsilon} \leq V(0)e^{-\varepsilon t} + \frac{\gamma}{\varepsilon} \in \mathscr{L}_\infty \tag{55}$$

The combination of (52), V_a, and Lemma 2 in [3] yields

$$\frac{\tau P}{2l} z^2(x,t) \leq \frac{\tau P}{2}\int_0^L z_x^2(x,t)\,dx \leq V_a \leq \frac{1}{\varepsilon_2}V_n \tag{56}$$

$$\frac{1}{2}\phi^2(t) \leq \frac{1}{\varepsilon_2}V_n \tag{57}$$

Substituting (55) into (56) and (57), respectively, leads to

$$\begin{aligned} & |z(x,t)| \leq \sqrt{\frac{2l}{\tau\varepsilon_2 P}\left[V_n(0)e^{-\varepsilon t} + \frac{\gamma}{\varepsilon}\right]}, \\ & \qquad \forall (x,t) \in [0,l] \times [0,+\infty) \end{aligned} \tag{58}$$

$$|\phi(t)| \leq \sqrt{\frac{2}{\varepsilon_2}\left[V_n(0)e^{-\varepsilon t} + \frac{\gamma}{\varepsilon}\right]}, \quad \forall t \in [0,+\infty) \tag{59}$$

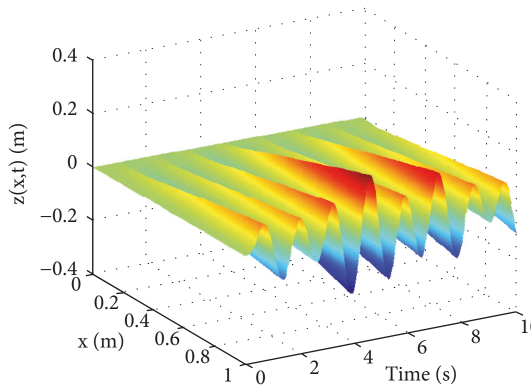

FIGURE 2: Deflection of the string without control.

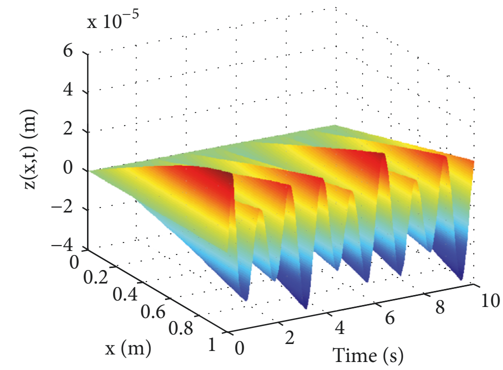

FIGURE 3: Deflection of the string with the proposed control.

From the aforementioned analysis, we further obtain

$$\lim_{t\longrightarrow\infty}|z(x,t)| \leq \sqrt{\frac{2l\gamma}{\tau\varepsilon_2\varepsilon P}}, \quad \forall x \in [0,l] \tag{60}$$

$$\lim_{t\longrightarrow\infty}|\varpi(t)| \leq \sqrt{\frac{2\gamma}{\varepsilon_2\varepsilon}} \tag{61}$$

Thus, the proof is completed. □

Remark 6. In this paper, the controller and observer design proceed on the basis of the infinite dimensional partial differential dynamics; hence there is absence of control spillover issue. In future studies, we will exploit modal method to conduct neural network or learning based control design for achieving the transient performance regulation [32–44].

4. Simulations

In this section, the finite difference method is exploited to simulate the performance of the proposed control in MATLAB software, which can provide a straightforward and accurate process to resolve partial differential equations with two independent variables. The parameters of the considered system are given as ρ=1.0kg/m, m=5.0kg, l=1.0m, c=1.0Ns/m^2, β=1.0Ns/m, P=100N, and $v(t)$=2 + sin(t). The initial conditions of the considered system are represented as $z(x,0) = z_t(x,0) = 0$. The external disturbance is described as $d(t) = \sin(t) + 3\sin(3t) + 5\sin(5t)$.

Figure 2 displays the spatiotemporal response of the string system under free vibration; namely, $u(t) = 0$, in the presence of external disturbance and input saturation. Figure 3 depicts the spatial time representation of the string system under the proposed control with the choice of the control design parameters $c_1 = c_2 = c_3$=100, c_4=10, μ=100, τ=0.1, ϕ_0=0.001, u_m=2, ϵ=0.005, and $\overline{\lambda}_1$=10. Figures 4 and 5 show the vibrational offset of the considered string system examined at x=1.0m and x=0.5m for controlled and uncontrolled response, respectively. Figure 6 displays the time responses of the external disturbance tracking. The time histories of the designed control command $u_0(t)$ and saturated control input $u(t)$ are, respectively, shown in Figures 7 and 8.

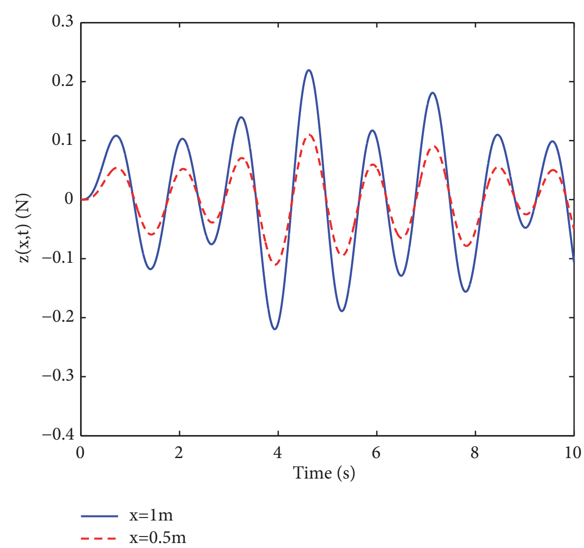

FIGURE 4: Two-dimensional deflection of the string without control.

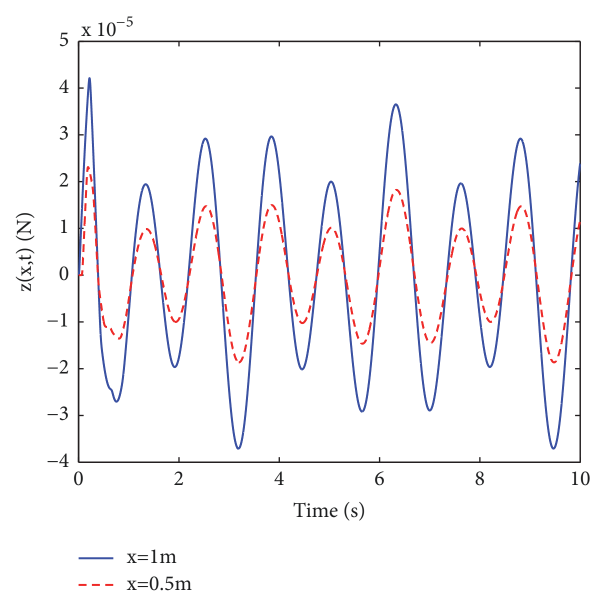

FIGURE 5: Two-dimensional deflection of the string with control.

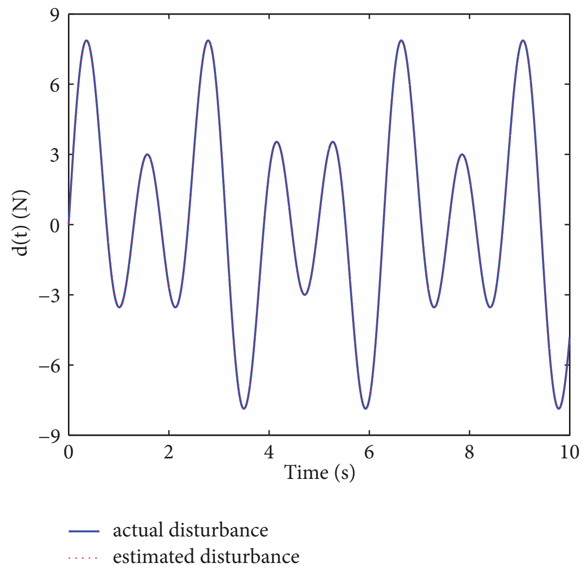

Figure 6: External disturbance tracking.

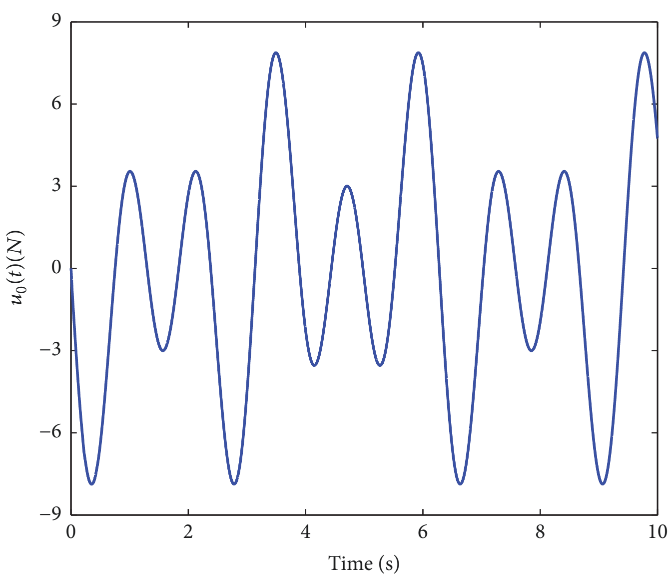

Figure 7: Designed control command $u_0(t)$.

We draw a conclusion from Figures 2–8 that the vibrational deflection of the string system is significantly suppressed when acting on the designed control, the estimated disturbance can better and faster track the actual disturbance, the designed control command $u_0(t)$ is mainly used for rejecting the external disturbance, and the restricted control input $u(t)$ is saturated in the domain [-2, 2] due to the existence of input saturation.

5. Conclusion

In this paper, the vibration boundary control issue for an axial string system subject to input restriction and external disturbance has been studied. The output feedback control has been developed to damp the vibration and stabilize the system around the equilibrium position synthesizing observer backstepping and high-gain observers. Besides, the auxiliary system and disturbance observer have been introduced to compensate the nonlinear input saturation effect and reject the external disturbance. Based on the rigorous Lyapunov analysis, the stability of the controlled system has been demonstrated without resorting to model reduction method. Numerical simulations have been conducted to clarify the performance of the proposed control approach. In the future, we will concentrate on the output and input restriction problem for axially moving systems and applying this control approach to the actual system is also an interesting topic.

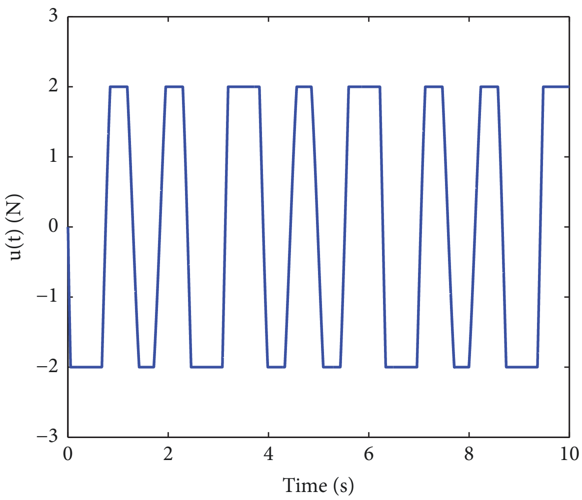

Figure 8: Saturated control input $u(t)$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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