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Research Article

Rational Waves and Complex Dynamics: Analytical Insights into a Generalized Nonlinear Schrödinger Equation with Distributed Coefficients

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In this paper, we first present a complex multirational exp-function ansatz for constructing explicit solitary wave solutions, N-wave solutions, and rouge wave solutions of nonlinear partial differential equations (PDEs) with complex coefficients. To illustrate the effectiveness of the complex multirational exp-function ansatz, we then consider a generalized nonlinear Schrödinger (gNLS) equation with distributed coefficients. As a result, some explicit rational exp-function solutions are obtained, including solitary wave solutions, N-wave solutions, and rouge wave solutions. Finally, we simulate some spatial structures and dynamical evolutions of the modules of the obtained solutions for more insights into these complex rational waves. It is shown that the complex multirational exp-function ansatz can be used for explicit solitary wave solutions, N-wave solutions, and rouge wave solutions of some other nonlinear PDEs with complex coefficients.

1. Introduction

In the real world, complex nonlinear phenomena are everywhere and nonlinear PDEs are often used to describe these nonlinear complexities. To gain more insights into the essence behind the nonlinear phenomena for further applications, people usually restore to the dynamical evolutions of exact wave solutions of nonlinear PDEs. It is well known that the celebrated Schrödinger wave equation possesses N-soliton solutions and is often used to describe quantum mechanical behavior. In the field of nonlinear mathematical physics, many analytical methods have been presented for exactly solving nonlinear PDEs, such as those in [1–19]. It is worth mentioning that the exp-function method [8] with a rational exp-function ansatz is an effective mathematical tool for constructing exact wave solutions.

In this paper, with a complex multirational exp-function ansatz, we shall construct and gain more insights into the rational solutions, including solitary wave solutions, N-wave solutions, and rouge wave solutions of the following gNLS equation with gain in the form used in nonlinear fiber optics [20–24]:

$$\mathrm{i}\psi_x + \frac{1}{2}\alpha(x)\psi_{tt} + \beta(x)|\psi|^2\psi = \mathrm{i}\gamma(x)\psi, \tag{1}$$

where $\psi = \psi(x,t)$ is a complex-valued function of the propagation distance x and the retarded time t, while $\alpha(x)$, $\beta(x)$, and $\gamma(x)$ are all differentiable functions of x, which denote the group velocity dispersion, nonlinearity, and distributed gain, respectively. If we set

$$\begin{aligned} \alpha(x) &= 2, \\ \beta(x) &= \pm 1, \\ \gamma(x) &= 0, \end{aligned} \tag{2}$$

then (1) can be reduced to the well-known NLS equation:

$$\mathrm{i}\psi_x + \psi_{tt} \pm |\psi|^2\psi = 0. \tag{3}$$

The rest of the paper is organized as follows. In Section 2, we give a description of the complex multirational exp-function ansatz used to construct explicit solitary wave solutions, N-wave solutions, and rouge wave solutions of nonlinear PDEs with complex coefficients. In Section 3, we use the introduced complex multirational exp-function ansatz to construct solitary wave solutions, N-wave solutions, and rouge wave solutions of the gNLS in (1). In Section 4, in order to gain more insights into the complex dynamics of the obtained wave solutions, we simulate the dynamical evolutions of some solitary wave solutions, N-wave solutions, and rouge wave solutions. In Section 5, we conclude this paper.

2. Complex Multirational Exp-Function Ansatz

For a given nonlinear PDE with complex coefficients, for example, the NLS in (3), we suppose that its complex multirational exp-function ansatz has the following form [9]:

$$\psi = \frac{\sum_{i_1=0}^{p_1}\sum_{i_2=0}^{p_2}\cdots\sum_{i_{2N}=0}^{p_{2N}} a_{i_1 i_2\cdots i_{2N}} e^{\sum_{g=1}^{N}(i_g\xi_g+i_{g+N}\xi_g^*)}}{\sum_{j_1=0}^{q_1}\sum_{j_2=0}^{q_2}\cdots\sum_{j_{2N}=0}^{q_{2N}} b_{j_1 j_2\cdots j_{2N}} e^{\sum_{g=1}^{N}(j_g\xi_g+j_{g+N}\xi_g^*)}}, \quad N \geq 1, \tag{4}$$

where $\xi_g = k_g t + c_g x + d_g$, $\xi_g^* = k_g^* t + c_g^* x + d_g^*$, $a_{i_1 i_2\cdots i_{2N}}$, $b_{j_1 j_2\cdots j_{2N}}$, c_g, and k_g are complex constants to be determined by substituting (4) into (3), d_g is an arbitrary complex constant, the real values of $p_1, p_2, \cdots, p_{2N}, q_1, q_2, \cdots, q_{2N}$ are integers determined by the process of homogeneous balance, and $*$ denotes the complex conjugate.

Special case 1 of (4): solitary wave ansatz:

$$\psi = \frac{a_0 + a_1 e^{\xi_1} + a_2 e^{2\xi_1}}{b_0 + b_1 e^{\xi_1} + b_2 e^{2\xi_1}}, \tag{5}$$

for the separate real and imaginary parts of the NLS in (3) in an indirect way.

Special case 2 of (4): N-wave ansatz:

When $N = 1$, (4) gives

$$\psi = \frac{\sum_{i_1=0}^{p_1}\sum_{i_2=0}^{p_2} a_{i_1 i_2} e^{i_1\xi_1+i_2\xi_1^*}}{\sum_{j_1=0}^{q_1}\sum_{j_2=0}^{q_2} b_{j_1 j_2} e^{i_1\xi_1+i_2\xi_1^*}}, \tag{6}$$

which can be used to construct single-wave solution of the NLS in (3) in a direct way.

When $N = 2$, (4) gives

$$\psi = \frac{\sum_{i_1=0}^{p_1}\sum_{i_2=0}^{p_2}\sum_{i_3=0}^{p_3}\sum_{i_4=0}^{p_4} a_{i_1 i_2 i_3 i_4} e^{\sum_{g=1}^{2}(i_g\xi_g+i_{g+2}\xi_g^*)}}{\sum_{j_1=0}^{q_1}\sum_{j_2=0}^{q_2}\sum_{j_3=0}^{q_3}\sum_{j_4=0}^{q_4} b_{j_1 j_2 j_3 j_4} e^{\sum_{g=1}^{2}(j_g\xi_g+j_{g+2}\xi_g^*)}}, \tag{7}$$

which can be used to construct double-wave solution of the NLS in (3) in a direct way.

When $N = 3$, (4) gives

$$\psi = \frac{\sum_{i_1=0}^{p_1}\sum_{i_2=0}^{p_2}\sum_{i_3=0}^{p_3}\sum_{i_4=0}^{p_4}\sum_{i_5=0}^{p_5}\sum_{i_6=0}^{p_6} a_{i_1 i_2 i_3 i_4 i_5 i_6} e^{\sum_{g=1}^{3} i_g(i_g\xi_g+i_{g+3}\xi_g^*)}}{\sum_{j_1=0}^{q_1}\sum_{j_2=0}^{q_2}\sum_{j_3=0}^{q_3}\sum_{j_4=0}^{q_4}\sum_{j_5=0}^{q_5}\sum_{j_6=0}^{q_6} b_{j_1 j_2 j_3 j_4 j_5 j_6} e^{\sum_{g=1}^{3}(j_g\xi_g+j_{g+3}\xi_g^*)}}, \tag{8}$$

which can be used for three-wave solution of the NLS in (3) in a direct way.

Special case 3 of (4): rouge wave ansatz:

$$\psi = \frac{a_0 + a_1 \cos(1/2)(\xi_1 - \xi_1^*) e^{\xi_1 - k_1 t} + a_2 e^{\xi_1+\xi_1^*}}{b_0 + b_1 \cos(1/2)(\xi_1 - \xi_1^*) e^{\xi_1 - k_1 t} + b_2 e^{\xi_1+\xi_1^*}}, \tag{9}$$

with the constraints $k_1 = -k_1^*$ and $c_1 = c_1^*$, which can be used for the NLS in (3) in a direct way.

3. Rational Exp-Function Solutions

In this section, we employ the rational exp-function ansatz (4) and its special cases (5)-(9) to construct rational solutions, including solitary wave solutions, N-wave solutions, and rouge wave solutions of the gNLS in (1).

3.1. Solitary Wave Solutions. Let us begin with the gNLS in (1). Firstly, we assume that $\alpha(x)$, $\beta(x)$, and $\gamma(x)$ are all real functions and let

$$\begin{aligned} \psi &= A e^{i\theta}, \\ A &= A(x,t), \\ \theta &= \theta(x,t), \end{aligned} \tag{10}$$

where A and θ are the amplitude and phase functions, respectively. With the help of (10), we separate the real and imaginary parts of (1) as follows:

$$-A\theta_x + \frac{1}{2}\alpha(x)\left(A_{tt} - A\theta_t^2\right) + \beta(x)A^3 = 0, \tag{11}$$

$$A_x + \frac{1}{2}\alpha(x)\left(2A_t\theta_t + A\theta_{tt}\right) - \gamma(x)A = 0. \tag{12}$$

Then we further suppose that

$$A = \frac{a_0(x) + a_1(x)e^{\xi}}{1 + b_1 e^{\xi}}, \quad \xi = p(x) + q(x)t, \tag{13}$$

$$\theta = f(x)t^2 + g(x)t + h(x), \tag{14}$$

where $a_0(x)$, $a_1(x)$, $p(x)$, $q(x)$, $f(x)$, $g(x)$, and $h(x)$ are undetermined real functions of x, while b_1 is a constant to be determined later. Substituting (13) and (14) into (11) and (12) and collecting all terms with the same order of $t^{\vartheta}e^{j\xi}(\vartheta = 0,1,2; j = 0,1,2,\cdots)$ together, we derive a set of nonlinear PDEs for $a_0(x)$, $a_1(x)$, $p(x)$, $q(x)$, $f(x)$, $g(x)$, and $h(x)$, from which we have

$$a_0 = \pm\frac{1}{2}q(x)\sqrt{-\frac{\alpha(x)}{\beta(x)}},$$
$$a_1 = \mp\frac{1}{2}b_1 q(x)\sqrt{-\frac{\alpha(x)}{\beta(x)}}, \tag{15}$$

$$f(x) = \frac{1}{f_0 + 2\int\alpha(x)\,\mathrm{d}x},$$
$$g(x) = \frac{g_0}{f_0 + 2\int\alpha(x)\,\mathrm{d}x}, \tag{16}$$

$$h(x) = h_0 + \frac{2g_0^2 + q_0^2}{8\left(f_0 + 2\int\alpha(x)\,\mathrm{d}x\right)}, \tag{17}$$

$$p(x) = \frac{g_0 q_0}{2\left(f_0 + 2\int\alpha(x)\,\mathrm{d}x\right)},$$
$$q(x) = \frac{q_0}{f_0 + 2\int\alpha(x)\,\mathrm{d}x}, \tag{18}$$

under the constraint

$$\gamma(x) = -\frac{\alpha(x)}{f_0 + 2\int\alpha(x)\,\mathrm{d}x} + \frac{\alpha'(x)}{2\alpha(x)} - \frac{\beta'(x)}{2\beta(x)}, \tag{19}$$

where b_1, f_0, h_0, g_0, and q_0 are arbitrary constants.

We therefore obtain a pair of rational exp-function solutions of the gNLS in (1):

$$\psi = \pm\frac{g_0 q_0\left(1 \mp b_1 e^{\xi}\right)}{2\left(f_0 + 2\int\alpha(x)\,\mathrm{d}x\right)\left(1 + b_1 e^{\xi}\right)}\sqrt{-\frac{\alpha(x)}{\beta(x)}} \cdot \exp\left\{i\left[\frac{2(2t+g_0)^2 + q_0^2}{8\left(f_0 + 2\int\alpha(x)\,\mathrm{d}x\right)} + h_0\right]\right\}, \tag{20}$$

where

$$\xi = \frac{q_0}{f_0 + 2\int\alpha(x)\,\mathrm{d}x} + \frac{g_0 q_0}{2\left(f_0 + 2\int\alpha(x)\,\mathrm{d}x\right)}t. \tag{21}$$

3.2. N-Wave Solutions. For convenience, we let

$$X = \frac{1}{2}\int\alpha(x)\,\mathrm{d}x,$$
$$T = t,$$
$$\beta(x) = \frac{1}{2}\alpha(x)\,e^{-\int\alpha(x)\mathrm{d}x},$$
$$\gamma(x) = \frac{1}{2}\alpha(x),$$
$$\psi = \lambda e^{(1+i\lambda^2)X}\overline{\psi}, \tag{22}$$

and we still write $\overline{\psi}$ as ψ; then (1) is converted into

$$i\psi_X + \psi_{TT} + \lambda^2\left(|\psi|^2 - 1\right)\psi = 0. \tag{23}$$

In what follows, we construct N-wave solutions of (23). To begin with the single-wave solution, we suppose that

$$\psi = \frac{1}{\lambda}e^{-i\lambda^2 X}\frac{a_0 + a_1 e^{\xi_1} + a_2 e^{\xi_1^*} + a_3 e^{\xi_1+\xi_1^*}}{1 + b_1 e^{\xi_1} + b_2 e^{\xi_1^*} + b_3 e^{\xi_1+\xi_1^*}},$$
$$\xi_1 = k_1 T + c_1 X + d_1. \tag{24}$$

Substituting (24) into (23) and equating each coefficient of the same order power of $e^{\theta\xi_1+\vartheta\xi_1^*}$ $(\theta, \vartheta = 0,1,2,\cdots)$ to zero yield a set of algebraic equations for a_0, a_1, a_2, a_3, b_1, b_2, b_3, c_1, and k_1. Solving the set of equations, we have

$$a_0 = 0,$$
$$a_2 = 0,$$
$$b_1 = 0,$$
$$b_2 = 0,$$
$$c_1 = ik_1^2,$$
$$b_3 = \frac{a_1 a_1^*}{2\left(k_1 + k_1^*\right)^2}, \tag{25}$$

and hence we obtain the single-wave solution:

$$\psi = \frac{1}{\lambda}e^{-i\lambda^2 X}\frac{a_1 e^{\xi_1}}{1 + a_1 a_1^* e^{\xi_1+\xi_1^*+A_{13}}},$$
$$\xi_1 = k_1 T + ik_1^2 X + d_1, \tag{26}$$

where a_1, k_1, and d_1 are arbitrary complex constants and

$$e^{A_{13}} = \frac{1}{2\left(k_1 + k_1^*\right)^2}. \tag{27}$$

For the double-wave solution, we next suppose that

$$\psi = \frac{1}{\lambda}e^{-i\lambda^2 X} \cdot \frac{a_1 e^{\xi_1} + a_2 e^{\xi_2} + a_3 e^{\xi_1+\xi_2+\xi_1^*} + a_4 e^{\xi_1+\xi_2+\xi_2^*}}{1 + b_1 e^{\xi_1+\xi_1^*} + b_2 e^{\xi_1+\xi_2^*} + b_3 e^{\xi_2+\xi_1^*} + b_4 e^{\xi_2+\xi_2^*} + b_5 e^{\xi_1+\xi_2+\xi_1^*+\xi_2^*}}, \tag{28}$$

where $\xi_1 = k_1 T + c_1 X + d_1$ and $\xi_2 = k_2 T + c_2 X + d_2$. Substituting (28) into (23) and equating each coefficient of the same order

power of $e^{\theta\xi_1+\vartheta\xi_2+\mu\xi_1^*+\rho\xi_2^*}(\theta,\vartheta,\mu,\rho=0,1,2,\cdots)$ to zero yield a set of algebraic equations, from which we have

$$a_3=\frac{a_1a_2a_1^*(k_1-k_2)^2}{2(k_1+k_1^*)^2(k_2+k_1^*)^2},$$
$$a_4=\frac{a_1a_2a_2^*(k_1-k_2)^2}{2(k_1+k_2^*)^2(k_2+k_2^*)^2}, \tag{29}$$

$$b_1=\frac{a_1a_1^*}{2(k_1+k_1^*)^2},$$
$$b_2=\frac{a_1a_2^*}{2(k_1+k_2^*)^2},$$
$$b_3=\frac{a_2a_1^*}{2(k_2+k_1^*)^2},$$
$$b_4=\frac{a_2a_2^*}{2(k_2+k_2^*)^2}, \tag{30}$$

$$b_5=\frac{a_1a_2a_1^*a_2^*(k_1-k_2)^2(k_1^*-k_2^*)^2}{4(k_1+k_1^*)^2(k_2+k_1^*)^2(k_1+k_2^*)^2(k_2+k_2^*)^2}, \tag{31}$$

$$c_1=\mathrm{i}k_1^2,$$
$$c_2=\mathrm{i}k_2^2, \tag{32}$$

and hence we obtain the double-wave solution as follows:

$$\psi=\frac{1}{\lambda}e^{-\mathrm{i}\lambda^2X}\frac{F(\xi_1,\xi_2,\xi_1^*,\xi_2^*)}{H(\xi_1,\xi_2,\xi_1^*,\xi_2^*)}, \tag{33}$$

where

$$\begin{aligned}F(\xi_1,\xi_2,\xi_1^*,\xi_2^*)&=a_1e^{\xi_1}+a_2e^{\xi_2}+a_1a_2a_1^*e^{\xi_1+\xi_2+\xi_1^*+A_{12}+A_{13}+A_{23}}\\&\quad+a_1a_2a_2^*e^{\xi_1+\xi_2+\xi_2^*+A_{12}+A_{14}+A_{24}},\end{aligned} \tag{34}$$

$$\begin{aligned}H(\xi_1,\xi_2,\xi_1^*,\xi_2^*)&=1+a_1a_1^*e^{\xi_1+\xi_1^*+A_{13}}+a_1a_2^*e^{\xi_1+\xi_2^*+A_{14}}\\&\quad+a_2a_1^*e^{\xi_2+\xi_1^*+A_{23}}+a_2a_2^*e^{\xi_2+\xi_2^*+A_{24}}\\&\quad+a_1a_2a_1^*a_2^*e^{\xi_1+\xi_2+\xi_1^*+\xi_2^*+A_{12}+A_{13}+A_{14}+A_{23}+A_{24}+A_{34}},\end{aligned} \tag{35}$$

$$\xi_1=k_1T+\mathrm{i}k_1^2X+d_1,$$
$$\xi_2=k_2T+\mathrm{i}k_2^2X+d_2, \tag{36}$$

$$e^{A_{12}}=2(k_1-k_2)^2,$$
$$e^{A_{j,l+2}}=\frac{1}{2(k_j+k_l^*)^2}, \tag{37}$$
$$(j,l=1,2),$$

$$e^{A_{34}}=2(k_1^*-k_2^*)^2, \tag{38}$$

and a_1, a_2, c_1, c_2, k_1, k_2, d_1, and d_2 are arbitrary complex constants.

Finally, we determine the three-wave solution of the following form:

$$\psi=\frac{1}{\lambda}e^{-\mathrm{i}\lambda^2X}\frac{F(\xi_1,\xi_2,\xi_3,\xi_1^*,\xi_2^*,\xi_3^*)}{H(\xi_1,\xi_2,\xi_3,\xi_1^*,\xi_2^*,\xi_3^*)}, \tag{39}$$

where $\xi_1=k_1T+c_1X+d_1$, $\xi_2=k_2T+c_2X+d_2$, $\xi_3=k_3T+c_3X+d_3$, and

$$\begin{aligned}&F(\xi_1,\xi_2,\xi_3,\xi_1^*,\xi_2^*,\xi_3^*)\\&\quad=a_1e^{\xi_1}+a_2e^{\xi_2}+a_3e^{\xi_3}+a_4e^{\xi_1+\xi_2+\xi_1^*}+a_5e^{\xi_1+\xi_2+\xi_2^*}\\&\qquad+a_6e^{\xi_1+\xi_2+\xi_3^*}+a_7e^{\xi_1+\xi_3+\xi_1^*}+a_8e^{\xi_1+\xi_3+\xi_2^*}\\&\qquad+a_9e^{\xi_1+\xi_3+\xi_3^*}+a_{10}e^{\xi_2+\xi_3+\xi_1^*}+a_{11}e^{\xi_2+\xi_3+\xi_2^*}\\&\qquad+a_{12}e^{\xi_2+\xi_3+\xi_3^*}+a_{13}e^{\xi_1+\xi_2+\xi_3+\xi_1^*+\xi_2^*}\\&\qquad+a_{14}e^{\xi_1+\xi_2+\xi_3+\xi_1^*+\xi_3^*}+a_{15}e^{\xi_1+\xi_2+\xi_3+\xi_2^*+\xi_3^*},\end{aligned} \tag{40}$$

$$\begin{aligned}&H(\xi_1,\xi_2,\xi_3,\xi_1^*,\xi_2^*,\xi_3^*)\\&\quad=1+b_1e^{\xi_1+\xi_1^*}+b_2e^{\xi_1+\xi_2^*}+b_3e^{\xi_1+\xi_3^*}+b_4e^{\xi_2+\xi_1^*}\\&\qquad+b_5e^{\xi_2+\xi_2^*}+b_6e^{\xi_2+\xi_3^*}+b_7e^{\xi_3+\xi_1^*}+b_8e^{\xi_3+\xi_2^*}\\&\qquad+b_9e^{\xi_3+\xi_3^*}+b_{10}e^{\xi_1+\xi_2+\xi_1^*+\xi_2^*}+b_{11}e^{\xi_1+\xi_2+\xi_1^*+\xi_3^*}\\&\qquad+b_{12}e^{\xi_1+\xi_2+\xi_2^*+\xi_3^*}+b_{13}e^{\xi_1+\xi_3+\xi_1^*+\xi_2^*}\\&\qquad+b_{14}e^{\xi_1+\xi_3+\xi_1^*+\xi_3^*}+b_{15}e^{\xi_1+\xi_3+\xi_2^*+\xi_3^*}\\&\qquad+b_{16}e^{\xi_2+\xi_3+\xi_1^*+\xi_2^*}+b_{17}e^{\xi_2+\xi_3+\xi_1^*+\xi_3^*}\\&\qquad+b_{18}e^{\xi_2+\xi_3+\xi_2^*+\xi_3^*}+b_{19}e^{\xi_1+\xi_2+\xi_3+\xi_1^*+\xi_2^*+\xi_3^*}.\end{aligned} \tag{41}$$

Similarly, we have

$$a_4 = \frac{a_1 a_2 a_1^* (k_1 - k_2)^2}{2(k_1 + k_1^*)^2 (k_2 + k_1^*)^2},$$
$$a_5 = \frac{a_1 a_2 a_2^* (k_1 - k_2)^2}{2(k_1 + k_2^*)^2 (k_2 + k_2^*)^2}, \tag{42}$$
$$a_6 = \frac{a_1 a_2 a_3^* (k_1 - k_2)^2}{2(k_1 + k_3^*)^2 (k_2 + k_3^*)^2},$$

$$a_7 = \frac{a_1 a_3 a_1^* (k_1 - k_3)^2}{2(k_1 + k_1^*)^2 (k_3 + k_1^*)^2},$$
$$a_8 = \frac{a_1 a_3 a_2^* (k_1 - k_3)^2}{2(k_1 + k_2^*)^2 (k_3 + k_2^*)^2}, \tag{43}$$
$$a_9 = \frac{a_1 a_3 a_3^* (k_1 - k_3)^2}{2(k_1 + k_3^*)^2 (k_3 + k_3^*)^2},$$

$$a_{10} = \frac{a_2 a_3 a_1^* (k_2 - k_3)^2}{2(k_2 + k_1^*)^2 (k_3 + k_1^*)^2},$$
$$a_{11} = \frac{a_2 a_3 a_2^* (k_2 - k_3)^2}{2(k_2 + k_2^*)^2 (k_3 + k_2^*)^2}, \tag{44}$$
$$a_{12} = \frac{a_2 a_3 a_3^* (k_2 - k_3)^2}{2(k_2 + k_3^*)^2 (k_3 + k_3^*)^2},$$

$$a_{13} = \frac{a_1 a_2 a_3 a_1^* a_2^* (k_1 - k_2)^2 (k_1 - k_3)^2 (k_2 - k_3)^2 (k_1^* - k_2^*)^2}{4(k_1 + k_1^*)^2 (k_2 + k_1^*)^2 (k_3 + k_1^*)^2 (k_1 + k_2^*)^2 (k_2 + k_2^*)^2 (k_3 + k_2^*)^2}, \tag{45}$$

$$a_{14} = \frac{a_1 a_2 a_3 a_1^* a_3^* (k_1 - k_2)^2 (k_1 - k_3)^2 (k_2 - k_3)^2 (k_1^* - k_3^*)^2}{4(k_1 + k_1^*)^2 (k_2 + k_1^*)^2 (k_3 + k_1^*)^2 (k_1 + k_3^*)^2 (k_2 + k_3^*)^2 (k_3 + k_3^*)^2}, \tag{46}$$

$$a_{15} = \frac{a_1 a_2 a_3 a_2^* a_3^* (k_1 - k_2)^2 (k_1 - k_3)^2 (k_2 - k_3)^2 (k_2^* - k_3^*)^2}{4(k_1 + k_2^*)^2 (k_2 + k_2^*)^2 (k_3 + k_2^*)^2 (k_1 + k_3^*)^2 (k_2 + k_3^*)^2 (k_3 + k_3^*)^2}, \tag{47}$$

$$b_1 = \frac{a_1 a_1^*}{2(k_1 + k_1^*)^2},$$
$$b_2 = \frac{a_1 a_2^*}{2(k_1 + k_2^*)^2}, \tag{48}$$
$$b_3 = \frac{a_1 a_3^*}{2(k_1 + k_3^*)^2},$$

$$b_4 = \frac{a_2 a_1^*}{2(k_2 + k_1^*)^2},$$
$$b_5 = \frac{a_2 a_2^*}{2(k_2 + k_2^*)^2}, \tag{49}$$
$$b_6 = \frac{a_2 a_3^*}{2(k_2 + k_3^*)^2},$$

$$b_7 = \frac{a_3 a_1^*}{2\left(k_3 + k_1^*\right)^2},$$

$$b_8 = \frac{a_3 a_2^*}{2\left(k_3 + k_2^*\right)^2}, \tag{50}$$

$$b_9 = \frac{a_3 a_3^*}{2\left(k_3 + k_3^*\right)^2},$$

$$b_{10} = \frac{a_1 a_2 a_1^* a_2^* \left(k_1 - k_2\right)^2 \left(k_1^* - k_2^*\right)^2}{4\left(k_1 + k_1^*\right)^2 \left(k_2 + k_1^*\right)^2 \left(k_1 + k_2^*\right)^2 \left(k_2 + k_2^*\right)^2}, \tag{51}$$

$$b_{11} = \frac{a_1 a_2 a_1^* a_3^* \left(k_1 - k_2\right)^2 \left(k_1^* - k_3^*\right)^2}{4\left(k_1 + k_1^*\right)^2 \left(k_2 + k_1^*\right)^2 \left(k_1 + k_3^*\right)^2 \left(k_2 + k_3^*\right)^2}, \tag{52}$$

$$b_{12} = \frac{a_1 a_2 a_2^* a_3^* \left(k_1 - k_2\right)^2 \left(k_2^* - k_3^*\right)^2}{4\left(k_1 + k_2^*\right)^2 \left(k_2 + k_2^*\right)^2 \left(k_1 + k_3^*\right)^2 \left(k_2 + k_3^*\right)^2}, \tag{53}$$

$$b_{13} = \frac{a_1 a_3 a_1^* a_2^* \left(k_1 - k_3\right)^2 \left(k_1^* - k_2^*\right)^2}{4\left(k_1 + k_1^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_1 + k_2^*\right)^2 \left(k_3 + k_2^*\right)^2}, \tag{54}$$

$$b_{14} = \frac{a_1 a_3 a_1^* a_3^* \left(k_1 - k_3\right)^2 \left(k_1^* - k_3^*\right)^2}{4\left(k_1 + k_1^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_1 + k_3^*\right)^2 \left(k_3 + k_3^*\right)^2}, \tag{55}$$

$$b_{15} = \frac{a_1 a_3 a_2^* a_3^* \left(k_1 - k_3\right)^2 \left(k_2^* - k_3^*\right)^2}{4\left(k_1 + k_2^*\right)^2 \left(k_3 + k_2^*\right)^2 \left(k_1 + k_3^*\right)^2 \left(k_3 + k_3^*\right)^2}, \tag{56}$$

$$b_{16} = \frac{a_2 a_3 a_1^* a_2^* \left(k_2 - k_3\right)^2 \left(k_1^* - k_2^*\right)^2}{4\left(k_2 + k_1^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_2 + k_2^*\right)^2 \left(k_3 + k_2^*\right)^2}, \tag{57}$$

$$b_{17} = \frac{a_2 a_3 a_1^* a_3^* \left(k_2 - k_3\right)^2 \left(k_1^* - k_3^*\right)^2}{4\left(k_2 + k_1^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_3 + k_3^*\right)^2}, \tag{58}$$

$$b_{18} = \frac{a_2 a_3 a_2^* a_3^* \left(k_2 - k_3\right)^2 \left(k_2^* - k_3^*\right)^2}{4\left(k_2 + k_2^*\right)^2 \left(k_3 + k_2^*\right)^2 \left(k_3 + k_2^*\right)^2 \left(k_3 + k_3^*\right)^2}, \tag{59}$$

$$b_{19} = \frac{a_1 a_2 a_3 a_1^* a_2^* a_3^* \left(k_1 - k_2\right)^2 \left(k_1 - k_3\right)^2 \left(k_2 - k_3\right)^2 \left(k_1^* - k_2^*\right)^2 \left(k_1^* - k_3^*\right)^2 \left(k_2^* - k_3^*\right)^2}{8\left(k_1 + k_1^*\right)^2 \left(k_2 + k_1^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_2 + k_1^*\right)^2 \left(k_2 + k_2^*\right)^2 \left(k_2 + k_3^*\right)^2 \left(k_3 + k_1^*\right)^2 \left(k_3 + k_2^*\right)^2 \left(k_3 + k_3^*\right)^2}, \tag{60}$$

$$c_1 = \mathrm{i}k_1^2,$$

$$c_2 = \mathrm{i}k_2^2, \tag{61}$$

$$c_3 = \mathrm{i}k_3^2.$$

With the help of (40)-(61), the three-wave solution (39) can be finally determined as follows:

$$\psi = \frac{1}{\lambda} \mathrm{e}^{-\mathrm{i}\lambda^2 X} \frac{\sum_{\mu=0,1} B_1(\mu) \prod_{j=1}^{6} a_j^{\mu_j} \mathrm{e}^{\sum_{j=1}^{6} \mu_j \xi_j + \sum_{j=1}^{6} \mu_j \mu_l A_{jl}}}{\sum_{\mu=0,1} B_2(\mu) \prod_{j=1}^{6} a_j^{\mu_j} \mathrm{e}^{\sum_{j=1}^{6} \mu_j \xi_j + \sum_{j=1}^{6} \mu_j \mu_l A_{jl}}}, \tag{62}$$

where

$$\xi_j = k_j T + \mathrm{i}k_j^2 X + d_j,$$

$$\xi_{N+j} = \xi_j^* = k_j^* T - \mathrm{i}k_j^{*2} X + d_j^*, \tag{63}$$

$$(j = 1, 2, 3),$$

$$e^{A_{jl}} = 2\left(k_j - k_l\right)^2, \quad \left(j < l = 2,3\right), \tag{64}$$

$$e^{A_{j,N+l}} = \frac{1}{2\left(k_j + k_l^*\right)^2}, \quad \left(j,l = 1,2,3\right), \tag{65}$$

$$e^{A_{N+j,N+l}} = e^{A_{j,l}^*} = 2\left(k_j^* - k_l^*\right)^2, \quad \left(j < l = 2,3\right), \tag{66}$$

$$a_{N+j} = a_j^*, \quad \left(j = 1,2,\cdots,N\right), \tag{67}$$

and the summation $\Sigma_{\mu=0,1}$ refers to all the combinations of $\mu_j = 0,1(j = 1,2,3)$; $B_1(\mu)$ and $B_2(\mu)$ denote that all the following conditions must hold:

$$\begin{aligned} \sum_{j=1}^{3}\mu_j &= \sum_{j=1}^{3}\mu_{3+j}, \\ \sum_{j=1}^{3}\mu_j &= \sum_{j=1}^{3}\mu_{3+j} + 1. \end{aligned} \tag{68}$$

Generally, introducing the notations

$$\begin{aligned} &\xi_j = k_jT + ik_j^2X + d_j, \\ &\xi_{N+j} = \xi_j^* = k_j^*T - ik_j^{*2}X + d_j^*, \\ &\qquad\qquad \left(j = 1,2,\cdots,N\right), \end{aligned} \tag{69}$$

$$\begin{aligned} &e^{A_{jl}} = 2\left(k_j - k_l\right)^2, \\ &\qquad\qquad \left(j < l = 2,3,\cdots,N\right), \end{aligned} \tag{70}$$

$$\begin{aligned} &e^{A_{j,N+l}} = \frac{1}{2\left(k_j + k_l^*\right)^2}, \\ &\qquad\qquad \left(j,l = 1,2,\cdots,N\right), \end{aligned} \tag{71}$$

$$\begin{aligned} &e^{A_{N+j,N+l}} = e^{A_{j,l}^*} = 2\left(k_j^* - k_l^*\right)^2, \\ &\qquad\qquad \left(j < l = 2,3,\cdots,N\right), \end{aligned} \tag{72}$$

$$\begin{aligned} &a_{N+j} = a_j^*, \\ &\qquad\qquad \left(j = 1,2,\cdots,N\right), \end{aligned} \tag{73}$$

we can obtain a uniform formula of the N-wave solution:

$$\begin{aligned} &\psi \\ &= \frac{1}{\lambda}e^{-i\lambda^2X}\frac{\sum_{\mu=0,1}B_1\left(\mu\right)\prod_{j=1}^{2N}a_j^{\mu_j}e^{\sum_{j=1}^{2N}\mu_j\xi_j+\sum_{j=1}^{2N}\mu_j\mu_lA_{jl}}}{\sum_{\mu=0,1}B_2\left(\mu\right)\prod_{j=1}^{2N}a_j^{\mu_j}e^{\sum_{j=1}^{2N}\mu_j\xi_j+\sum_{j=1}^{2N}\mu_j\mu_lA_{jl}}}, \end{aligned} \tag{74}$$

where the summation $\Sigma_{\mu=0,1}$ refers to all the combinations of $\mu_j = 0,1(j = 1,2,\cdots,N)$; $B_1(\mu)$ and $B_2(\mu)$ denote that all the following conditions must hold:

$$\begin{aligned} \sum_{j=1}^{N}\mu_j &= \sum_{j=1}^{N}\mu_{N+j}, \\ \sum_{j=1}^{N}\mu_j &= \sum_{j=1}^{N}\mu_{N+j} + 1. \end{aligned} \tag{75}$$

3.3. Rouge Wave Solutions. To construct rouge wave solutions, we rewrite (9) as

$$\psi = \frac{a_0 + a_1\cos kTe^{cX+d} + a_2e^{2cX+2d}}{1 + b_1\cos kTe^{cX+d} + b_2e^{2cX+2d}}, \tag{76}$$

and we substitute (76) into (23); then we equate each coefficient of the same order power of $\cos^{\mu}kTe^{\theta(cX+d)}(\mu,\theta = 0,1,2,\cdots)$ to zero; a set of algebraic equations is derived. Solving the set of equations, we have

$$\begin{aligned} a_0 &= 1, \\ a_1 &= \frac{\lambda\sqrt{2b_2\left(2\lambda^2 - k^2\right)}}{\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}}, \\ a_2 &= \frac{b_2\left(\lambda^2 - k^2 \mp ik\sqrt{2\lambda^2 - k^2}\right)}{\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}}, \end{aligned} \tag{77}$$

$$\begin{aligned} b_1 &= \frac{\sqrt{2b_2}\left(2\lambda^2 - k^2\right)}{\lambda}, \\ c &= \pm k\sqrt{2\lambda^2 - k^2}, \end{aligned} \tag{78}$$

and

$$\begin{aligned} a_0 &= 1, \\ a_1 &= -\frac{\lambda\sqrt{2b_2\left(2\lambda^2 - k^2\right)}}{\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}}, \\ a_2 &= \frac{b_2\left(\lambda^2 - k^2 \mp ik\sqrt{2\lambda^2 - k^2}\right)}{\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}}, \end{aligned} \tag{79}$$

$$\begin{aligned} b_1 &= -\frac{\sqrt{2b_2}\left(2\lambda^2 - k^2\right)}{\lambda}, \\ c &= \pm k\sqrt{2\lambda^2 - k^2}. \end{aligned} \tag{80}$$

We, therefore, obtain two pairs of rational exp-function wave solutions as follows:

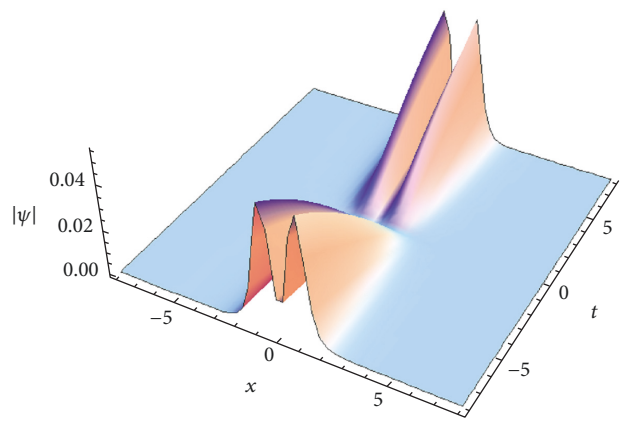


Figure 1: Spatial structure of the module of solution (20) with $q_0 = 0.5$.

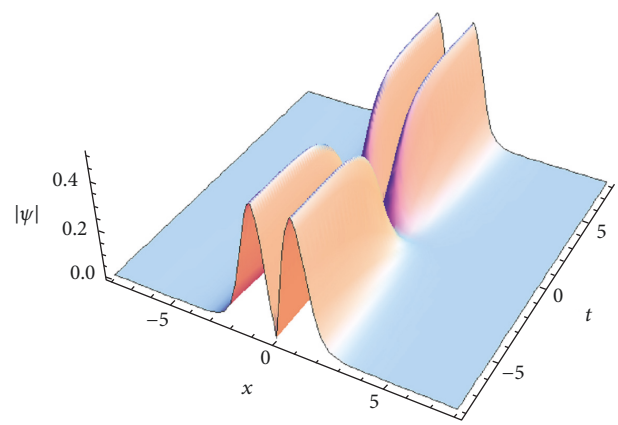


Figure 2: Spatial structure of the module of solution (20) with $q_0 = 3$.

$$\psi = \frac{1 + \left(\lambda\sqrt{2b_2(2\lambda^2 - k^2)}/\left(\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}\right)\right)\cos kT e^{\pm k\sqrt{2\lambda^2-k^2}X+d} + \left(b_2\left(\lambda^2 - k^2 \mp ik\sqrt{2\lambda^2 - k^2}\right)/\left(\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}\right)\right)e^{\pm 2k\sqrt{2\lambda^2-k^2}X+2d}}{1 + \left(\sqrt{2b_2(2\lambda^2 - k^2)}/\lambda\right)\cos kT e^{\pm k\sqrt{2\lambda^2-k^2}X+d} + b_2 e^{\pm 2k\sqrt{2\lambda^2-k^2}X+2d}}, \tag{81}$$

$$\psi = \frac{1 - \left(\lambda\sqrt{2b_2(2\lambda^2 - k^2)}/\left(\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}\right)\right)\cos kT e^{\pm k\sqrt{2\lambda^2-k^2}X+d} + \left(b_2\left(\lambda^2 - k^2 \mp ik\sqrt{2\lambda^2 - k^2}\right)/\left(\lambda^2 - k^2 \pm ik\sqrt{2\lambda^2 - k^2}\right)\right)e^{\pm 2k\sqrt{2\lambda^2-k^2}X+2d}}{1 - \left(\sqrt{2b_2(2\lambda^2 - k^2)}/\lambda\right)\cos kT e^{\pm k\sqrt{2\lambda^2-k^2}X+d} + b_2 e^{\pm 2k\sqrt{2\lambda^2-k^2}X+2d}}, \tag{82}$$

It is easy to see that when $b_2 = 1$, $d = 0$, and $\lambda < 0$ the molecular and denominator of solution (81) tend to zeros, respectively. We differentiate (81) with respect to k twice and let $k \longrightarrow 0$; then the limits of solution (81) give two rouge wave solutions:

$$\psi = \frac{9 - 8i\lambda^2 t - 12\lambda^4 t^2 + 2\lambda^2 x^2}{1 + 4\lambda^4 t^2 + 2\lambda^2 x^2}, \tag{83}$$

$$\psi = \frac{-3 - 24i\lambda^2 t + 4\lambda^4 t^2 + 2\lambda^2 x^2}{1 + 4\lambda^4 t^2 + 2\lambda^2 x^2}. \tag{84}$$

In a similar way, when $b_2 = 1$, $d = 0$, and $\lambda > 0$, the limits of solution (82) give two rouge wave solutions, which are the same as those in (83) and (84), respectively.

4. Complex Dynamics

To gain more insights into the solutions obtained in Section 3, we investigate the dynamical evolutions of some obtained solutions. Firstly, we select $b_1 = 1$, $f_0 = 1$, $g_0 = 0.1$, $h_0 = 2$, $\alpha(x) = x^3$, and $\beta(x) = -x$; then the modules of solution

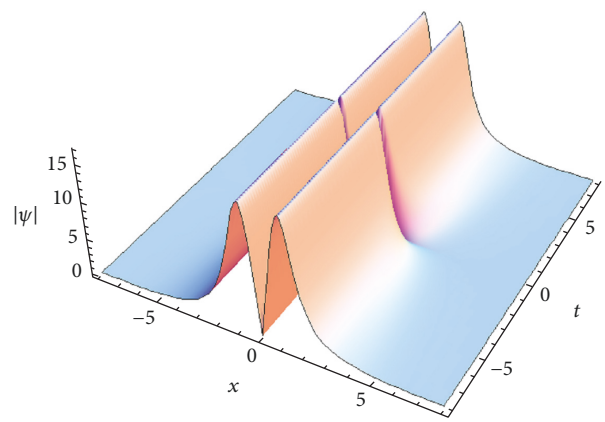


Figure 3: Spatial structure of the module of solution (20) with $q_0 = 100$.

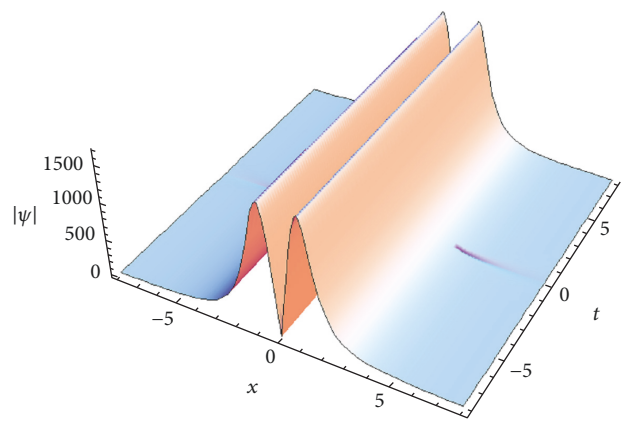


Figure 4: Spatial structure of the module of solution (20) with $q_0 = 10000$.

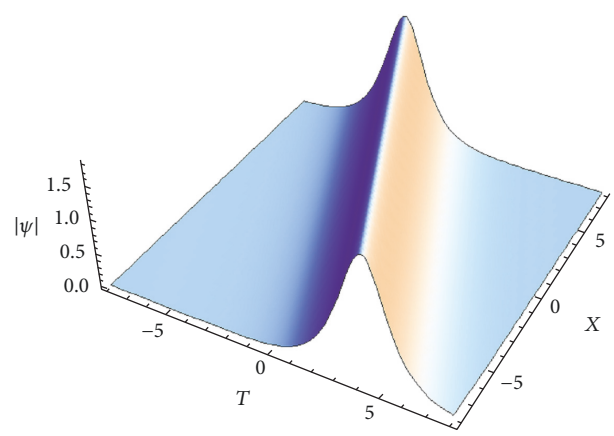


Figure 5: Spatial structure of the module of solution (26).

(20) with "+" branch and different values of q_0 are shown in Figures 1–4, respectively. It is shown in Figures 1–4 that when the other parameters are fixed, the larger the value of q_0 is, the smaller the influence on M-shape wave will be.

Secondly, we consider solutions (26), (33), and (62). In Figure 5, the module of the single-wave solution (26) is shown by selecting $a_1 = 1+\mathrm{i}$, $k_1 = 1-0.2\mathrm{i}$, and $\lambda = 1$. We simulate the module of the double-wave solution (33) in Figure 6, where $a_1 = 0.5$, $a_2 = 1$, $k_1 = 1$, $k_2 = 0.2$, and $\lambda = 1$. Selecting $a_1 = 1$, $a_2 = 0.5$, $a_3 = 1.5$, $k_1 = 1$, $k_2 = 0.5$, $k_3 = -0.25$, and $\lambda = 1$, we show the module of the three-wave solution (62) in Figure 7.

Thirdly, in Figures 8–17, we simulate some modules of one branch of solution (81) by selecting $\lambda = 1$ and different values of k. We can see from Figures 8–17 that the value

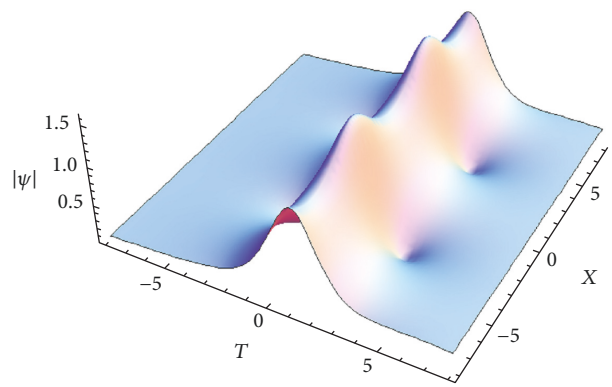


Figure 6: Spatial structure of the module of solution (33).

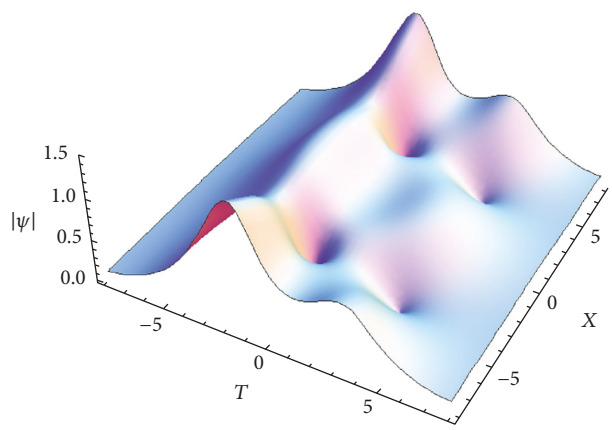


Figure 7: Spatial structure of the module of solution (62).

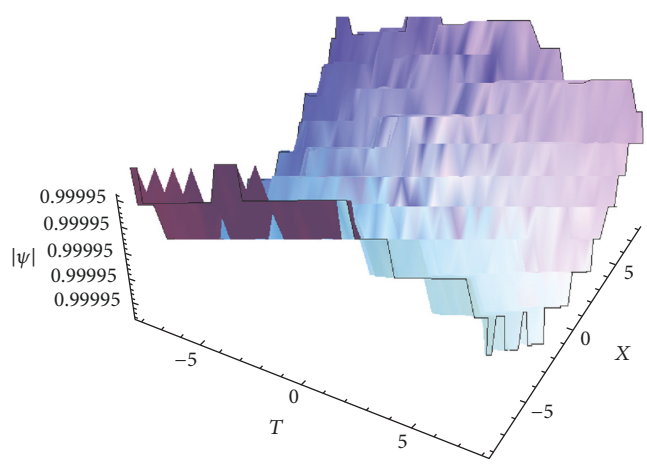


Figure 8: Spatial structure of the module of solution (81) with $k = 0.01$.

of k has influenced the spatial structures of the module of solution (81) and can also lead to the periodicity and singularity.

Finally, we simulate the rouge wave solutions (83) and (84). In Figure 18, a rouge wave structure determined by the module of solution (83) is shown by selecting $\lambda = -1$. We show the contour of the rouge wave structure and dynamical evolutions determined by the module of solution (83) in Figures 19 and 20, respectively. At the same time, in Figure 21, we show another rouge wave structure determined by the

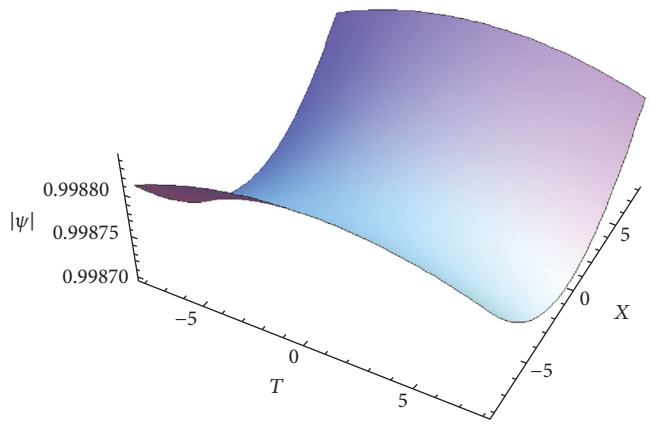


Figure 9: Spatial structure of the module of solution (81) with $k = 0.05$.

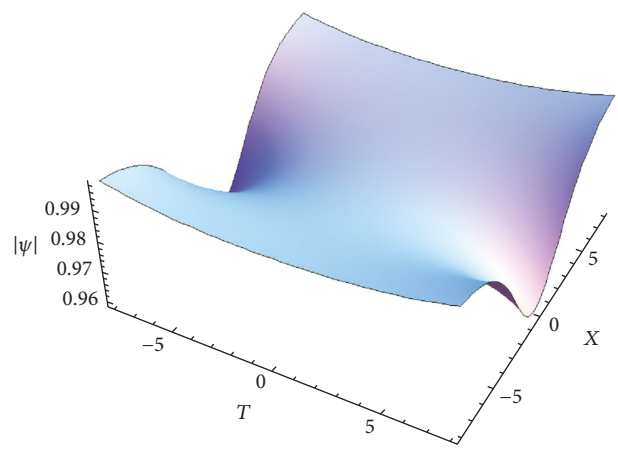


Figure 10: Spatial structure of the module of solution (81) with $k = 0.2$.

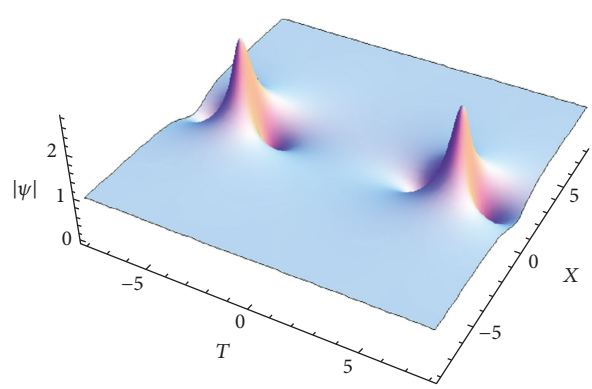


Figure 11: Spatial structure of the module of solution (81) with $k = 0.6$.

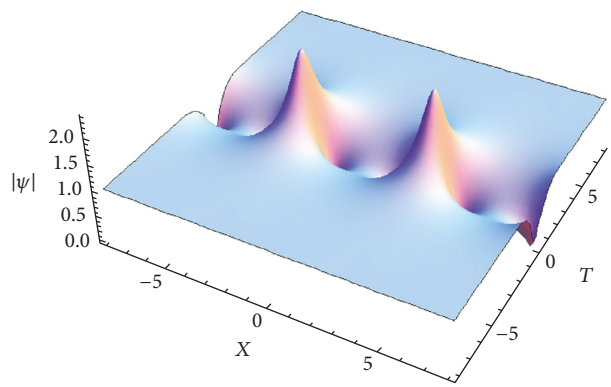


Figure 12: Spatial structure of the module of solution (81) with $k = 1$.

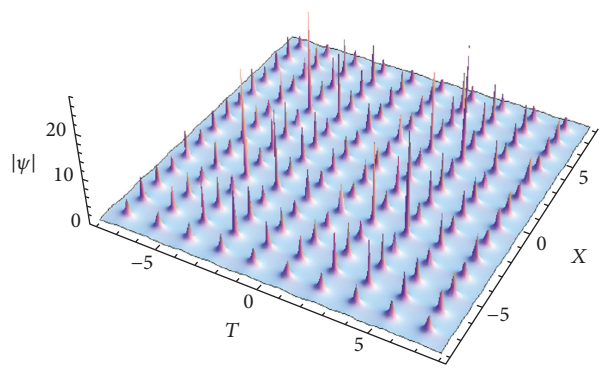


Figure 13: Spatial structure of the module of solution (81) with $k = 2$.

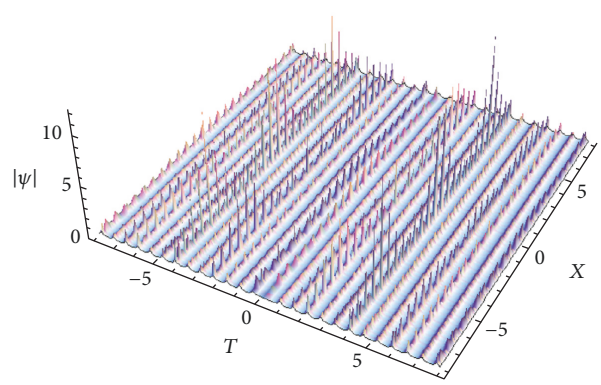


Figure 14: Spatial structure of the module of solution (81) with $k = 5$.

module of solution (84) by selecting $\lambda = -1$. In Figures 22 and 23, the contour of the rouge wave structure and dynamical evolutions determined by the module of solution (84) are shown.

5. Conclusion

In summary, we have obtained explicit solitary wave solutions, N-wave solutions, and rouge wave solutions of the gNLS in (1) benefiting from the complex multirational exp-function ansatz (4) presented in this paper for nonlinear PDEs with complex coefficients. To the best of our knowledge, the exp-function method and its improvements [8–11] have not been used for the gNLS in (1) and the obtained solutions with free parameters have not been reported in the literatures. In 2001, Serkin and Belyaeva derived a new and more general NLS equation [25]:

$$\begin{aligned} \mathrm{i}q_t = {} & \frac{1}{2}D(t)\,q_{xx} + \beta R(t)\,|q|^2\,q - 2\alpha(t)\,xq \\ & + \frac{\mathrm{i}}{2}\frac{W[R(t), D(t)]}{R(t)\,D(t)}q - \mathrm{i}V(t)\,q_t - 2\Gamma(t)\,q, \end{aligned} \tag{85}$$

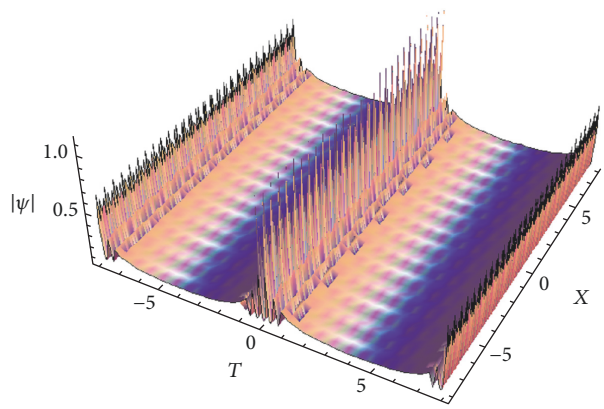


Figure 15: Spatial structure of the module of solution (81) with $k = 10$.

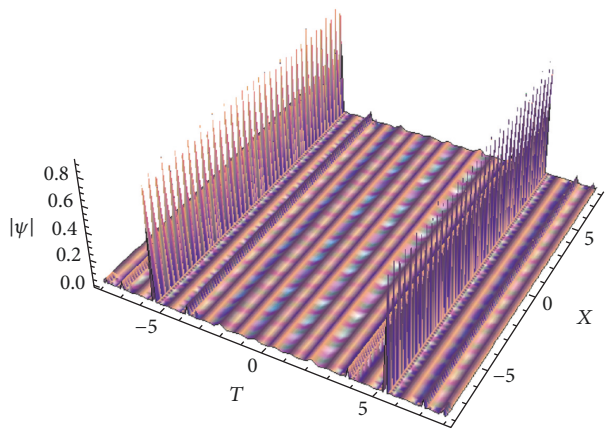


Figure 16: Spatial structure of the module of solution (81) with $k = 100$.

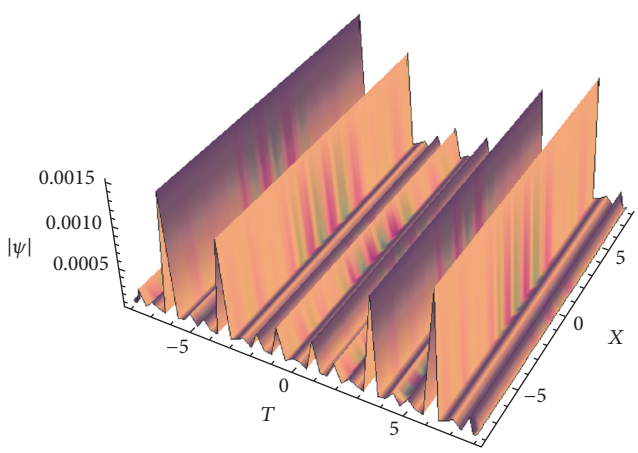


Figure 17: Spatial structure of the module of solution (81) with $k = 10000$.

as the condition for integrability of a pair of linear differential equations; here the Wronskian $W[R(t), D(t)] = R(t)\mathrm{d}D(t)/\mathrm{d}t - D(t)\mathrm{d}R(t)/\mathrm{d}t$. Since (85) is Lax-integrable [25] and the gNLS in (1) can be reduced from (85), the Lax-integrability of the gNLS in (1) is obvious. As for the NLS equations with varying coefficients, their Lax-representations, and other investigations, we can refer to a lot of references such as those in [25–38]. Compared with the inverse scattering method [1], Hirota's bilinear method [2], and Darboux transformation [3], the complex multirational exp-function ansatz (4) for constructing solitary wave solutions, N-wave solutions, and rouge wave solutions does not carry out the processes of bilinearization and spectral problems. In spite of this, the approach needs a lot of calculations and even has a lot of uncertainties. Recently, nonlinear PDEs with fractional derivatives and their applications have

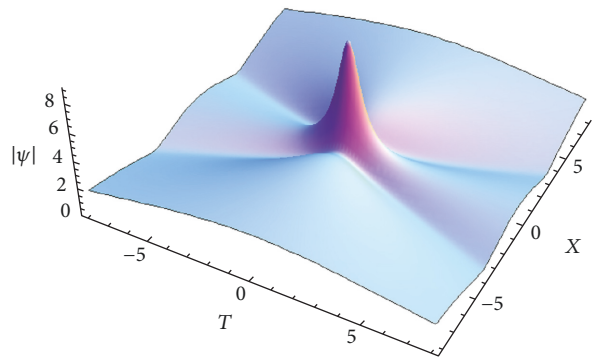


Figure 18: Rouge wave structure determined by the module of solution (83).

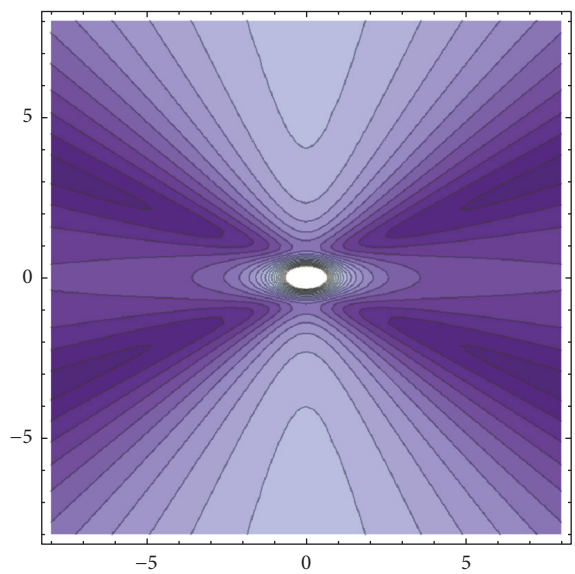
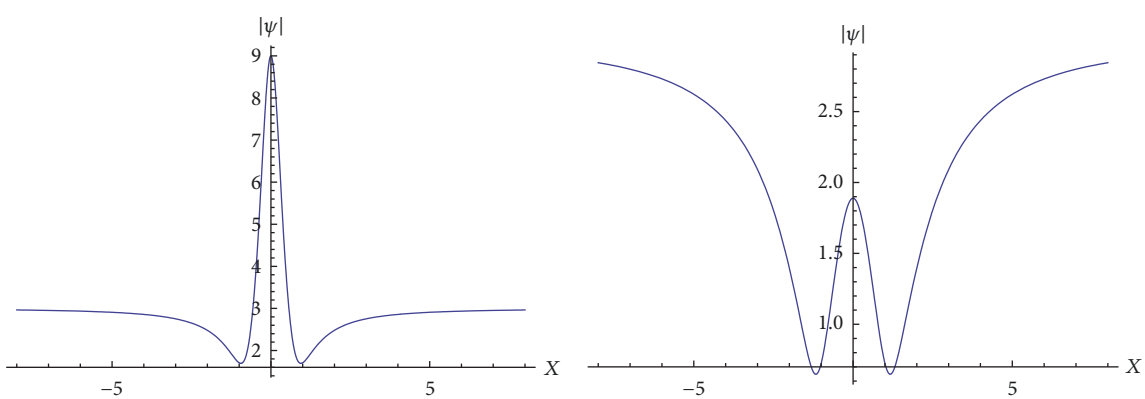


Figure 19: Contour of the rouge wave structure determined by the module of solution (83).



(a) $T = 0$ (b) $T = 2$

Figure 20: Dynamical evolutions of the rouge wave structure determined by the module of solution (83).

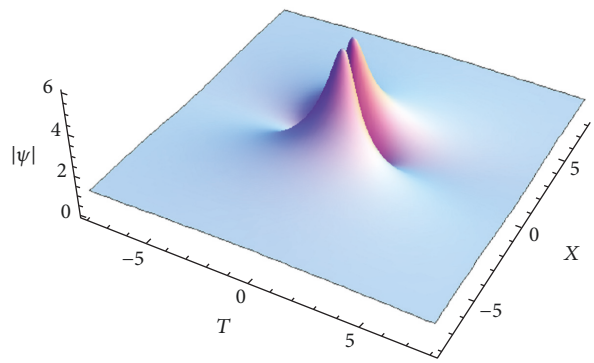


Figure 21: Rouge wave structure determined by the module of solution (84).

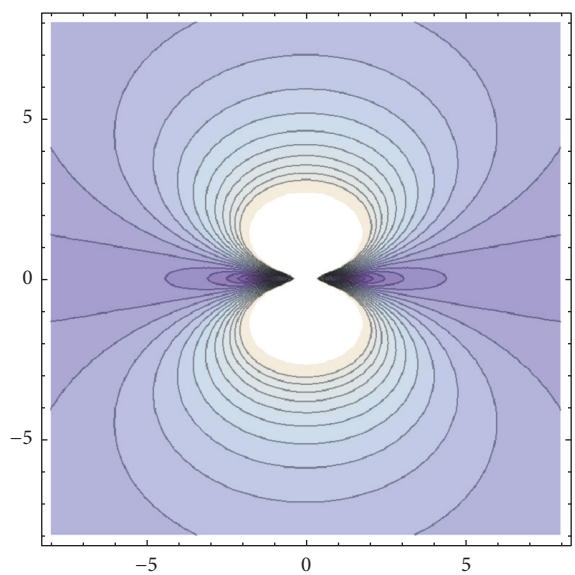
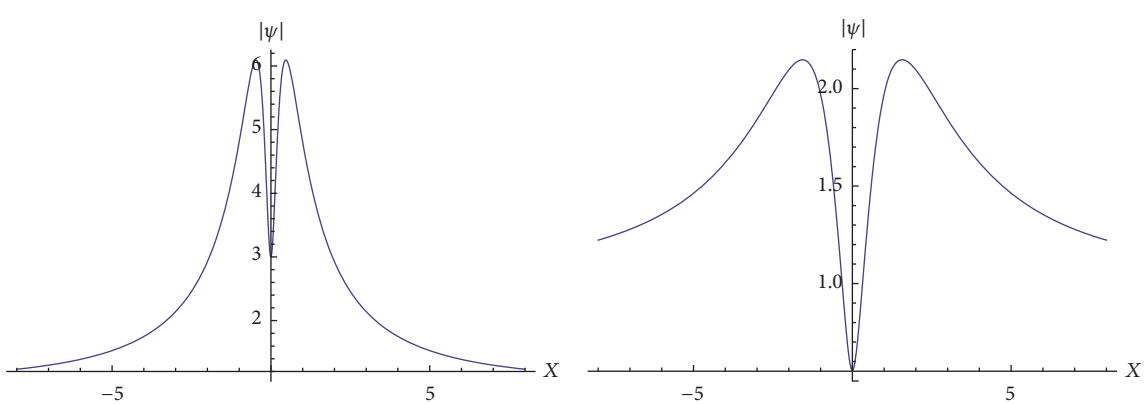


Figure 22: Contour of the rouge wave structure determined by the module of solution (84).



(a) $T = 0$ (b) $T = 2$

Figure 23: Dynamical evolutions of the rouge wave structure determined by the module of solution (84).

attracted much attention [39–53]. How to extend the complex multirational exp-function ansatz (4) to such fractional PDEs and the NLS equations with varying coefficients in [25–38] is worthy of study.

Data Availability

The data in the manuscript are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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