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Research Article

Mean Square Exponential Stability of Stochastic Complex-Valued Neural Networks with Mixed Delays

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This paper investigates the mean square exponential stability problem of a class of complex-valued neural networks with stochastic disturbance and mixed delays including both time-varying delays and continuously distributed delays. Under different assumption conditions concerning stochastic disturbance term from the existing ones, some sufficient conditions are derived for assuring the mean square exponential stability of the equilibrium point of the system based on the vector Lyapunov function method and *Itô* differential-integral theorem. The obtained results not only generalize the existing ones, but also reduce the conservatism of the previous stability results about complex-valued neural networks with stochastic disturbances. Two numerical examples with simulation results are given to verify the feasibility of the proposed results.

1. Introduction

In recent years, the dynamical behavior analysis of various neural networks models defined in complex number domain has attracted more and more attention due to their extensive applications in many fields, such as filtering, speech synthesis, remote sensing, and signal processing, which cannot be solved comprehensively with only their counterparts defined in real number domain [1, 2].

It is well known that delays arise because of the processing of information in both biological and man-made neural networks in view of a practical point [3, 4]. Besides, the neural networks usually have a spatial extent due to the presences of a multitude of parallel pathway with a variety of axon sizes and lengths. There will be a distribution of conduction velocities along these pathways and a distribution of propagation be designed with discrete delays. Therefore, the more appropriate way is to incorporate continuously distributed delays [3–7]. In [8–11], the authors have studied several kinds of complex-valued neural networks with continuously distributed delays. Some significant results were obtained for assuring the stability of the proposed systems in [8–11].

In fact, most real models of neural networks are affected by many external and internal perturbations which are of great uncertainty, such as impulsive disturbances [5, 9–15], Markovian jumping parameters [16–19], and parameter uncertainties [20–22]. As Haykin [23] points out, in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. One approach to the mathematical incorporation of such effects is to use probabilistic threshold models. In current papers, neural networks model with external random perturbations is to view as nonlinear dynamical systems with white noise (perturbations of Brownian motion) [3, 24]. Therefore, it is of real significance to consider that stochastic effects to the stability of neural networks with continuously distributed delays.

As far as we know, the existing stability results concerning various neural networks are mainly applied to judge the neural networks in real number domain, such as [3, 7, 12–14, 16, 17, 24–31]. Recently, there have been some literatures with regard to the complex-valued neural networks with stochastic disturbances [32–34]. In [32], the authors established a class of stochastic memristor-based complex-valued neural networks with time varying delays. In [33], the robust state estimation problem was investigated for a class of complex-valued neural networks involving parameter uncertainties, constant time delays, and stochastic disturbances by resorting to the sampled data information from the available output measurements. However, the infinitely

distributed delays have not been considered in the model of complex-valued neural networks studied in [32, 33]. In [34], the passivity analysis was conducted for a class of stochastic memristor-based complex-valued recurrent neural networks with discrete time delays and continuously distributed delays by applying both the scalar Lyapunov function method and the LMI method. The stochastic weighted coefficients in the model [34] were assumed to be constants in real number domain, which was conservatism and needed to be generalized in the complex number domain further. It is well known that the synchronization problem of chaotic neural networks can be translated into the stability problem of the corresponding error system of driving system and driven system. In [35], the authors concerned the problem of finite-time synchronization for a class of complex-valued neural networks with stochastic disturbances and mixed delays. However, the stochastic terms in the driving and driven systems were only assumed to be bounded directly, which was a shortage of generality. Besides, the results in [35] are unable to judge the stability of the corresponding systems.

Separating the model of complex-valued neural networks into its real and imaginary parts is a routine approach to study the dynamical behavior of the systems, such as [2, 8, 11, 15, 32–35]. The complex-valued activation functions were supposed to be with existence, continuity, and boundedness of the partial derivatives of the activation functions about the real and imaginary parts of the state variables [2, 8, 11, 15, 32]. As pointed in [33, 36–38], the assumption concerning activation functions in [2, 8, 11, 15, 32] were with additional restriction that the partial derivatives of its real parts and imaginary parts were required to satisfy the existence and continuity.

To the best of authors' knowledge, there is no research concerning the exponential stability in the mean square sense for the complex-valued neural networks with mixed delays (including both time-varying delays and continuously distributed delays) and stochastic disturbances. Therefore, in this paper we will establish some new conditions for assuring the stability of the mentioned systems. The main advantages and contributions can be listed as follows. (a) The stochastic neural networks with mixed delays in complex number domain are proposed. (b) Different assumption conditions concerning stochastic disturbance term from the existing ones are given in this paper, which is of less conservatism. (c) Both stochastic disturbances and interval parameter uncertainties are considered in the addressed systems. (d) Some sufficient conditions with simple matrix forms are obtained for ensuring the mean square exponential stability and robust exponential stability in the mean square sense of the systems, which are easy to be checked in practice.

2. Notations and Model Descriptions

In this paper, we consider a class of complex-valued neural networks as follows:

$$\begin{aligned} \mathrm{d}z_k(t) = \Bigg\{ & -d_k z_k(t) + \sum_{j=1}^{n} \Bigg[a_{kj} f_j\left(z_j(t)\right) \\ & + b_{kj} f_j\left(z_j\left(t-\tau_{kj}(t)\right)\right) \\ & + p_{kj} \int_{-\infty}^{t} \theta_{kj}(t-s) f_j\left(z_j(s)\right) \mathrm{d}s \Bigg] + J_k \Bigg\} \mathrm{d}t \\ & + \sum_{j=1}^{n} \sigma_{kj}\left(z_j(t), z_j\left(t-\tau_{kj}(t)\right)\right) \mathrm{d}w_j(t), \end{aligned} \tag{1}$$

In (1), $z_k \in \mathbf{C}$ represents the k-th neuron state, $k = 1, 2, \ldots\ldots, n$, where $\mathbf{C}$ denotes a complex number set, and n denotes the neuron number. $\boldsymbol{A} = (a_{kj})_{n\times n} \in \mathbf{C}^{n\times n}$, $\boldsymbol{B} = (b_{kj})_{n\times n} \in \mathbf{C}^{n\times n}$, and $\boldsymbol{P} = (p_{kj})_{n\times n} \in \mathbf{C}^{n\times n}$ represent the connection weighted matrices, respectively. $\boldsymbol{J} = (J_1, J_2, \ldots\ldots, J_n)^{\mathrm{T}} \in \mathbf{C}^n$ is the external input vector, where $(.)^{\mathrm{T}}$ denotes the transpose of the vector. $\boldsymbol{f}(\boldsymbol{z}(t)) = (f_1(z_1(t)), f_2(z_2(t)), \ldots\ldots, f_n(z_n(t)))^{\mathrm{T}} \in \mathbf{C}^n$ represents the activation function. $\boldsymbol{D} = \mathrm{diag}(d_1, d_2, \ldots\ldots, d_n) \in \mathbf{R}^{n\times n}$ with $d_k > 0$ $(k = 1, 2, \ldots\ldots, n)$ denotes the neuron self-feedback coefficient matrix, where $\mathbf{R}$ denote a real number set. σ_{kj} $(j, k = 1, 2, \ldots\ldots, n)$ denotes the weighted function of stochastic disturbances and $\boldsymbol{w}(t) = [w_1(t), w_2(t), \ldots\ldots, w_n(t)]^{\mathrm{T}}$ is the Brown motion defined on a complete probability space $(\Omega, F, \{F_t\}_{t\geq 0}, \mathrm{P})$ with a natural filtration $\{F_t\}_{t\geq 0}$ generated by $\{\boldsymbol{w}(s) : 0 \leq s \leq t\}$.

It is assumed that $z_k(s) = \varphi_k(s)$ is the initial condition of system (1), where $\varphi_k(s)$ is continuous function mapping from $(-\infty, 0]$ to $\mathbf{C}$, $k = 1, 2, \ldots\ldots, n$.

Let $\mathbf{z}^{\#} = (z_1^{\#}, z_2^{\#}, \ldots\ldots, z_n^{\#})^{\mathrm{T}}$ be the equilibrium point of (1), where $z_k^{\#} = x_k^{\#} + \mathrm{i}y_k^{\#}$ $(k = 1, 2, \ldots\ldots, n)$ and i denotes the imaginary unit; i.e., $\mathrm{i} = \sqrt{-1}$.

Let $z_k = \mathrm{Re}(z_k) + \mathrm{Im}(z_k)\mathrm{i} = x_k + \mathrm{i}y_k$, where Re(.)and Im(.) represent the real part and imaginary part of a complex number, respectively. Furthermore, the activation functions $f_k(z_k)$ are assumed to be expressed as (2) by separating them into their real parts and imaginary parts,

$$f_k(z_k) = f_k^R(x_k, y_k) + \mathrm{i}f_k^I(x_k, y_k), \quad k = 1, 2, \ldots, n, \tag{2}$$

where $f_k^R(x_k, y_k) : \mathbf{R}^2 \longrightarrow \mathbf{R}$ and $f_k^I(x_k, y_k) : \mathbf{R}^2 \longrightarrow \mathbf{R}$ represent the real part and the imaginary part of $f_k(z_k)$, respectively.

Next, some assumptions concerning (1) are given to obtain the stability results.

Assumption 1. Suppose that $\tau_{kj}(t)$ $(k, j = 1, 2, \ldots\ldots, n)$ is a bounded function with $\tau = \max_{1\leq k,j\leq n}\sup_{t\geq 0}\tau_{kj}(t)$.

Assumption 2. Suppose that $\theta_{kj}(.)$: $[0, +\infty) \longrightarrow [0, +\infty)$ are piecewise continuous functions and satisfy

$$\int_0^{+\infty} \exp(\beta s)\, \theta_{kj}(s)\, \mathrm{d}s = \mu_{kj}(\beta), \quad k, j = 1, 2, \ldots, n, \tag{3}$$

where $\mu_{kj}(\beta)$ is continuous on $[0, \delta)$, and $\mu_{kj}(0) = 1$.

Let $|.|$ denote the module of a complex number.

Assumption 3. It is assumed that $f_k(.)$ with the form of (2) satisfies the mean value theorem of multivariable functions. That is, for any given $x_k, x'_k, y_k, y'_k \in \mathbf{R}$, there exist positive constants l_k^{RR}, l_k^{RI}, l_k^{IR}, and l_k^{II} such that

$$\begin{aligned}&\left|f_k^R(x_k, y_k) - f_k^R(x'_k, y'_k)\right| \\ &\quad \le l_k^{RR}\left|x_k - x'_k\right| + l_k^{RI}\left|y_k - y'_k\right|,\end{aligned} \tag{4}$$

$$\begin{aligned}&\left|f_k^I(x_k, y_k) - f_k^I(x'_k, y'_k)\right| \\ &\quad \le l_k^{IR}\left|x_k - x'_k\right| + l_k^{II}\left|y_k - y'_k\right|.\end{aligned} \tag{5}$$

Let $L^{RR} = \mathrm{diag}(l_1^{RR}, l_2^{RR}, \ldots\ldots, l_n^{RR})$, $L^{RI} = \mathrm{diag}(l_1^{RI}, l_2^{RI}, \ldots\ldots, l_n^{RI})$, $L^{IR} = \mathrm{diag}(l_1^{IR}, l_2^{IR}, \ldots\ldots, l_n^{IR})$, $L^{II} = \mathrm{diag}(l_1^{II}, l_2^{II}, \ldots\ldots, l_n^{II})$.

Remark 4. In [2, 8, 11, 15, 32], the activation functions were supposed to satisfy that the partial derivatives of their real parts and imaginary parts were required to be with the existence and continuity. In fact it is an additional restriction on the activation functions in complex number domain. In this paper the mentioned restriction is no longer required.

Assumption 5. Assume that the weighted functions of stochastic disturbances σ_{kj} $(k, j = 1, 2, \ldots\ldots, n)$ can be separated into their real parts and imaginary parts with following forms:

$$\begin{aligned}&\sigma_{kj}^R = \sigma_{kj}^R\left(x_j(t), y_j(t), x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right): \\ &\quad \mathbf{R}^4 \longrightarrow \mathbf{R}, \\ &\sigma_{kj}^I = \sigma_{kj}^I\left(x_j(t), y_j(t), x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right): \\ &\quad \mathbf{R}^4 \longrightarrow \mathbf{R},\end{aligned} \tag{6}$$

where $\sigma_{kj}^R(0,0,0,0) = \sigma_{kj}^I(0,0,0,0) = 0$. Suppose that there exist nonnegative constants ω_{kj}^{RR}, ω_{kj}^{RI}, ω_{kj}^{IR}, and ω_{kj}^{II} such that inequalities (5) and (7) hold:

$$\begin{aligned}&\left(\sigma_{kj}^R\left(x_j(t), y_j(t), x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right)\right)^2 \\ &\quad - \left(\sigma_{kj}^R\left(x'_j(t), y'_j(t), x'_j\left(t - \tau_{kj}(t)\right),\right.\right. \\ &\quad \left.\left. y'_j\left(t - \tau_{kj}(t)\right)\right)\right)^2 \le \left(\omega_{kj}^{RR}\right)^2\left[\left(x_j(t) - x'_j(t)\right)^2\right. \\ &\quad \left. + \left(x_j\left(t - \tau_{kj}(t)\right) - x'_j\left(t - \tau_{kj}(t)\right)\right)^2\right] + \left(\omega_{kj}^{RI}\right)^2 \\ &\quad \cdot\left[\left(y_j(t) - y'_j(t)\right)^2 + \left(y_j\left(t - \tau_{kj}(t)\right)\right.\right. \\ &\quad \left.\left. - y'_j\left(t - \tau_{kj}(t)\right)\right)^2\right],\end{aligned} \tag{7}$$

$$\begin{aligned}&\left(\sigma_{kj}^I\left(x_j(t), y_j(t), x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right)\right)^2 \\ &\quad - \left(\sigma_{kj}^I\left(x'_j(t), y'_j(t), x'_j\left(t - \tau_{kj}(t)\right),\right.\right. \\ &\quad \left.\left. y'_j\left(t - \tau_{kj}(t)\right)\right)\right)^2 \le \left(\omega_{kj}^{IR}\right)^2\left[\left(x_j(t) - x'_j(t)\right)^2\right. \\ &\quad \left. + \left(x_j\left(t - \tau_{kj}(t)\right) - x'_j\left(t - \tau_{kj}(t)\right)\right)^2\right] + \left(\omega_{kj}^{II}\right)^2 \\ &\quad \cdot\left[\left(y_j(t) - y'_j(t)\right)^2 + \left(y_j\left(t - \tau_{kj}(t)\right)\right.\right. \\ &\quad \left.\left. - y'_j\left(t - \tau_{kj}(t)\right)\right)^2\right].\end{aligned} \tag{8}$$

Let $\boldsymbol{W}^{RR} = ((\omega_{kj}^{RR})^2)_{n\times n}$, $\boldsymbol{W}^{RI} = ((\omega_{kj}^{RI})^2)_{n\times n}$, $\boldsymbol{W}^{IR} = ((\omega_{kj}^{IR})^2)_{n\times n}$, $\boldsymbol{W}^{II} = ((\omega_{kj}^{II})^2)_{n\times n}$.

Next, we will separate (1) into its real part and imaginary part as follows:

$$\begin{aligned}\mathrm{d}x_k(t) = &\left\{-d_k x_k(t) + \sum_{j=1}^n\left[a_{kj}^R f_j^R\left(x_j(t), y_j(t)\right) - a_{kj}^I f_j^I\left(x_j(t), y_j(t)\right)\right]\right. \\ &+ \sum_{j=1}^n\left[b_{kj}^R f_j^R\left(x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right) - b_{kj}^I f_j^I\left(x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right)\right] \\ &\left. + \sum_{j=1}^n\int_{-\infty}^t \theta_{kj}(t-s)\left[p_{kj}^R f_j^R\left(x_j(s), y_j(s)\right) - p_{kj}^I f_j^I\left(x_j(s), y_j(s)\right)\right]\mathrm{d}s + J_k^R\right\}\mathrm{d}t \\ &+ \sum_{j=1}^n \sigma_{kj}^R\left(x_j(t), y_j(t), x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right)\mathrm{d}\omega_j(t), \\ \mathrm{d}y_k(t) = &\left\{-d_k y_k(t) + \sum_{j=1}^n\left[a_{kj}^R f_j^I\left(x_j(t), y_j(t)\right) + a_{kj}^I f_j^R\left(x_j(t), y_j(t)\right)\right]\right. \\ &+ \sum_{j=1}^n\left[b_{kj}^R f_j^I\left(x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right) + b_{kj}^I f_j^R\left(x_j\left(t - \tau_{kj}(t)\right), y_j\left(t - \tau_{kj}(t)\right)\right)\right]\end{aligned} \tag{9}$$

$$+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\left[p_{kj}^{R}f_{j}^{I}\left(x_{j}(s),y_{j}(s)\right)+p_{kj}^{I}f_{j}^{R}\left(x_{j}(s),y_{j}(s)\right)\right]\mathrm{d}s$$

$$\left.+J_{k}^{I}\right\}\mathrm{d}t\sum_{j=1}^{n}\sigma_{kj}^{I}\left(x_{j}(t),y_{j}(t),x_{j}\left(t-\tau_{kj}(t)\right),y_{j}\left(t-\tau_{kj}(t)\right)\right)\mathrm{d}w_{j}(t), \tag{10}$$

where $k = 1, 2, \ldots\ldots, n$. Let $\boldsymbol{A}^R = (a_{kj}^R)_{n\times n}$ and $\boldsymbol{A}^I = (a_{kj}^I)_{n\times n}$, $\boldsymbol{B}^R = (b_{kj}^R)_{n\times n}$ and $\boldsymbol{B}^I = (b_{kj}^I)_{n\times n}$, $\boldsymbol{P}^R = (p_{kj}^R)_{n\times n}$ and $\boldsymbol{P}^I = (p_{kj}^I)_{n\times n}$ be the real part and imaginary part of matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{P}$, respectively. Let $\boldsymbol{J}^R = (J_1^R, J_2^R, \ldots\ldots, J_n^R)^{\mathrm{T}}$ and $\boldsymbol{J}^I = (J_1^I, J_2^I, \ldots\ldots, J_n^I)^{\mathrm{T}}$ be the real part and imaginary part of external input $\boldsymbol{J}$.

For a complex number vector $\boldsymbol{z} \in \mathbf{C}^n$, let $|\boldsymbol{z}| = (|z_1|, |z_2|, \ldots\ldots, |z_n|)^{\mathrm{T}}$ and $\|\boldsymbol{z}\| = \sqrt{\sum_{k=1}^{n}|z_k|^2}$.

Definition 6. The equilibrium point $\boldsymbol{z}^{\#} = (z_1^{\#}, z_2^{\#}, \ldots\ldots, z_n^{\#})^{\mathrm{T}}$ of (1) is said to be exponentially stable in the mean square sense if, for all $\boldsymbol{J} \in \mathbf{C}^n$, there exist constants $\lambda > 0$ and $\Gamma > 0$ such that

$$\mathrm{E}\left(\left\|\boldsymbol{z}(t)-\boldsymbol{z}^{\#}\right\|^{2}\right) \leq \Gamma \sup_{s\in(-\infty,0]}\mathrm{E}\left(\left\|\boldsymbol{\varphi}(s)-\boldsymbol{z}^{\#}\right\|^{2}\right)\exp(-\lambda t), \quad t \geq 0, \tag{11}$$

where

$$\begin{aligned}\mathrm{E}\left(\left\|\boldsymbol{z}(t)-\boldsymbol{z}^{\#}\right\|^{2}\right) &= \left[\mathrm{E}\left(\left|z_1(t)-z_1^{\#}\right|^{2}\right),\right.\\ &\quad \left.\mathrm{E}\left(\left|z_2(t)-z_2^{\#}\right|^{2}\right),\ldots,\mathrm{E}\left(\left|z_n(t)-z_n^{\#}\right|^{2}\right)\right]^{\mathrm{T}};\\ \mathrm{E}\left(\left\|\boldsymbol{\varphi}(s)-\boldsymbol{z}^{\#}\right\|^{2}\right) &= \left[\mathrm{E}\left(\left|\varphi_1(s)-z_1^{\#}\right|^{2}\right),\right.\\ &\quad \left.\mathrm{E}\left(\left|\varphi_2(s)-z_2^{\#}\right|^{2}\right),\ldots,\mathrm{E}\left(\left|\varphi_n(s)-z_n^{\#}\right|^{2}\right)\right]^{\mathrm{T}}.\end{aligned} \tag{12}$$

Lemma 7 (see [8]). *Let $\boldsymbol{A} = (a_{kj})_{n\times n}$ be a real number matrix with $a_{kj} \leq 0$, $(k \neq j)$. The following statements are equivalent:*

(i) $\boldsymbol{A}$ is an M-matrix.

(ii) The real parts of all eigenvalues of $\boldsymbol{A}$ are positive.

(iii) There exists a positive vector $\boldsymbol{\varsigma} \in \mathbf{R}^n$ such that $\boldsymbol{A\varsigma} > \boldsymbol{0}$.

Lemma 8 (see [39]). *Let $y(t)$ be an Itô process given by*

$$\mathrm{d}y(t) = u\,\mathrm{d}t + v\,\mathrm{d}w(t). \tag{13}$$

Let $V(t, y) : [0, \infty) \times \mathbf{R} \longrightarrow \mathbf{R}$ be again an Itô process with second derivative with respect to y; then

$$\mathrm{d}V(t,y) = \frac{\partial V}{\partial t}\mathrm{d}t + \frac{\partial V}{\partial y}\mathrm{d}y + \frac{1}{2}\frac{\partial^2 V}{\partial y^2}(\mathrm{d}y)^2, \tag{14}$$

where $(\mathrm{d}y)^2 = (\mathrm{d}y)(\mathrm{d}y)$ is computed according to the rules $\mathrm{d}t.\mathrm{d}t=\mathrm{d}t.\mathrm{d}w(t)=\mathrm{d}w(t).\mathrm{d}t=0$ and $\mathrm{d}w(t).\mathrm{d}w(t)=\mathrm{d}t$.

Remark 9. By substituting (13) into (14) and considering the computation rule in Lemma 8, we can obtain the following equation, which will be used in the process of the proof for theorems follow-up.

$$\mathrm{d}V(t,y) = \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial y}u + \frac{1}{2}\frac{\partial^2 V}{\partial y^2}v^2\right)\mathrm{d}t + \frac{\partial V}{\partial y}v\,\mathrm{d}w(t). \tag{15}$$

3. Main Results on Mean Square Exponential Stability

For simplification, let $\boldsymbol{x} = [x_1, x_2, \ldots\ldots, x_n]^{\mathrm{T}}$, $\boldsymbol{y} = [y_1, y_2, \ldots\ldots, y_n]^{\mathrm{T}}$, $\boldsymbol{\alpha} = [\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{y}^{\mathrm{T}}]^{\mathrm{T}}$.

In what follows, we will study the mean square exponential stability of the equilibrium point of (1). For analysis convenience, we translate the coordinate of (1). Let $\tilde{z}_k = z_k - z_k^{\#} = \tilde{x}_k + \mathrm{i}\tilde{y}_k$, $k = 1, 2, \ldots\ldots, n$.

By the translation, (1) can be changed as follows, where $k, j = 1, 2, \ldots\ldots, n$,

$$\begin{aligned}\mathrm{d}\tilde{x}_k(t) &= \left\{-d_k\tilde{x}_k(t) + \sum_{j=1}^{n}\left[a_{kj}^{R}g_j^{R}\left(\tilde{x}_j(t),\right.\right.\right.\\ &\quad \left.\tilde{y}_j(t)\right) - a_{kj}^{I}g_j^{I}\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\Big] + \sum_{j=1}^{n}\left[b_{kj}^{R}g_j^{R}\left(\tilde{x}_j\left(t\right.\right.\right.\\ &\quad \left.-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\Big) - b_{kj}^{I}g_j^{I}\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\\ &\quad \left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\Big] + \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\left[p_{kj}^{R}g_j^{R}\left(\tilde{x}_j(s),\right.\right.\\ &\quad \left.\left.\left.\tilde{y}_j(s)\right) - p_{kj}^{I}g_j^{I}\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right]\mathrm{d}s\right\}\mathrm{d}t\\ &\quad + \sum_{j=1}^{n}\tilde{\sigma}_{kj}^{R}\left(\tilde{x}_j(t),\right.\\ &\quad \left.\tilde{y}_j(t),\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\mathrm{d}w_j(t),\end{aligned} \tag{16}$$

$$
\begin{aligned}
&\mathrm{d}\tilde{y}_k(t) \\
&= \Bigg\{-d_k\tilde{y}_k(t) + \sum_{j=1}^{n}\Big[a_{kj}^{R}g_j^{I}\big(\tilde{x}_j(t), \\
&\tilde{y}_j(t)\big) + a_{kj}^{I}g_j^{R}\big(\tilde{x}_j(t), \tilde{y}_j(t)\big)\Big] + \sum_{j=1}^{n}\Big[b_{kj}^{R}g_j^{I}\big(\tilde{x}_j\big(t \\
&- \tau_{kj}(t)\big), \tilde{y}_j\big(t-\tau_{kj}(t)\big)\big) + b_{kj}^{I}g_j^{R}\big(\tilde{x}_j\big(t-\tau_{kj}(t)\big), \\
&\tilde{y}_j\big(t-\tau_{kj}(t)\big)\big)\Big] + \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\Big[p_{kj}^{R}g_j^{I}\big(\tilde{x}_j(s), \\
&\tilde{y}_j(s)\big) + p_{kj}^{I}g_j^{R}\big(\tilde{x}_j(s), \tilde{y}_j(s)\big)\Big]\,\mathrm{d}s\Bigg\}\,\mathrm{d}t \\
&+ \sum_{j=1}^{n}\tilde{\sigma}_{kj}^{I}\big(\tilde{x}_j(t), \\
&\tilde{y}_j(t), \tilde{x}_j\big(t-\tau_{kj}(t)\big), \tilde{y}_j\big(t-\tau_{kj}(t)\big)\big)\,\mathrm{d}w_j(t),
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
&\tilde{\boldsymbol{\alpha}} = \left((\tilde{\boldsymbol{x}})^{\mathrm{T}}, (\tilde{\boldsymbol{y}})^{\mathrm{T}}\right)^{\mathrm{T}}; \\
&g_j^{R}\left(\tilde{x}_j, \tilde{y}_j\right) = f_j^{R}\left(x_j, y_j\right) - f_j^{R}\left(x_j^{\#}, y_j^{\#}\right); \\
&g_j^{I}\left(\tilde{x}_j, \tilde{y}_j\right) = f_j^{I}\left(x_j, y_j\right) - f_j^{I}\left(x_j^{\#}, y_j^{\#}\right); \\
&\tilde{\sigma}_{kj}^{R}\left(\tilde{x}_j(t), \tilde{y}_j(t), \tilde{x}_j\left(t-\tau_{kj}(t)\right), \tilde{y}_j\left(t-\tau_{kj}(t)\right)\right) \\
&\quad = \sigma_{kj}^{R}\left(x_j(t), y_j(t), x_j\left(t-\tau_{kj}(t)\right), y_j\left(t-\tau_{kj}(t)\right)\right) \\
&\quad - \sigma_{kj}^{R}\left(x_j^{\#}(t), y_j^{\#}(t), x_j^{\#}\left(t-\tau_{kj}(t)\right), y_j^{\#}\left(t-\tau_{kj}(t)\right)\right); \\
&\tilde{\sigma}_{kj}^{I}\left(\tilde{x}_j(t), \tilde{y}_j(t), \tilde{x}_j\left(t-\tau_{kj}(t)\right), \tilde{y}_j\left(t-\tau_{kj}(t)\right)\right) \\
&\quad = \sigma_{kj}^{I}\left(x_j(t), y_j(t), x_j\left(t-\tau_{kj}(t)\right), y_j\left(t-\tau_{kj}(t)\right)\right) \\
&\quad - \sigma_{kj}^{I}\left(x_j^{\#}(t), y_j^{\#}(t), x_j^{\#}\left(t-\tau_{kj}(t)\right), y_j^{\#}\left(t-\tau_{kj}(t)\right)\right).
\end{aligned}
\tag{18}
$$

Moreover, the initial conditions of (1) are of the forms $\psi_k(s) = \varphi_k(s) - z_k^{\#}$, $k = 1, 2, \ldots\ldots, n$.

Obviously, if the zero solution of the system being composed of (16) and (17) is exponentially stable in the mean square sense, then the equilibrium point of the system being composed of (9) and (10) is also exponentially stable in the mean square sense.

Next, we will give some sufficient conditions for assuring the mean square exponential stability of the equilibrium point $\boldsymbol{z}^{\#}$ of (1). First of all, we let

$$
\begin{aligned}
&\widetilde{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D} \end{bmatrix}, \\
&\widetilde{\boldsymbol{L}} = \begin{bmatrix} \boldsymbol{L}^{RR} + \boldsymbol{L}^{RI} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L}^{IR} + \boldsymbol{L}^{II} \end{bmatrix}, \\
&\widehat{\boldsymbol{L}} = \begin{bmatrix} \boldsymbol{L}^{RR} & \boldsymbol{L}^{RI} \\ \boldsymbol{L}^{IR} & \boldsymbol{L}^{II} \end{bmatrix}, \\
&\boldsymbol{W} = \begin{bmatrix} \boldsymbol{W}^{RR} & \boldsymbol{W}^{RI} \\ \boldsymbol{W}^{IR} & \boldsymbol{W}^{II} \end{bmatrix}, \\
&\widetilde{\boldsymbol{Q}} = \begin{bmatrix} \left|\boldsymbol{A}^{R}\right| + \left|\boldsymbol{B}^{R}\right| + \left|\boldsymbol{P}^{R}\right| & \boldsymbol{0} \\ \boldsymbol{0} & \left|\boldsymbol{A}^{I}\right| + \left|\boldsymbol{B}^{I}\right| + \left|\boldsymbol{P}^{I}\right| \end{bmatrix}, \\
&\widehat{\boldsymbol{Q}} = \begin{bmatrix} \left|\boldsymbol{A}^{R}\right| + \left|\boldsymbol{B}^{R}\right| + \left|\boldsymbol{P}^{R}\right| & \left|\boldsymbol{A}^{I}\right| + \left|\boldsymbol{B}^{I}\right| + \left|\boldsymbol{P}^{I}\right| \\ \left|\boldsymbol{A}^{I}\right| + \left|\boldsymbol{B}^{I}\right| + \left|\boldsymbol{P}^{I}\right| & \left|\boldsymbol{A}^{R}\right| + \left|\boldsymbol{B}^{R}\right| + \left|\boldsymbol{P}^{R}\right| \end{bmatrix}.
\end{aligned}
\tag{19}
$$

Theorem 10. *If Assumptions 1–5 are satisfied and the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is an M-matrix, then the equilibrium point $\boldsymbol{z}^{\#}$ of (1) is of mean square exponential stability for any external input $\boldsymbol{J} \in \mathbf{C}^n$.*

Proof. Because $2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is an M matrix, according to Lemma 7 there exists a positive vector $\boldsymbol{\Lambda} = (\varsigma_1, \varsigma_2, \ldots, \varsigma_{2n})$ such that the following inequalities hold, where $\varsigma_{k'} = \beta_k$ and $k = k'$, when $1 \le k' \le n$; $\varsigma_{k'} = \gamma_k$ and $k = k' - n$ when $n + 1 \le k' \le 2n$:

$$
\begin{aligned}
&\Bigg\{-2d_k + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right| + \left|b_{kj}^{R}\right| + \left|p_{kj}^{R}\right|\right)\left(l_j^{RR} + l_j^{RI}\right) \\
&\quad + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right| + \left|p_{kj}^{I}\right|\right)\left(l_j^{IR} + l_j^{II}\right)\Big]\Bigg\}\beta_k + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right|\right. \\
&\quad \left. + \left|b_{kj}^{R}\right| + \left|p_{kj}^{R}\right|\right)l_j^{RR} + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right| + \left|p_{kj}^{I}\right|\right)l_j^{IR} \\
&\quad + 2\left(\omega_{kj}^{RR}\right)^2\Big]\beta_j + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right| + \left|b_{kj}^{R}\right| + \left|p_{kj}^{R}\right|\right)l_j^{RI} \\
&\quad + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right| + \left|p_{kj}^{I}\right|\right)l_j^{II} + 2\left(\omega_{kj}^{RI}\right)^2\Big]\gamma_j < 0, \\
&\Bigg\{-2d_k + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right| + \left|b_{kj}^{R}\right| + \left|p_{kj}^{R}\right|\right)\left(l_j^{IR} + l_j^{II}\right) \\
&\quad + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right| + \left|p_{kj}^{I}\right|\right)\left(l_j^{RR} + l_j^{RI}\right)\Big]\Bigg\}\gamma_k \\
&\quad + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right| + \left|b_{kj}^{R}\right| + \left|p_{kj}^{R}\right|\right)l_j^{IR} + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right|\right. \\
&\quad \left. + \left|p_{kj}^{I}\right|\right)l_j^{RR} + 2\left(\omega_{kj}^{IR}\right)^2\Big]\beta_j + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right| + \left|b_{kj}^{R}\right|\right. \\
&\quad \left. + \left|p_{kj}^{R}\right|\right)l_j^{II} + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right| + \left|p_{kj}^{I}\right|\right)l_j^{RI} + 2\left(\omega_{kj}^{II}\right)^2\Big]\gamma_j \\
&\quad < 0.
\end{aligned}
\tag{20}
$$

Construct the following functions:

$$\begin{aligned}
F_k^R(\varepsilon) = \Bigg\{ & -2d_k + \varepsilon \\
& + \sum_{j=1}^{n} \Big[\left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right| \right) \left(l_j^{RR} + l_j^{RI} \right) \\
& + \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right| \right) \left(l_j^{IR} + l_j^{II} \right) \Big] \Bigg\} \beta_k + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| \\
& + \left|b_{kj}^R\right| \exp(\varepsilon\tau) + \left|p_{kj}^R\right| \mu_{kj}(\varepsilon) \Big) l_j^{RR} + \Big(\left|a_{kj}^I\right| \\
& + \left|b_{kj}^I\right| \exp(\varepsilon\tau) + \left|p_{kj}^I\right| \mu_{kj}(\varepsilon) \Big) l_j^{IR} + \left(\omega_{kj}^{RR}\right)^2 \\
& + \exp(\varepsilon\tau) \left(\omega_{kj}^{RR}\right)^2 \Big] \beta_j + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| \exp(\varepsilon\tau) \\
& + \left|p_{kj}^R\right| \mu_{kj}(\varepsilon) \Big) l_j^{RI} + \Big(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| \exp(\varepsilon\tau) \\
& + \left|p_{kj}^I\right| \mu_{kj}(\varepsilon) \Big) l_j^{II} + \exp(\varepsilon\tau) \left(\omega_{kj}^{RI}\right)^2 + \left(\omega_{kj}^{RI}\right)^2 \Big] \gamma_j, \\
F_k^I(\varepsilon) = \Bigg\{ & -2d_k + \varepsilon \\
& + \sum_{j=1}^{n} \Big[\left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right| \right) \left(l_j^{IR} + l_j^{II} \right) \\
& + \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right| \right) \left(l_j^{RR} + l_j^{RI} \right) \Big] \Bigg\} \gamma_k + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| \\
& + \left|b_{kj}^R\right| \exp(\varepsilon\tau) + \left|p_{kj}^R\right| \mu_{kj}(\varepsilon) \Big) l_j^{IR} + \Big(\left|a_{kj}^I\right| \\
& + \left|b_{kj}^I\right| \exp(\varepsilon\tau) + \left|p_{kj}^I\right| \mu_{kj}(\varepsilon) \Big) l_j^{RR} + \exp(\varepsilon\tau) \left(\omega_{kj}^{IR}\right)^2 \\
& + \left(\omega_{kj}^{IR}\right)^2 \Big] \beta_j + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| \exp(\varepsilon\tau) \\
& + \left|p_{kj}^R\right| \mu_{kj}(\varepsilon) \Big) l_j^{II} + \Big(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| \exp(\varepsilon\tau) \\
& + \left|p_{kj}^I\right| \mu_{kj}(\varepsilon) \Big) l_j^{RI} + \exp(\varepsilon\tau) \left(\omega_{kj}^{II}\right)^2 + \left(\omega_{kj}^{II}\right)^2 \Big] \gamma_j.
\end{aligned} \tag{21}$$

Because $F_k^R(\varepsilon)$ and $F_k^I(\varepsilon)$ are continuous with respect to ε, and $F_k^R(0) < 0$, $F_k^I(0) < 0$, there exist constants $\lambda_1 > 0$ and $\lambda_2 > 0$, such that $F_k^R(\lambda_1) < 0$ and $F_k^I(\lambda_2) < 0$, $k = 1, 2, \ldots, n$. Obviously, there exists a positive constant $\lambda > 0$ satisfying $\lambda \le \min\{\lambda_1, \lambda_2\}$ such that $F_k^R(\lambda) < 0$ and $F_k^I(\lambda) < 0$ hold; that is,

$$\begin{aligned}
F_k^R(\lambda) = \Bigg\{ & -2d_k + \lambda \\
& + \sum_{j=1}^{n} \Big[\left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right| \right) \left(l_j^{RR} + l_j^{RI} \right) \\
& + \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right| \right) \left(l_j^{IR} + l_j^{II} \right) \Big] \Bigg\} \beta_k + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| \\
& + \left|b_{kj}^R\right| \exp(\lambda\tau) + \left|p_{kj}^R\right| \mu_{kj}(\lambda) \Big) l_j^{RR} + \Big(\left|a_{kj}^I\right| \\
& + \left|b_{kj}^I\right| \exp(\lambda\tau) + \left|p_{kj}^I\right| \mu_{kj}(\lambda) \Big) l_j^{IR} + \left(\omega_{kj}^{RR}\right)^2 \\
& + \exp(\lambda\tau) \left(\omega_{kj}^{RR}\right)^2 \Big] \beta_j + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| \exp(\lambda\tau) \\
& + \left|p_{kj}^R\right| \mu_{kj}(\lambda) \Big) l_j^{RI} + \Big(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| \exp(\lambda\tau) \\
& + \left|p_{kj}^I\right| \mu_{kj}(\lambda) \Big) l_j^{II} + \exp(\lambda\tau) \left(\omega_{kj}^{RI}\right)^2 + \left(\omega_{kj}^{RI}\right)^2 \Big] \gamma_j \\
& < 0,
\end{aligned} \tag{22}$$

$$\begin{aligned}
F_k^I(\lambda) = \Bigg\{ & -2d_k + \lambda \\
& + \sum_{j=1}^{n} \Big[\left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right| \right) \left(l_j^{IR} + l_j^{II} \right) \\
& + \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right| \right) \left(l_j^{RR} + l_j^{RI} \right) \Big] \Bigg\} \gamma_k \\
& + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| \exp(\lambda\tau) + \left|p_{kj}^R\right| \mu_{kj}(\lambda) \Big) l_j^{IR} \\
& + \Big(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| \exp(\lambda\tau) + \left|p_{kj}^I\right| \mu_{kj}(\lambda) \Big) l_j^{RR} \\
& + \exp(\lambda\tau) \left(\omega_{kj}^{IR}\right)^2 + \left(\omega_{kj}^{IR}\right)^2 \Big] \beta_j + \sum_{j=1}^{n} \Big[\Big(\left|a_{kj}^R\right| \\
& + \left|b_{kj}^R\right| \exp(\lambda\tau) + \left|p_{kj}^R\right| \mu_{kj}(\lambda) \Big) l_j^{II} + \Big(\left|a_{kj}^I\right| \\
& + \left|b_{kj}^I\right| \exp(\lambda\tau) + \left|p_{kj}^I\right| \mu_{kj}(\lambda) \Big) l_j^{RI} + \exp(\lambda\tau) \\
& \cdot \left(\omega_{kj}^{II}\right)^2 + \left(\omega_{kj}^{II}\right)^2 \Big] \gamma_j < 0.
\end{aligned} \tag{23}$$

Choose a candidate vector Lyapunov function in what follows:

$$V_{k'}\left(t, \tilde{\alpha}_{k'}(t)\right) = \frac{1}{2} \exp(\lambda t) \left|\tilde{\alpha}_{k'}(t)\right|^2, \quad k' = 1, 2, \ldots, 2n; \tag{24}$$

i.e.,

$$\begin{aligned}
V_{k'}\left(t, \tilde{\alpha}_{k'}(t)\right) &= \frac{1}{2} \exp(\lambda t) \left(\tilde{x}_{k'}(t)\right)^2 \\
&= \frac{1}{2} \exp(\lambda t) \left(\tilde{x}_k(t)\right)^2, \\
& \qquad 1 \le k' \le n, \ k' = k
\end{aligned}$$

$$V_{k'}\left(t,\tilde{\alpha}_{k'}(t)\right)=\frac{1}{2}\exp(\lambda t)\left(\tilde{y}_{k'-n}(t)\right)^2=\frac{1}{2}\exp(\lambda t)\left(\tilde{y}_k(t)\right)^2,\quad n+1\le k'\le 2n,\ k=k'-n, \tag{25}$$

where $k=1,2,\ldots\ldots,n$.

Let $V_{k'}(t)$ denote $V_{k'}(t,\tilde{\alpha}_{k'}(t))$ if there is no confusion, $k'=1,2,\ldots\ldots,2n$.

(i) When $1\le k'\le n$, according to $It\hat{o}$ formula (15) and considering the real parts (16) of (1), we get

$$\begin{aligned}
\mathrm{d}V_{k'}\left(t,\tilde{\alpha}_{k'}(t)\right)&=\mathrm{d}\left(\frac{1}{2}\exp(\lambda t)\left(\tilde{x}_k(t)\right)^2\right)\\
&=\left\{\frac{1}{2}\lambda\exp(\lambda t)\left(\tilde{x}_k(t)\right)^2+\exp(\lambda t)\tilde{x}_k(t)\right.\\
&\cdot\left\{-d_k\tilde{x}_k(t)+\sum_{j=1}^{n}\left[a_{kj}^R g_j^R\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\right.\right.\\
&-a_{kj}^I g_j^I\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\Big]+\sum_{j=1}^{n}\Big[b_{kj}^R g_j^R\\
&\cdot\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\\
&-b_{kj}^I g_j^I\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\Big]\\
&+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\left[p_{kj}^R g_j^R\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right.\\
&\left.\left.-p_{kj}^I g_j^I\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right]\mathrm{d}s\right\}+\frac{1}{2}\exp(\lambda t)\sum_{j=1}^{n}\left(\tilde{\sigma}_{kj}^R\right.\\
&\cdot\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\\
&\left.\left.\left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right)^2\right\}\mathrm{d}t+\exp(\lambda t)\tilde{x}_k(t)\sum_{j=1}^{n}\left[\tilde{\sigma}_{kj}^R\right.\\
&\cdot\left(\tilde{x}_j(t),\tilde{y}_j(t),\ \tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\\
&\left.\left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right]\mathrm{d}w_j(t).
\end{aligned} \tag{26}$$

Next, we will transform $It\hat{o}$ differential form of (26) into $It\hat{o}$ integral form. By using stochastic differential equation theory [39], we can integrate on both sides of (26) from t to $t+\delta$ for any $\delta>0$ and compute the expectation of (26):

$$\begin{aligned}
&\mathrm{E}\left(V_{k'}(t+\delta)\right)-\mathrm{E}\left(V_{k'}(t)\right)\\
&=\mathrm{E}\left(\int_t^{t+\delta}\left\{\frac{1}{2}\lambda\exp(\lambda t)\left(\tilde{x}_k(t)\right)^2+\exp(\lambda t)\right.\right.\\
&\cdot\tilde{x}_k(t)\left\{-d_k\tilde{x}_k(t)+\sum_{j=1}^{n}\left[a_{kj}^R g_j^R\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\right.\right.\\
&-a_{kj}^I g_j^I\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\Big]+\sum_{j=1}^{n}\Big[b_{kj}^R g_j^R\\
&\cdot\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\\
&-b_{kj}^I g_j^I\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\Big]\\
&+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\left[p_{kj}^R g_j^R\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right.\\
&\left.\left.-p_{kj}^I g_j^I\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right]\mathrm{d}s\right\}+\frac{1}{2}\exp(\lambda t)\sum_{j=1}^{n}\left(\tilde{\sigma}_{kj}^R\right.\\
&\cdot\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\right.\\
&\left.\left.\left.\cdot\left(t-\tau_{kj}(t)\right),\ \tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right)^2\right\}\mathrm{d}t\right)\\
&+\mathrm{E}\left(\int_t^{t+\delta}\exp(\lambda t)\tilde{x}_k(t)\sum_{j=1}^{n}\left[\tilde{\sigma}_{kj}^R\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\right.\right.\right.\\
&\left.\left.\left.\cdot\left(t-\tau_{kj}(t)\right),\ \tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right]\mathrm{d}w_j(t)\right).
\end{aligned} \tag{27}$$

It is well known that the expectation of $It\hat{o}$ process is a continuous function. According to the properties of $It\hat{o}$ integral [39] and Dini derivation, (27) can be deduced as follows:

$$\begin{aligned}
&\mathrm{D}^+\mathrm{E}\left(V_{k'}(t)\right)\\
&=\mathrm{E}\left(\frac{1}{2}\right.\\
&\cdot\lambda\exp(\lambda t)\left(\tilde{x}_k(t)\right)^2+\exp(\lambda t)\tilde{x}_k(t)\left\{-d_k\tilde{x}_k(t)\right.\\
&+\sum_{j=1}^{n}\left[a_{kj}^R g_j^R\left(\tilde{x}_j(t),\right.\right.\\
&\left.\tilde{y}_j(t)\right)-a_{kj}^I g_j^I\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\Big]\\
&+\sum_{j=1}^{n}\Big[b_{kj}^R g_j^R\left(\tilde{x}_j\left(t\right.\right.\\
&\left.-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\Big)-b_{kj}^I g_j^I\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\\
&\left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\Big]
\end{aligned}$$

$$+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\left[p_{kj}^{R}g_{j}^{R}\left(\tilde{x}_{j}(s),\tilde{y}_{j}(s)\right)-p_{kj}^{I}g_{j}^{I}\left(\tilde{x}_{j}(s),\tilde{y}_{j}(s)\right)\right]\mathrm{d}s\Bigg\}$$
$$+\frac{1}{2}\exp(\lambda t)\sum_{j=1}^{n}\left(\tilde{\sigma}_{kj}^{R}\left(\tilde{x}_{j}(t),\tilde{y}_{j}(t),\tilde{x}_{j}\left(t-\tau_{kj}(t)\right),\tilde{y}_{j}\left(t-\tau_{kj}(t)\right)\right)\right)^{2}\Bigg). \tag{28}$$

Considering Assumptions 1–5, (28) can be deduced furthermore as follows:

$$\begin{aligned}
\mathrm{D}^{+}\mathrm{E}\left(V_{k'}(t)\right)\leq{}&\Bigg\{-2d_{k}+\lambda\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|+\left|b_{kj}^{R}\right|+\left|p_{kj}^{R}\right|\right)\left(l_{j}^{RR}+l_{j}^{RI}\right)\right.\\
&\left.+\left(\left|a_{kj}^{I}\right|+\left|b_{kj}^{I}\right|+\left|p_{kj}^{I}\right|\right)\left(l_{j}^{IR}+l_{j}^{II}\right)\right]\Bigg\}\mathrm{E}\left(V_{k}(t)\right)\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_{j}^{RR}+\left|a_{kj}^{I}\right|l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right]\mathrm{E}\left(V_{j}(t)\right)\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_{j}^{RI}+\left|a_{kj}^{I}\right|l_{j}^{II}\right)+\left(\omega_{kj}^{RI}\right)^{2}\right]\mathrm{E}\left(V_{j+n}(t)\right)\\
&+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right|l_{j}^{RR}+\left|b_{kj}^{I}\right|l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right]\exp(\lambda\tau)\\
&\cdot\mathrm{E}\left(V_{j}\left(t-\tau_{kj}(t)\right)\right)+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right|l_{j}^{RI}+\left|b_{kj}^{I}\right|l_{j}^{II}\right)\right.\\
&\left.+\left(\omega_{kj}^{RI}\right)^{2}\right]\exp(\lambda\tau)\mathrm{E}\left(V_{j+n}\left(t-\tau_{kj}(t)\right)\right)\\
&+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\exp(\lambda(t-s))\left(\left|p_{kj}^{R}\right|l_{j}^{RR}\right.\\
&\left.+\left|p_{kj}^{I}\right|l_{j}^{IR}\right)\mathrm{E}\left(V_{j}(s)\right)\mathrm{d}s+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\\
&\cdot\exp(\lambda(t-s))\left(\left|p_{kj}^{R}\right|l_{j}^{RI}+\left|p_{kj}^{I}\right|l_{j}^{II}\right)\\
&\cdot\mathrm{E}\left(V_{j+n}(s)\right)\mathrm{d}s,\quad k=1,2,\ldots,n.
\end{aligned}\tag{29}$$

The detailed deducing of (29) can be referred in the Appendix part of this paper.

Define the curve

$$\varsigma=\left\{\boldsymbol{\eta}(\chi):\eta_{k'}=\varsigma_{k'}\chi,\ \chi>0,\ k'=1,2,\ldots,2n\right\}\tag{30}$$

and the sets

$$\Omega(\boldsymbol{\eta})=\left\{\boldsymbol{h}:\mathbf{0}\leq\boldsymbol{h}\leq\boldsymbol{\eta},\ \boldsymbol{\eta}\in\boldsymbol{\varsigma}\right\}.\tag{31}$$

It is obvious that we have $\Omega(\boldsymbol{\eta}(\chi))\supset\Omega(\boldsymbol{\eta}(\chi'))$ as $\chi>\chi'$.

In what follows, we will prove the mean square exponential stability of (16). That is, there exist constants $\Gamma>0$ and $\lambda>0$ such that $|\tilde{\boldsymbol{x}}(t)|\leq\Gamma\|\boldsymbol{\psi}\|\exp(-\lambda t)$ hold, where $\|\boldsymbol{\psi}\|=\sup_{s\in(-\infty,0]}\|\boldsymbol{\psi}(s)\|$, $t\geq0$.

Let $\overline{\pi}=\max_{1\leq k\leq n}\{\beta_k,\gamma_k\}$, $\underline{\pi}=\min_{1\leq k\leq n}\{\beta_k,\gamma_k\}$, and $\chi_0=\delta\|\boldsymbol{\psi}\|/\underline{\pi}$, where $\delta>1$ is a constant; then

$$\left\{\mathrm{E}(\boldsymbol{V}(s)):\mathrm{E}(\boldsymbol{V}(s))=\frac{1}{2}\exp(\lambda s)\mathrm{E}\left((\tilde{\boldsymbol{\alpha}}(s))^{2}\right),\ -\infty<s\leq0\right\}\subset\Omega\left(\boldsymbol{\eta}_{0}(\chi_{0})\right);\tag{32}$$

i.e., when $-\infty<s\leq0$, we have $\mathrm{E}(V_j(s))=(1/2)\exp(\lambda s)\mathrm{E}((x_j(s))^2)<\beta_j\chi_0$, $\mathrm{E}(V_{j+n}(s))=(1/2)\exp(\lambda s)\mathrm{E}((\tilde{y}_j(s))^2)<\gamma_j\chi_0$, $j=1,2,\ldots\ldots,n$.

Furthermore, it can be obtained that $\mathrm{E}(V_j(t))<\beta_j\chi_0$, $\mathrm{E}(V_{j+n}(t))<\gamma_j\chi_0$, $j=1,2,\ldots\ldots,n$. If they are not true, then there exist some k and $t_1>0$ such that $\mathrm{E}(V_{k'}(t_1))=\beta_k\chi_0$, $\mathrm{D}^{+}\mathrm{E}(V_{k'}(t_1))\geq0$; $\mathrm{E}(V_j(t_1))<\beta_j\chi_0$, $\mathrm{E}(V_{j+n}(t_1))\leq\gamma_j\chi_0$, $j\neq k'$, $j=1,2,\ldots\ldots,n$. Substituting them into (29) and considering (22), we have

$$\begin{aligned}
\mathrm{D}^{+}\mathrm{E}\left(V_{k'}(t_1)\right)\leq{}&\Bigg\{-2d_{k}+\lambda\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|+\left|b_{kj}^{R}\right|+\left|p_{kj}^{R}\right|\right)\left(l_{j}^{RR}+l_{j}^{RI}\right)\right.\\
&\left.+\left(\left|a_{kj}^{I}\right|+\left|b_{kj}^{I}\right|+\left|p_{kj}^{I}\right|\right)\left(l_{j}^{IR}+l_{j}^{II}\right)\right]\Bigg\}\mathrm{E}\left(V_{k}(t_1)\right)\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_{j}^{RR}+\left|a_{kj}^{I}\right|l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right]\mathrm{E}\left(V_{j}(t_1)\right)\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_{j}^{RI}+\left|a_{kj}^{I}\right|l_{j}^{II}\right)+\left(\omega_{kj}^{RI}\right)^{2}\right]\mathrm{E}\left(V_{j+n}(t_1)\right)\\
&+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right|l_{j}^{RR}+\left|b_{kj}^{I}\right|l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right]\exp(\lambda\tau)\\
&\cdot\mathrm{E}\left(V_{j}\left(t_1-\tau_{kj}(t_1)\right)\right)+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right|l_{j}^{RI}+\left|b_{kj}^{I}\right|l_{j}^{II}\right)\right.\\
&\left.+\left(\omega_{kj}^{RI}\right)^{2}\right]\exp(\lambda\tau)\mathrm{E}\left(V_{j+n}\left(t_1-\tau_{kj}(t_1)\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\int_{-\infty}^{t_1}\theta_{kj}(t_1-s)\exp(\lambda(t_1-s))\left(\left|p_{kj}^{R}\right|l_j^{RR}\right.\\
&\left.+\left|p_{kj}^{I}\right|l_j^{IR}\right)\mathrm{E}\left(V_j(s)\right)\mathrm{d}s+\sum_{j=1}^{n}\int_{-\infty}^{t_1}\theta_{kj}(t_1-s)\\
&\cdot\exp(\lambda(t_1-s))\left(\left|p_{kj}^{R}\right|l_j^{RI}+\left|p_{kj}^{I}\right|l_j^{II}\right)\\
&\cdot\mathrm{E}\left(V_{j+n}(s)\right)\mathrm{d}s\le\left\{-2d_k+\lambda\right.\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|+\left|b_{kj}^{R}\right|+\left|p_{kj}^{R}\right|\right)\left(l_j^{RR}+l_j^{RI}\right)\right.\\
&\left.\left.+\left(\left|a_{kj}^{I}\right|+\left|b_{kj}^{I}\right|+\left|p_{kj}^{I}\right|\right)\left(l_j^{IR}+l_j^{II}\right)\right]\right\}\beta_k\chi_0\\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_j^{RR}+\left|a_{kj}^{I}\right|l_j^{IR}\right)+\left(\omega_{kj}^{RR}\right)^2+\exp(\lambda\tau)\right.\\
&\cdot\left[\left(\left|b_{kj}^{R}\right|l_j^{RR}+\left|b_{kj}^{I}\right|l_j^{IR}\right)+\left(\omega_{kj}^{RR}\right)^2\right]+\left(\left|p_{kj}^{R}\right|l_j^{RR}\right.\\
&\left.\left.+\left|p_{kj}^{I}\right|l_j^{IR}\right)\mu_{kj}(\lambda)\right]\beta_j\chi_0+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_j^{RI}+\left|a_{kj}^{I}\right|l_j^{II}\right.\right.\\
&\left.+\left(\omega_{kj}^{RI}\right)^2\right)+\exp(\lambda\tau)\left[\left(\left|b_{kj}^{R}\right|l_j^{RI}+\left|b_{kj}^{I}\right|l_j^{II}\right)\right.\\
&\left.\left.+\left(\omega_{kj}^{RI}\right)^2\right]+\left(\left|p_{kj}^{R}\right|l_j^{RI}+\left|p_{kj}^{I}\right|l_j^{II}\right)\mu_{kj}(\lambda)\right]\gamma_j\chi_0<0.
\end{aligned}
\tag{33}
$$

This is a contradiction with the assumption $\mathrm{D}^{+}\mathrm{E}(V_{k'}(t_1))\ge 0$; i.e., for all $t\ge 0$, we have $\mathrm{E}(V_k(t))<\beta_k\chi_0$, $\mathrm{E}(V_{k+n}(t))<\gamma_k\chi_0$, $k=1,2,\ldots\ldots,n$. That is, $\mathrm{E}(V_{k'}(t))<\varsigma_{k'}\chi_0$, $k'=1,2,\ldots\ldots,n$.

(ii) When $n+1\le k'\le 2n$, according to *Itô* formula (15) and considering the imaginary parts (17) of (1), we get

$$
\begin{aligned}
\mathrm{d}V_{k'}(t,\tilde{\alpha}_{k'}(t))=&\left\{\frac{1}{2}\lambda\exp(\lambda t)\left(\tilde{y}_k(t)\right)^2+\exp(\lambda t)\right.\\
&\cdot\tilde{y}_k(t)\left\{-d_k\tilde{y}_k(t)\right.\\
&+\sum_{j=1}^{n}\left[a_{kj}^{R}g_j^{I}\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)+a_{kj}^{I}g_j^{R}\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\right]\\
&+\sum_{j=1}^{n}\left[b_{kj}^{R}g_j^{I}\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right.\\
&\left.+b_{kj}^{I}g_j^{R}\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right]\\
&+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\left[p_{kj}^{R}g_j^{I}\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right.\\
&\left.\left.+p_{kj}^{I}g_j^{R}\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right]\mathrm{d}s\right\}+\frac{1}{2}\sum_{j=1}^{n}\left(\tilde{\sigma}_{kj}^{I}\right.\\
&\cdot\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\\
&\left.\left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right)^2\Big\}\mathrm{d}t+\exp(\lambda t)\,\tilde{y}_k(t)\sum_{j=1}^{n}\left[\tilde{\sigma}_{kj}^{I}\right.\\
&\cdot\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\\
&\left.\left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right]\mathrm{d}w_j(t).
\end{aligned}
\tag{34}
$$

Next, we will transform Itô differential form of (34) into *Itô* integral form. By using stochastic differential equation theory [39], we can integrate on both sides of (34) from t to $t+\delta$ for any $\delta>0$ and compute the expectation of (34), we get

$$
\begin{aligned}
&\mathrm{E}\left(V_{k'}(t+\delta)\right)-\mathrm{E}\left(V_{k'}(t)\right)\\
&=\mathrm{E}\left(\int_t^{t+\delta}\left\{\frac{1}{2}\lambda\exp(\lambda t)\left(\tilde{y}_k(t)\right)^2+\exp(\lambda t)\right.\right.\\
&\cdot\tilde{y}_k(t)\left\{-d_k\tilde{y}_k(t)+\sum_{j=1}^{n}\left[a_{kj}^{R}g_j^{I}\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)+a_{kj}^{I}g_j^{R}\left(\tilde{x}_j(t),\tilde{y}_j(t)\right)\right]+\sum_{j=1}^{n}\left[b_{kj}^{R}g_j^{I}\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)+b_{kj}^{I}g_j^{R}\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right]+\sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t\right.\\
&-s)\left[p_{kj}^{R}g_j^{I}\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right.\\
&\left.\left.\left.+p_{kj}^{I}g_j^{R}\left(\tilde{x}_j(s),\tilde{y}_j(s)\right)\right]\mathrm{d}s\right\}+\frac{1}{2}\sum_{j=1}^{n}\left(\tilde{\sigma}_{kj}^{I}\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\left(t-\tau_{kj}(t)\right),\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right)^2\right\}\mathrm{d}t\right)\\
&+\mathrm{E}\left(\int_t^{t+\delta}\exp(\lambda t)\right.\\
&\cdot\tilde{y}_k(t)\sum_{j=1}^{n}\left[\tilde{\sigma}_{kj}^{I}\left(\tilde{x}_j(t),\tilde{y}_j(t),\tilde{x}_j\left(t-\tau_{kj}(t)\right),\right.\right.\\
&\left.\left.\left.\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\right]\mathrm{d}w_j(t)\right).
\end{aligned}
\tag{35}
$$

Noticing that the expectation of $It\hat{o}$ process is a continuous function, according to the properties of $It\hat{o}$ integral and Dini derivation, (35) can be deduced for all $k = 1, 2, \ldots\ldots, n$, $k' = n+1, n+2, \ldots\ldots, 2n$ as follows:

$$\begin{aligned}
D^+E(V_{k'}(t)) = E\Bigg(&\frac{1}{2}\lambda\exp(\lambda t)(\tilde{y}_k(t))^2 + \exp(\lambda t) \\
&\cdot \tilde{y}_k(t)\Bigg\{-d_k\tilde{y}_k(t) + \sum_{j=1}^{n}\left[a_{kj}^R g_j^I\left(\tilde{x}_j(t), \tilde{y}_j(t)\right)\right. \\
&+ a_{kj}^I g_j^R\left(\tilde{x}_j(t), \tilde{y}_j(t)\right)\Big] + \sum_{j=1}^{n}\Big[b_{kj}^R g_j^I \\
&\cdot\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right), \tilde{y}_j\left(t-\tau_{kj}(t)\right)\right) \\
&+ b_{kj}^I g_j^R\left(\tilde{x}_j\left(t-\tau_{kj}(t)\right), \tilde{y}_j\left(t-\tau_{kj}(t)\right)\right)\Big] \\
&+ \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\Big[p_{kj}^R g_j^I\left(\tilde{x}_j(s), \tilde{y}_j(s)\right) + p_{kj}^I g_j^R \\
&\cdot\left(\tilde{x}_j(s), \tilde{y}_j(s)\right)\Big]\,ds\Bigg\} + \frac{1}{2} \\
&\cdot\sum_{j=1}^{n}\Big(\tilde{\sigma}_{kj}^I\left(\tilde{x}_j(t), \tilde{y}_j(t), \tilde{x}_j\left(t-\tau_{kj}(t)\right), \tilde{y}_j\right. \\
&\cdot\left.\left(t-\tau_{kj}(t)\right)\right)\Big)^2\Bigg).
\end{aligned} \tag{36}$$

Considering Assumptions 1–5, the following inequality can be obtained by adopting the similar analysis of (29):

$$\begin{aligned}
D^+E(V_{k'}(t)) \le \Bigg\{&-2d_k + \lambda \\
&+ \sum_{j=1}^{n}\left[\left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right|\right)\left(l_j^{IR} + l_j^{II}\right)\right. \\
&\left.+ \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right|\right)\left(l_j^{RR} + l_j^{RI}\right)\right]\Bigg\} E(V_{k'}(t)) \\
&+ \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^R\right| l_j^{IR} + \left|a_{kj}^I\right| l_j^{RR} + \left(\omega_{kj}^{IR}\right)^2\right)E(V_j(t)) \\
&+ \left(\left|a_{kj}^R\right| l_j^{II} + \left|a_{kj}^I\right| l_j^{RI} + \left(\omega_{kj}^{II}\right)^2\right)E(V_{j+n}(t))\Big] \\
&+ \sum_{j=1}^{n}\exp(\lambda\tau)\Big[\left(\left|b_{kj}^R\right| l_j^{IR} + \left|b_{kj}^I\right| l_j^{RR} + \left(\omega_{kj}^{IR}\right)^2\right) \\
&\cdot E\left(V_j\left(t-\tau_{kj}(t)\right)\right) + \left(\left|b_{kj}^R\right| l_j^{II} + \left|b_{kj}^I\right| l_j^{RI}\right. \\
&\left.+ \left(\omega_{kj}^{II}\right)^2\right)E\left(V_{j+n}\left(t-\tau_{kj}(t)\right)\right)\Big] + \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t \\
&- s)\exp(\lambda(t-s))\Big[\left(\left|p_{kj}^R\right| l_j^{IR} + \left|p_{kj}^I\right| l_j^{RR}\right)E(V_j(s)) \\
&+ \left(\left|p_{kj}^R\right| l_j^{II} + \left|p_{kj}^I\right| l_j^{RI}\right)E(V_{j+n}(s))\Big]\,ds.
\end{aligned} \tag{37}$$

By similar analysis with part (i) previously, we can conclude that $E(V_k(t)) < \beta_k\chi_0$ and $E(V_{k'}(t)) < \gamma_k\chi_0$, where $1 \le k \le n$, $n+1 \le k' \le 2n$, $t \ge 0$.

In what follows, we will prove the mean square exponential stability of system (17). That is to say, there exist constants $\Gamma > 0$ and $\lambda > 0$ such that $|\tilde{\boldsymbol{y}}(t)| \le \Gamma\|\boldsymbol{\psi}\|\exp(-\lambda t)$ hold, where $\|\boldsymbol{\psi}\| = \sup_{s\in(-\infty,0]}\|\boldsymbol{\psi}(s)\|$, $t \ge 0$.

We claim that $E(V_j(t)) < \beta_j\chi_0$ and $E(V_{j+n}(t)) < \gamma_j\chi_0$, $j = 1, 2, \ldots\ldots, n$. If they are not true, then there exist some $n+1 \le \hat{k} \le 2n$ and time instant $t_2 > 0$ such that $E(V_{\hat{k}}(t_2)) = \gamma_{\hat{k}-n}\chi_0$, $D^+E(V_{\hat{k}}(t_2)) \ge 0$; $E(V_j(t_2)) < \beta_j\chi_0$, $E(V_{j+n}(t_2)) \le \gamma_j\chi_0$, $j \ne \hat{k}-n$, $j = 1, 2, \ldots\ldots, n$. Substituting them into (37) and considering (23), we have

$$\begin{aligned}
D^+E(V_{\hat{k}}(t_2)) \le \Bigg\{&\Big[-2d_k + \lambda + \left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right|\right) \\
&\cdot\left(l_j^{IR} + l_j^{II}\right) + \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right|\right)\left(l_j^{RR} + l_j^{RI}\right)\Big] \\
&\cdot E(V_{\hat{k}}(t_2)) + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^R\right| l_j^{IR} + \left|a_{kj}^I\right| l_j^{RR} + \left(\omega_{kj}^{IR}\right)^2\right) \\
&\cdot E(V_j(t_2)) + \left(\left|a_{kj}^R\right| l_j^{II} + \left|a_{kj}^I\right| l_j^{RI} + \left(\omega_{kj}^{II}\right)^2\right) \\
&\cdot E(V_{j+n}(t_2))\Big] + \sum_{j=1}^{n}\exp(\lambda\tau)\Big[\left(\left|b_{kj}^R\right| l_j^{IR} + \left|b_{kj}^I\right| l_j^{RR}\right. \\
&\left.+ \left(\omega_{kj}^{IR}\right)^2\right)E\left(V_j\left(t_2-\tau_{kj}(t_2)\right)\right) + \left(\left|b_{kj}^R\right| l_j^{II} + \left|b_{kj}^I\right|\right. \\
&\left.\cdot l_j^{RI} + \left(\omega_{kj}^{II}\right)^2\right)E\left(V_{j+n}\left(t_2-\tau_{kj}(t_2)\right)\right)\Big] \\
&+ \sum_{j=1}^{n}\int_{-\infty}^{t_2}\theta_{kj}(t_2-s)\exp(\lambda(t_2-s))\Big[\left(\left|p_{kj}^R\right| l_j^{IR}\right. \\
&\left.+ \left|p_{kj}^I\right| l_j^{RR}\right)E(V_j(s)) + \left(\left|p_{kj}^R\right| l_j^{II} + \left|p_{kj}^I\right| l_j^{RI}\right) \\
&\cdot E(V_{j+n}(s))\Big]\,ds\Bigg\} \le \Bigg\{-2d_k + \lambda \\
&+ \sum_{j=1}^{n}\left[\left(\left|a_{kj}^R\right| + \left|b_{kj}^R\right| + \left|p_{kj}^R\right|\right)\left(l_j^{IR} + l_j^{II}\right)\right. \\
&\left.+ \left(\left|a_{kj}^I\right| + \left|b_{kj}^I\right| + \left|p_{kj}^I\right|\right)\left(l_j^{RR} + l_j^{RI}\right)\right]\Bigg\}\gamma_{\hat{k}-n}\chi_0
\end{aligned}$$

$$+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_{j}^{IR}+\left|a_{kj}^{I}\right|l_{j}^{RR}+\left(\omega_{kj}^{IR}\right)^{2}\right)+\exp\left(\lambda\tau\right)\right.$$
$$\cdot\left(\left|b_{kj}^{R}\right|l_{j}^{IR}+\left|b_{kj}^{I}\right|l_{j}^{RR}+\left(\omega_{kj}^{IR}\right)^{2}\right)+\mu_{kj}\left(\lambda\right)\left(\left|p_{kj}^{R}\right|l_{j}^{IR}\right.$$
$$\left.\left.+\left|p_{kj}^{I}\right|l_{j}^{RR}\right)\right]\beta_{j}\chi_{0}+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|l_{j}^{II}+\left|a_{kj}^{I}\right|l_{j}^{RI}\right.\right.$$
$$\left.+\left(\omega_{kj}^{II}\right)^{2}\right)+\exp\left(\lambda\tau\right)\left(\left|b_{kj}^{R}\right|l_{j}^{II}+\left|b_{kj}^{I}\right|l_{j}^{RI}+\left(\omega_{kj}^{II}\right)^{2}\right)$$
$$\left.+\mu_{kj}\left(\lambda\right)\left(\left|p_{kj}^{R}\right|l_{j}^{II}+\left|p_{kj}^{I}\right|l_{j}^{RI}\right)\right]\gamma_{j}\chi_{0}<0. \tag{38}$$

This is a contradiction with the assumption $\mathrm{D}^{+}\mathrm{E}(V_{\tilde{k}}(t_2)) \geq 0$. Therefore, for all $t \geq 0$, we have $\mathrm{E}(V_k(t)) < \beta_k\chi_0$ and $\mathrm{E}(V_{k'}(t)) < \gamma_{k'}\chi_0$. That is, $\mathrm{E}(V_{k'}(t)) < \varsigma_{k'}\chi_0$, $k' = 1, 2, \ldots\ldots, 2n$.

To sum up, it follows from (i) and (ii) that $\mathrm{E}(V_{k'}(t)) < \varsigma_{k'}\chi_0$ hold for all $k' = 1, 2, \ldots\ldots, 2n$; i.e., $(1/2)\exp(-\lambda t)\mathrm{E}(|\tilde{x}_k(t)|^2) < \beta_k\chi_0$, $(1/2)\exp(\lambda t)\mathrm{E}(|\tilde{y}_k(t)|^2) < \gamma_k\chi_0$, $k = 1, 2, \ldots\ldots, n$.

Furthermore, for $k = 1, 2, \ldots\ldots, n$, we get

$$\mathrm{E}\left(\left|\tilde{x}_k\left(t\right)\right|^2\right) < \frac{2\delta\beta_k\exp\left(-\lambda t\right)\left\|\mathrm{E}\left(\boldsymbol{\psi}\left(s\right)\right)\right\|^2}{\underline{\pi}},$$
$$\mathrm{E}\left(\left|\tilde{y}_k\left(t\right)\right|^2\right) < \frac{2\delta\gamma_k\exp\left(-\lambda t\right)\left\|\mathrm{E}\left(\boldsymbol{\psi}\left(s\right)\right)\right\|^2}{\underline{\pi}}. \tag{39}$$

Let $\Gamma = 2\delta\overline{\pi}/\underline{\pi}$. We further have

$$\mathrm{E}\left(\left|\tilde{x}_k\left(t\right)\right|^2\right) < \Gamma\exp\left(-\lambda t\right)\left\|\mathrm{E}\left(\boldsymbol{\psi}\left(s\right)\right)\right\|^2,$$
$$\mathrm{E}\left(\left|\tilde{y}_k\left(t\right)\right|^2\right) < \Gamma\left\|\mathrm{E}\left(\boldsymbol{\psi}\left(s\right)\right)\right\|^2\exp\left(-\lambda t\right), \tag{40}$$
$$k = 1, 2, \ldots, n.$$

According to Definition 6, the zero solution of the complex-valued neural networks being composed of the real part (16) and the imaginary part (17) is exponentially stable in the mean square sense. Namely, the equilibrium point of the complex-valued neural networks being composed of the real part (9) and the imaginary part (10) is exponentially stable in the mean square sense. The proof is completed. □

Remark 11. When there is no stochastic disturbance in the complex-valued neural networks described by (1), model (1) in this paper is the same as model (1) in [8]. Theorem 2 in [8] can be obtained directly by using the same analysis in Theorem 10. The obtained results of this paper include the ones in [8], which means that they are with more generality.

Remark 12. The authors in [32] studied a class of delayed memristor-based complex-valued neural networks with stochastic disturbances. The continuously distributed delays were not considered in [32]. Moreover, it can be seen from Assumption 5 of this paper that the stochastic weighted functions $\sigma^R(.)$ and $\sigma^I(.)$ are with regard to both the real part and the imaginary part of neuro state, respectively. Obviously, Assumption 2 concerning stochastic disturbances in [32] was with more constraint because $\sigma^R(.)$ was supposed to the function only with regard to the real part of neuro state and $\sigma^I(.)$ with regard to imaginary part of neuro state. Therefore, the research of this paper extends the corresponding work in [32].

Motivated by the ideal of Assumption 2 in [32], some other assumption conditions on stochastic disturbances are proposed as follows.

Assumption 13. It is assumed that the weighted function of stochastic disturbances σ_{kj} $(k, j = 1, 2, \ldots\ldots, n)$ can be separated into its real part and imaginary part with the following forms:

$$\sigma_{kj}^{R} = \sigma_{kj}^{R}\left(x_j\left(t\right), x_j\left(t-\tau_{kj}\left(t\right)\right)\right) : \mathbf{R}^2 \longrightarrow \mathbf{R},$$
$$\sigma_{kj}^{I} = \sigma_{kj}^{I}\left(y_j\left(t\right), y_j\left(t-\tau_{kj}\left(t\right)\right)\right) : \mathbf{R}^2 \longrightarrow \mathbf{R}, \tag{41}$$

where $\sigma_{kj}^{R}(0,0) = \sigma_{kj}^{I}(0,0) = 0$. Suppose that there exist nonnegative constants ω_{kj}^{RR} and ω_{kj}^{II} such that the following inequalities hold:

$$\left\{\sigma_{kj}^{R}\left(x_j\left(t\right), x_j\left(t-\tau_{kj}\left(t\right)\right)\right)\right\}^2$$
$$-\left\{\sigma_{kj}^{R}\left(x'_j\left(t\right), x'_j\left(t-\tau_{kj}\left(t\right)\right)\right)\right\}^2 \leq \left(\omega_{kj}^{RR}\right)^2$$
$$\cdot\left[\left(x_j\left(t\right)-x'_j\left(t\right)\right)^2\right.$$
$$\left.+\left(x_j\left(t-\tau_{kj}\left(t\right)\right)-x'_j\left(t-\tau_{kj}\left(t\right)\right)\right)^2\right],$$
$$\left\{\sigma_{kj}^{I}\left(y_j\left(t\right), y_j\left(t-\tau_{kj}\left(t\right)\right)\right)\right\}^2$$
$$-\left\{\sigma_{kj}^{I}\left(y'_j\left(t\right), y'_j\left(t-\tau_{kj}\left(t\right)\right)\right)\right\}^2 \leq \left(\omega_{kj}^{II}\right)^2$$
$$\cdot\left[\left(y_j\left(t\right)-y'_j\left(t\right)\right)^2\right.$$
$$\left.+\left(y_j\left(t-\tau_{kj}\left(t\right)\right)-y'_j\left(t-\tau_{kj}\left(t\right)\right)\right)^2\right]. \tag{42}$$

Let $\boldsymbol{W}^{RR} = ((\omega_{kj}^{RR})^2)_{n\times n}$, $\boldsymbol{W}^{II} = ((\omega_{kj}^{II})^2)_{n\times n}$, and $\boldsymbol{W}' = \begin{bmatrix} \boldsymbol{W}^{RR} & 0 \\ 0 & \boldsymbol{W}^{II} \end{bmatrix}$.

Corollary 14. *If Assumptions 1–3 and Assumption 13 are satisfied, and the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{L}} - 2\boldsymbol{W}'$ is an M-matrix, then the equilibrium point $\boldsymbol{z}^{\#}$ of (1) is of mean square exponential stability for any external input $\boldsymbol{J} \in \mathbf{C}^n$.*

Remark 15. In order to compare the result to one in [32] from the computational point of view, we assume that $\sigma_{kj}^{R} = 0.2x_j(t) + 0.4x_j(t-\tau_{kj}(t))$ and $\sigma_{kj}^{I} = 0.3y_j(t) + 0.1y_j(t-\tau_{kj}(t))$. By computation, the parameters of Assumption 2 in [32] are $\delta_k^2 = 0.12$, $\hbar_k^2 = 0.24$, $\mu_k^2 = 0.12$, and $\delta_k^2 = 0.04$,

respectively. The parameters in Assumption 13 of this paper are $(\omega_{kj}^{RR})^2 = \max\{\delta_k^2, \hbar_k^2\} = 0.24$ and $(\omega_{kj}^{II})^2 = \max\{\mu_k^2, \nu_k^2\} = 0.12$. Although the expression form of Assumption 2 in [32] is different from Assumption 13 in this paper, it can be seen from the proof of Theorem 10 about values of a_k and b_k that the obtained stability conditions concerning the stochastic terms in [32] are the same as ours. Besides, the matrix for judging stability in Corollary 14 in this paper is of more simple and compact expression, which is easier to calculate by MATLAB software.

Remark 16. There are some errors inevitably between theoretical and practical systems for the unexpected factors, such as sensing error, modeling error, parameter aging, and channel strength. Therefore, it is necessary to suppose that parameter uncertainties of system in given intervals can overcome those problems. We call the system with randomly occurring parameter uncertainties interval system. In [18, 20–22, 33, 40], the authors investigated some interval neural networks in complex number domain. However, the stochastic disturbances were not considered in [18, 20–22] and the continuously distributed delays were not considered in [33, 40].

Next, we will give a theorem for judging the robust exponential stability in the mean square sense for a class of interval complex-valued neural networks with mixed delays and stochastic disturbances.

Assumption 17. The weighted matrices defined in complex number domain of (1) are supposed to be in the following intervals:

$$\begin{aligned}
\boldsymbol{A}_I &= \left\{|\boldsymbol{A}| = \left(\left|a_{kj}\right|\right)_{n\times n} : \left|\underset{\sim}{\boldsymbol{A}}\right| \le |\boldsymbol{A}| \le \left|\widetilde{\boldsymbol{A}}\right|,\ i.e.\ \left|\underset{\sim}{a}_{kj}\right| \right.\\
&\quad \left. \le \left|a_{kj}\right| \le \left|\widetilde{a}_{kj}\right|,\ k, j = 1, 2, \ldots, n\right\},\\
\boldsymbol{B}_I &= \left\{|\boldsymbol{B}| = \left(\left|b_{kj}\right|\right)_{n\times n} : \left|\underset{\sim}{\boldsymbol{B}}\right| \le |\boldsymbol{B}| \le \left|\widetilde{\boldsymbol{B}}\right|,\ i.e.\ \left|\underset{\sim}{b}_{kj}\right| \right.\\
&\quad \left. \le \left|b_{kj}\right| \le \left|\widetilde{b}_{kj}\right|,\ k, j = 1, 2, \ldots, n\right\},\\
\boldsymbol{P}_I &= \left\{|\boldsymbol{P}| = \left(\left|p_{ij}\right|\right)_{n\times n} : \left|\underset{\sim}{\boldsymbol{P}}\right| \le |\boldsymbol{P}| \le \left|\widetilde{\boldsymbol{P}}\right|,\ i.e.\ \left|\underset{\sim}{p}_{kj}\right| \right.\\
&\quad \left. \le \left|p_{kj}\right| \le \left|\widetilde{p}_{kj}\right|,\ k, j = 1, 2, \ldots, n\right\}.
\end{aligned} \tag{43}$$

Let

$$\begin{aligned}
\widetilde{\mathbf{Q}}_1 &= \begin{bmatrix} \left|\widetilde{\boldsymbol{A}}^R\right| + \left|\widetilde{\boldsymbol{B}}^R\right| + \left|\widetilde{\boldsymbol{P}}^R\right| & \mathbf{0} \\ \mathbf{0} & \left|\widetilde{\boldsymbol{A}}^I\right| + \left|\widetilde{\boldsymbol{B}}^I\right| + \left|\widetilde{\boldsymbol{P}}^I\right| \end{bmatrix},\\
\widehat{\mathbf{Q}}_2 &= \begin{bmatrix} \left|\widetilde{\boldsymbol{A}}^R\right| + \left|\widetilde{\boldsymbol{B}}^R\right| + \left|\widetilde{\boldsymbol{P}}^R\right| & \left|\widetilde{\boldsymbol{A}}^I\right| + \left|\widetilde{\boldsymbol{B}}^I\right| + \left|\widetilde{\boldsymbol{P}}^I\right| \\ \left|\widetilde{\boldsymbol{A}}^I\right| + \left|\widetilde{\boldsymbol{B}}^I\right| + \left|\widetilde{\boldsymbol{P}}^I\right| & \left|\widetilde{\boldsymbol{A}}^R\right| + \left|\widetilde{\boldsymbol{B}}^R\right| + \left|\widetilde{\boldsymbol{P}}^R\right| \end{bmatrix}.
\end{aligned} \tag{44}$$

Definition 18. The equilibrium point $\boldsymbol{z}^\# = (z_1^\#, z_2^\#, \ldots, z_n^\#)^{\mathrm{T}}$ of (1) is said to be robustly exponentially stable in the mean square sense if, for all $\boldsymbol{A} \in \boldsymbol{A}_I$, $\boldsymbol{B} \in \boldsymbol{B}_I$, $\boldsymbol{P} \in \boldsymbol{P}_I$ and $\boldsymbol{J} \in \mathbf{C}^n$, there exist constants $\lambda > 0$ and $\Gamma > 0$ such that

$$\begin{aligned}
&\mathrm{E}\left(\left\|\boldsymbol{z}(t) - \boldsymbol{z}^\#\right\|^2\right)\\
&\quad \le \Gamma \sup_{s\in(-\infty,0]} \mathrm{E}\left(\left\|\boldsymbol{\varphi}(s) - \boldsymbol{z}^\#\right\|^2\right) \exp(-\lambda t), \quad t \ge 0,
\end{aligned} \tag{45}$$

where

$$\begin{aligned}
&\mathrm{E}\left(\left\|\boldsymbol{z}(t) - \boldsymbol{z}^\#\right\|^2\right) = \left[\mathrm{E}\left(\left|z_1(t) - z_1^\#\right|^2\right),\right.\\
&\quad \left.\mathrm{E}\left(\left|z_2(t) - z_2^\#\right|^2\right), \ldots, \mathrm{E}\left(\left|z_n(t) - z_n^\#\right|^2\right)\right]^{\mathrm{T}};\\
&\mathrm{E}\left(\left\|\boldsymbol{\varphi}(s) - \boldsymbol{z}^\#\right\|^2\right) = \left[\mathrm{E}\left(\left|\varphi_1(s) - z_1^\#\right|^2\right),\right.\\
&\quad \left.\mathrm{E}\left(\left|\varphi_2(s) - z_2^\#\right|^2\right), \ldots, \mathrm{E}\left(\left|\varphi_n(s) - z_n^\#\right|^2\right)\right]^{\mathrm{T}}.
\end{aligned} \tag{46}$$

Theorem 19. *If Assumptions 1–5 and Assumption 17 are satisfied, and the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\mathbf{Q}}_1\widetilde{\boldsymbol{L}} - \widehat{\mathbf{Q}}_2\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is an M-matrix, then the equilibrium point $\boldsymbol{z}^\#$ of (1) is of robust exponential stability in the mean square sense for any external input $\boldsymbol{J} \in \mathbf{C}^n$.*

Proof. Because $2\widetilde{\boldsymbol{D}} - \widetilde{\mathbf{Q}}_1\widetilde{\boldsymbol{L}} - \widehat{\mathbf{Q}}_2\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is an M-matrix, according to Lemma 7 there exists positive vector $\boldsymbol{\Lambda} = (\varsigma_1, \varsigma_2, \ldots, \varsigma_{2n})$ such that the following inequalities hold, where $\varsigma_{k'} = \beta_k$ and $= k'$, when $1 \le k' \le n$; $\varsigma_{k'} = \gamma_k$ and $k = k' - n$ when $n + 1 \le k' \le 2n$,

$$\begin{aligned}
&\left\{-2d_k + \sum_{j=1}^{n}\left[\left(\left|\widetilde{a}_{kj}^R\right| + \left|\widetilde{b}_{kj}^R\right| + \left|\widetilde{p}_{kj}^R\right|\right)\left(l_j^{RR} + l_j^{RI}\right)\right.\right.\\
&\quad \left.\left. + \left(\left|\widetilde{a}_{kj}^I\right| + \left|\widetilde{b}_{kj}^I\right| + \left|\widetilde{p}_{kj}^I\right|\right)\left(l_j^{IR} + l_j^{II}\right)\right]\right\}\beta_k + \sum_{j=1}^{n}\left[\left(\left|\widetilde{a}_{kj}^R\right|\right.\right.\\
&\quad \left. + \left|\widetilde{b}_{kj}^R\right| + \left|\widetilde{p}_{kj}^R\right|\right) l_j^{RR} + \left(\left|\widetilde{a}_{kj}^I\right| + \left|\widetilde{b}_{kj}^I\right| + \left|\widetilde{p}_{kj}^I\right|\right) l_j^{IR}\\
&\quad \left. + 2\left(\omega_{kj}^{RR}\right)^2\right]\beta_j + \sum_{j=1}^{n}\left[\left(\left|\widetilde{a}_{kj}^R\right| + \left|\widetilde{b}_{kj}^R\right| + \left|\widetilde{p}_{kj}^R\right|\right) l_j^{RI}\right.\\
&\quad \left. + \left(\left|\widetilde{a}_{kj}^I\right| + \left|\widetilde{b}_{kj}^I\right| + \left|\widetilde{p}_{kj}^I\right|\right) l_j^{II} + 2\left(\omega_{kj}^{RI}\right)^2\right]\gamma_j < 0,\\
&\left\{-2d_k + \sum_{j=1}^{n}\left[\left(\left|\widetilde{a}_{kj}^R\right| + \left|\widetilde{b}_{kj}^R\right| + \left|\widetilde{p}_{kj}^R\right|\right)\left(l_j^{IR} + l_j^{II}\right)\right.\right.\\
&\quad \left.\left. + \left(\left|\widetilde{a}_{kj}^I\right| + \left|\widetilde{b}_{kj}^I\right| + \left|\widetilde{p}_{kj}^I\right|\right)\left(l_j^{RR} + l_j^{RI}\right)\right]\right\}\gamma_k\\
&\quad + \sum_{j=1}^{n}\left[\left(\left|\widetilde{a}_{kj}^R\right| + \left|\widetilde{b}_{kj}^R\right| + \left|\widetilde{p}_{kj}^R\right|\right) l_j^{IR} + \left(\left|\widetilde{a}_{kj}^I\right| + \left|\widetilde{b}_{kj}^I\right|\right.\right.
\end{aligned}$$

$$+\left|\widetilde{p}_{kj}^{I}\right|\right) l_j^{RR} + 2\left(\omega_{kj}^{IR}\right)^2\Big]\beta_j + \sum_{j=1}^{n}\Big[\left(\left|\widetilde{a}_{kj}^{R}\right| + \left|\widetilde{b}_{kj}^{R}\right|\right.$$
$$+\left|\widetilde{p}_{kj}^{R}\right|\right) l_j^{II} + \left(\left|\widetilde{a}_{kj}^{I}\right| + \left|\widetilde{b}_{kj}^{I}\right| + \left|\widetilde{p}_{kj}^{I}\right|\right) l_j^{RI} + 2\left(\omega_{kj}^{II}\right)^2\Big]\gamma_j$$
$$< 0. \tag{47}$$

The rest of proof is almost similar with the proof of Theorem 10 only by replacing all parameters $|a_{kj}^R|, |b_{kj}^R|, |p_{kj}^R|, |a_{kj}^I|, |b_{kj}^I|, |p_{kj}^I|$ by $|\widetilde{a}_{kj}^R|, |\widetilde{b}_{kj}^R|, |\widetilde{p}_{kj}^R|, |\widetilde{a}_{kj}^I|, |\widetilde{b}_{kj}^I|, |\widetilde{p}_{kj}^I|$, respectively, so it is omitted here. The proof is completed. □

When the stochastic disturbances satisfy Assumption 13, it is easy to obtain the following Corollary 20 for (1) with the mentioned parameter uncertainties above.

Corollary 20. *If Assumptions 1–3 and Assumptions 13 and 17 are satisfied and the matrix* $2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}_1\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}_2\widehat{\boldsymbol{L}} - 2\boldsymbol{W}'$ *is an M-matrix, then the equilibrium point* $\boldsymbol{z}^{\#}$ *of (1) is of robust exponential stability in the mean square sense for any external input* $\boldsymbol{J} \in \mathbf{C}^n$.

Remark 21. When there is only time-varying delays or continuously distributed delays in (1), some corresponding corollaries can be obtained directly by letting $\boldsymbol{P} = \boldsymbol{0}$ or $\boldsymbol{B} = \boldsymbol{0}$ in the matrices $\widetilde{\boldsymbol{Q}}$, $\widehat{\boldsymbol{Q}}$, $\widetilde{\boldsymbol{Q}}_1$, $\widehat{\boldsymbol{Q}}_2$ defined in Theorem 10, Corollary 14, Theorem 19, and Corollary 20, respectively.

4. Numerical Examples

In this section, two numerical examples with simulation figures will be given to illustrate the obtained results of Theorems 10 and 19, respectively.

Example 1. Consider the following system:

$$\begin{aligned} dz_k(t) = \Bigg\{ & -d_k z_k(t) + \sum_{j=1}^{2}\Big[a_{kj} f_j\left(z_j(t)\right) \\ & + b_{kj} f_j\left(z_j\left(t-\tau_{kj}(t)\right)\right) \\ & + p_{kj}\int_{-\infty}^{t}\theta_{kj}(t-s) f_j\left(z_j(s)\right) ds\Big] + J_k\Bigg\} dt \\ & + \sum_{j=1}^{2}\sigma_{kj}\left(z_j(t), z_j\left(t-\tau_{kj}(t)\right)\right) dw_j(t) \\ & \qquad k = 1, 2, \end{aligned} \tag{48}$$

In (48), let $\boldsymbol{D} = \operatorname{diag}(3, 2)$; the interconnected matrices are

$$\boldsymbol{A} = \begin{bmatrix} -0.3-0.2i & 0.8+0.5i \\ -0.4-0.15i & 0.7+0.8i \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0.9+0.5i & -0.7+0.9i \\ 0.8-0.3i & -0.3+0.6i \end{bmatrix},$$

$$\boldsymbol{P} = \begin{bmatrix} 0.5+0.6i & -0.2+0.3i \\ 0.7-0.7i & -0.4+0.9i \end{bmatrix}. \tag{49}$$

The activation functions are $f_1(z_1) = 1.2((1-e^{-x_1})/(1+e^{-x_1})) + i(0.8/(1+e^{-y_1}))$ and $f_2(z_2) = 0.5((1-e^{-y_2})/(1+e^{-y_2})) + i(1.5/(1+e^{-x_2}))$. The real part and imaginary part of stochastic weighted function are $\sigma_{kj}^R = 0.3x_j(t) + 0.2y_j(t) + 0.1x_j(t-\tau_{kj}(t)) + 0.15y_j(t-\tau_{kj}(t))$ and $\sigma_{kj}^I = 0.15x_j(t) + 0.1y_j(t) + 0.25x_j(t-\tau_{kj}(t)) + 0.2y_j(t-\tau_{kj}(t))$, respectively.

By computing, we have

$$\boldsymbol{L}^{RR} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\boldsymbol{L}^{RI} = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix},$$

$$\boldsymbol{L}^{IR} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$\boldsymbol{L}^{II} = \begin{bmatrix} 0.375 & 0 \\ 0 & 0 \end{bmatrix};$$

$$\boldsymbol{W}^{RR} = \begin{bmatrix} 0.225 & 0.225 \\ 0.225 & 0.225 \end{bmatrix},$$

$$\boldsymbol{W}^{RI} = \begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.15 \end{bmatrix},$$

$$\boldsymbol{W}^{IR} = \begin{bmatrix} 0.175 & 0.175 \\ 0.175 & 0.175 \end{bmatrix},$$

$$\boldsymbol{W}^{II} = \begin{bmatrix} 0.122 & 0.122 \\ 0.122 & 0.122 \end{bmatrix},$$

$$\widetilde{\boldsymbol{D}} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix},$$

$$\widetilde{\boldsymbol{L}} = \begin{bmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.375 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix},$$

$$\widehat{\boldsymbol{L}} = \begin{bmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 \\ 0 & 0 & 0.375 & 0 \\ 0 & 0.2 & 0 & 0 \end{bmatrix},$$

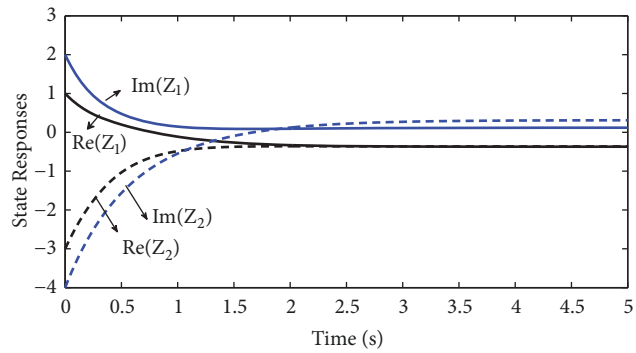

FIGURE 1: State curves of the real parts and imaginary parts in (48) without stochastic term.

$$\boldsymbol{W} = \begin{bmatrix} 0.225 & 0.225 & 0.15 & 0.15 \\ 0.225 & 0.225 & 0.15 & 0.15 \\ 0.175 & 0.175 & 0.122 & 0.122 \\ 0.175 & 0.175 & 0.122 & 0.122 \end{bmatrix},$$

$$\widetilde{\boldsymbol{Q}} = \begin{bmatrix} 1.6 & 1.7 & 0 & 0 \\ 1.9 & 1.4 & 0 & 0 \\ 0 & 0 & 1.3 & 1.7 \\ 0 & 0 & 0.85 & 2.3 \end{bmatrix},$$

$$\widehat{\boldsymbol{Q}} = \begin{bmatrix} 1.6 & 1.7 & 1.3 & 1.7 \\ 1.9 & 1.4 & 0.85 & 2.3 \\ 1.3 & 1.7 & 1.6 & 1.7 \\ 0.85 & 2.3 & 1.9 & 1.4 \end{bmatrix}. \tag{50}$$

It can be obtained that

$$2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{L}} - 2\boldsymbol{W} = \begin{bmatrix} 3.63 & -1.12 & -0.79 & -0.73 \\ -2.73 & 2.74 & -0.62 & -0.65 \\ -1.13 & -0.69 & 4.67 & 1.01 \\ -0.86 & -0.63 & -1.28 & 2.72 \end{bmatrix}. \tag{51}$$

The eigenvalue matrix of the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is diag(0.2552, 3.1280, 4.9570, 5.4193). It follows from Lemma 7 that the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{Q}}\widetilde{\boldsymbol{L}} - \widehat{\boldsymbol{Q}}\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is an M-matrix. According to Theorem 10, it can be concluded that the equilibrium point of (48) is exponentially stable in the mean square sense.

Let the delays in (48) be $\tau_{1j} = 0.3 + 0.1\sin t$ and $\tau_{2j} = 0.2 + 0.1\cos t$, $t \geq 0$. Let $\theta_{kj}(t-s) = \exp(-(t-s))$, $k, j = 1, 2$, $t \geq 0$, $s \in (-\infty, 0]$. It is assumed that the initial condition of (48) are $z_1(s) = 1 + 2\mathrm{i}$ and $z_2(s) = -3 - 4\mathrm{i}$, $s \in (-\infty, 0]$.

The numerical simulations of (48) are shown in Figures 1–5. In order to make comparison, Figures 1 and 3 show the state curves of the real part and the imaginary part of (48) without/with stochastic disturbances, respectively. Figures 2 and 4 show the amplitude curves of neuro states of (48) without/with stochastic disturbances, respectively. Figure 5 shows the norm curves of neuro states of (48) both with stochastic disturbance and without stochastic disturbances. From the simulation results, it can be seen that the equilibrium point of (48) is stable. The simulation results verify the correctness of Theorem 10 in this paper.

Example 2. Consider the following system:

$$\begin{aligned} \mathrm{d}z_k(t) = \Bigg\{ & -d_k z_k(t) + \sum_{j=1}^{2} \Big[a_{kj} f_j\big(z_j(t)\big) \\ & + b_{kj} f_j\big(z_j\big(t - \tau_{kj}(t)\big)\big) \\ & + p_{kj} \int_{-\infty}^{t} \theta_{kj}(t-s) f_j\big(z_j(s)\big)\,\mathrm{d}s \Big] + J_k \Bigg\}\,\mathrm{d}t \\ & + \sum_{j=1}^{2} \sigma_{kj}\big(z_j(t), z_j\big(t - \tau_{kj}(t)\big)\big)\,\mathrm{d}w_j(t) \\ & k = 1, 2, \end{aligned} \tag{52}$$

In (52), let $\boldsymbol{D} = \mathrm{diag}(8, 5)$. The interval interconnected matrices are supposed to be

$$\begin{aligned} \boldsymbol{A}_I &= \begin{bmatrix} [-1.5, 1] + [1.6, 1.9]\,\mathrm{i} & [-2, 1] + [1.7, 2]\,\mathrm{i} \\ [1.5, 1.8] + [-1.4, 1]\,\mathrm{i} & [-1.4, 1.5] + [1.9, 2.2]\,\mathrm{i} \end{bmatrix}, \\ \boldsymbol{B}_I &= \begin{bmatrix} [1.4, 1.8] + [1.8, 2]\,\mathrm{i} & [-1.3, 1.6] + [1, 1.8]\,\mathrm{i} \\ [-1.3, 1] + [1.1, 1.6]\,\mathrm{i} & [1.7, 1.9] + [-2, -1.2]\,\mathrm{i} \end{bmatrix}, \\ \boldsymbol{P}_I &= \begin{bmatrix} [0, 1.5] + [1.1, 1.6]\,\mathrm{i} & [0.2, 1.4] + [-1.1, 1.5]\,\mathrm{i} \\ [-1, 2] + [-1.3, 1.9]\,\mathrm{i} & [0, 1.7] + \left[-1.1, \sqrt{2}\right]\mathrm{i} \end{bmatrix}. \end{aligned} \tag{53}$$

The rest of the assumptions concerning the activation functions and stochastic weighted functions are the same as ones given in Example 1.

By computation, we have

$$\widetilde{\boldsymbol{D}} = \begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix},$$

$$\widetilde{\boldsymbol{Q}}_1 = \begin{bmatrix} 4.8 & 5 & 0 & 0 \\ 5.1 & 5.1 & 0 & 0 \\ 0 & 0 & 5.5 & 5.3 \\ 0 & 0 & 4.9 & 5.61 \end{bmatrix},$$

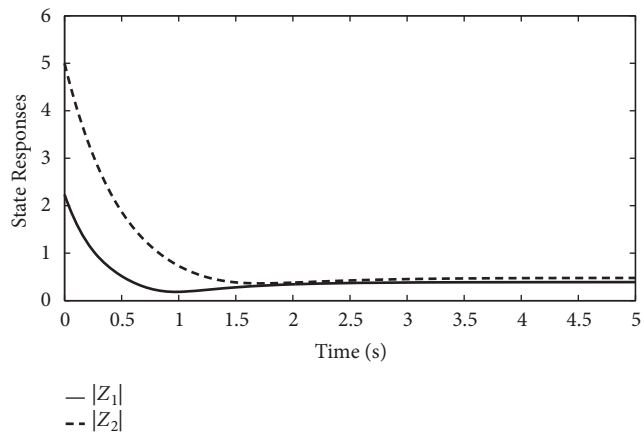

Figure 2: State module curves of (48) without stochastic term.

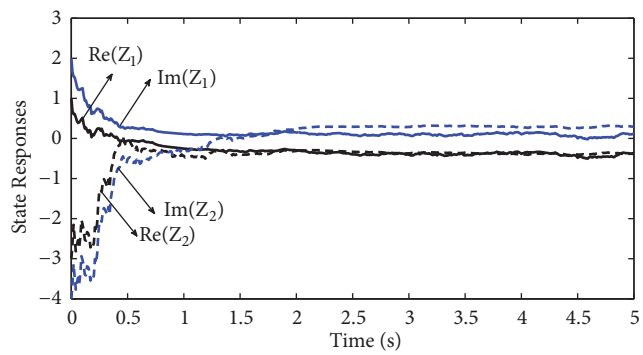

Figure 3: State curves of the real parts and imaginary parts in (48) with stochastic term.

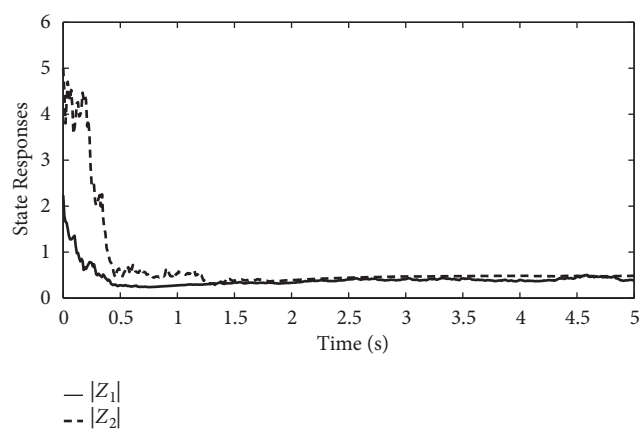

Figure 4: State module curves of (48) with stochastic term.

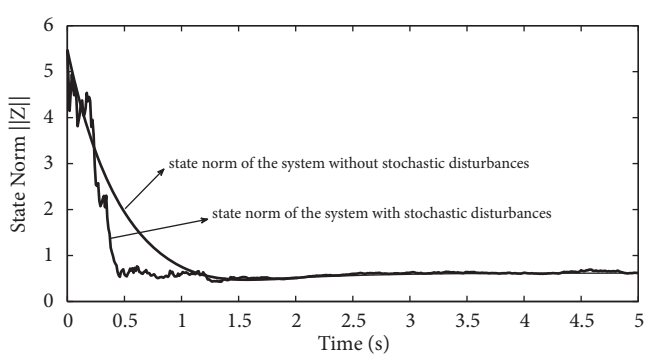

Figure 5: State norm curves of (48) with/without stochastic term.

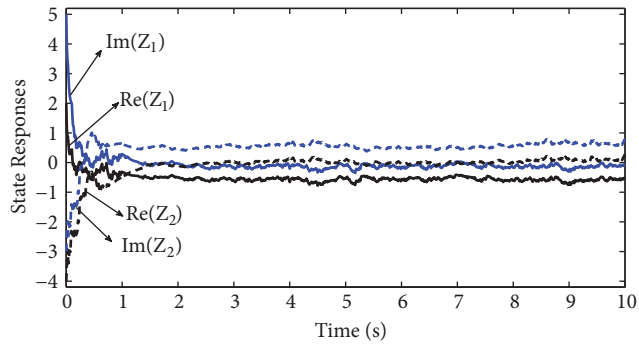

Figure 6: State curves of the real parts and imaginary parts in (52) with stochastic term.

$$\widehat{\mathbf{Q}}_2 = \begin{bmatrix} 4.8 & 5 & 5.5 & 5.3 \\ 5.1 & 5.1 & 4.9 & 5.61 \\ 5.5 & 5.3 & 4.8 & 5.0 \\ 4.9 & 5.61 & 5.1 & 5.1 \end{bmatrix}. \tag{54}$$

It can be obtained that

$$2\widetilde{\boldsymbol{D}} - \widetilde{\mathbf{Q}}_1\widetilde{\boldsymbol{L}} - \widehat{\mathbf{Q}}_2\widehat{\boldsymbol{L}} - 2\boldsymbol{W} = \begin{bmatrix} 9.79 & -2.76 & -2.36 & 1.55 \\ -6.57 & 7.15 & -2.14 & -1.58 \\ -3.65 & -1.35 & 11.89 & -2.63 \\ -3.29 & -1.37 & -3.99 & 7.23 \end{bmatrix}. \tag{55}$$

Because the eigenvalue matrix of the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\mathbf{Q}}_1\widetilde{\boldsymbol{L}} - \widehat{\mathbf{Q}}_2\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is $\mathrm{diag}(0.84, 14.47, 12.62, 8.14)$, it follows from Lemma 7 that the matrix $2\widetilde{\boldsymbol{D}} - \widetilde{\mathbf{Q}}_1\widetilde{\boldsymbol{L}} - \widehat{\mathbf{Q}}_2\widehat{\boldsymbol{L}} - 2\boldsymbol{W}$ is an M-matrix. Therefore, the equilibrium point of (52) is robustly exponentially stable in the mean square sense in accordance with Theorem 19.

In order to finish the simulation, let the delays in (52) be $\tau_{1j} = 0.8 + 0.2\sin t$ and $\tau_{2j} = 0.1 + 0.6\cos t$, $t \geq 0$. Let $\theta_{kj}(t - s) = \exp(-(t - s))$, $k, j = 1, 2$, $t \geq 0$, $s \in (-\infty, 0]$. It is assumed that the initial conditions of (52) are $z_1(s) = 2 + 5\mathrm{i}$ and $z_2(s) = -4 - 3\mathrm{i}$, $s \in (-\infty, 0]$.

Let

$$\boldsymbol{A} = \begin{bmatrix} -1.5 + 1.9\mathrm{i} & -2 + 2\mathrm{i} \\ 1.8 - 1.4\mathrm{i} & 1.5 + 2.2\mathrm{i} \end{bmatrix},$$
$$\boldsymbol{B} = \begin{bmatrix} 1.8 + 2\mathrm{i} & 1.6 + 1.8\mathrm{i} \\ -1.3 + 1.6\mathrm{i} & 1.9 - 2\mathrm{i} \end{bmatrix}, \tag{56}$$
$$\boldsymbol{P} = \begin{bmatrix} 1.5 + 1.6\mathrm{i} & 1.4 + 1.5\mathrm{i} \\ 2 + 1.9\mathrm{i} & 1.7 + \sqrt{2}\mathrm{i} \end{bmatrix}.$$

It is obvious that the matrices $\boldsymbol{A} \in \boldsymbol{A}_I$, $\boldsymbol{B} \in \boldsymbol{B}_I$, and $\boldsymbol{P} \in \boldsymbol{P}_I$.

Figures 6–8 show the state curves of the real part and the imaginary part, the amplitude curves of neuro states, and the norm curves of neuro states of (52), respectively. From the simulation results, it can be seen that the equilibrium point of (52) is stable. The simulation results verify the correctness of Theorem 19.

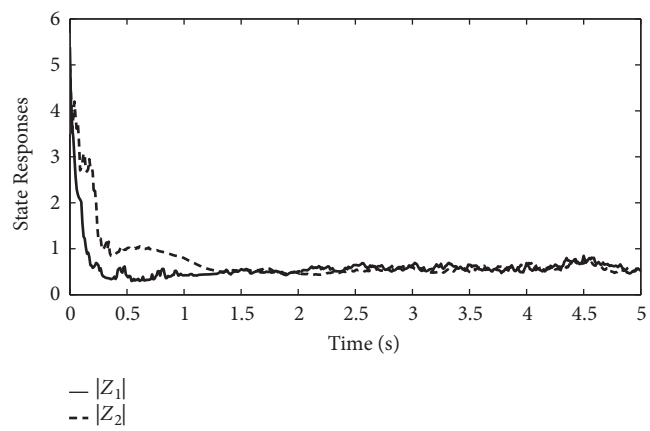

Figure 7: State module curves of (52) with stochastic term.

5. Conclusions and Future Work

Some sufficient conditions have been proposed for assuring the mean square exponential stability of a class of complex-valued neural networks with stochastic disturbances and mixed delays based on the vector Lyapunov function method and *Itô* differential-integral theorem. The results established in this paper generalize the existing ones and reduce the conservatism of the previous results about complex-valued neural networks with stochastic disturbances. Two numerical examples are given at last to show the correctness and feasibility of the proposed results. Based on the study method presented in this paper and the idea of driving-response conception [35, 41], we will attempt to study the synchronization problem for a class of chaotic complex-valued neural networks with mixed delays and stochastic disturbances in the near future. Besides, numerous references concerning neural networks with impulsive effect have been emerged in the past two decades; see [5, 9–15, 22, 40, 42]. It will be an interesting research to consider impulsive effect from the view of both disturbance and control when scholars study the

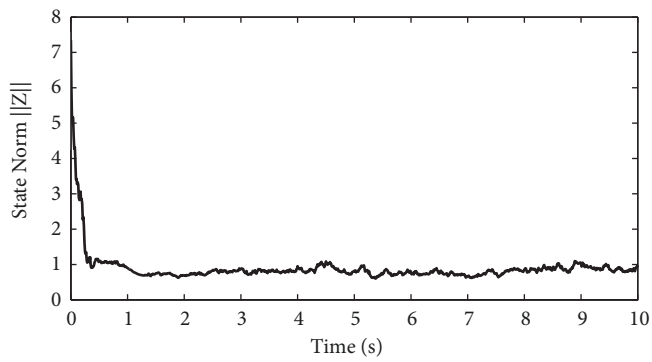

Figure 8: State norm curves of (52).

synchronization problem of various chaotic complex-valued neural networks.

Appendix

The detailed deducing of inequality (29) in Theorem 10 is as follows.

$$\begin{aligned}
\mathrm{D}^{+}\mathrm{E}\left(V_{k'}(t)\right) \le \mathrm{E}\Bigg(& \frac{1}{2}\lambda \exp(\lambda t)\left(\tilde{x}_k(t)\right)^2 + \exp(\lambda t) \\
&\cdot \Bigg\{ -d_k\left|\tilde{x}_k(t)\right|^2 + \left|\tilde{x}_k(t)\right| \sum_{j=1}^{n}\Big[\left|a_{kj}^{R}\right|\Big(l_j^{RR}\left|\tilde{x}_j(t)\right| \\
&+ l_j^{RI}\left|\tilde{y}_j(t)\right|\Big) + \left|a_{kj}^{I}\right|\Big(l_j^{IR}\left|\tilde{x}_j(t)\right| + l_j^{II}\left|\tilde{y}_j(t)\right|\Big)\Big] \\
&+ \left|\tilde{x}_k(t)\right| \sum_{j=1}^{n}\Big[\left|b_{kj}^{R}\right|\Big(l_j^{RR}\left|\tilde{x}_j\left(t-\tau_{kj}(t)\right)\right| \\
&+ l_j^{RI}\left|\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right|\Big) + \left|b_{kj}^{I}\right| \\
&\cdot \Big(l_j^{IR}\left|\tilde{x}_j\left(t-\tau_{kj}(t)\right)\right| + l_j^{II}\left|\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right|\Big)\Big] \\
&+ \left|\tilde{x}_k(t)\right| \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\Big[\left|p_{kj}^{R}\right|\Big(l_j^{RR}\left|\tilde{x}_j(s)\right| \\
&+ l_j^{RI}\left|\tilde{y}_j(s)\right|\Big) + \left|p_{kj}^{I}\right| \\
&\cdot \Big(l_j^{IR}\left|\tilde{x}_j(s)\right| + l_j^{II}\left|\tilde{y}_j(s)\right|\Big)\Big]\,\mathrm{d}s\Bigg\} + \frac{1}{2}\exp(\lambda t) \\
&\cdot \sum_{j=1}^{n}\Big[\left(\omega_{kj}^{RR}\right)^2\Big(\left|\tilde{x}_j(t)\right|^2 + \left|\tilde{x}_j\left(t-\tau_{kj}(t)\right)\right|^2\Big) \\
&+ \left(\omega_{kj}^{RI}\right)^2\Big(\left|\tilde{y}_j(t)\right|^2 + \left|\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right|^2\Big)\Big]\Bigg) \\
\le \mathrm{E}\Bigg(& \frac{1}{2}\exp(\lambda t)\left(-2d_k+\lambda\right)\left(\tilde{x}_k(t)\right)^2 + \frac{1}{2}\exp(\lambda t) \\
&\times \Bigg[\sum_{j=1}^{n}\left(\left|a_{kj}^{R}\right| l_j^{RR} + \left|a_{kj}^{I}\right| l_j^{IR}\right)\left(\left|\tilde{x}_k(t)\right|^2 + \left|\tilde{x}_j(t)\right|^2\right) \\
&+ \sum_{j=1}^{n}\left(\left|a_{kj}^{R}\right| l_j^{RI} + \left|a_{kj}^{I}\right| l_j^{II}\right)\left(\left|\tilde{x}_k(t)\right|^2 + \left|\tilde{y}_j(t)\right|^2\right) \\
&+ \sum_{j=1}^{n}\left(\left|b_{kj}^{R}\right| l_j^{RR} + \left|b_{kj}^{I}\right| l_j^{IR}\right)\Big(\left|\tilde{x}_k(t)\right|^2 \\
&+ \left|\tilde{x}_j\left(t-\tau_{kj}(t)\right)\right|^2\Big) + \sum_{j=1}^{n}\left(\left|b_{kj}^{R}\right| l_j^{RI} + \left|b_{kj}^{I}\right| l_j^{II}\right) \\
&\cdot \Big(\left|\tilde{x}_k(t)\right|^2 + \left|\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right|^2\Big) \\
&+ \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\Big(\left|p_{kj}^{R}\right| l_j^{RR} + \left|p_{kj}^{I}\right| l_j^{IR}\Big)\Big(\left|\tilde{x}_k(t)\right|^2 \\
&+ \left|\tilde{x}_j(s)\right|^2\Big)\,\mathrm{d}s + \sum_{j=1}^{n}\int_{-\infty}^{t}\theta_{kj}(t-s)\Big(\left|p_{kj}^{R}\right| l_j^{RI} \\
&+ \left|p_{kj}^{I}\right| l_j^{II}\Big)\Big(\left|\tilde{x}_k(t)\right|^2 + \left|\tilde{y}_j(s)\right|^2\Big)\,\mathrm{d}s\Bigg] + \frac{1}{2}\exp(\lambda t) \\
&\cdot \sum_{j=1}^{n}\Big[\left(\omega_{kj}^{RR}\right)^2\Big(\left|\tilde{x}_j(t)\right|^2 + \left|\tilde{x}_j\left(t-\tau_{kj}(t)\right)\right|^2\Big) \\
&+ \left(\omega_{kj}^{RI}\right)^2\Big(\left|\tilde{y}_j(t)\right|^2 + \left|\tilde{y}_j\left(t-\tau_{kj}(t)\right)\right|^2\Big)\Big]\Bigg) \\
= \Bigg\{& -2d_k + \lambda + \sum_{j=1}^{n}\Big[\left(\left|a_{kj}^{R}\right| + \left|b_{kj}^{R}\right| + \left|p_{kj}^{R}\right|\right)\Big(l_j^{RR} \\
&+ l_j^{RI}\Big) + \left(\left|a_{kj}^{I}\right| + \left|b_{kj}^{I}\right| + \left|p_{kj}^{I}\right|\right)\Big(l_j^{IR} + l_j^{II}\Big)\Big]\Bigg\} \\
&\cdot \mathrm{E}\left(\frac{1}{2}\exp(\lambda t)\left|\tilde{x}_k(t)\right|^2\right)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right| l_{j}^{RR}+\left|a_{kj}^{I}\right| l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right] \\
&\cdot \mathrm{E}\left(\frac{1}{2} \exp(\lambda t)\left|\tilde{x}_{j}(t)\right|^{2}\right) \\
&+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right| l_{j}^{RI}+\left|a_{kj}^{I}\right| l_{j}^{II}\right)+\left(\omega_{kj}^{RI}\right)^{2}\right] \\
&\cdot \mathrm{E}\left(\frac{1}{2} \exp(\lambda t)\left|\tilde{y}_{j}(t)\right|^{2}\right) \\
&+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right| l_{j}^{RR}+\left|b_{kj}^{I}\right| l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right] \\
&\cdot \exp\left(\lambda \tau_{kj}(t)\right) \mathrm{E}\left(\frac{1}{2} \exp\left(\lambda\left(t-\tau_{kj}(t)\right)\right)\right. \\
&\left.\cdot\left|\tilde{x}_{j}\left(t-\tau_{kj}(t)\right)\right|^{2}\right) \\
&+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right| l_{j}^{RI}+\left|b_{kj}^{I}\right| l_{j}^{II}\right)+\left(\omega_{kj}^{RI}\right)^{2}\right] \exp\left(\lambda \tau_{kj}(t)\right) \\
&\cdot \mathrm{E}\left(\frac{1}{2} \exp\left(\lambda\left(t-\tau_{kj}(t)\right)\right)\left|\tilde{y}_{j}\left(t-\tau_{kj}(t)\right)\right|^{2}\right) \\
&+\sum_{j=1}^{n} \int_{-\infty}^{t} \theta_{kj}(t-s) \exp(\lambda(t-s))\left(\left|p_{kj}^{R}\right| l_{j}^{RR}\right. \\
&\left.+\left|p_{kj}^{I}\right| l_{j}^{IR}\right) \mathrm{E}\left(\frac{1}{2} \exp(\lambda s)\left|\tilde{x}_{j}(s)\right|^{2}\right) \mathrm{d}s \\
&+\sum_{j=1}^{n} \int_{-\infty}^{t} \theta_{kj}(t-s) \exp(\lambda(t-s))\left(\left|p_{kj}^{R}\right| l_{j}^{RI}\right. \\
&\left.+\left|p_{kj}^{I}\right| l_{j}^{II}\right) \mathrm{E}\left(\frac{1}{2} \exp(\lambda s)\left|\tilde{y}_{j}(s)\right|^{2}\right) \mathrm{d}s \\
\leq &\left\{-2 d_{k}+\lambda+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right|+\left|b_{kj}^{R}\right|+\left|p_{kj}^{R}\right|\right)\left(l_{j}^{RR}\right.\right.\right. \\
&\left.\left.\left.+l_{j}^{RI}\right)+\left(\left|a_{kj}^{I}\right|+\left|b_{kj}^{I}\right|+\left|p_{kj}^{I}\right|\right)\left(l_{j}^{IR}+l_{j}^{II}\right)\right]\right\} \\
&\cdot \mathrm{E}\left(V_{k}(t)\right)+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right| l_{j}^{RR}+\left|a_{kj}^{I}\right| l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right] \\
&\cdot \mathrm{E}\left(V_{j}(t)\right)+\sum_{j=1}^{n}\left[\left(\left|a_{kj}^{R}\right| l_{j}^{RI}+\left|a_{kj}^{I}\right| l_{j}^{II}\right)+\left(\omega_{kj}^{RI}\right)^{2}\right] \\
&\cdot \mathrm{E}\left(V_{j+n}(t)\right) \\
&+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right| l_{j}^{RR}+\left|b_{kj}^{I}\right| l_{j}^{IR}\right)+\left(\omega_{kj}^{RR}\right)^{2}\right] \exp(\lambda \tau) \\
&\cdot \mathrm{E}\left(V_{j}\left(t-\tau_{kj}(t)\right)\right) \\
&+\sum_{j=1}^{n}\left[\left(\left|b_{kj}^{R}\right| l_{j}^{RI}+\left|b_{kj}^{I}\right| l_{j}^{II}\right)+\left(\omega_{kj}^{RI}\right)^{2}\right] \exp(\lambda \tau) \\
&\cdot \mathrm{E}\left(V_{j+n}\left(t-\tau_{kj}(t)\right)\right) \\
&+\sum_{j=1}^{n} \int_{-\infty}^{t} \theta_{kj}(t-s) \exp(\lambda(t-s))\left(\left|p_{kj}^{R}\right| l_{j}^{RR}\right. \\
&\left.+\left|p_{kj}^{I}\right| l_{j}^{IR}\right) \mathrm{E}\left(V_{j}(s)\right. \mathrm{d}s \\
&+\sum_{j=1}^{n} \int_{-\infty}^{t} \theta_{kj}(t-s) \exp(\lambda(t-s))\left(\left|p_{kj}^{R}\right| l_{j}^{RI}\right. \\
&\left.+\left|p_{kj}^{I}\right| l_{j}^{II}\right) \mathrm{E}\left(V_{j+n}(s)\right. \mathrm{d}s.
\end{aligned}
\tag{A.1}
$$

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] A. Hirose, "Recent progress in applications of complex-valued neural networks," in *Proceedings of the 10th International Conference on Artificiality Intelligence Soft Computing II*, pp. 42–46, 2010.

[2] J. Hu and J. Wang, "Global stability of complex-valued recurrent neural networks with time-delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 6, pp. 853–865, 2012.

[3] B. Li and D. Xu, "Mean square asymptotic behavior of stochastic neural networks with infinitely distributed delays," *Neurocomputing*, vol. 72, no. 13–15, pp. 3311–3317, 2009.

[4] C. Aouiti, E. Assali, J. Cao, and A. Alsaedi, "Global exponential convergence of neutral-type competitive neural networks with multi-proportional delays, distributed delays and time-varying delay in leakage delays," *International Journal of Systems Science*, vol. 49, no. 10, pp. 2202–2214, 2018.

[5] C. Aouiti and E. A. Assali, "Stability analysis for a class of impulsive bidirectional associative memory (BAM) neural networks with distributed delays and leakage time-varying delays," *Neural Processing Letters*, 2018.

[6] A. Arbi, F. Cherif, C. Aouiti, and A. Touati, "Dynamics of new class of Hopfield neural networks with time-varying and distributed delays," *Acta Mathematica Scientia*, vol. 36, no. 3, pp. 891–912, 2016.

[7] L. Li and J. Jian, "Exponential p-convergence analysis for stochastic BAM neural networks with time-varying and infinite distributed delays," *Applied Mathematics and Computation*, vol. 266, pp. 860–873, 2015.

[8] X. Xu, J. Zhang, and J. Shi, "Exponential stability of complex-valued neural networks with mixed delays," *Neurocomputing*, vol. 128, pp. 483–490, 2014.

[9] X. Xu, J. Zhang, Q. Xu, Z. Chen, and W. Zheng, "Impulsive disturbances on the dynamical behavior of complex-valued cohen-grossberg neural networks with both time-varying delays and continuously distributed delays," *Complexity*, vol. 2017, Article ID 3826729, 12 pages, 2017.

[10] Q. Song, H. Yan, Z. Zhao, and Y. Liu, "Global exponential stability of impulsive complex-valued neural networks with both asynchronous time-varying and continuously distributed delays," *Neural Networks*, vol. 81, pp. 1–10, 2016.

[11] X. Xu, J. Zhang, and J. Shi, "Dynamical behaviour analysis of delayed complex-valued neural networks with impulsive effect," *International Journal of Systems Science*, vol. 48, no. 4, pp. 686–694, 2017.

[12] W. Zhang, Y. Tang, W. K. Wong, and Q. Miao, "Stochastic stability of delayed neural networks with local impulsive effects," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 26, no. 10, pp. 2336–2345, 2015.

[13] C. Aouiti, "Oscillation of impulsive neutral delay generalized high-order Hopfield neural networks," *Neural Computing and Applications*, vol. 29, no. 9, pp. 477–495, 2018.

[14] C. Aouiti, "Neutral impulsive shunting inhibitory cellular neural networks with time-varying coefficients and leakage delays," *Cognitive Neurodynamics*, vol. 10, no. 6, pp. 573–591, 2016.

[15] L. Wang, Q. Song, Y. Liu, Z. Zhao, and F. E. Alsaadi, "Global asymptotic stability of impulsive fractional-order complex-valued neural networks with time delay," *Neurocomputing*, vol. 243, pp. 49–59, 2017.

[16] P. Balasubramaniam and C. Vidhya, "Exponential stability of stochastic reaction-diffusion uncertain fuzzy neural networks with mixed delays and Markovian jumping parameters," *Expert Systems with Applications*, vol. 39, no. 3, pp. 3109–3115, 2012.

[17] C. D. Zheng, Q. H. Shan, H. Zhang, and Z. Wang, "On stabilization of stochastic Cohen-Grossberg neural networks with mode-dependent mixed time-delays and Markovian switching," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 5, pp. 800–811, 2013.

[18] X. Xu, Q. Xu, Y. Peng, J. Zhang, and Y. Xu, "Stochastic exponential robust stability of delayed complex-valued neural networks with markova jumping parameters," *IEEE Access*, vol. 6, no. 1, pp. 839–849, 2018.

[19] T. Radhika, G. Nagamani, Q. Zhu, S. Ramasamy, and R. Saravanakumar, "Further results on dissipativity analysis for Markovian jump neural networks with randomly occurring uncertainties and leakage delays," *Neural Computing and Applications*, vol. 30, no. 11, pp. 3565–3579, 2018.

[20] Q. Song, Q. Yu, Z. Zhao, Y. Liu, and F. E. Alsaadi, "Boundedness and global robust stability analysis of delayed complex-valued neural networks with interval parameter uncertainties," *Neural Networks*, vol. 103, pp. 55–62, 2018.

[21] W. Gong, J. Liang, C. Zhang, and J. Cao, "Nonlinear measure approach for the stability analysis of complex-valued neural networks," *Neural Processing Letters*, vol. 44, no. 2, pp. 539–554, 2016.

[22] Y. Tan, S. Tang, J. Yang, and Z. Liu, "Robust stability analysis of impulsive complex-valued neural networks with time delays and parameter uncertainties," *Journal of Inequalities and Applications*, vol. 2017, no. 215, 15 pages, 2017.

[23] S. Haykin, *Neural Networks*, Prentice-Hall, New Jersey, NJ, USA, 1994.

[24] X. Xu, J. Zhang, and W. Zhang, "Mean square exponential stability of stochastic neural networks with reaction-diffusion terms and delays," *Applied Mathematics Letters*, vol. 24, no. 1, pp. 5–11, 2011.

[25] Q. Zhou and L. Wan, "Exponential stability of stochastic delayed Hopfield neural networks," *Applied Mathematics and Computation*, vol. 199, no. 1, pp. 84–89, 2008.

[26] Y. Chen and W. X. Zheng, "Stability analysis of time-delay neural networks subject to stochastic perturbations," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 2122–2134, 2013.

[27] Y. Song, W. Sun, and F. Jiang, "Mean-square exponential input-to-state stability for neutral stochastic neural networks with mixed delays," *Neurocomputing*, vol. 205, pp. 195–203, 2016.

[28] Q. Zhu, J. Cao, and R. Rakkiyappan, "Exponential input-to-state stability of stochastic Cohen-Grossberg neural networks with mixed delays," *Nonlinear Dynamics*, vol. 79, no. 2, pp. 1085–1098, 2015.

[29] K. Zhong, S. Zhu, and Q. Yang, "Further results for global exponential stability of stochastic memristor-based neural networks with time-varying delays," *International Journal of Systems Science*, vol. 47, no. 15, pp. 3573–3580, 2015.

[30] Y. Cui, Y. Liu, W. Zhang, and F. E. Alsaadi, "Stochastic stability for a class of discrete-time switched neural networks with stochastic noise and time-varying mixed delays," *International Journal of Control, Automation, and Systems*, vol. 16, no. 1, pp. 158–167, 2018.

[31] L. Liu, A. Wu, Z. Zeng, and T. Huang, "Global mean square exponential stability of stochastic neural networks with retarded and advanced argument," *Neurocomputing*, vol. 247, pp. 156–164, 2017.

[32] D. Liu, S. Zhu, and W. Chang, "Global exponential stability of stochastic memristor-based complex-valued neural networks with time delays," *Nonlinear Dynamics*, vol. 90, no. 2, pp. 915–934, 2017.

[33] W. Gong, J. Liang, X. Kan, L. Wang, and A. Dobaie, "Robust state estimation for stochastic complex-valued neural networks with sampled-data," *Neural Computing & Applications*, vol. 31, Supplement 1, pp. 523–542, 2019.

[34] J. Guo, Z. Meng, and Z. Xiang, "Passivity analysis of stochastic memristor-based complex-valued recurrent neural networks

with mixed time-varying delays," *Neural Processing Letters*, vol. 47, no. 3, pp. 1097–1113, 2018.

[35] C. Zhou, W. Zhang, X. Yang, C. Xu, and J. Feng, "Finite-time synchronization of complex-valued neural networks with mixed delays and uncertain perturbations," *Neural Processing Letters*, vol. 46, no. 1, pp. 271–291, 2017.

[36] J. Hu and J. Wang, "Global exponential periodicity and stability of discrete-time complex-valued recurrent neural networks with time-delays," *Neural Networks*, vol. 66, pp. 119–130, 2015.

[37] J. Jian and P. Wan, "Lagrange α-exponential stability and α-exponential convergence for fractional-order complex-valued neural networks," *Neural Networks*, vol. 91, pp. 1–10, 2017.

[38] J. Jian and P. Wan, "Global exponential convergence of fuzzy complex-valued neural networks with time-varying delays and impulsive effects," *Fuzzy Sets and Systems*, vol. 338, pp. 23–39, 2018.

[39] B. Ksendal, *Stochastic Differential Equations*, Springer, Berlin, Germany, sixth edition, 2003.

[40] X. Xu, H. Xue, Y. Peng, Q. Xu, and J. Yang, "Robust exponential stability of switched complex-valued neural networks with interval parameter uncertainties and impulses," *Complexity*, vol. 2018, Article ID 4981812, 12 pages, 2018.

[41] Q. Xu, X. Xu, S. Zhuang, J. Xiao, C. Song, and C. Che, "New complex projective synchronization strategies for drive-response networks with fractional complex-variable dynamics," *Applied Mathematics and Computation*, vol. 338, pp. 552–566, 2018.

[42] C. Aouiti, I. B. Gharbia, J. Cao, and A. Alsaedi, "Dynamics of impulsive neutral-type BAM neural networks," *Journal of The Franklin Institute*, vol. 356, no. 4, pp. 2294–2324, 2019.