Effects of the Sharing Economy on Sequential Innovation Products

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1. Introduction

Extensive research on the sharing economy has been conducted in recent years. Sharing is a phenomenon as old as humankind. It is based on a broad array of consumption issues ranging from sharing household resources versus atomized family possessions to file sharing versus intellectual property rights [1]. With the widespread adoption of mobile communications and the development of technological advances in online sharing platforms, the sharing economy has seen phenomenal growth in recent years [2]. The term sharing economy refers to the economic activities in which participants share access to products and services through online market places [3]. The sharing economy is exhibiting an increasing trend in consumer behavior, changing the way in which products and services are provided and consumed [4]. In the US and China, it encompasses product-sharing platforms, such as Turo, Zipcar, Pley, and Mobike, as well as service-sharing platforms such as Uber, Airbnb, TaskRabbit, DiDi, and 58 tongcheng. Whether product sharing or services sharing, its essence lies in waste utilization, sustainability, and secondary rental markets. Does the sharing market affect the traditional market? Some traditional manufacturers think that sharing has an impact on the traditional market and have begun to cooperate with sharing platforms. For example, General Motors has worked with RelayRides to make it easier for owners to rent out their underused vehicles [5]. BMW has also encountered a similar situation, in which it has entered into the U.S. car-sharing market [6]. In this study, we focus on product sharing rather than services sharing, and analyze the impact of product sharing on the manufacturer that sells sequentially upgraded products in the traditional market. It is worth noting that sharing products involves mainly idle goods of consumers, rather than investment goods. When consumers rent out idle goods, they give up the good’s right of use.

Due to speedy technological development and a segment of enthusiastic consumers, many manufacturers introduce upgraded products frequently and view the frequent product introductions as an important means of increasing their market share. Frequent product introductions are common in industries such as cellular phones, computers, consumer electronics, and automobiles. More frequent product introductions result in more frequent product rollovers—the process of phasing out the old generation while introducing a new one to the market [7,8]. In reality, there exists a dual product
In this paper, we explore the following two questions. (i) How will product sharing affect the manufacturer that offers sequential innovation products? (ii) Which product made by the manufacturer will benefit or suffer from product sharing? To answer these questions, we develop a two-period model in which a monopoly manufacturer sells an old product and introduces a new product in each period. In the same period, the patron (we call them “owner” below) who bought the product for self-use from the manufacturer in the previous period rents it out in this period. We assume that the market comprises strategic consumers. The consumer's usage value for the product may vary over time. In each usage period, when consumers have a need to use it, they can decide to buy the sequential product from the manufacturer or to rent the sharing product from the sharing platform. When owners' purchased products are lying idle, they might consider renting them out or not renting them out. We will analyze the trilateral game between the manufacturer, the sharing platform and consumers, to research the impacts of product sharing on the manufacturer's strategy of sequential innovation products and the consumer's purchase decision. We will analyze the impacts of product sharing on the stakeholders in the sharing market.

This paper bridges the gap between the sharing economy and the product rollover. Our results reveal that the sharing market increases or decreases the manufacturer's profit, and this is mainly determined by the moral hazard cost and the salvage value of sharing products. Furthermore, the sharing market has an insignificant effect on the upgraded products, but there is a bumping-down effect on the old product's sales. Finally, the effect of the sharing market on the revenue of the owner and the sharing platform depends on the risk of moral hazard, and it also affects the manufacturer's product rollover strategy. The detailed findings of this research are summarized as follows. First, there exists an upper limit and a lower limit for the moral hazard cost. Above the upper limit, the owner will not rent out her product, while below the lower limit, the manufacturer will offer a single product rollover instead of a dual product rollover. Second, the moral hazard cost for sharing has positive impacts on the old product's demand, but reverse impacts on the upgraded product's price and demand for it, the sharing product's price and demand for it, and the old product's price. Third, there exists a threshold for the moral hazard cost in the sharing market by the platform pricing. Below the threshold, the existence of a sharing market will benefit the manufacturer compared with when there is no sharing market. Above it, the sharing market will reduce the manufacturer's profit compared with when there is no sharing market. Above it, the sharing market will benefit the manufacturer. With owner pricing, by contrast, the sharing market will always benefit the manufacturer no matter whether the salvage value is low or high.

The remainder of the paper is organized as follows. We review the related literature in Section 2. Section 3 introduces the model and highlights the assumptions. Section 4 models the impact of product sharing on the dual product rollover and provides insight into the influence of moral hazard cost and salvage value on the stakeholders in the sharing market. Possible extensions of the model are outlined in Section 5. Finally, Section 6 summarizes the paper and illustrates its contribution and directions for future research. All proofs of results are included in the Appendix.

2. Literature Review

In this section, we give a brief review of the literature pertinent to our research on the sharing economy and product rollover.

2.1. Research on the Sharing Economy. Scholars have conducted literature reviews on the sharing economy [1, 11, 12]. The term sharing economy was first used by Professor Lawrence Lessig at Harvard Law School in 2008 [13]. Belk [14] assessed that there were two commonalities in sharing economies: (1) their use of temporary access to nonownership models of utilizing consumer products and (2) their reliance on the Internet, particularly Web 2.0, to bring this about. Scholars who researched the sharing economy mainly took Airbnb and Uber as cases. Guttentag [15], Laurell and Sandström [16], Bashir and Verma [17], Laurell [18], Ferrell et al. [3], Guttentag and Smith [19], and Kim et al. [20] regarded Airbnb and Uber as forms of sharing economies to disrupt the traditional accommodation and transportation market through the lens of disruptive innovation theory. As typical disruptive innovations, Airbnb and Uber often were cheaper than traditional industries [15, 21]. Gibbs et al. (2017) examined the impact of a variety of variables on the rates published for Airbnb listings in five large metropolitan areas in Canada. The results showed that physical characteristics, location, and host characteristics significantly affected the price [22]. Wang and Nicolau [23] investigated the price determinants of the sharing economy based on accommodation rentals. Moreover, this study identified the factors determining the price of sharing-economy-based accommodation, which differed from those determining conventional hotel prices. The abovementioned literature discussed the influential factors on the sharing product's price and suggested that the sharing economy should translate to a cheaper provision of products until the end of the price wars [24]. Most of the aforementioned literature was empirical with case analysis on the sharing economy. Fang et al. [25] introduced a two-sided market model that comprised a sharing platform, a set of product owners who were interested in sharing products, and a set of product renters. The results highlighted that revenue-maximizing prices lead to more sharing compared to welfare-maximizing pricing [25]. Kung and Zhong [26] presented
a game-theoretic study featuring network externality and sharing economy to investigate three pricing strategies in platform delivery (membership-based pricing, transaction-based pricing, and cross-subsidization), analyzed the optimal prices under these three strategies, and showed that all the three strategies resulted in the same numbers of shoppers, consumers, and profits in equilibrium. However, the above two studies discussed the platform’s price strategies but did not consider competition between the sharing market and the traditional market. Fraiberger and Sundararajan [27] used US automobile industry data and peer-to-peer car rental data from Getaround to study the welfare and distributional effects of a peer-to-peer rental market. The conclusions showed that the peer-to-peer rental markets changed the consumption mixes significantly, substituted the rental for ownership and lowered the used-good prices while increasing the consumer surplus. Because of that, consumers may also trade their durable assets in secondary markets: Jiang and Tian [28] examined the impact of consumer-to-consumer sharing of products where a monopolist manufacturer sold the product directly to consumers. The results showed that the manufacturer’s unit cost and the product-sharing transaction cost played a critical role in determining the impact of the sharing market. Tian and Jiang [29] also found that product sharing affected the distribution channel. The analysis revealed that there existed a threshold for the capacity cost coefficient above which product sharing would increase the manufacturer’s optimal capacity and below which it would reduce the manufacturer’s optimal capacity. Our research differs from the aforementioned literature, in that we mainly study the impact of product sharing on the manufacturer that offers dual product rollover.

2.2. Research on Product Rollover. The concept of “product rollover” is not new at all, and its importance and necessity have been investigated in the early literature [7, 8, 30, 31]. As a result, how to deal with used products has always been a core issue, and many studies have focused on the role of the secondary market, remanufacturing, and trade-in programs. Therefore, in recent years, the impacts of the secondary market, remanufacturing, and trade-in on product rollover have aroused the interest of academic researchers. First, Levinthal and Purohit [32] incorporated the role of the second-hand market for used durables and the effects of anticipated obsolescence on the demand for the current generation of the product; they found that, for modest levels of product improvement, the firm’s optimal policy was to phase out sales of the old product while for large improvements, the buy-back policy was more profitable. Yin et al. [33] analyzed how the sequential addition of retail and peer-to-peer used goods markets affected manufacturers’ product upgrades; they found that the addition of the retail used goods market resulted in less frequent product upgrades and the subsequent emergence of the peer-to-peer used goods market actually increased the frequency of product upgrades. Xiong et al. [34] focused on the effect of manufacturer’s upgraded products on the sales of used products in the secondary market; they found that when upgrades were typically small or moderate, the upgrading of new products could increase a third-party entrant’s profitability in the secondary market, but it did not benefit the third-party entrant when upgrades were typically large. Second, the previous literature has discussed that remanufacturing might cannibalize the new product’s sales [35, 36]. Some works, until recently, showed that there was an increasing trend of companies selling products through nontraditional channels or e-channels [37, 38]. Agrawal et al. [39] investigated how the presence of remanufactured products influenced the perceived value of new products, and this effect was different when products were remanufactured by an OEM or a third-party remanufacturer; they also found that the presence of OEM-remanufactured products had a negative effect on the perceived value of new products but that the presence of third-party remanufactured products had a positive effect. Based on this study, Li et al. [40] showed that both the life cycle phase and the consumers’ perception had an impact on the OEM’s decision whether to allow the third-party remanufacturer in the remanufacturing business. Third, Ray et al. [41] researched trade-in strategies for durable and remanufacturable products; they characterized the roles that the durability, the return revenues, the age profile, and the relative size of the two customer segments played in shaping the optimal prices and the amount of trade-in rebates offered. Both Yin & Tang [42] and Yin et al. [43] examined the effect of the trade-in programs on a firm that sold two successive-generation products; they found that trade-in programs could benefit the firm significantly, particularly under given conditions. Zhu et al. [44] studied a firm that made new products in the first period and collected used products through trade-in, along with new product sales, in the second period; they found that, to increase new product sales, the trade-in firm even agreed to sacrifice the collecting of its direct revenue. Xiao [45] proposed an exchange program to manage demand in the presence of product rollover and developed an exchange-discount-sharing mechanism to coordinate the supply chain.

There are three important conceptual differences from the existing literature in our paper. First, a resale transaction in the abovementioned markets (secondary, remanufacturing, and trade-in) involves the permanent transfer of product ownership from the seller to the buyer, whereas a sharing transaction in the sharing market involves a temporary transfer of use right from the owner to the renter only for the particular sharing period (e.g., one afternoon, one day, and one week) [28]. Second, the owner still owns the product’s continuation value for future periods (i.e., any future usage value and the salvage value) in the sharing market, but the seller does not in the traditional market. Third, the manufacturer and the patrons are considered indirect competitors along with the sharing economy because some potential buyers in the previous period may switch from buying to renting from the sharing market.

3. Problem Formulation

3.1. Assumptions. A brief description of the two-period model is illustrated in Figure 1. We suppose that a monopoly manufacturer launches a product that can be sold for only two periods $t \in \{t, t + 1\}$, where $t=1, 2, 3, \ldots$ To obtain growth, renewal, and competitive advantage, the manufacturer may
introduce sequential upgraded products. We assume that the upgraded product's quality and price remain unchanged in each period, but the old product's quality and price are relatively lower. In period \( t \), the manufacturer introduces Generation \( n \) of quality \( q \), marginal cost \( c_1 \), and price \( p_1 \). In period \( t+1 \), Generation \( n \) continues to be sold in the market. Due to not being updated in this period, it remains an old product, and the quality of Generation \( n \) is down to \( q q \) where \( q \in (0,1) \) represents durability. \( q \) represents the depreciation degree of product performance because of technological advances [46, 47]. There are two special cases. If \( q = 0 \), the old product has no use to consumers; the manufacturer adopts a single product rollover strategy. If \( q = 1 \), there is no difference in utility to consumers between the two generations. The marginal cost of Generation \( n \) is \( c_2 \) and the price is \( p_2 \). Considering about the material upgrade of products, let \( c_2 \leq c_1 \). In period \( t+1 \), the manufacturer launches Generation \( n+1 \) of quality \( q \), marginal cost \( c_1 \) and price \( p_1 \). Therefore, there are three products \( j \in \{ n, n+1, s \} \) on the market in every period except for the first period (\( t = 1 \)), where \( n = 1, 2, 3, \ldots \). In period \( t+1 \), as shown in Figure 1, Generation \( n+1 \) represents the upgraded product made by the manufacturer. Generation \( n \) represents the old product, which remains in the market together with Generation \( n+1 \). \( s \) represents the sharing product from owners who bought for self-use in the previous period and rent it out in this period.

Furthermore, in the sharing market, when products purchased by consumers in the previous period remain idle, owners who bought Generation \( n \) in period \( t \) for self-use rent it out through the third-party platform in period \( t+1 \). Because the sharing product has been used by the owner, its quality is lower than the old product Generation \( n \) in period \( t+1 \) [46]. Let the sharing product's quality be \( \beta \rho q \), where \( \beta \in (0,1) \) represents the renter's acceptance of used products [48], with the price \( p_s \) and the moral hazard cost \( m \). Here, \( \beta \) refers to the performance loss, component wear, or depreciation of used products. There are two special cases. If \( \beta = 0 \), the used product has no use to consumers, so sharing does not occur. If \( \beta = 1 \), there is no difference in utility between the old product and the upgraded product. The sharing economy, in essence, is reusing access to underutilized products, which prioritizes utilization and accessibility over ownership [11]. Therefore, in a sharing transaction, a moral hazard problem requires some extra cost [49, 50]. For example, when other things are equal, a consumer will assess a higher moral hazard cost when sharing an expensive high-quality product than sharing a cheap low-quality product. In this paper, we denote the owner's moral hazard cost of the sharing product by \( m \).

For owners of the sharing platform, if they rent out their sharing products, they will earn a rental fee. Whether the owners pay a certain fee to the sharing platform depends on the charging mode of the platform. Some sharing platforms, such as Uber and Didi, make a price for the sharing product and charge a percentage of the rental fee to owners. In other platforms, such as 58 tongcheng and Zhuanzhuan, the price of the sharing products is set by the owners and the platforms do not charge any fee to the owners. In this paper, we suppose that the owners need to pay the sharing platform a percentage fee, denoted by \( \lambda \in [0, 1) \) of the rental fee. Typically, in practice, the sharing platform collects the rental fee from the consumer. By keeping a fixed \( \lambda \) fraction of that fee as service charge, it will give the remaining fraction (1-\( \lambda \)) to the owner [28]). We assume that the rental fee \( \lambda \) is a constant and the same for each product from the sharing platform.

The timing of events in the core model is as follows. First, in the beginning of period \( t \), the manufacturer chooses its price \( p_s \) for Generation \( n \). Second, consumers decide whether to buy the product. Third, in the beginning of period \( t+1 \), the manufacturer chooses its price \( p_1 \) for Generation \( n+1 \) and price \( p_s \) for Generation \( n \). Fourth, the owner decides whether to share her underutilized product on the sharing platform. The sharing platform decides the price of the sharing product \( p_s \) according to the sharing market demand (the scenario that the owner sets \( p_s \) will be discussed in the extension). The revenues of the sharing platform and the owners are \( \lambda p_s \) and \( (1-\lambda)p_s \) respectively. Finally, according to the utility maximization, consumers decide whether to buy the old product Generation \( n \) or the upgraded product Generation \( n+1 \), or to rent the sharing product \( s \). Note that, after the sharing transaction, the product is returned from the renter to the original owner, who will obtain the salvage value \( \epsilon \) at the end of the second usage period.

**3.2. Consumers’ Strategic Options.** There are three products \( j \in \{ n, n+1, s \} \) in the market. Without loss of generality, the market size is normalized as one in each period throughout the paper. Three products are vertically differentiated by their quality level and price level. Consumers purchase the product that provides them with the highest surplus. Following Mussa and Rosen [51] and Tirole [52], we consider a consumer utility model where the net utility a consumer with valuation \( \theta \sim U[0, 1] \) receives from purchasing (renting) a product \( j \), with

\[
\text{Figure 1: Consumer utilities over two periods.}
\]
price and quality level is assumed to be of the form $U_{ij}$ (where $i \in \{t, t+1\}$ and $j \in \{n, n+1, s\}$). Each consumer’s per-period usage value from the product may vary over time. Therefore, consumers’ net utility in different periods is as follows.

First, a consumer purchases the product Generation $n$ in period $t$, and then the consumer can choose from three options:

1. Use it in both period $t$ and $t+1$.
   \[ U_{t,n}^1 = \theta q - p_1 + \epsilon \]  
   where $\epsilon$ represents the salvage value of the product at the end of the second usage periods.

2. Use it in period $t$ and rent it out in period $t+1$.
   \[ U_{t,n}^2 = \theta q - p_1 + \delta[(1-\lambda)p_s - m] + \epsilon \]  
   We assume that there exists a perfect sharing market. Consumers are strategic, that is, forward looking; when they decide whether to buy the product in period $t$, they rationally anticipate the possibility of the sharing product in period $t+1$. A discount rate of the product is $\delta \in [0, 1]$.

3. Rent it out in period $t$ and use it in period $t+1$.
   \[ U_{t,n}^3 = r_1 + \theta \beta q - p_1 + \epsilon \]  
   where $r_i (i=1, 2)$ is the sharing price of the new product; we have $r_1 > p_1$.

Second, the consumer purchases the product Generation $n$ in period $t+1$.

4. Use it in period $t+1$.
   \[ U_{t+1,n}^4 = \theta \phi q - p_2 + \epsilon \]  

5. Rent it out in period $t+1$.
   \[ U_{t+1,n}^5 = r_2 - p_2 + \epsilon \]  

Third, the consumer does not purchase any product, and then, the consumer can choose from two options.

6. Share the product Generation $n$ in period $t+1$.
   \[ U_{t+1,s}^6 = \theta \beta \phi q - p_s \]  

7. Do not purchase or share any product, and the consumer’s net utility is zero.

We summarize the notations in this paper as shown in Table 1.

### 4. Analysis

#### 4.1. Consumer’s Options Analysis

Our analysis relies on the assumptions listed in the previous section. Let us first analyze consumer’s options (1) and (2). When $U_{t,n}^1 = \theta q - p_1 + \epsilon \geq 0$, we obtain $\theta \geq (p_1 - \epsilon)/q$ and substitute it in $U_{t,n}^2$. It is easy to obtain $U_{t,n}^2 \geq 0$. That is, the consumer who uses Generation $n$ in two periods is willing to use it in period $t$ and rent it out in period $t+1$. This implies that option (1) is dominated by option (2). We analyze only option (2) and define such consumers’ net utility as $U_{t,n}^2$.

Second, we analyze consumer’s options (3) and (4). Consumer’s option (3) shows that the consumer has usage demand only in period $t+1$, which indicates if and only if $U_{t,n}^1 > U_{t+1,n}^2$, the consumer will purchase products in period $t$. That is, $r_1 > (1-\beta)\theta \phi q + p_1 - p_2$ from the view of product owners. Otherwise, she will purchase or rent products in period $t+1$. However, from the view of product renters, consumers’ net utility of renting products exceeds that of purchasing products, because only then can the user rent the owner’s product. That is, $\theta q - r_2 > \theta q - p_1 + \delta[(1-\lambda)p_s - m] + \epsilon$. Further, we easily obtain $r_1 < p_1 - \delta[(1-\lambda)p_s - m] - \epsilon$. It is easy to prove that the supply and demand of sharing products in period $t$ are contradictory. That is, the formula $U_{t,n}^3 > U_{t+1,n}^4$ is not valid. Therefore, we obtain $U_{t,n}^3 \leq U_{t+1,n}^4$, which implies that option (3) is dominated by option (4). We analyze only option (4) and define such consumers’ net utility as $U_{t,n}^4$.

Third, we analyze consumer’s options (5) and (6). From the view of product owners, when $U_{t,n}^1 > U_{t+1,n}^4$, we obtain $r_2 \geq p_2 - \epsilon$. However, from the view of product renters, consumers’ net utility of renting the new Generation $n$ exceeds that of renting the old Generation $n$. That is, $\theta q - r_2 > U_{t+1,n}^4 = \theta \beta \phi q - p_2$. Further, we easily obtain $r_2 > (1-\beta)\theta \phi q + p_2 - \theta \phi q$ (for $p_2 < \theta \phi q$). It is easy to obtain $p_2 - \epsilon \leq r_2 < \theta \phi q - p_2 - \epsilon$. At this point, that is, $U_{t+1,n}^4 \geq 0$, the retailer prefers to buy rather than rent products. Therefore, the consumer does not choose option (5). We define the consumer’s option (6) as $U_{t+1,s}$.

The above analysis implies that no consumer is willing to rent out the new product. It also shows that all consumers will prefer purchasing the product from the manufacturer rather than renting it from the sharing market.

#### 4.2. No Sharing Market (N)

We first consider the benchmark case of there being no sharing market. This case can occur, for example, if the moral hazard cost for sharing is prohibitively high (e.g., $m \geq q$). We use a superscript $N$ to represent the equilibrium outcome in the current case of there being no sharing market. The product’s optimal equilibrium solutions are as shown in the Appendix.

#### 4.3. Sharing Market (S)

We now examine the case in which there exists a sharing market. We consider only the nontrivial case of $m \in [0, (1-\lambda)q)$, since there will be no sharing transactions in equilibrium if $m \geq (1-\lambda)q$, and the results will be the same as if no sharing market exists. In this section, we first consider a scenario in which the third-party platform sets the price for the sharing product. Another scenario in which the owner sets the price will be analyzed in Section 5. We formulate demand functions for the product Generation $n$ in different periods in the presence of a sharing market.

In period $t$, there exist Generation $n$, Generation $n-1$, and sharing product $s$ in the market. When $U_{t,n} \geq 0$ and $U_{t,n-1} \geq U_{t,1}$ (In period $t$, when $p_1 < (p_2 + \delta[(1-\lambda)p_s - m])/q$, the consumer’s net utility $U_{t,n}$ of purchasing Generation $n$ is higher than the consumer’s net utility $U_{t,n-1}$ of purchasing the old Generation $n-1$, so they do not purchase the old product Generation $n-1$. When $p_2 < p_1/\beta + \epsilon$, consumers rent the sharing product $s$, without buying any product. Therefore,
In period $t$, the manufacturer will adopt the pricing strategy with $p_1 \geq (p_2 + \delta(1 - \lambda) p_s - m)/(1 - \phi)\lambda$ and $p_2 \geq p_s/\beta + \epsilon$. We obtain $U_{t,n} \geq U_{t,n-1} \geq U_{t,n-2}$, and consumers will purchase the upgraded product Generation $n$. Then, we obtain $\theta \geq (p_1 - p_2 - \delta(1 - \lambda) p_s - m)/(1 - \phi)\lambda$ and let $\theta_1 = (p_1 - p_2 - \delta(1 - \lambda) p_s - m)/(1 - \phi)\lambda$, so the demand function of the upgraded product Generation $n$ in period $t$ is described as follows:

$$d_{t,n} = \int_{\theta_1}^{1} f(\theta) d\theta = 1 - \frac{p_1 - p_2 - \delta(1 - \lambda) p_s - m}{(1 - \phi)\lambda}$$ (7)

In period $t+1$, there exist Generation $n+1$, Generation $n$ and sharing product $s$ in the market. When $U_{t+1,n} \geq 0$, $U_{t+1,n-1} \geq U_{t+1,n}$ and $U_{t+1,n} \geq U_{t+1,n+1}$, consumers will purchase the old product Generation $n$. Then, we obtain $(p_2 - p_s - \epsilon)/(1 - \beta)\phi q < \theta < \theta_1$ and let $\theta_2 = (p_2 - p_s - \epsilon)/(1 - \beta)\phi q$. Therefore, the demand function of the old product Generation $n$ in period $t+1$ is given as follows:

$$d_{t+1,n} \geq \int_{\theta_2}^{1} f(\theta) d\theta = \frac{p_1 - p_2 - \delta(1 - \lambda) p_s - m}{(1 - \phi)\lambda} - \frac{p_2 - p_s - \epsilon}{(1 - \beta)\phi q}$$ (8)

In period $t+1$, when $U_{t+1,n} \geq 0$, $U_{t+1,n} \geq U_{t+1,n-1}$ and $U_{t+1,n} \geq U_{t+1,n+1}$, consumers will rent sharing products from owners. Then, we obtain $p_s/\beta \phi q \leq \theta < \theta_2$ and let $\theta_s = p_s/\beta \phi q$, so the demand function of the sharing product $s$ in period $t+1$ is obtained as follows:

$$d_{t+1,s} = \int_{\theta_s}^{1} f(\theta) d\theta = \frac{p_2 - p_s - \epsilon}{(1 - \beta)\phi q} - \frac{p_s}{\beta \phi q}$$ (9)

From the foregoing assumptions, we know that owners' sharing products in period $t+1$ are bought from the manufacturer in period $t$, and thus $d_{t+1,s} \leq d_{t,n}$. Otherwise, if $d_{t+1,s} > d_{t,n}$, the demand of sharing products in period $t+1$ exceeds the manufacturer's product supplies in period $t$. When $\theta < \theta_s$, consumers will not purchase or share any product and the consumer’s net utility is zero.

We formulate profit functions of the manufacturer, owner and platform in the presence of a sharing market as follows. The manufacturer's total profits in two periods are expressed as

$$\pi_M = (p_1 - c_1) d_{t,n} + \delta(p_2 - c_2) d_{t+1,n} = (p_1 - c_1) \left\{ 1 - \frac{p_1 - p_2 - \delta(1 - \lambda) p_s - m}{(1 - \phi)\lambda} \right\}$$ (10)

$$+ \delta(p_2 - c_2) \left\{ \frac{p_1 - p_2 - \delta(1 - \lambda) p_s - m}{(1 - \phi)\lambda} - \frac{p_2 - p_s - \epsilon}{(1 - \beta)\phi q} \right\}$$

<table>
<thead>
<tr>
<th>$t$, $n$</th>
<th>Positive integer, $t=1, 2, 3, \ldots, n=1, 2, 3, \ldots$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Two usage periods, $i=t$ (first period), $t+1$ (second period).</td>
</tr>
<tr>
<td>$j$</td>
<td>Products, $j=n$ (Generation $n$ product), $n+1$ (Generation $n+1$ product), $s$ (sharing product).</td>
</tr>
<tr>
<td>$q$</td>
<td>Upgraded product’s quality.</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Upgraded product’s marginal cost.</td>
</tr>
<tr>
<td>$p_1$</td>
<td>Upgraded product’s price.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Product’s durability, $\phi \in (0,1)$.</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Old product’s marginal cost.</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Old product’s price.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Renter’s acceptance of used products, $\beta \in (0,1)$.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Sharing platform’s percentage fee, $\lambda \in [0, 1]$.</td>
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<tr>
<td>$\epsilon$</td>
<td>Salvage value of product.</td>
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<tr>
<td>$\theta$</td>
<td>Consumer’s valuation for quality, $\theta \sim U[0, 1]$.</td>
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<tr>
<td>$U_{ij}$</td>
<td>Consumer’s utility of choosing product $j$ in period $i$.</td>
</tr>
<tr>
<td>$d_{ij}$</td>
<td>Demand of product $j$ in period $i$.</td>
</tr>
<tr>
<td>$\pi_M$</td>
<td>Manufacturer’s total profit for two periods.</td>
</tr>
<tr>
<td>$\pi_p$</td>
<td>Third-party platform’s profit.</td>
</tr>
<tr>
<td>$\pi_o$</td>
<td>Owner’s earnings.</td>
</tr>
<tr>
<td>$N$</td>
<td>Equilibrium outcomes in the case of there being no sharing market, denoted as a superscript $N$.</td>
</tr>
<tr>
<td>$S$</td>
<td>Equilibrium outcomes in the case of there being a sharing market with the platform pricing, denoted as a superscript $S$.</td>
</tr>
<tr>
<td>$SO$</td>
<td>Equilibrium outcomes in the case of there being a sharing market with the owner pricing, denoted as a superscript $SO$.</td>
</tr>
</tbody>
</table>
If the owner rents product out in period $t+1$, her earnings are expressed as

$$\pi_O = [(1 - \lambda) p_s - m] \cdot d_{t+1,s}$$

$$= [(1 - \lambda) p_s - m] \left[ \frac{p_s - p - \varepsilon}{(1 - \beta) \varphi q} - \frac{p_s}{\beta \varphi q} \right]$$  \hspace{1cm} (11)$$

and the platform's revenues are expressed as

$$\pi_p = \lambda p_s d_{t+1,s} - c_p = \lambda p_s \left[ \frac{p_s - p - \varepsilon}{(1 - \beta) \varphi q} - \frac{p_s}{\beta \varphi q} \right] - c_p$$  \hspace{1cm} (12)$$

Without loss of generality, we assume $\delta = 1$. For the sharing platform, there is a certain operation and management cost $c_p$, which has no effect on the conclusion of this paper. Without loss of generality, we normalize $c_p$ to zero.

To ensure the third-party platform's profit maximization, we obtain the equilibrium price $p_s$ of the sharing product, the optimal platform's profit, and the equilibrium owner's earning as follows in Lemma 1.

**Lemma 1.** The equilibrium price of the sharing product is $p^*_s = \beta (p_s - \varepsilon)/2$. The optimal platform's profit at equilibrium $p_s$ is $\pi_p = \lambda \beta (p_s - \varepsilon)^2 / 4q(1 - \beta) \varphi q$, and the owner's equilibrium earning is $\pi_O = (p_s - \varepsilon) [\beta \varphi (p_s (1 - \lambda) + \epsilon (2 - \beta + \beta \lambda) - 2m)] / 4q(1 - \beta)$.

For the proof of Lemma 1, see the Appendix.

Lemma 1 shows that the sharing product's price, the platform's profit, and the owner's earning are positively correlated with the old product's price $p_2$ and the renter's acceptance of used products $\beta$, but irrelevant to the upgraded product's price $p_1$. First, the higher the old product's price by the manufacturer in the sharing market is, the higher the sharing product's price, the platform's revenues, and the owner's earnings are. Second, the less consumers care that the sharing product has been used by the owner, the higher the sharing product's price, the platform's revenues, and the owner's earnings are. Third, when the product's salvage value is low and the owner keeps the product relatively new, consumers who tend to use rather than own will be more likely to rent the product in the sharing market. This is because, on the one hand, the existence of the sharing market prompts some consumers to switch from buying old products to renting sharing products with the consumer's surplus $U_{r, t+1, s} \geq U_{r, t+1, n} \geq 0$. On the other hand, the sharing market lowers the barriers for other consumers to rent sharing products with the consumer's surplus $U_{t+1, s} \geq 0 \geq U_{t+1, n}$.

We use a superscript $S$ to represent the equilibrium outcomes in the current case of the sharing market. The stakeholders' equilibrium solutions are as shown in the Appendix.

We obtain the feedback equilibrium solutions by using backwards induction. However, those equilibrium solutions are verified under the following conditions. First, for each equilibrium, the competitive player's decisions are positive. Second, the concavity conditions ensuring that the extremum are interior maximum for the two games are satisfied. We obtain that there exists a boundary upper limit $\overline{m}$,

$$\overline{m} = \beta (\lambda - 1) \left[ 2\varepsilon (2 - \beta) (1 - \varphi) + \varphi (1 - \beta) \right]$$

$$\cdot \left[ c_1 \beta (1 - \lambda) - q (1 - \varphi) (4 + \beta (1 - \lambda)) \right]$$

$$+ c_2 \left[ \beta (2 + \varphi (\beta + \lambda - \beta \lambda - 3)) - 4 (1 - \varphi) \right] \times (2 (2 - \beta) (1 - \varphi))^{-1}$$  \hspace{1cm} (13)$$

When $m \in [0, \overline{m}]$, each equilibrium solution of the two cases is feasible. When $m > \overline{m}$, the owner will not rent out her product. Because if the moral hazard cost for sharing is prohibitively high, the owner will be more likely to need the extra cost to restore the product and less likely to offer the rental option to other consumers. Furthermore, there exists an $\underline{m}$, such that if $m < \underline{m}$, the manufacturer will offer a single product rollover instead of a dual product rollover. Because if the moral hazard cost for sharing is prohibitively low, the owner will prefer to rent out the product to other consumers. However, the expansion of the sharing market will finally squeeze the old product's sales in the traditional market and cause the manufacturer to only introduce upgraded products (single product rollover).

$$\underline{m} = \left\{ \begin{array}{ll}
\varepsilon (\beta - 2) [4 - 4\varphi + \beta^2 \varphi (\lambda - 1)] & \\
- \beta (2 + \varphi (\lambda - 3)) - 2c_2 (\beta - 2) [2 + \beta^2 \varphi (\lambda - 1)] & \\
- \beta (1 + \lambda \varphi) + \varphi (\beta - 1) & \\
\cdot \left[ c_1 (\beta - 2) (-4 + \beta (\lambda - 1)) \\
+ q \beta (\lambda - 1) (2 + \beta^2 \varphi (\lambda - 1) - \beta (1 + \lambda \varphi)) \right] \right\}$$

$$\times (\varphi (\beta - 2) (\beta - 1) [\beta (\lambda - 1) - 4])^{-1}$$  \hspace{1cm} (14)$$

We easily obtain the Lemma 2.

**Lemma 2.** There exist one upper limit $\overline{m}$ and one lower limit $\underline{m}$ for the moral hazard cost. Above $\overline{m}$, the owner will not rent out hers product and below $\underline{m}$, the manufacturer will offer a single product rollover instead of a dual product rollover.

For the proof of Lemma 2, see the Appendix.

**4.4. Impact of the Sharing Market.** We have analyzed the equilibrium outcomes for the two cases based on whether the sharing market exists. We can now examine the impact of sharing products on the manufacturer's prices, demands and profits. We study how the three key factors—the moral hazard cost $m$, the platform's fee $\lambda$, and the salvage value $\varepsilon$—affect the economic impact of stakeholders in the sharing market.

**Proposition 3.** There exists a boundary value $\overline{m}$, and when $m \in [0, \overline{m}]$, the upgraded product's price $p_1$, the old product's price $p_2$, the sharing product's price $p_s$, the manufacturer's profit $\pi^*_p$, and the sharing product's demand $d^S_{t,s}$ will increase, but the old product's demand $d^O_{t+1,s}$ will decrease.
The proofs of all the propositions in this paper are included in the Appendix.

According to Proposition 3, unlike in the previous literature, our main findings are as follows. First, a lower moral hazard cost in the sharing market will actually lead to a price rise by the manufacturer, whether the price is an old product or an upgraded product. This is because the moral hazard cost in the sharing market is reduced; the owner’s future expected revenue will increase so that the owner is willing to pay a higher price for the products from the manufacturer. In other words, in this paper, the results imply that the owner’s increased incentive for sharing a purchased product can induce the manufacturer to raise rather than lower its price \( p_1 \). In the case of quality loss remaining the same, the old product’s price \( p_2 \) in the subsequent sales period will be higher with an increase in the upgraded product’s price \( p_1 \).\footnote{In Proposition 4, since \( m^* \) is a long expression that depends on the model’s parameters \( q, \lambda, \varepsilon, c_1, c_2, \varphi \), and \( \beta \), we show it in the Appendix.} This is contrary to our general intuition. Our general intuition is that the less the moral hazard cost in the sharing market is, the more the owner will be likely to offer in the sharing product to other consumers who may now be less likely to buy products from the manufacturer. Then, the manufacturer will lower the price to attract consumers to purchase.

Second, the product’s price increases in the traditional market, and the sharing product’s price \( p_1 \) will also increase in the sharing market. This is also contrary to our general intuition. One may intuit that when the moral hazard cost decreases, the sharing product’s price will drop proportionately. However, as Proposition 3 shows, the low moral hazard cost brings a high price for the sharing product rather than a low one. This explains why although lower costs and transaction fees make the sharing economy hugely popular among entrepreneurs and consumers \([53]\), premium price strategies are still key factors for sharing products’ triumphs instead of price wars \([24]\).

Third, when the risk of the moral hazard in the sharing market decreases, more people are willing to participate in the sharing product. It is easy to obtain an increase in the sharing product’s demand \( d_{S,1,1} \). In the case of the increasing demand of sharing products, the owner will be more likely to buy upgraded products from the manufacturer and increase the upgraded product’s demand \( d_{S,n} \). The old product’s demand \( d_{S,1,n} \) will be harmed because of attack from the increase in \( d_{S,1,1} \) and \( d_{S,n} \). Nevertheless, the existence of a sharing market increases the owner’s potential sharing earnings that in turn benefit the upgraded product’s profits. The benefits of upgraded products compensate for the losses of old products. Therefore, the manufacturer’s total profit \( \pi_M^S \) over the two periods will increase as the moral hazard cost decreases.

Proposition 4. There exists a threshold \( m^* \in [0, M] \) for the moral hazard cost. Below it \( i.e., m \in [0, m^*] \), the existence of a sharing market will benefit the manufacturer, but above it \( i.e., m \in (m^*, M] \), the sharing market will harm the manufacturer.

Proposition 5. There exists a threshold \( \bar{\varepsilon} \in [0, \varphi q] \) for the salvage value. Below it \( i.e., \varepsilon \in [0, \bar{\varepsilon}] \), the sharing market will reduce the manufacturer’s profit \( \pi_M^S < \pi_M^N \). Above it \( i.e., \varepsilon \in (\bar{\varepsilon}, \varphi q]\), the sharing market will benefit the manufacturer \( \pi_M^S > \pi_M^N \).

In Proposition 5, since \( \bar{\varepsilon} \) is a long expression that depends on the model’s parameters \( q, \lambda, \varepsilon, c_1, c_2, \varphi \), and \( \beta \), it is shown in the Appendix. In this paper, we assume that the product’s salvage value is lower than the manufacturer’s marginal cost and consumers’ willingness to pay for the sharing product. To delineate Proposition 5, let us use concrete examples, documented in Table 3, with \( q=1, \lambda=0.2, \varphi=0.5, \) and \( \beta=0.5 \).
Moreover, we divide the manufacturer's products \((c_1 \& c_2)\) into low-cost, middle-cost, and high-cost according to different values of the marginal cost. Then, we assign values for the salvage value \(e\) and the moral hazard cost \(m\) with a fixed proportion (e.g., 100% or 50%).

As we see in Table 3, the manufacturer's profit is less affected by the product cost and more affected by the salvage value. Then, we find the threshold \(\bar{e} \in [0, q\eta]\) for the salvage value. Proposition 5 illustrates that a high salvage value rather than a high product cost benefits the owner in the presence of a sharing market. However, a sharing product with a high production cost and low residual value may reduce the manufacturer's profit. This is because one owner who purchased a product with high salvage value for self-use in the first period consumes less use-cost. Then, she rents out the unutilized product to other consumers in the second period and obtains earnings. The sharing market increases the manufacturer’s sales in return. Products with higher salvage values include houses and cars, etc.

In addition to the moral hazard cost \(m\) and the salvage value \(e\), the economic impact of stakeholders in the sharing market are also affected by the sharing platform’s percentage fee \(\lambda\). By similar analysis and intuition, it is easily obtained that the sharing platform’s percentage fee \(\lambda\) is directly proportional to the sharing price. We also easily obtain that a decrease in the sharing platform’s percentage fee will benefit the owners and the manufacturer, but cut the sharing platform’s profit.

5. An Extension to Sharing-Product Pricing

In response to the existence of a sharing market, the sharing product’s price is set by not only the third-party platform (Airbnb, Uber and DiDi) but also the owner (TaskRabbit and Ganji). In this section, we consider a scenario in which the owner sets the price for the sharing product. To ensure the owner’s earning maximization, we obtain the equilibrium price of the sharing product as follows in Lemma 6. We use a superscript \(SO\) to represent the equilibrium outcomes in the current case of the sharing market by the owner pricing.

**Lemma 6.** In the case of the owner pricing, the equilibrium price of the sharing product is \(p_{SO}^e = \beta(p_2 - e)/2 + m/2(1 - \lambda)\).

For the proof of Lemma 6, see the Appendix.

Lemma 6 shows that, in the case of owner pricing, the sharing product’s price is positively correlated with the old product’s price \(p_2\) and the renter’s acceptance of used products \(\beta\), but irrelevant to the upgraded product’s price \(p_1\). Lemma 6 explains that, compared to the platform pricing, the owner also considers the moral hazard cost \(m\) in the pricing in addition to \(p_2\) and \(\beta\). By comparing Lemma 6 with Lemma 1, we obtain Lemma 7 as follows.

**Lemma 7.** The sharing product’s price set by the owner is higher than that set by the platform. The excess is \(m/2(1 - \lambda)\).

Lemma 7 shows that the excess price reflects the owner’s expected moral hazard loss in the case of pricing by herself.

**Table 2: Examples with a low-cost product.**

<table>
<thead>
<tr>
<th>(\pi_M)</th>
<th>(\pi_O)</th>
<th>(\pi_P)</th>
<th>(p_1)</th>
<th>(p_2)</th>
<th>(p_3)</th>
<th>(d_{1,0})</th>
<th>(d_{1,1,0})</th>
<th>(d_{1,1,1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>No sharing</td>
<td>0.1225</td>
<td>-</td>
<td>-</td>
<td>0.65</td>
<td>0.325</td>
<td>-</td>
<td>0.35</td>
<td>0.4875</td>
</tr>
<tr>
<td>Sharing</td>
<td>(m=0)</td>
<td>0.13616</td>
<td>0.0577001</td>
<td>0.00419013</td>
<td>0.650168</td>
<td>0.304698</td>
<td>0.5011745</td>
<td>0.39094</td>
</tr>
<tr>
<td>(m=0.02)</td>
<td>0.128498</td>
<td>0.0492951</td>
<td>0.0041627</td>
<td>0.63943</td>
<td>0.304027</td>
<td>0.5010067</td>
<td>0.370805</td>
<td>0.017141</td>
</tr>
<tr>
<td>(m=0.04)</td>
<td>0.121284</td>
<td>0.0409441</td>
<td>0.00413535</td>
<td>0.628691</td>
<td>0.303356</td>
<td>0.5008389</td>
<td>0.350671</td>
<td>0.0392617</td>
</tr>
</tbody>
</table>

Note: \(\pi_M=0.0406667\) and \(m^*=0.0365482\) for above concrete examples.

**Table 3: Examples with different costs of products.**

<table>
<thead>
<tr>
<th>(\varepsilon)</th>
<th>(m)</th>
<th>(\pi_M^S)</th>
<th>(\pi_M^N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-cost: (c_1=0.2, c_2=0.1)</td>
<td>0.05</td>
<td>0.025</td>
<td>0.146206</td>
</tr>
<tr>
<td>(\varepsilon=0.1)</td>
<td>0.05</td>
<td>0.13616</td>
<td>0.16</td>
</tr>
<tr>
<td>Middle-cost: (c_1=0.5, c_2=0.25)</td>
<td>0.125</td>
<td>0.0625</td>
<td>0.059957</td>
</tr>
<tr>
<td>(\varepsilon=0.25)</td>
<td>0.125</td>
<td>0.0959522</td>
<td>0.0625</td>
</tr>
<tr>
<td>High-cost: (c_1=0.8, c_2=0.4)</td>
<td>0.2</td>
<td>0.05</td>
<td>0.00729866</td>
</tr>
<tr>
<td>(\varepsilon=0.4)</td>
<td>0.1</td>
<td>0.0838926</td>
<td>0.01</td>
</tr>
<tr>
<td>(m=0.1)</td>
<td>0.025</td>
<td>0.00285235</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.0144295</td>
<td>0.0197651</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.0855755</td>
<td>0.119128</td>
<td></td>
</tr>
</tbody>
</table>


Table 4: Examples with a low-cost product (owner pricing).

<table>
<thead>
<tr>
<th>( \pi_M )</th>
<th>( \pi_O )</th>
<th>( \pi_P )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_s )</th>
<th>( d_{1,2} )</th>
<th>( d_{1,1,2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No sharing</td>
<td>0.1225</td>
<td>-</td>
<td>-</td>
<td>0.65</td>
<td>0.325</td>
<td>-</td>
<td>0.35</td>
</tr>
<tr>
<td>( m = 0 )</td>
<td>0.177852</td>
<td>0.0167605</td>
<td>0.00419013</td>
<td>0.6504689</td>
<td>0.304698</td>
<td>0.0511745</td>
<td>0.39094</td>
</tr>
<tr>
<td>Sharing (owner pricing)</td>
<td>0.185447</td>
<td>0.0105952</td>
<td>0.312752</td>
<td>0.6565879</td>
<td>0.38255</td>
<td>0.0291946</td>
<td>0.325503</td>
</tr>
<tr>
<td>( m = 0.04 )</td>
<td>0.191118</td>
<td>0.00583757</td>
<td>0.374161</td>
<td>0.657886</td>
<td>0.374161</td>
<td>0.0634228</td>
<td>0.241611</td>
</tr>
</tbody>
</table>

Table 5: Examples with different costs of products (owner pricing).

<table>
<thead>
<tr>
<th>( \pi_S ) ( \pi_O ) ( \pi_M )</th>
<th>( \pi_M )</th>
<th>( \pi_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-cost: ( c_1 = 0.2, c_2 = 0.1 )</td>
<td>( \varepsilon = 0.05 )</td>
<td>( m = 0.025 )</td>
</tr>
<tr>
<td></td>
<td>( m = 0.05 )</td>
<td>0.166957</td>
</tr>
<tr>
<td></td>
<td>( m = 0.1 )</td>
<td>0.20742405</td>
</tr>
<tr>
<td>Middle-cost: ( c_1 = 0.5, c_2 = 0.25 )</td>
<td>( \varepsilon = 0.125 )</td>
<td>( m = 0.0625 )</td>
</tr>
<tr>
<td></td>
<td>( m = 0.125 )</td>
<td>0.103349</td>
</tr>
<tr>
<td></td>
<td>( m = 0.25 )</td>
<td>0.23621</td>
</tr>
<tr>
<td>High-cost: ( c_1 = 0.8, c_2 = 0.4 )</td>
<td>( \varepsilon = 0.2 )</td>
<td>( m = 0.2 )</td>
</tr>
<tr>
<td></td>
<td>( m = 0.4 )</td>
<td>0.188758</td>
</tr>
<tr>
<td></td>
<td>( \varepsilon = 0.4 )</td>
<td>( m = 0.4 )</td>
</tr>
</tbody>
</table>

This is because that the owner incurs the risk of renting out the product. The risk represents the extra cost needed for the owner to restore the sharing product to its pre-sharing condition. The owner will set a higher price in response to this risk.

Then, we obtain the stakeholders’ equilibrium solutions as shown in the Appendix.

Foregoing feedback equilibrium solutions are obtained by using backwards induction (See the Appendix for details). We obtain that there exists a boundary upper limit \( m \),

\[
\beta (\lambda - 1) \left\{ 2 \varepsilon (\beta - 2) (p - 1) + \varphi (\beta - 1) \right\} \\
+ \beta^2 (-1 + \lambda) \varphi - \beta (1 + \lambda \varphi) \right\} + \varphi (\beta - 1) \\
\cdot \left[ c_1 (\beta - 2) (\beta (\lambda - 1) - 4) \right] \\
+ q \beta (\lambda - 1) \left( 2 + \varphi \beta^2 (\lambda - 1) - \beta (1 + \lambda \varphi) \right) \right\} \times (4) \\
+ \varphi \beta^3 (1 - \lambda)^2 - 4 \lambda \varphi + \varphi \beta^2 \left( 1 + \lambda - 2 \lambda^2 \right) - \beta 2 \\
- \varphi \left( 5 \lambda + \lambda^2 - 4 \right) \right\}^{-1} \\
(15)
\]

When \( m \in [0, m] \), each equilibrium solution of the two cases is feasible. When \( m > m \), the owner will not rent out her product. There also exists a \( m \) such that if \( m < m \), the manufacturer will offer a single product rollover instead of a dual product rollover.

\[
\bar{m} = \beta (\lambda - 1) \left\{ 2 \varepsilon (\beta - 2) (p - 1) + \varphi (\beta - 1) \right\} \\
+ \beta^2 (-1 + \lambda) \varphi - \beta (1 + \lambda \varphi) \right\} + \varphi (\beta - 1) \\
\cdot \left[ c_1 (\beta - 2) (\beta (\lambda - 1) - 4) \right] \\
+ q \beta (\lambda - 1) \left( 2 + \varphi \beta^2 (\lambda - 1) - \beta (1 + \lambda \varphi) \right) \right\} \times (4) \\
+ \varphi \beta^3 (1 - \lambda)^2 - 4 \lambda \varphi + \varphi \beta^2 \left( 1 + \lambda - 2 \lambda^2 \right) - \beta 2 \\
- \varphi \left( 5 \lambda + \lambda^2 - 4 \right) \right\}^{-1} \\
(16)
\]

Next, we study how the two important factors—the moral hazard cost \( m \) and the salvage value \( \varepsilon \)—affect the economic impact of stakeholders in the presence of a sharing market by owner pricing. Referring to Proposition 3 and Table 2, we obtain Table 4. With the platform pricing, there exists a boundary value \( \bar{m} \). The difference is that when \( m \in [0, \bar{m}] \), the existence of a sharing market will always be a triple-win situation for the manufacturer, the sharing platform and the owners. This is because the high sharing product’s price weakens the sharing market and benefits the traditional market.

In the same way, referring to Proposition 5 and Table 3, we obtain Table 5. Different from Proposition 5, Table 5 shows that the existence of the sharing market by owner pricing will always benefit the manufacturer whether the salvage value is low or high.

6. Discussion and Conclusions

The sharing economy has seen phenomenal growth in recent years, with the widespread adoption of mobile communications and the development of technological advances in...
online sharing platforms. The sharing economy has changed the way in which products or services are provided and consumed. The emergence of a sharing economy has affected consumers and traditional manufacturers. We focused on products sharing and analyzed its impacts on the manufacturer that offers sequential innovation products. To the best of our knowledge, this was the first effort to consider the impacts of the sharing economy on the product rollover. We have provided a two-period model in which a monopoly manufacturer sold an old product and introduced a new product in each period. In the same period, an owner who previously bought a product for self-use from the manufacturer in the first period rent it out through the sharing platform in the second period. The market comprises strategic consumers. We have examined the customer’s purchasing and sharing decisions, and analyzed how these three key factors—the moral hazard cost, the platform’s fee and the salvage value—in the product-sharing market affect the stakeholders.

Our results revealed that the sharing market increased or decreased the manufacturer’s profit, and this was mainly determined by the moral hazard cost and the salvage value of sharing products. Furthermore, the sharing market has an insignificant effect on the upgraded products, but there is a bumping-down effect on old product’s sales. Finally, the effect of the sharing market on the revenue of the owner and the sharing platform depended on the risk of moral hazard, and it also affected the manufacturer’s product rollover strategy. We have shown several detailed findings.

First, there existed one upper limit and one lower limit for the moral hazard cost. Above the upper limit, the owner would not rent out her product, and below the lower limit, the manufacturer would offer a single product rollover instead of a dual product rollover. On the one hand, if the moral hazard cost for sharing was prohibitively high, the owner would be more likely to need the extra cost to restore the product and less likely to offer the rental option to other consumers. On the other hand, if the moral hazard cost for sharing was prohibitively low, the owner would prefer to rent out the product to other consumers. However, the expansion of the sharing market would finally squeeze the old product’s sales in the traditional market and cause the manufacturer to only introduce upgraded products (single product rollover).

Second, the moral hazard cost for sharing has positive impacts on the old product’s demand, but reverse impacts on the upgraded product’s price and the demand for it, the sharing product’s price and the demand for it, and the old product’s price. The intuition was that the less the moral hazard cost in the sharing market was, the more the owner would be likely to offer the sharing product to other consumers who might be less likely to buy the products from the manufacturer. Then, the manufacturer would lower the price to attract consumers to purchase. However, our analysis showed that a lower moral hazard cost in the sharing market would actually lead to a price increase by the manufacturer. Because the moral hazard cost in the sharing market was reduced, the owner’s future expected revenue would be increased so that the owner was willing to pay a higher price for the product from the manufacturer. Low moral hazard cost also brought the sharing product a high rather than a low price. It explained why although lower costs and transaction fees made the sharing economy hugely popular among entrepreneurs and consumers [53], premium price strategies instead of price wars were key factors of sharing products’ triumphs [24].

Third, there existed a threshold for the moral hazard cost in the sharing market by platform pricing. Below it, the existence of the sharing market would benefit the manufacturer compared with when there was no sharing market. Above it, the sharing market would harm the manufacturer. With owner pricing, by contrast, the sharing market would always benefit the manufacturer compared with when there was no sharing market. Our analysis showed that once the sharing market emerged, the third-party platform should incent owners to reduce the risk of moral hazard. Because the moral hazard cost was low enough, sharing products among consumers was a triple-win situation for the manufacturer, the sharing platform and the owners. In practice, the sharing platform has tried to reduce the moral hazard cost for owners by offering insurance coverage or enabling owners to rate the renters after transactions, which would to some extent alleviate the moral hazard problem [28]. This provided an opportunity to boost the sharing market for manufacturer and consumers.

Fourth, there existed a threshold for the salvage value in the sharing market by platform pricing. Below it, the sharing market would reduce the manufacturer’s profit compared with when there was no sharing market. Above it, the sharing market would benefit the manufacturer. With owner pricing, in contrast, the sharing market would always benefit the manufacturer whether the salvage value was low or high. Our results illustrated that a high salvage value rather than a high product cost benefited the manufacturer in the presence of a sharing market. Because one owner who purchased a product with high salvage value for self-use in the first period consumed less use-cost, then she rented out the unutilized product to other consumers in the second period and obtained earnings. Finally, the sharing market increased the manufacturer’s sales in return.

Certainly, the research may be extended in the following three directions. First, this paper considered only a monopoly manufacturer that sold vertically differentiated products and one sharing platform that offered the sharing products. It will be interesting to introduce competitive manufacturers and several sharing platforms in future research. Second, to build the analytical model conveniently in this paper, we assumed that the product was promoted moderately and the upgraded product’s price or quality remained unchanged in each period. It will be interesting to consider the case of rapid innovation in which the product is improved promptly in present value terms [46]. Third, this paper analyzed only the impact of sharing product on the manufacturer that offered a dual product rollover. It will be interesting to discuss the impact on a single product rollover.
Appendix

A. Strategies and Equilibrium Solutions

A.1. The Product's Optimal Equilibrium Solutions with No Sharing Market. We use a superscript $N$ to represent the equilibrium outcome in the current case of there being no sharing market.

\[
\begin{align*}
p_N^1 &= \frac{c_1 + q}{2} \\
p_N^2 &= \frac{c_2 + q}{2} \\
d_{t,n}^{N-1} &= \frac{1}{2} + \frac{c_2 - c_1}{2(1 - \varphi) q} \\
d_{t+1,n}^{N-1} &= \frac{\frac{q c_1 - c_2}{2(1 - \varphi) q}}{2} \\
\pi_M^N &= \frac{c_2^2 - 2\varphi c_1 c_2 + \varphi \left[ c_1^2 - 2(1 - \varphi) q c_1 + (1 - \varphi) q^2 \right]}{4(1 - \varphi) q \varphi}.
\end{align*}
\]

A.2. The Product's Optimal Solutions with the Sharing Market by the Platform Pricing. We use a superscript $S$ to represent the equilibrium outcomes in the current case of there being the sharing market by the platform pricing.

\[
\begin{align*}
p_S^1 &= \left\{ 4c_1 (2 - \beta) + 8(q + \epsilon - m) + \beta [4m - 4q + c_2 (2 - \beta) (1 - \lambda) + 2e (1 - \lambda) \\
& + \varphi c_1 [\beta + \beta^2 (1 - \lambda)^2 - \beta \lambda^2 + 2(1 + \lambda)] - 8 \right\} \\
& - \varphi (8(q + \epsilon) + 2m[3 - \beta (1 - \lambda) - \lambda] - 4) \\
& + \beta (c_2 \beta + e (2 - \beta)) + 2\lambda [2e \\
& + \beta [c_2 - 2c_2 \beta - \epsilon (3 - 2\beta)] - 2\lambda^2 \beta (1 - \beta) (c_2 - \epsilon) - 6e + 4q (1 - \beta) (1 - \lambda) + 4\varphi^2 q \beta (\beta + \lambda \\
& - \beta \lambda)] \times \left[ (8 - 2\beta) + \varphi [8\beta \\
& - \beta^2 (1 - \beta) (1 - \lambda)^2 - 16\beta] \right]^{-1} \\
p_S^2 &= \left\{ \beta^3 \varphi (1 - \lambda)^2 + 8(1 - \varphi) (e + \varphi q) + \varphi \beta^2 (1 - \lambda) (2c_1 + 2m - 2q - \epsilon e + 2q \varphi) \\
& + 2\beta [2e (1 - \varphi) \\
& + \varphi [c_1 (1 - \lambda) + m (1 - \lambda) + q (3 + \lambda) (1 - \varphi)]] \\
& + 2c_2 [4 - 4\varphi - \beta^2 (1 - \lambda) - \beta [2 - \varphi (3 - \lambda)]] \right\} \\
& \times \left[ 16(1 - \varphi) - 8\beta (1 - \varphi) - \varphi \beta^2 (1 - \beta) (1 - \lambda)^2 \right]^{-1} \\
\pi_M^S &= \{ - (2 - \beta)^3 \epsilon^2 (1 - \varphi) - c_2^2 (2 - \beta) \left[ 2 \\
& + \varphi \beta^2 (1 - \lambda) - \beta (1 + \lambda \varphi)] - \epsilon \varphi (1 - \beta) \\
& \cdot \left\{ (c_1 + m) (2 - \beta) (1 - \lambda) + q [8 \\
& - 2\beta (3 - \lambda) (1 - \varphi) - 8 \varphi + \varphi \beta^3 (1 - \lambda)^2 \\
& + \beta^2 (1 - \lambda) [1 + \varphi (2 - \lambda)] + 2\varphi (1 - \beta) \\
& \cdot \left[ - (c_1^2 + m^2) (2 - \beta) + m q \left( 4 - 4\varphi \right) \\
& - \beta^2 (1 - \lambda) (1 - \beta) \right] \right\}^{-1} \}
\end{align*}
\]
A.3. The Product’s Optimal Solutions with the Sharing Market by the Owner Pricing in the Extension.

We use a superscript SO to represent the equilibrium outcomes in the current case of there being the sharing market by the owner pricing.

\( p_{SO}^{1} = \{c_{2} \beta^{2} - 2c_{2} \beta - 3m \beta - 8e + 6 \epsilon \beta - \beta^{2} \epsilon - 4m \lambda + 4c_{2} \beta \lambda + 3m \beta \lambda - 2c_{2} \beta^{2} \lambda + 8e \lambda - 8 \epsilon \lambda + 2 \beta \epsilon \lambda - 2c_{2} \beta \lambda^{2} + \epsilon \lambda^{2} - \{m(\beta (1 - \lambda) - 4)\}[\beta + \lambda (1 - \beta)] + \varphi (1 - \lambda) [8e - \beta [6e + [2e + \beta (c_{2} - \epsilon) (1 - \lambda)]] \lambda (1 - \beta)] - c_{1} (1 - \lambda) [8 - 4 \epsilon \beta + \varphi - 8] + \beta [\beta + \beta^{2} (1 - \lambda) - 2 \beta \lambda^{2} + 2 (1 + \lambda)]\} - 4q (1 - \lambda) (1 - \varphi) [2 - \beta [1 + \varphi \beta (1 - \lambda) + \lambda \varphi]] \times (1 - \lambda) [16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^{2} (1 - \lambda)^{2} (1 - \beta)]^{-1}

\( p_{SO}^{2} = \{m [16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^{2} (1 - \lambda)^{2} (1 - \beta)]^{-1} \}

\( d_{SO}^{S} = \{8q - 4m + 8q - 4c_{1} (2 - \beta) + c_{2} (2 - \beta) [4 + \beta (1 - \lambda)] + \beta [3m - 4q - \epsilon (2 - \beta) (1 - \lambda)] + 2q \varphi [4 - \beta (1 - \lambda) - \lambda] \times (q [16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^{2} (1 - \lambda)^{2} (1 - \beta)]^{-1}

\( \pi_{SO}^{S} = \beta \lambda \{2e (2 - \beta) (1 - \varphi) + (1 - \beta)

\cdot \{[\beta (c_{1} + m) (1 - \lambda)] + q (1 - \varphi) (4 + \beta (1 - \lambda)] + c_{2} [4 (1 - \varphi) + \beta \varphi (3 - \lambda) - 2 \beta]

+ \beta^{2} (\varphi - \lambda \varphi)\} \times (\varphi q (1 - \beta) [16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^{2} (1 - \lambda)^{2} (1 - \beta)]^{-1}

\text{(A.2)}
\[ a_{2,1,2}^S = \left\{ -2c_2 (2 - \beta) (1 - \lambda) \left[ 2 - \varphi \beta^2 (1 - \lambda) - \beta (1 + \lambda \varphi) \right] + m \left\{ 4 + \varphi \beta^2 (1 - \lambda)^2 - 4 \lambda \varphi + \beta^2 (1 + \lambda - 2 \lambda^2) \varphi \\ + \beta \left[ -2 + \varphi (4 - 5 \lambda + \lambda^2) \right] \right\} \right\} (1 - \lambda) \left\{ -\epsilon (2 - \beta) \left[ 4 - 4 \varphi - \varphi \beta^2 (1 - \lambda) - \beta (2 + (-3 + \lambda) \varphi) \right] - \varphi (1 - \beta) \right\} \right\}
\]
\[ b_1 (2 - \beta) \left\{ 4 + \varphi \beta^2 (1 - \lambda) - \beta (1 + \lambda \varphi) \right\} \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]
\[ a_{2,1,2}^S = \left\{ -2m (4 - 3 \beta) (1 - \varphi) + \beta (1 - \lambda) \left\{ -2 \epsilon (2 - \beta) (1 - \varphi) + \varphi (1 - \beta) \left\{ c_1 (1 - \lambda) + q (4 + \beta (1 - \lambda)) \right\} (1 - \varphi) \right\} \right\} + c_2 \left\{ 4 - 4 \varphi - \beta^2 (1 - \lambda) \varphi - (2 - \varphi (3 - \lambda)) \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]
\[ \pi^S_{SO} = \left\{ -m^2 \left\{ 1 + \varphi (1 - \lambda - \varphi [2 - \beta (1 - \lambda)] \lambda] - c_2 (2 - \beta) (1 - \lambda)^2 (2 - \beta + \lambda \varphi \beta (1 - \lambda) + \lambda \varphi) \right\} - m (1 - \lambda) \left\{ -\epsilon (2 - \beta) (1 - \lambda) - \lambda \left\{ 3 + \beta (1 - \lambda) \lambda] \lambda] + q (4 + \beta (1 - \lambda)) \beta (1 - \lambda) + \lambda \varphi \right\} + \epsilon \left\{ 4 - 4 \varphi \\ + \beta \left[ -2 + \varphi (4 - 5 \lambda + \lambda^2) \right] \right\} \right\} \right\} \right\} + (1 - \lambda) \left\{ -\epsilon (2 - \beta) (1 - \lambda) \right\} \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]
\[ \pi^S_{SE} = \left\{ -m^2 \left\{ 1 + \varphi (1 - \lambda - \varphi [2 - \beta (1 - \lambda)] \lambda] - c_2 (2 - \beta) (1 - \lambda)^2 (2 - \beta + \lambda \varphi \beta (1 - \lambda) + \lambda \varphi) \right\} - m (1 - \lambda) \left\{ -\epsilon (2 - \beta) (1 - \lambda) - \lambda \left\{ 3 + \beta (1 - \lambda) \lambda] \lambda] + q (4 + \beta (1 - \lambda)) \beta (1 - \lambda) + \lambda \varphi \right\} + \epsilon \left\{ 4 - 4 \varphi \\ + \beta \left[ -2 + \varphi (4 - 5 \lambda + \lambda^2) \right] \right\} \right\} \right\} \right\} + (1 - \lambda) \left\{ -\epsilon (2 - \beta) (1 - \lambda) \right\} \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]
\[ \pi^S_{CE} = \left\{ 2m (4 - 3 \beta) (1 - \varphi) - \beta (1 - \lambda) \left\{ -2 \epsilon (2 - \beta) (1 - \varphi) + \varphi (1 - \beta) \left\{ -c_1 (1 - \lambda) + q (1 - \varphi) (4 + \beta (1 - \lambda)) \right\} \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]
\[ \pi^S_{OE} = \left\{ 2m (4 - 3 \beta) (1 - \varphi) - \beta (1 - \lambda) \left\{ -2 \epsilon (2 - \beta) (1 - \varphi) + \varphi (1 - \beta) \left\{ -c_1 (1 - \lambda) + q (1 - \varphi) (4 + \beta (1 - \lambda)) \right\} \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]
\[ \pi^S_{PO} = \left\{ 2m (4 - 3 \beta) (1 - \varphi) - \beta (1 - \lambda) \left\{ -2 \epsilon (2 - \beta) (1 - \varphi) + \varphi (1 - \beta) \left\{ -c_1 (1 - \lambda) + q (1 - \varphi) (4 + \beta (1 - \lambda)) \right\} \right\} \right\} \times \left( \varphi q (1 - \beta) (1 - \lambda) \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \varphi \beta^2 (1 - \lambda)^2 (1 - \beta) \right] \right) \right\}^{-1}
\]

\[ (A.3)\]

B. Proofs

Proof of the Product’s Optimal Solutions with No Sharing Market. We obtain feedback equilibrium solutions using backwards induction for this game. This game is a benchmark situation, denoted hereafter by the superscript \( N \), where there was no sharing market. Next, we present the method for obtaining the equilibrium for each game.

The manufacturer chooses the upgraded product’s price \( p_1 \) and the old product’s price \( p_2 \), in order to maximize its profits. Therefore, the problem that manufacturer is facing can be written as follows:

\[ \text{max } \Pi_M = \text{max } \left\{ \left( p_1 - c_1 \right) \left[ 1 - \frac{p_1 - p_2}{(1 - \varphi) q} \right] \right\} \]

(B.1)

The manufacturers’ second-period profit functions are strictly concave in their decision variables in this period, \( p_1 \) and \( p_2 \).
Proof of the Product's Optimal Solutions with the Sharing Market by the Platform Pricing. We now proof the case in which there exists a sharing market. This game is denoted hereafter by the superscript $S$. We obtain feedback equilibrium solutions using backwards induction.

First, the platform chooses the rental price of sharing products to maximize its profits. Therefore, the problem can be written as

$$\max_{p_1^S} \pi_p = \max_{p_1^S} \left\{ \lambda \cdot p_1^S \left[ \frac{p_2^S - p_1^S - \epsilon}{\beta \phi q} - \frac{p_1^S}{\beta \phi q} \right] \right\}$$  \hspace{1cm} (B.4)

Differentiating $\pi_p$ with respect to $p_1^S$, we easily obtain $\frac{\partial^2 \pi_p}{\partial p_1^S} = -2\lambda/\beta \phi q(1 - \beta) < 0$. So $\pi_p$ is concave in $p_1^S$, there is an optimal $p_1^* = \frac{\beta (p_2 - \epsilon)}{2}$ that maximizes the platform's profit, and the first derivative condition is $\frac{\partial \pi_p}{\partial p_1^S} = 0$; we obtain

$$p_1^* = \frac{\beta (p_2 - \epsilon)}{2}$$  \hspace{1cm} (B.5)

Substitute it in (11) and (12). Then we obtain

$$\pi_0 = \left( p_2 - \epsilon \right) \left[ \beta p_2 (1 - \lambda) + \epsilon (2 - \beta + \beta \lambda) - 2m \right]$$

$$\pi_p = \frac{\lambda \beta (p_2 - \epsilon)^2}{4q (1 - \beta) \phi}$$  \hspace{1cm} (B.6)

Second, the manufacturer chooses products' price $p_1$ and $p_2$ to maximize its profits. Therefore, the problem that the manufacturer is facing can be written as

$$\max_{p_1, p_2} \pi_M = \max_{p_1, p_2} \left\{ \left(p_1 - c_1\right) \cdot \left\{ 1 - \frac{p_1 - p_2 - \delta \left[ (1 - \lambda) p_2 - m \right]}{(1 - \phi) q} \right\} \right\}$$

$$\frac{\partial p_1^*}{\partial m} = \frac{2 \phi \beta^2 (1 - \lambda) - 8 (1 - \phi) + 2 \beta [2 - \phi (3 - \lambda)]}{16 (1 - \phi) - 8 \beta (1 - \phi) - \phi \beta^2 (1 - \lambda)^2 + \phi \beta^3 (1 - \lambda)^2} < 0$$

$$\frac{\partial p_2^*}{\partial m} = -\frac{2 (1 - \beta) \beta (1 - \lambda) \phi}{16 (1 - \phi) - 8 \beta (1 - \phi) - \phi \beta^2 (1 - \lambda)^2 + \phi \beta^3 (1 - \lambda)^2} < 0$$

$$\frac{\partial \pi_M}{\partial m} = \frac{(1 - \beta) \beta^2 (1 - \lambda) \phi}{16 (1 - \phi) - 8 \beta (1 - \phi) - \beta^2 (1 - \lambda)^2 \phi + \beta^3 (1 - \lambda)^2 \phi} < 0$$

where $p_2 = (1/2)\beta (p_2 - \epsilon)$.

The manufacturer's profit function is strictly concave in their decision variables in this period. Letting $\partial \pi_M / \partial p_1 = 0$ and $\partial \pi_M / \partial p_2 = 0$, we obtain $p_1^S$ and $p_2^S$. Then substituting them in (7)-(12) and (B.5), we obtain the product's optimal solutions with sharing market by the platform pricing.

Proof finished.

Proof of Lemma 2. To ensure the owner to rent out her unutilized product, that is, $(1 - \lambda) p_2 - m > 0$, we obtain that there exists an upper limit for the moral hazard cost.

$$m = \beta (\lambda - 1) [2 \epsilon (2 - \beta) (1 - \phi) + \phi (1 - \beta)$$

$$\cdot \left\{ c_1 \beta (1 - \lambda) - q (1 - \phi) [4 + \beta (1 - \lambda)] \right\}$$

$$+ c_2 \left\{ 2 + \phi (\beta + \lambda - \beta \lambda - 3) - 4 (1 - \phi) \right\} \times (8 (2 - \beta) (1 - \phi)^{-1}$$

To ensure the manufacturer to offer the old product, that is, $d_{t+1,n} > 0$, we obtain that there exists a lower limit for the moral hazard cost.

$$m = \left\{ \epsilon (\beta - 2) [4 - 4 \phi + \beta^2 \phi (\lambda - 1)$$

$$- \beta (2 + \phi (\lambda - 3))] - 2 c_2 (\beta - 2) [2$$

$$+ \beta^2 \phi (\lambda - 1) - \beta (1 + \lambda \phi) \right\} \phi (\beta - 1)$$

$$\cdot \left\{ c_1 (\beta - 2) (-4 + \beta (\lambda - 1))$$

$$+ q \beta (\lambda - 1) (2 + \beta^3 \phi (\lambda - 1) - \beta (1 + \lambda \phi)) \right\} \times (\phi (\beta - 2) (\beta - 1) [\beta (\lambda - 1) - 4])^{-1}$$

Proof of Proposition 3. For the equilibrium options, taking the derivative with respect to $m$ gives us
\[
\frac{\partial \pi^S_M}{\partial m} = -q \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \beta^2 (1 - \lambda)^2 \varphi + \beta^3 (1 - \lambda)^2 \varphi \right] < 0
\]

\[
\frac{\partial \pi^S_{i+1,n}}{\partial m} = q \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) - \beta^2 (1 - \lambda)^2 \varphi + \beta^3 (1 - \lambda)^2 \varphi \right] > 0
\]

\[
\frac{\partial \pi^S_M}{\partial m} = -\frac{(2 - \beta) [4q - \beta \epsilon (1 - \lambda) - 4c_1 - 4m + c_2 (4 + \beta - \beta \lambda)] + 2\varphi q [4 + \beta (\beta + \lambda - \beta \lambda - 3)]}{q [16 (1 - \varphi) - 8 \beta (1 - \varphi) - \beta^2 (1 - \lambda)^2 \varphi + \beta^3 (1 - \lambda)^2 \varphi]} < 0
\]

\[
\frac{\partial \pi^S_i}{\partial m} = -\frac{\beta (1 - \lambda)}{q [16 (1 - \varphi) - 8 \beta (1 - \varphi) - \beta^2 (1 - \lambda)^2 \varphi + \beta^3 (1 - \lambda)^2 \varphi]} < 0
\]

\[
\frac{\partial \pi^S_{i+1,n}}{\partial m} = -\left\{-c_2 \left[ 16 (1 - \varphi) - 8 \beta (1 - \varphi) (1 - \lambda)^2 \right] - 8 (2 - \beta) \right\} \left( 4 - 2\beta + \varphi \left[ -4 + \beta \left[ 3 - \beta (1 - \lambda) - \lambda \right] \right] \right\} - \varphi (1 - \beta)
\]

\[
\cdot \left\{ 8c_1 (2 - \beta) + 16m (2 - \beta) (1 - \varphi) - \varphi c_1 [16 - \beta \left[ 8 + \beta (1 - \beta) (1 - \lambda)^2 \right]] \right\}
\]

\[
- q \left[ 4 + \beta (1 - \lambda) \right] (1 - \varphi) \left[ 16 (1 - \varphi) - \beta \left[ 8 - \varphi (8 + (1 - \beta) \beta (1 - \lambda)^2) \right] \right] + \varphi \left[ -64 (1 - \varphi) - \varphi^2 \beta^5 (1 - \lambda)^3 \right]
\]

\[
+ \varphi (1 - \beta) \left[ 8 (2 - \beta) \right]
\]

\[
= \left\{ \beta [8 - (1 - \beta) (1 - \lambda)^2] - 16 \right\} \left[ 4 + \varphi \left[ 3 - \lambda (8 - \lambda) \right] \right] \}
\]

\[
\cdot \left( \varphi q (1 - \beta) \right) \left[ 8 (2 - \beta) \right]
\]

\[
\left[ \beta [8 - (1 - \beta) (1 - \lambda)^2] - 16 \right] \left[ 4 + \varphi \left[ 3 - \lambda (8 - \lambda) \right] \right] \}
\]

\[
(B.10)
\]

Proof finished.

Proof of Proposition 4. Let \( \Delta \pi_M = \pi^S_M - \pi^N_M \). If \( \Delta \pi_M > 0 \), a sharing market benefits the manufacturer; if not, a sharing market harms the manufacturer. We easily obtain that there exists a \( m^* \) making \( \Delta \pi_M = 0 \). Since \( m^* \) is a long expression, we omit it in this paper.

Proof of Proposition 5. Similarly, we easily obtain that there exists \( \varepsilon^* \) making \( \Delta \pi_M = 0 \). Since \( m^* \) is a long expression, we omit it too.

Proof of the Product’s Optimal Solutions with the Sharing Market by the Owner Pricing in the Extension. We now proof the case in which there exists a sharing market, where the owner chooses their price of the sharing products. This game is denoted hereafter by the superscript \( SO \).

First, the owner chooses the rental price of sharing products to maximize its profits. Therefore, the platform’s problem can be written as

\[
\max_{\hat{p}_s} \pi^O_{SO} = \max_{\hat{p}_s} \left\{ [(1 - \lambda) \cdot \hat{p}_s - m] \cdot \frac{\hat{p}_s - \hat{p}_s - \varepsilon}{(1 - \beta) \varphi q} - \frac{\hat{p}_s}{\beta \varphi q} \right\} \}
\]

Differentiating \( \pi^O \) with respect to \( p_s \), we easily obtain \( \frac{\partial^2 \pi^O}{\partial p_s^2} = -2\lambda / \beta \varphi q (1 - \beta) < 0 \). So \( \pi^O \) is concave in \( p_s \), there is an optimal \( p_s^* \) that maximizes the platform’s profit, and the first derivative condition is \( \partial \pi^O / \partial p_s = 0 \); we obtain

\[
p_s^* = \frac{\beta (p_s - \varepsilon)}{2} + \frac{m}{2 (1 - \lambda)}
\]

The solution to problem (B.11) gives the owner’s reaction functions and the rent price of the sharing products \( p_s \), and the owner’s profit is a strictly concave function of its decision variables, and the react function can be derived.
Second, the manufacturer chooses products’ price \( p_1 \) and \( p_2 \) to maximize its profits. Therefore, the problem that manufacturer is facing can be written as

\[
\max_{p_1, p_2} \pi_M = \max_{p_1, p_2} \left( p_1 - c_1 \right) \left[ 1 - \frac{p_1 - p_2 - \delta (1 - \lambda) p_1 - m}{(1 - \varphi) q} \right] + \delta \left( p_2 - c_2 \right)
\]

where \( p_1 = \beta (p_2 - \epsilon) / 2 + m / 2 (1 - \lambda) \).

The manufacturer’s profit function is strictly concave in their decision variables in this period. Letting \( \partial \pi_M / \partial p_1 = 0 \) and \( \partial \pi_M / \partial p_2 = 0 \), we obtain \( p_1^{SO} \) and \( p_2^{SO} \). Then substituting them in (7)–(12) and (B.11), we obtain the product’s optimal solutions with sharing market by the owner pricing.

Proof finished.

Proof of the Lemma 6. According to the above proof, we can easily obtain \( p_1^{SO} \).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

Zhenfeng Liu, Jian Feng, and Jinfeng Wang designed the study, performed the research, analyzed data, and wrote the paper.

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