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Research Article

Synchronization in pth Moment for Stochastic Chaotic Neural Networks with Finite-Time Control

Yuhua Xu,[1] **Jinmeng Wang**,[1] **Wuneng Zhou**,[2] **and Xin Wang**[3]

[1]*School of Finance, Nanjing Audit University, Jiangsu 211815, China*
[2]*College of Information Science and Technology, Donghua University, Shanghai 201620, China*
[3]*Zhejiang Business Technology Institute, Zhejiang 315012, China*

Correspondence should be addressed to Jinmeng Wang; jinmengwang@163.com

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Finite-time synchronization in pth moment is considered for time varying stochastic chaotic neural networks. Compared with some existing results about finite-time mean square stability of stochastic neural network, we obtain some useful criteria of finite-time synchronization in pth moment for chaotic neural networks based on finite-time nonlinear feedback control and finite-time adaptive feedback control, which are efficient and easy to implement in practical applications. Finally, a numerical example is given to illustrate the validity of the derived synchronization conditions.

1. Introduction

In the past few decades, most of the studies on control of dynamical network were the infinite time control [1–9]. However, in practical engineering, the control of dynamical network is often required in a finite time. From the point of view of the time optimization of the controlled system, finite time convergent of dynamical system is the time optimal control method. Besides the advantages of the optimal convergence performance, the finite-time control of dynamical system has better robust performance and antidisturbance performance compared with the nonfinite-time control of complex system, because of the fractional power in the finite time controller. It makes the finite-time control theory more and more important, and many researchers have studied the finite-time control of dynamical network in detail [10–13].

Moreover, there is inevitable delay in neural networks. The generation of time delays may affect the stability of neural network. Therefore, it is of great practical value and theoretical significance to study synchronization control of dynamic neural networks with time delays. In addition, neural networks with delays are easily affected by stochastic disturbances [14]. At present, some authors studied the finite-time stability of stochastic neural networks [15, 16], and some authors discussed the pth moment stability of stochastic neural networks [11, 17–19]. In these studies, LMI and a matrix equality constraint conditions often were applied to establish sufficient conditions about finite-time mean square asymptotic stability of the stochastic neural network [20–23]. As the LMI software cannot handle large-sized problems and it is not numerically stable, adaptive finite-time control method can be more useful for applying to finite-time mean square asymptotic stability of the stochastic neural network, and the finite-time stability of stochastic chaotic neural networks in pth moment is less studied by the existing works. As the quality of the output signal of the neural network can be measured by the speed of the output signal converging to 0 in pth moment. Therefore, it is of great value to study the finite-time stability in pth moment for stochastic neural networks.

Motivated by the existing works, we considered finite-time synchronization in pth moment for time varying delayed stochastic neural networks in this paper, and we will establish synchronization criterion in pth moment for stochastic delayed neural networks in finite time.

2. Problem Formulation

Consider the following chaotic neural networks:

$$\dot{u}_i(t) = Cu_i(t) + Af(u_i(t)) + Bf(u_i(t-\tau(t))) + \sum_{j=1}^{N} d_{ij}\Gamma u_j(t-\tau(t)), \tag{1}$$

where $u_i = (u_{i1}, u_{i2}, \dots, u_{in})^T \in R^n$, $f(u_i(t)) = (f_1(u_{i1}(t)), f_2(u_{i2}(t)), \dots, f_n(u_{in}(t)))^T$, $C = \operatorname{diag}\{c_1, c_2, \dots, c_n\}$, $A = (a_{pq})_{n\times n}$, $B = (b_{pq})_{n\times n}$, and $\Gamma = (\lambda_{ij})_{n\times n}$. $D = (d_{ij})_{N\times N}$, $d_{ij} \geq 0$, for $i \neq j$ and $d_{ii} = -\sum_{j=1,j\neq i}^{N} d_{ij}$, $i = 1, 2, \dots N$. $\tau(t)$ satisfies $0 < \tau(t) \leq \overline{\tau}$, $\dot{\tau}(t) \leq \widehat{\tau} < 1$, where $\overline{\tau}, \widehat{\tau}$ are constants.

Let $u(t) = ((u_1(t))^T, (u_2(t))^T, \dots, (u_N(t))^T)^T \in R^{nN}$ and $F(u(t)) = (f(u_1(t))^T, f(u_2(t))^T, \dots, f(u_N(t))^T)^T$; then

$$\begin{aligned} du(t) = {} & [(I_N \otimes C)u(t) + (I_N \otimes A)F(u(t)) \\ & + (I_N \otimes B)F(u(t-\tau(t))) \\ & + (D\otimes\Gamma)u(t-\tau(t))]\,dt. \end{aligned} \tag{2}$$

The corresponding response chaotic neural network is

$$\begin{aligned} dv(t) = {} & [(I_N \otimes C)v(t) + (I_N \otimes A)F(v(t)) \\ & + (I_N \otimes B)F(v(t-\tau(t))) + (D\otimes\Gamma)v(t-\tau(t)) \\ & + r(t)]\,dt + \sigma(t, v(t) - u(t), v(t-\tau(t)) \\ & - u(t-\tau(t)))\,d\omega(t), \end{aligned} \tag{3}$$

where $v(t)$ is the state vector, $\omega(t) = (\omega_1(t), \omega_2(t), \dots, \omega_n(t))^T$ is an n-dimensional Brown moment, $\sigma : R_+ \times R^n \times R^n \longrightarrow R^{n\times n}$ is the noise intensity matrix, and $r(t) = (r_1(t), r_2(t), \dots, r_n(t))^T$ is the controller.

Let the errors $w_i(t) = v_i(t) - u_i(t)$, $w(t) = ((w_1(t))^T, (w_2(t))^T, \dots, (w_N(t))^T)^T \in R^{nN}$, and $w(t-\tau(t)) = w_\tau(t)$; then

$$\begin{aligned} dw(t) = {} & [(I_N \otimes C)w(t) \\ & + (I_N \otimes A)(F(v(t)) - F(u(t))) \\ & + (I_N \otimes B)(F(v(t-\tau(t))) - F(u(t-\tau(t)))) \\ & + (D\otimes\Gamma)w_\tau(t) + r(t)]\,dt \\ & + \sigma(t, w(t), w_\tau(t))\,d\omega(t), \end{aligned} \tag{4}$$

Definition 1. The trivial solution $w(t, \xi(s))$ of the error system (4) is said to be stability in pth moment if

$$\limsup_{t\longrightarrow\infty} \frac{1}{t}\log\left(\mathbf{E}\left|w(t,\xi(s))\right|^p\right) < 0, \tag{5}$$

where $p \geq 2$, $p \in Z$.

Obviously, Definition 1 is stability in mean square for $p = 2$.

Definition 2 (finite-time stable). For the error system (4), if there exists a constant $t^* > 0$ such that

$$\lim_{t\longrightarrow t^*} |w(t)| = 0, \tag{6}$$

and $|w(t)| \equiv 0$, if $t \geq t^*$, then error system (4) is called stable in finite time.

Assumption 3. Suppose f satisfies the following conditions:

$$\left|f(v_i(t)) - f(u_i(t))\right| \leq \eta(v_i(t) - u_i(t)), \quad \eta \in R. \tag{7}$$

Assumption 4. Let $\chi, \gamma > 0$; then

$$trace\left(\sigma^T(t, w, w_\tau)\,\sigma(t, w, w_\tau)\right) \leq \left(\chi|w|^2 + \gamma\left|w_\tau\right|^2\right). \tag{8}$$

Lemma 5 (see [24]). *Letting $x \in R^n$ and $y \in R^n$, then*

$$x^Ty + y^Tx \leq \varsigma x^Tx + \varsigma^{-1}y^Ty, \quad \varsigma > 0. \tag{9}$$

Lemma 6 (see [25]). *If $V(t, u(t))$ and $\mathbf{L}V(t, u(t))$ are bounded on $t \in [\tau_1, \tau_2]$ with probability 1, then*

$$\mathbf{E}V(\tau_2, u(\tau_2)) - \mathbf{E}V(\tau_1, u(\tau_1)) = \mathbf{E}\int_{\tau_1}^{\tau_2} \mathbf{L}V(s, u(s), u_\tau(s))\,ds, \tag{10}$$

where $\tau_2 \geq \tau_1 \geq 0$.

Lemma 7 (see [26]). *For some constants ψ, β,*

$$\Phi(t) \leq \psi + \beta\int_0^t \Phi(s)\,ds, \quad \forall 0 \leq t \leq T, \tag{11}$$

then

$$\Phi(t) \leq \psi\exp(\beta t), \quad \forall 0 \leq t \leq T. \tag{12}$$

Lemma 8 (see [27]). *For $du(t) = f(u)dt + g(u)d\omega(t)$, $T_0(x_0) = \inf\{T \geq 0 : x(t, x_0) = 0,\ \forall t \geq T\}$. Assume that $du(t) = f(u)dt + g(u)d\omega(t)$ has the unique global solution, if there is a positive definite, twice continuously differentiable, radially unbounded Lyapunov function $V : R^n \longrightarrow R^+$, and real numbers $k > 0, 0 < \rho < 1$, such that*

$$\mathbf{L}V(u) \leq -k(V(u))^\rho, \tag{13}$$

then the origin of system $du(t) = f(u)dt + g(u)d\omega(t)$ is globally stochastically finite-time stable.

3. Main Results

Theorem 9. *Under Assumptions 3 and 4, $p \geq 2$ and meet the following conditions:*

$$\kappa\left|w_\tau\right|^p \leq \pi|w|^p, \tag{14}$$

$$\pi > \frac{1}{1-\widehat{\tau}}\kappa, \tag{15}$$

where $\pi = p(H - \Sigma_0 - \eta_1\Sigma_1 - (1/2)\eta_2\Sigma_2 - (1/2)\Theta - (1/2)(p-1)[\chi + ((p-2)/p)\gamma])$, $\Sigma_0 = \lambda_{\max}(I_N \otimes C)$, $\Sigma_1 = \lambda_{\max}(I_N \otimes A)$,

$\Sigma_2 = \lambda_{\max}((I_N \otimes B)(I_N \otimes B)^T)$, $\Theta = \lambda_{\max}((D \otimes \Gamma)(D \otimes \Gamma)^T)$, *and* $\kappa = ((p-1)\gamma + (1/2)\eta_2 + 1/2)$.

Then error system (4) is finite-time stability in pth moment by nonlinear feedback controller

$$r(t) = \begin{cases} -Hw(t) - \alpha \operatorname{sign}(w(t)) |w(t)|^{\theta}, & \text{if } |w(t)| \neq 0 \\ 0, & \text{if } |w(t)| = 0, \end{cases} \tag{16}$$

where H is a positive constant which is to be determined, $0 < \theta < 1$, and $\alpha = 1/p|w(t)|^{p-2}$.

Proof. We prove Theorem 9 in two steps.

The First Step. Error system (4) is finite-time stability under nonlinear feedback controller (16).

Let

$$V(t,w) = |w|^p. \tag{17}$$

By computing $\mathbf{L}V(t,w(t))$,

$$\begin{aligned} \mathbf{L}V(t,w,w_\tau) &= V_t(t,w) + V_w(t,w)\big[(I_N \otimes C)w(t) \\ &\quad + (I_N \otimes A)(F(v(t)) - F(u(t))) + (I_N \otimes B) \\ &\quad \cdot (F(v(t-\tau(t))) - F(u(t-\tau(t)))) + (D \otimes \Gamma) \\ &\quad \cdot w_\tau(t) + r(t)\big] + \frac{1}{2} trace\left(\sigma^T(t,w,w_\tau) V_{ww}(t,w)\right. \\ &\quad \left.\cdot \sigma(t,w,w_\tau)\right) = p|w|^{p-2} w^T \big[(I_N \otimes C)w(t) \\ &\quad + (I_N \otimes A)(F(v(t)) - F(u(t))) + (I_N \otimes B) \\ &\quad \cdot (F(v(t-\tau(t))) - F(u(t-\tau(t)))) + (D \otimes \Gamma) \\ &\quad \cdot w_\tau(t) - Hw(t) - \alpha \operatorname{sign}(w(t))|w(t)|^{\theta}\big] + \frac{1}{2} \\ &\quad \cdot trace\left(\sigma^T(t,w,w_\tau)\, p(p-1)|w|^{p-2} \sigma(t,w,w_\tau)\right). \end{aligned} \tag{18}$$

By using Assumptions 3 and 4 and Lemma 5,

$$\begin{aligned} \mathbf{L}V &\leq p|w|^{p-2} w^T \big[(I_N \otimes C)w(t) + \eta_1 (I_N \otimes A) w(t) \\ &\quad + \eta_2 (I_N \otimes B) w_\tau + (D \otimes \Gamma) w_\tau(t) - Hw(t) \\ &\quad - \alpha \operatorname{sign}(w(t))|w(t)|^{\theta}\big] + \frac{1}{2} p(p-1)|w|^{p-2} \\ &\quad \cdot \left(\chi |w|^2 + \gamma |w_\tau|^2\right). \\ &= p|w|^{p-2} w^T \big[(I_N \otimes C)w(t) + \eta_1 (I_N \otimes A) w(t) \\ &\quad - Hw(t) - \alpha \operatorname{sign}(w(t))|w(t)|^{\theta}\big] + p|w|^{p-2} \\ &\quad \cdot w^T \left[\eta_2 (I_N \otimes B) w_\tau + (D \otimes \Gamma) w_\tau(t)\right] + \frac{1}{2} p\big(p \\ &\quad - 1\big) \gamma |w|^{p-2} |w_\tau|^2 + \frac{1}{2} p(p-1)\chi |w|^p \leq p|w|^{p-2} \\ &\quad \cdot w^T \big[(I_N \otimes C)w(t) + \eta_1 (I_N \otimes A) w(t) - Hw(t) \\ &\quad - \alpha \operatorname{sign}(w(t))|w(t)|^{\theta}\big] + \frac{1}{2} p(p-1)\gamma |w|^{p-2} \\ &\quad \cdot |w_\tau|^2 + \frac{1}{2} p(p-1)\chi|w|^p + p|w|^{p-2} \\ &\quad \cdot \Big[\eta_2 \left(\frac{1}{2} w^T (I_N \otimes B)(I_N \otimes B)^T w + \frac{1}{2} w_\tau^T w_\tau\right) \\ &\quad + \frac{1}{2} w^T (D \otimes \Gamma)(D \otimes \Gamma)^T w + \frac{1}{2} w_\tau^T w_\tau\Big] \leq p|w|^{p-2} \\ &\quad \cdot w^T \big[(I_N \otimes C)w(t) + \eta_1 (I_N \otimes A) w(t) - Hw(t) \\ &\quad - \alpha \operatorname{sign}(w(t))|w(t)|^{\theta}\big] + \frac{1}{2} p(p-1) \\ &\quad \cdot \gamma \left(\frac{p-2}{p}|w|^p + \frac{2}{p}|w_\tau|^p\right) + \frac{1}{2} p(p-1)\chi|w|^p \\ &\quad + p|w|^{p-2} \\ &\quad \cdot \Big[\eta_2 \left(\frac{1}{2} w^T (I_N \otimes B)(I_N \otimes B)^T w + \frac{1}{2} w_\tau^T w_\tau\right) \\ &\quad + \frac{1}{2} w^T (D \otimes \Gamma)(D \otimes \Gamma)^T w + \frac{1}{2} w_\tau^T w_\tau\Big] \leq p\Big(\Sigma_0 \\ &\quad + \eta_1 \Sigma_1 + \frac{1}{2}\eta_2 \Sigma_2 + \frac{1}{2}\Theta - H \\ &\quad + \frac{1}{2}(p-1)\left(\chi + \frac{p-2}{p}\gamma\right)\Big)|w|^p + \Big((p-1)\gamma \\ &\quad + \frac{1}{2}\eta_2 + \frac{1}{2}\Big)|w_\tau|^p \\ &\quad - p|w|^{p-2} w^T \alpha \operatorname{sign}(w(t))|w(t)|^{\theta}. \end{aligned} \tag{19}$$

Meanwhile,

$$\begin{aligned} &-\alpha p |w(t)|^{p-2} w^T(t) \operatorname{sign}(w(t))|w(t)|^{\theta} \\ &\quad = -\alpha p |w(t)|^{p-2} \left[\operatorname{sign}(w(t))|w(t)|^{\theta}\right]^T w(t) \\ &\quad = -\alpha p |w(t)|^{p-2} \left|w^T(t)\right| |w(t)|^{\theta} \\ &\quad \leq -\alpha p |w(t)|^{p-2} \left|w^T(t) w(t)\right|^{(\theta+1)/2}. \end{aligned} \tag{20}$$

Therefore,

$$\begin{aligned} \mathbf{L}V &\leq p\Big(\Sigma_0 + \eta_1 \Sigma_1 + \frac{1}{2}\eta_2 \Sigma_2 + \frac{1}{2}\Theta - H \\ &\quad + \frac{1}{2}(p-1)\left(\chi + \frac{p-2}{p}\gamma\right)\Big)|w|^p + \Big((p-1)\gamma \\ &\quad + \frac{1}{2}\eta_2 + \frac{1}{2}\Big)|w_\tau|^p - \alpha p |w(t)|^{p-2} \\ &\quad \cdot \left|w^T(t) w(t)\right|^{(\theta+1)/2} = -\pi |w|^p + \kappa |w_\tau|^p \\ &\quad - \alpha p |w(t)|^{p-2} \left|w^T(t) w(t)\right|^{(\theta+1)/2}. \end{aligned} \tag{21}$$

where $\pi = p(H-\Sigma_0-\eta_1\Sigma_1-(1/2)\eta_2\Sigma_2-(1/2)\Theta-(1/2)(p-1)[\chi+((p-2)/p)\gamma])$, $\Sigma_0=\lambda_{\max}(I_N\otimes C)$, $\Sigma_1=\lambda_{\max}(I_N\otimes A)$, $\Sigma_2=\lambda_{\max}((I_N\otimes B)(I_N\otimes B)^T)$, $\Theta=\lambda_{\max}((D\otimes\Gamma)(D\otimes\Gamma)^T)$, and $\kappa=((p-1)\gamma+(1/2)\eta_2+1/2)$.

From $\kappa|w_\tau|^p\le\pi|w|^p$,

$$\begin{aligned}\mathbf{L}V &\le -\alpha p\,|w(t)|^{p-2}\left|w^T(t)\,w(t)\right|^{(\theta+1)/2}\\ &= -\left(|w(t)|^p\right)^{(\theta+1)/p} = -\left(V(t,w)\right)^{(\theta+1)/p}.\end{aligned}\tag{22}$$

By using Lemma 8, the error system (4) is stability in finite-time.

The Second Step. The stability in pth moment of error system (4) is achieved under nonlinear feedback controller (16).

From (21), we have

$$\begin{aligned}\mathbf{L}V &\le -\pi|w|^p+\kappa\left|w_\tau\right|^p\\ &\quad -\alpha p\,|w(t)|^{p-2}\left|w^T(t)\,w(t)\right|^{(\theta+1)/2}\\ &\le -\pi|w|^p+\kappa\left|w_\tau\right|^p,\end{aligned}\tag{23}$$

so

$$\begin{aligned}\mathbf{E}|w|^p &\le \mathbf{E}V(w(t)) \le \mathbf{E}V(0)+\mathbf{E}\int_0^t \mathbf{L}V ds\\ &\le \mathbf{E}V(0)+\mathbf{E}\int_0^t\left(-\pi|w|^p+\kappa\left|w_\tau\right|^p\right)ds.\end{aligned}\tag{24}$$

From the literature [12]

$$\begin{aligned}\int_0^t\left|w_\tau\right|^p ds &\le \frac{\overline{\tau}}{1-\widehat{\tau}}\max_{\overline{\tau}\le s\le 0}|\xi(s)|^p\\ &\quad+\frac{1}{1-\widehat{\tau}}\int_0^t|w(s)|^p ds,\end{aligned}\tag{25}$$

so

$$\begin{aligned}\mathbf{E}|w|^p &\le \left(\mathbf{E}V(0)+\kappa\frac{\overline{\tau}}{1-\widehat{\tau}}\max_{\overline{\tau}\le s\le 0}\left|\mathbf{E}\xi(s)\right|^p\right)\\ &\quad+\int_0^t\left(\kappa\frac{1}{1-\widehat{\tau}}-\delta\right)\mathbf{E}|w|^p ds.\end{aligned}\tag{26}$$

From Lemma 8,

$$\begin{aligned}\mathbf{E}|w|^p &\le \left(\mathbf{E}V(0)+\kappa\frac{\overline{\tau}}{1-\widehat{\tau}}\max_{\overline{\tau}\le s\le 0}\left|\mathbf{E}\xi(s)\right|^p\right)\\ &\quad\cdot\exp\left(-\left(\delta-\kappa\frac{1}{1-\widehat{\tau}}\right)t\right).\end{aligned}\tag{27}$$

Therefore

$$\lim_{t\longrightarrow\infty}\sup\frac{1}{t}\log\left(\mathbf{E}\left|w(t,\xi)\right|^p\right)\le-\left(\pi-\kappa\frac{1}{1-\widehat{\tau}}\right)<0.\tag{28}$$

The stability in pth moment of error system (4) is achieved by using Definition 1.

Combining the first step and the second step, one can get that the fixed-time stability in pth moment of error system (4) is finally realized by nonlinear feedback controller (16). □

In fact, H may be very large to ensure the stability of error systems (4), and it can be meaningless in some practical applications. Therefore, adaptive feedback control is selected by the following Theorem 10.

Theorem 10. *Under Assumptions 3 and 4, $p\ge 2$ and meet the following conditions:*

$$\kappa\left|w_\tau\right|^p\le\pi^*|w|^p,\tag{29}$$

$$\pi^*>\frac{1}{1-\widehat{\tau}}\kappa,\tag{30}$$

where $\pi^*=p(k_0-\Sigma_0-\eta_1\Sigma_1-(1/2)\eta_2\Sigma_2-(1/2)\Theta-(1/2)(p-1)[\chi+((p-2)/p)\gamma])$, $\Sigma_0=\lambda_{\max}(I_N\otimes C)$, $\Sigma_1=\lambda_{\max}(I_N\otimes A)$, $\Sigma_2=\lambda_{\max}((I_N\otimes B)(I_N\otimes B)^T)$, $\Theta=\lambda_{\max}((D\otimes\Gamma)(D\otimes\Gamma)^T)$, $\kappa=((p-1)\gamma+(1/2)\eta_2+1/2)$, and H is positive constant.

Then, the stability in pth moment of error system (4) is completed by the following adaptive feedback controller:

$$r_i(t)=\begin{cases}-k_iw_i(t)-\alpha\,\mathrm{sign}\left(w_i(t)\right)\left|w_i(t)\right|^\theta, & \text{if } \left|w_i(t)\right|\ne 0\\ 0, & \text{if } \left|w_i(t)\right|\ne 0,\end{cases}\tag{31}$$

$$\dot{k}_i=p\left|w_i\right|^{p-2}\left(w_i\right)^2-\kappa\,\mathrm{sign}\left(k_i(t)-k_0\right)\left|k_i(t)-k_0\right|^\theta,\tag{32}$$

where k_0 is a positive constant which is to be determined, $0<\theta<1$ and $\alpha=1/p|w(t)|^{p-2}$.

Proof. Similar to the proofs of Theorem 9.

The First Step. The finite-time stability of error system (4) is achieved under adaptive feedback controller (31).

Let

$$V(t,w)=|w|^p+\sum_{i=1}^N\frac{1}{2}\left(k_i-k_0\right)^2.\tag{33}$$

Computing $\mathbf{L}V(t,w(t))$,

$$\begin{aligned}\mathbf{L}V\left(t,w,w_\tau\right) &= V_t(t,w)+V_w(t,w)\left[\left(I_N\otimes C\right)w(t)\right.\\ &\quad+\left(I_N\otimes A\right)\left(F(v(t))-F(u(t))\right)\\ &\quad+\left(I_N\otimes B\right)\left(F(v(t-\tau(t)))-F(u(t-\tau(t)))\right)\\ &\quad\left.+(D\otimes\Gamma)\,w_\tau(t)+r(t)\right]+\frac{1}{2}\\ &\quad\cdot trace\left(\sigma^T\left(t,w,w_\tau\right)V_{ww}(t,w)\,\sigma\left(t,w,w_\tau\right)\right)\\ &= \sum_{i=1}^N\left(k_i-k_0\right)\dot{k}_i+p|w|^{p-2}w^T\left[\left(I_N\otimes C\right)w(t)\right.\\ &\quad+\left(I_N\otimes A\right)\left(F(v(t))-F(u(t))\right)\\ &\quad+\left(I_N\otimes B\right)\left(F(v(t-\tau(t)))-F(u(t-\tau(t)))\right)\\ &\quad\left.+(D\otimes\Gamma)\,w_\tau(t)+r(t)\right]+\frac{1}{2}\\ &\quad\cdot trace\left(\sigma^T\left(t,w,w_\tau\right)p(p-1)|w|^{p-2}\sigma\left(t,w,w_\tau\right)\right)\end{aligned}$$

$$
\begin{aligned}
&= p|w|^{p-2} w^T \Big[(I_N \otimes C) w(t) \\
&+ (I_N \otimes A)(F(v(t)) - F(u(t))) \\
&+ (I_N \otimes B)(F(v(t-\tau(t))) - F(u(t-\tau(t)))) \\
&+ (D \otimes \Gamma) w_\tau(t) - k_0 w(t) - \alpha \operatorname{sign}(w(t)) |w(t)|^\theta \Big] \\
&- \sum_{i=1}^{N} (k_i - k_0)\left(\kappa \operatorname{sign}(k_i(t) - k_0)|k_i(t) - k_0|^\theta\right) \\
&+ \frac{1}{2} \\
&\cdot trace\left(\sigma^T(t, w, w_\tau) p(p-1)|w|^{p-2} \sigma(t, w, w_\tau)\right) \\
&\le p|w|^{p-2} w^T \Big[(I_N \otimes C) w(t) + \eta_1 (I_N \otimes A) w(t) \\
&+ \eta_2 (I_N \otimes B) w_\tau + (D \otimes \Gamma) w_\tau(t) - k_0 w(t) \\
&- \alpha \operatorname{sign}(w(t)) |w(t)|^\theta \Big] + \frac{1}{2} p(p-1)|w|^{p-2} \\
&\cdot \left(\chi |w|^2 + \gamma |w_\tau|^2\right) \\
&- \sum_{i=1}^{N} (k_i - k_0)\left(\kappa \operatorname{sign}(k_i(t) - k_0)|k_i(t) - k_0|^\theta\right) \\
&\le p\Bigg(\Sigma_0 + \eta_1 \Sigma_1 + \frac{1}{2}\eta_2 \Sigma_2 + \frac{1}{2}\Theta - k_0 \\
&+ \frac{1}{2}(p-1)\left(\chi + \frac{p-2}{p}\gamma\right)\Bigg)|w|^p + \Bigg((p-1)\gamma \\
&+ \frac{1}{2}\eta_2 + \frac{1}{2}\Bigg)|w_\tau|^p - \alpha p |w(t)|^{p-2} \\
&\cdot \left|w^T(t) w(t)\right|^{(\theta+1)/2} - \sum_{i=1}^{N} \kappa |k_i(t) - k_0|^{\theta+1} \\
&= -\pi^* |w|^p + \kappa |w_\tau|^p - \alpha p |w(t)|^{p-2} \\
&\cdot \left|w^T(t) w(t)\right|^{(\theta+1)/2} - \sum_{i=1}^{N} \kappa |k_i(t) - k_0|^{\theta+1}.
\end{aligned} \tag{34}
$$

From $\kappa|w_\tau|^p \le \pi^*|w|^p$, have

$$
\begin{aligned}
&\mathbf{L}V \\
&\le -\alpha p|w(t)|^{p-2}\left|w^T(t) w(t)\right|^{(\theta+1)/2} \\
&- \sum_{i=1}^{N} \kappa |k_i(t) - k_0|^{\theta+1} \\
&= -\left(\left(|w(t)|^p\right)^{(\theta+1)/p} + \sum_{i=1}^{N} \kappa \left(|k_i(t) - k_0|^p\right)^{(\theta+1)/p} \right) \\
&= -\left(\left(|w(t)|^p\right)^{(\theta+1)/p} + \sum_{i=1}^{N} \left(\frac{1}{2}|k_i(t) - k_0|^2\right)^{(\theta+1)/p} \right) \\
&\le -\left(|w(t)|^p + \sum_{i=1}^{N} \frac{1}{2}|k_i(t) - k_0|^2 \right)^{(\theta+1)/p} \\
&= -(V(t, w))^{(\theta+1)/p},
\end{aligned} \tag{35}
$$

where $\kappa = ((1/2)(1/|k_i(t) - k_0|^{p-2}))^{(\theta+1)/p}$.

By using Lemma 8, error system (4) is stability in finite-time.

The Second Step. The stability in pth moment of error system (4) is achieved under adaptive feedback controller (31).

Form (34), we have

$$
\begin{aligned}
\mathbf{L}V &\le -\pi^* |w|^p + \kappa |w_\tau|^p \\
&\quad - \alpha p |w(t)|^{p-2} \left|w^T(t) w(t)\right|^{(\theta+1)/2} \\
&\quad - \sum_{i=1}^{N} \kappa |k_i(t) - k_0|^{\theta+1} \le -\pi^* |w|^p + \kappa |w_\tau|^p.
\end{aligned} \tag{36}
$$

The remaining reasoning is similar to the proof process of Theorem 9; we have

$$
\lim_{t \to \infty} \sup \frac{1}{t} \log\left(\mathbf{E}|w(t, \xi)|^p\right) \le -\left(\pi^* - \kappa \frac{1}{1 - \hat{\tau}}\right) < 0. \tag{37}
$$

By using Definition 1, error system (4) is stability in pth moment.

Combining the first step and the second step, error system (4) is finite-time stability in pth moment by adaptive feedback controller (31). □

Remark 11. It can be seen from Theorems 9 and 10 that the greater the noise intensity, the greater the control intensity, so the noisy environment may affect robustness.

Remark 12. In [20–23], research efforts have concentrated on studying finite-time mean stability of stochastic networks based on LMI. In this paper, finite-time nonlinear feedback control and finite-time adaptive feedback control are considered, which can be simpler than LMI.

4. Illustrative Example

In the following, we present an example to illustrate the usefulness of Theorem 10.

The drive systems are

$$
\begin{aligned}
\dot{u}_i(t) &= -C u_i(t) + A f(u_i(t)) + B f(u_i(t - \tau(t))) \\
&\quad + \sum_{j=1}^{4} d_{ij} \Gamma u_j(t - \tau(t)),
\end{aligned} \tag{38}
$$

where

$$
C = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},
$$

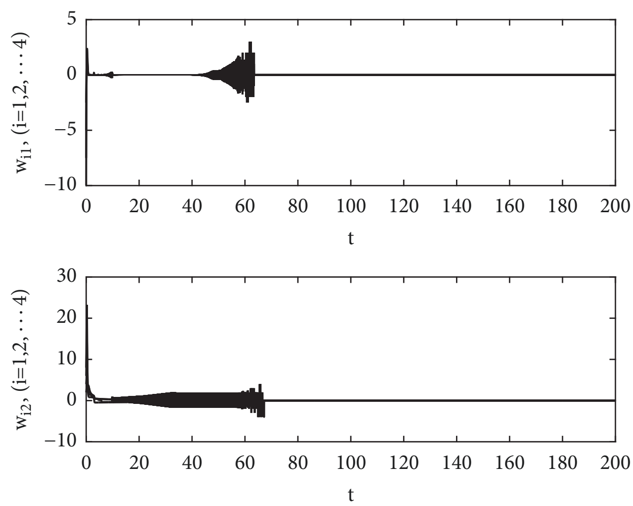

FIGURE 1: Synchronization errors.

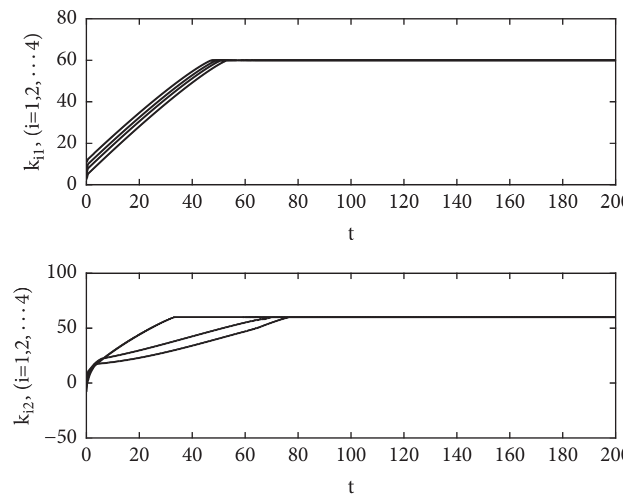

FIGURE 2: Dynamic curve of the feedback gain.

$$A = \begin{pmatrix} 3 & 8 \\ 0.3 & 2.5 \end{pmatrix},$$

$$B = \begin{pmatrix} -5 & 0.3 \\ 0.3 & -5 \end{pmatrix},$$

$$f(x) = \tanh(x),$$

$$\Gamma = I,$$

$$\tau(t) = 0.1,$$

$$\mathrm{D} = \begin{pmatrix} -1 & 2 & -1 & 0 \\ 2 & -1.5 & 0.5 & -1 \\ -1 & 0.5 & 0.5 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}. \tag{39}$$

The response systems are

$$\begin{aligned} \dot{v}_i(t) = {} & -Cv_i(t) + Af(v_i(t)) + Bf(v_i(t-\tau(t))) \\ & + \sum_{j=1}^{4} d_{ij}\Gamma v_j(t-\tau(t)) + 0.1w_i(t) \\ & + 0.1w_i(t-\tau(t)) + r_i(t). \end{aligned} \tag{40}$$

The adaptive controllers are

$$\begin{aligned} r_i(t) &= -k_i w_i(t) - \alpha\,\mathrm{sign}(w_i(t))\,|w_i(t)|^{\theta}, \\ \dot{k}_i &= p\,|w_i|^{p-2}(w_i)^2 \\ &\quad - \kappa\,\mathrm{sign}(k_i(t) - k_0)\,|k_i(t) - k_0|^{\theta}, \end{aligned} \tag{41}$$

where $p = 2$, $\alpha = 0.5$, $k_0 = 60$, $\theta = 0.15$, and $\kappa = (0.5)^{0.575}$.

We let $u_i(0) = (-0.5i, i)$, $v_i(0) = (2i, -i)$, and $k_i(0) = (3i, -2i)$. Figure 1 shows synchronization errors. Figure 2 shows dynamic curve of the feedback gain. Let $E(t) = \sqrt{\sum_{i=1}^{4}\|v_i(t) - u_i(t)\|^2/4}$. Obviously, when $E(t) \longrightarrow 0$, finite-time synchronization of neural networks is realized (See Figure 3). All numerical simulations illustrate the effectiveness of Theorem 10.

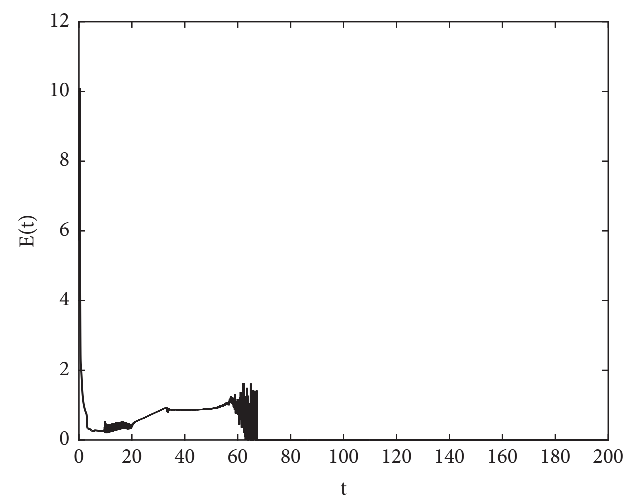

FIGURE 3: Dynamic curve of $E(t)$.

5. Conclusion

In this paper, we have discussed finite-time synchronization in pth moment for time-varying stochastic chaotic neural networks via finite-time nonlinear feedback control and finite-time adaptive feedback control, which are efficient and easy to implement in practical applications. Numerical simulations demonstrate the effectiveness of the main results obtained in this paper. Our future work is to study adaptive finite-time pining control in pth moment for time varying stochastic chaotic neural networks, which may help to save control cost and has more practical application in practical engineering.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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