Research Article

Multiobjective Optimal Control for Hydraulic Turbine Governing System Based on an Improved MOGWO Algorithm

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Received 3 January 2019; Revised 9 April 2019; Accepted 22 April 2019; Published 28 May 2019

Academic Editor: Matilde Santos

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Hydraulic turbine governing system (HTGS) which mostly contains PID controller is important automatic control equipment of hydroelectric unit [9]. The HTGS can adjust the output power of hydroelectric unit by changing the opening of guide. Therefore the control quality of HTGS influences the regulation quality of hydroelectric unit directly.

1. Introduction

With the increase of people’s consciousness of environmental protection, more and more renewable energy has been applied to replace the traditional energy, such as wind power [1, 2] and solar power [3]. As wind power and solar power are connected with the large-scale power grid, people want to maximally utilize renewable energy on the premise that power balance is maintained [4–6]. However, the output of wind power and solar power is not steady due to the intermittent and fluctuating nature. Therefore the power grid needs regulating equipment to keep power balance. As the hydroelectric unit is easier for changing the output power, it has been widely used as power and frequency regulating equipment in power grid [7, 8].

Many research works have been done to improve the control quality; these works can be mainly divided into two categories: proposed new control models instead of PID controller and optimized control parameters of HTGS. In [10], a fractional order PID controller, whose order of derivative portion and integral portion is not integer, is proposed for HTGS. The fractional order PID controller provides more flexibility in achieving control objective. In [11], a fuzzy-PID controller is designed to improve control quality of HTGS. In [12], the sliding mode variable structure control strategy led to HTGS which has better robustness and adaptability than PID controller. In [13], a new adaptive inverse control method based on the learning characteristic of neural network was proposed for HTGS to improve the dynamic and stationary performance. All the above control models are useful, but they ignore that it is hard to change the control device to reach the control models for most operating hydroelectric units. Thus control parameter optimization seems more suitable to improve control quality for actual situation. To optimize PID controller, some popular optimization algorithms, including genetic algorithm (GA) [14], particle swarm optimization...
(PSO) [15], gravitational search algorithm (GSA) [16, 17],
and ant lion optimization (ALO) [18], have been success-
fully applied in parameter optimization of HTGS. All these
algorithms optimize the PID parameters by optimizing one
objective function which means the optimal parameters are
under one single operating condition; few of the researches
have taken multiple operation conditions into consideration.
The optimal results may not be suitable for variation of
operation conditions of HTGS. Therefore, the study on
parameter optimal control of HTGS under multiple operation
conditions is meaningful and challengeable.

Parameter optimal control of HTGS under multiple
operation conditions actually refers to the multiobjective
optimization problems. Some multiobjective optimization
algorithms have been proposed and successfully applied in
various applications in decades [19–25], such as nondom-
inated sorting genetic algorithm II (NSGA-II) [19], multi-
objective evolutionary algorithm based on decomposition
(MOEA/D) [21], the strength Pareto evolutionary algorithm
(PESA-II) [22], multiobjective particle swarm optimization
(MOPSO) [23], and the multiobjective grey wolf optimizer
(MOGWO) [25]. Among these multiobjective optimization
algorithms, MOGWO algorithm has proved to be effective in
multiobjective optimization problems than other algorithms
[25]. However, as the guidance of lead wolves is much greater
than the random factor, this algorithm is easy to fall into local
optimum and has poor stability. Therefore it is necessary to
enhance the searching ability of MOGWO.

The main contribution of this paper is the design and
optimization of HTGS under multiple operation conditions
by using an improved MOGWO algorithm. Firstly, the
problem in optimal control of HTGS is explained and mul-
tiobjective optimal function is proposed. Secondly, a novel
MOGWO algorithm based on searching factor (sMOGWO)
is proposed to optimize the multiobjective problem. The
sMOGWO is expected to improve the searching ability of
MOGWO from two aspects: adding searching step to search
more no-domain solutions nearby the wolves and adjusting
control parameters to keep exploration ability in later period.

The remaining part of this paper is organized as follows:
the HTGS model and its control problem are discussed in
Section 2. In Section 3, the weakness of MOGWO algo-

The PID controller has been widely used in HTGS. In
this paper, the structure of the PID controller model and
electrohydraulic servo system is set as in Figure 1. Many non-
linear factors have been considered in the electrohydraulic
servo system, such as the dead zone and the relay device
limiting, where \( x_i \) is the frequency giving; \( x_{di} \) is the dis-
bance frequency; \( x \) is the frequency of the hydrogenerator
unit; \( k_p, k_d, k_i \) are the proportionality coefficient, differential

2. HTGS Model and Its Control Problems

2.1. The HTGS Model. The HTGS is essential equipment of
the hydropower station. The HTGS contains PID controller,
electrohydraulic servo system, hyroturbine, and hydrogen-
generator.

\[
m_t = m_i (y, h, \omega)
\]

\[
q = q (y, h, \omega)
\]

where \( m \) is the hydraulic machinery power, \( q \) is discharge of
hydraulic turbine, \( y \) represents the gate opening, \( \omega \) represents
speed of hydraulic turbine, and \( h \) represents the water head.

In this paper, the little fluctuation in transient of hydraulic
power station is researched; the model of hydraulic turbine
can be described as liner model, as follows:

\[
m_t = e_x \omega + e_y y + e_h h
\]

\[
q = e_{qx} \omega + e_{qy} y + e_{qh} h
\]

where \( e_h \) is the transfer coefficient of turbine torque on the
water head, \( e_x \) is the transfer coefficient of turbine torque on
the guide leaf opening, \( e_y \) is the transfer coefficient of turbine
torque on the speed, \( e_{qx} \) is the transfer coefficient of turbine
flow on the speed, \( e_{qy} \) is the transfer coefficient of turbine
flow on guide leaf opening, and \( e_{qt} \) is the transfer coefficient of
turbine flow on the water head.

In water diversion system, the water hammer and pipe
wall can be considered as rigid under small fluctuations.
The transfer function of rigid water diversion system can be
expressed as

\[
G_r (s) = - \frac{1}{T_w s}
\]

where \( T_w \) is the water inertia time constant.

In this paper, the core problem is the dynamic response of
the HTGS. Hence, the dynamic speed of the generator is
considered. The model of the generator can be described as the
following transfer function:

\[
\frac{x}{m_i} = \frac{1}{T_a s + e_g}
\]

where \( x \) represents the frequency of the hydrogenerator unit,
\( T_a \) represents the inertia time constant, and \( e_g \) is the adaptive
control coefficient.

As in the above description of the mathematical model,
the hydrogenerator unit model can be described as the block
diagram in Figure 2.
2.2. Optimal Control of HTGS and Problems Analysis. The performance indexes of the optimal control of the HTGS are the most direct measure to evaluate the control quality. Therefore, the optimal control of the HTGS is the optimal control of performance indexes. The objective functions of optimal control are formed by a certain performance index or several performance indexes in most researches. The performance indexes in HTGS can be mainly divided into the following two categories [26]:

(i) Performance indexes for the transition process mainly include adjusting time, rising time, steady state error, and overshoot.

(ii) Performance indexes of error functional integral mainly include integral time absolute error (ITAE), integral absolute error (IAE), integral square time absolute error (ISTAE), integral square time square error (ISTSE), integral time square error (ITSE), and integral square error (ISE).

The performance indexes of error functional integral are comprehensive indexes which make optimal control easier to meet the control requirements. The ITAE is one of the most widely used which has the characteristics of steady adjustment and small overshoot.

Many researches have been done for optimal control of HTGS by optimizing ITAE. However most of the researches are for one single operation condition. As the hydrogenerator usually plays the roles of generating electricity, peak modulation, frequency modulation, and voltage modulation, there is a variety of operation conditions and frequent changes. The optimal control parameters may not be suitable for all the operation conditions.

Figure 1: PID control device and electric-hydraulic servo system.

Here, we will give an example to explain this problem. Parameters of two classical operation conditions of the above HGTS model are given as follows:

No-load: $T_{y_1}=0.02s$, $T_y=0.1s$, $T_w=0.66s$, $T_{1v}=0.28$, $T_a=8.51s$, $b_p=0.01$, $e_g=1$, $e_0=0.508$, $e_g=0.903$, $e_c=-0.242$, $e_G=0.634$, $e_{ph}=-0.396$, $e_{ph}=0.261$.

On-load: $T_{y_1}=0.02s$, $T_y=0.1s$, $T_w=0.66s$, $T_{1v}=0.28$, $T_a=8.51s$, $b_p=0$, $e_g=1$, $e_0=1.34$, $e_g=0.926$, $e_c=-1.184$, $e_G=0.371$, $e_{ph}=0.974$, $e_{ph}=0.308$.

5% step disturbance is set for HGTS. The optimal control parameters of HTGS under no-load condition are acquired by optimizing ITAE. The parameters are $K_p=14.665$, $K_i=3.026$, and $K_d=0$. The control transient process is shown in Figure 3. The control effect is well. However, if the parameters are used for the on-load condition, it is an ineffective control strategy.

The result indicates that the optimal control parameters of one single operation condition cannot be suitable for all the operation conditions. Accordingly, the optimal control of HTGS is a multiobjective optimization problem in fact. In this paper, we have proposed a novel control parameters optimization objective function. Different operation conditions are considered for optimal control. The control transient processes are shown in Figure 3. The ITAE of both conditions are the objective functions. The objective function can be described as follows:

$$\min J_1 = ITAE_{no-load} = f_1(K_p, K_i, K_d, y)$$

$$J_2 = ITAE_{on-load} = f_2(K_p, K_i, K_d, y)$$

subject to

$$0 \leq K_p \leq 15$$

$$0 \leq K_i \leq 15$$

$$0 \leq K_d \leq 5$$

$$0 \leq y \leq 1$$

where $f_1$ and $f_2$ represent the relationship between the parameters and the ITAE. Equation (6) represents the range of parameters.

3. An Improved MOGWO Algorithm

MOGWO algorithm as a new intelligent optimization algorithm has a better convergence rate than other intelligent optimization algorithms. Therefore, it has received great attention and wide application since it has been proposed.
3.1. Grey Wolf Optimizer and MOGWO Algorithm. The grey wolf optimizer (GWO) algorithm proposed by Mirjalili et al. in 2014 is a new intelligent optimization algorithm mimicking the hierarchies and hunting strategies of wolves [27]. In GWO, each grey wolf is treated as a potential solution. The grey wolves at the best solution, the second best solution, and the third best solution are treated as $\alpha$, $\beta$, and $\delta$ wolves. The rest of the grey wolves are treated as $\omega$ wolves. The equations that simulate the guidance of hunting are as follows:

$$D = |C \cdot X_p(t) - X(t)| \quad (7)$$

$$X(t + 1) = X_p(t) - A \cdot D \quad (8)$$

where $X$ represents the position vector of a grey wolf, $X_p$ represents the position vector of the prey, and $t$ represents the current iteration. $A$ and $C$ are coefficient vectors subject to the following equations:

$$A = 2 \cdot a \cdot r_1 - a \quad (9)$$

$$C = 2 \cdot r_2 \quad (10)$$

where $r_1$ and $r_2$ are random vectors from 0 to 1. $a$ is control parameter from 2 to 0 and it will linearly decrease as the number of iterations increases.

In 2016, Mirjalili proposed a multiobjective GWO algorithm named ‘MOGWO algorithm’ for multiobjective problems. There are two major changes in MOGWO: using the external population Archive to store the current non-dominated solutions and proposing a selection strategy for multiobjective optimization.

3.2. sMOGWO Algorithm. Although the MOGWO algorithm has a better convergence rate, it is easy to fall into local optimum and has poor stability. The main reasons are summarized as follows:

(i) The algorithm has great randomness only when initializing the position of wolves. Even though there is random factor in algorithm, when the position of wolves is updated, the effect of the guidance of lead wolves is much greater than the random factor. Thus the algorithm is highly dependent on the initial value, and self-regulation ability is weak.

(ii) The MOGWO algorithms select $\alpha$, $\beta$, and $\delta$ wolves from Archive. However, if Archive set falls into local optimum, the algorithms can hardly skip the local optimum. Therefore, the exploration ability of the algorithm needs to be improved at later period of the optimization.

As the guidance of lead wolves has too much influence, the grey wolves always blindly follow the lead wolves and the nondominated solutions around the lead wolves. The grey wolves always ignore the nondominated solutions beside them. Actually, the grey wolves often pass nearby other nondominant solutions when following the lead wolves. If the grey wolves have the ability of independent searching, the global optimization ability of the algorithm will be greatly improved.

Consequently, in this paper, we proposed to add the searching factor in MOGWO named ‘sMOGWO algorithm’. After the position updating of wolves, the wolves will have the searching step. Each wolf will search the nearby position randomly; if the nearby position is better than the current position, the wolf will move to the new position. The step can be described as the following mathematical expressions:

$$\Delta (x_1, x_2, \cdots, x_{\text{dim}}) = \begin{cases} x_k = r \cdot (u_b k - l_b k), & k = \text{rand} \{1, 2, \cdots, \text{dim}\} \\ x_i = 0, & i = 1, 2, \cdots, \text{dim} \text{ and } i \neq k \end{cases} \quad (11)$$

$$X_{\text{searching}} = X + \Delta \quad (12)$$

$$X_{\text{new}} = X_{\text{searching}}, \text{ if } f(X_{\text{searching}}) > f(X) \quad (13)$$

$$X_{\text{new}} = X, \text{ otherwise}$$

where $\Delta$ is the searching step of the wolf, $r$ is a random number between -0.5 and 0.5, $u_b k$ and $l_b k$ are the upper and lower boundary of $x_k$, dim is the dimension of wolf, $X$ is the
The gray wolves and running parameters are initialized

The objective function values are calculated. The Archive set is established.

Control parameters and coefficient vectors are calculated. The lead wolves are selected.

The position of wolves is updated

The wolves get into the searching step

The objective function values of each individual in wolves are calculated. The Archive set is updated

The updated Archive population is regrouped. The extra element is deleted

If $it=$Maxit

No

Yes

Optimization results of the algorithm

The flow chart of the sMOGWO algorithm.

Figure 4: The flow chart of the sMOGWO algorithm.

The position of the wolf, $X_{new}$, is the new position after searching, and $>$ represents the dominance relationship.

In the original algorithm, the control parameter $a$ linearly decreases as the number of iterations increases to keep high exploration ability at early period and high exploitation ability at later period. However, there is higher demands for exploration ability in multiobjective optimization. Therefore, in this paper, the control parameter $a$ is also modified. The adjustment strategy of control parameter $a$ must be subject to the following:

$$a = 2 - 2 \cdot f \left( \frac{it}{Maxit} \right)$$

where $Maxit$ is the maximum number of iterations and $f(x)$ is a monotone increasing function, in original algorithm $f(x) = x$.

The higher the value of $a$, the stronger the exploration ability of the algorithm, so the $f(x)$ should be a concave function among $[0, 1]$. Here, we proposed to use the following equation for control parameter $a$.

$$a = 2 - 2 \cdot \left( \frac{it}{Maxit} \right)^2$$  (15)

The flow chart of the sMOGWO algorithm is shown in Figure 4.

The steps of the sMOGWO algorithm are as follows.

**Step 1.** Archive size is $N_A$, population size of grey wolves is $N$, number of grids per dimension is $dim$, inflation rate is $a$, and maximum number of iterations is $Maxit$. Leader selection pressure and deletion selection pressure are also set. The grey wolves and running parameters are initialized.

**Step 2.** The objective function values of each individual wolf are calculated. The Archive set is established, and the Archive population is grouped into iterative process.

**Step 3.** The $\vec{A}$, $\vec{C}$, and $a$ are calculated according to (9), (10), and (15); then the lead wolves are selected.

**Step 4.** The position of wolves is updated according to (7) and (8).
Step 5. The wolves get into the searching step according to (11), (12), and (13).

Step 6. The objective function values of each individual in wolves are calculated. The nondominant solutions of wolves are compared to the individuals in the Archive population one by one, and the Archive set is updated.

Step 7. The updated Archive set is regrouped and the number of individuals in the Archive population is checked. If the number of individuals exceeds the maximum of population, the extra solutions will be deleted.

Step 8. The algorithm ends when the maximum number of iterations is reached. All individuals in the Archive population are the optimization results of the algorithm. Otherwise, the algorithm returns back to Step 3.

3.3. Computational Complexity Analysis. The computational complexity of MOGWO and sMOGWO is analyzed in this part.

The computational complexity of each iteration (Step 3 to Step 7) of the iterative process is analyzed one by one. The number of individuals in the current Archive set is \( A \), the number of individuals in the grey wolf population is \( N \), and the dimension of each individual is \( \text{dim} \).

In Step 3, lead wolves are selected by calculating the roulette probability for each individual in the Archive set. The computational complexity can be expressed as \( O(A) \).

In Step 4, the computational complexity of position updating can be expressed as \( O(N \times \text{dim}) \).

In Step 5, the objective function values need be calculated by searching step, and the computational complexity can be expressed as \( O(N \times \text{dim}) \).

In Step 6, the nondominant solutions of wolves are compared to the individuals in the Archive set one by one to update the Archive set. The computational complexity is \( O(A \times N \times \text{dim}) \).

In Step 7, the Archive sets are grouped and sorted, and the computational complexity is \( O(A^2) \).

The computational complexity of sMOGWO in each iteration is \( O(A^2) + O(A \times N \times \text{dim}) \). The MOGWO includes all the above steps expect Step 5, and the computational complexity of MOGWO is also \( O(A^2) + O(A \times N \times \text{dim}) \). The proposed modified method has the same computational complexity as the original method. In addition, although an additional Step 5 is added to the sMOGWO algorithm, it can effectively reduce Archive set with a large number of similar nondominant solutions, which has great influence on the computational complexity. In fact, the proposed method can reduce the optimization time.

4. Simulation Verification

In this section, simulation verification is present to demonstrate the advantage of the proposed sMOGWO algorithm. The proposed algorithm is compared to the MOGWO algorithm [25] and some other popular multiobjective optimal algorithms [28]: MOPSO, NSGA-III, MOEA/D, and SPEA2.

4.1. Test Problems. UF series test problems in CEC 2009 are multimode test functions [29], and there are a lot of locally optimal solutions, so it is suitable for test of multiobjective optimal algorithms. Three typical test problems are chosen as
Table 1: The parameters of different multiobjective optimization algorithms.

<table>
<thead>
<tr>
<th>Description</th>
<th>sMOGWO</th>
<th>MOGWO</th>
<th>MOPSO</th>
<th>NSGA-III</th>
<th>MOEA/D</th>
<th>SPEA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum number of iterations</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Population size</td>
<td>(nPop=100)</td>
<td>(nPop=100)</td>
<td>(nPop=100)</td>
<td>(nPop=100)</td>
<td>(nPop=100)</td>
<td>(nPop=100)</td>
</tr>
<tr>
<td>Archive size</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>(\alpha=0.1)</td>
<td>(\alpha=0.1)</td>
<td>(\alpha=0.1)</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Number of grids per dimension</td>
<td>(nGrid=10)</td>
<td>(nGrid=10)</td>
<td>(nGrid=10)</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Leader selection pressure</td>
<td>(\beta=4)</td>
<td>(\beta=4)</td>
<td>(\beta=4)</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Deletion selection pressure</td>
<td>(\gamma=2)</td>
<td>(\gamma=2)</td>
<td>(\gamma=2)</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Inertia weight</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>(\omega_{damp}=0.99)</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Inertia weight damping rate</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>(\omega_{damp}=0.99)</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>(mu=0.1)</td>
<td>(mu=0.1)</td>
<td>(mu=0.1)</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Number of Neighbors</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>10</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

Table 2: Statistical results of GD for different algorithms.

<table>
<thead>
<tr>
<th>GD</th>
<th>sMOGWO</th>
<th>MOGWO</th>
<th>MOPSO</th>
<th>NSGA-III</th>
<th>MOEA/D</th>
<th>SPEA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>UF3 Mean</td>
<td>0.0210</td>
<td>0.0542</td>
<td>0.0853</td>
<td>0.0268</td>
<td>0.0561</td>
<td>0.0254</td>
</tr>
<tr>
<td>Med</td>
<td>0.0211</td>
<td>0.0540</td>
<td>0.0737</td>
<td>0.0255</td>
<td>0.0571</td>
<td>0.0252</td>
</tr>
<tr>
<td>Max</td>
<td>0.0253</td>
<td>0.0900</td>
<td>0.2322</td>
<td>0.0716</td>
<td>0.0829</td>
<td>0.0342</td>
</tr>
<tr>
<td>Min</td>
<td>0.0180</td>
<td>0.0315</td>
<td>0.0425</td>
<td>0.0166</td>
<td>0.0299</td>
<td>0.0116</td>
</tr>
<tr>
<td>UF4 Mean</td>
<td>0.0239</td>
<td>0.0331</td>
<td>0.0361</td>
<td>0.0516</td>
<td>0.0409</td>
<td>0.0485</td>
</tr>
<tr>
<td>Med</td>
<td>0.0241</td>
<td>0.0330</td>
<td>0.0363</td>
<td>0.0521</td>
<td>0.0439</td>
<td>0.0475</td>
</tr>
<tr>
<td>Max</td>
<td>0.0257</td>
<td>0.0394</td>
<td>0.0466</td>
<td>0.0622</td>
<td>0.0526</td>
<td>0.0618</td>
</tr>
<tr>
<td>Min</td>
<td>0.0234</td>
<td>0.0259</td>
<td>0.0324</td>
<td>0.0415</td>
<td>0.0320</td>
<td>0.0340</td>
</tr>
<tr>
<td>UF7 Mean</td>
<td>0.0078</td>
<td>0.0267</td>
<td>0.0468</td>
<td>0.0319</td>
<td>0.0802</td>
<td>0.0313</td>
</tr>
<tr>
<td>Med</td>
<td>0.0077</td>
<td>0.0278</td>
<td>0.0424</td>
<td>0.0316</td>
<td>0.0584</td>
<td>0.0301</td>
</tr>
<tr>
<td>Max</td>
<td>0.0091</td>
<td>0.1012</td>
<td>0.0881</td>
<td>0.0726</td>
<td>0.1751</td>
<td>0.0608</td>
</tr>
<tr>
<td>Min</td>
<td>0.0072</td>
<td>0.0148</td>
<td>0.0202</td>
<td>0.0140</td>
<td>0.0182</td>
<td>0.0082</td>
</tr>
</tbody>
</table>

follows: UF3 whose Pareto boundary is concave shape, UF4 whose Pareto boundary is convex shape, UF7 whose Pareto boundary is line. The three UF problems are as follows:

**UF3 problem**, \(n=30\), the search space is \([0, 1]^n\):

\[
\begin{align*}
\text{min} & \quad f_1 = x_1 \\
& + \frac{2}{|J_1|} \left( 4 \sum_{j \in J_1} y_j^2 - 2 \prod_{j \in J_1} \cos \left( \frac{20y_j\pi}{\sqrt{j}} \right) \right) \\
& + 2 \\
\end{align*}
\]

subject to \(y_j = x_j - x_1^{0.5(1+3(j-2)/(n-2))}\), \(j = 2, \ldots, n\),

\(J_1 = \{ j \mid j \text{ is odd and } 2 \leq j \leq n \}\)

\(J_2 = \{ j \mid j \text{ is even and } 2 \leq j \leq n \}\)

(16)

**UF4 problem**, \(n=30\), the search space is \([0, 1] \times [-2, 2]^{n-1}\):

\[
\begin{align*}
\text{min} & \quad f_1 = x_1 + \frac{2}{|J_1|} \sum_{j \in J_1} h(y_j) \\
& \quad f_2 = 1 - x_1^2 + \frac{2}{|J_1|} \sum_{j \in J_1} h(y_j) \\
\text{subject to} & \quad y_j = x_j - \sin \left( 6\pi x_1 + \frac{j\pi}{n} \right), \\
& \quad j = 2, \ldots, n, \\
& \quad h(t) = \frac{|t|}{1 + e^{2|t|}} \\
& \quad J_1 = \{ j \mid j \text{ is odd and } 2 \leq j \leq n \} \\
& \quad J_2 = \{ j \mid j \text{ is even and } 2 \leq j \leq n \} \\
\end{align*}
\]

(17)
Table 3: Statistical results of SP for different algorithms.

<table>
<thead>
<tr>
<th></th>
<th>SP</th>
<th>sMOGWO</th>
<th>MOGWO</th>
<th>MOPSO</th>
<th>NSGA-III</th>
<th>MOEA/D</th>
<th>SPEA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>UF3</td>
<td>Mean</td>
<td>0.5719</td>
<td>1.0807</td>
<td>0.9091</td>
<td>1.0130</td>
<td>1.0233</td>
<td>0.9992</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.53456</td>
<td>1.1129</td>
<td>0.9073</td>
<td>1.0000</td>
<td>1.0196</td>
<td>1.0002</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.7215</td>
<td>1.4434</td>
<td>1.0411</td>
<td>1.0850</td>
<td>1.0747</td>
<td>1.0002</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.4490</td>
<td>0.6002</td>
<td>0.7810</td>
<td>0.9991</td>
<td>1.0000</td>
<td>0.9971</td>
</tr>
<tr>
<td>UF4</td>
<td>Mean</td>
<td>0.6098</td>
<td>0.8207</td>
<td>0.7923</td>
<td>0.9991</td>
<td>1.2974</td>
<td>1.5026</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.6217</td>
<td>0.8055</td>
<td>0.7707</td>
<td>0.9940</td>
<td>1.2924</td>
<td>1.4754</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.6551</td>
<td>1.0963</td>
<td>0.9160</td>
<td>1.0616</td>
<td>1.4337</td>
<td>1.9870</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.5428</td>
<td>0.6724</td>
<td>0.6908</td>
<td>0.9464</td>
<td>1.1553</td>
<td>1.1513</td>
</tr>
<tr>
<td>UF7</td>
<td>Mean</td>
<td>0.6646</td>
<td>0.8731</td>
<td>0.8649</td>
<td>1.0961</td>
<td>1.0697</td>
<td>0.9995</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0.6646</td>
<td>0.8392</td>
<td>0.8386</td>
<td>1.0850</td>
<td>1.0015</td>
<td>0.9996</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>0.7011</td>
<td>1.2232</td>
<td>1.0611</td>
<td>1.2400</td>
<td>1.5393</td>
<td>1.0760</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>0.6186</td>
<td>0.5954</td>
<td>0.7292</td>
<td>0.9972</td>
<td>1.0000</td>
<td>0.9553</td>
</tr>
</tbody>
</table>

UF7 problem, n=30, the search space is $[0, 1] \times [-1, 1]^{n-1}$:

$$\min f_1 = \sqrt{x_1} + \frac{2}{|J_1|} \sum_{j \in J_1} y_j^2$$

$$f_2 = 1 - \sqrt{x_1} + \frac{2}{|J_2|} \sum_{j \in J_2} y_j^2$$

subject to $y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n})$, $j = 2, \ldots, n$,

$J_1 = \{j \mid j \text{ is odd and } 2 \leq j \leq n\}$

$J_2 = \{j \mid j \text{ is even and } 2 \leq j \leq n\}$.

4.2. Parameters Setting. The principles and the operation modes of the algorithms are different. In order to make the algorithm contrast, the same maximum number of iterations, population size, and Archive size are set up. The parameters of different multiobjective optimization algorithms are shown in Table 1.

4.3. Performance Evaluation Indexes. The Generational Distance (GD) and the Spacing (SP) [30] have been used to evaluation the performance of each algorithm.

$$GD = \frac{1}{N} \sqrt{\sum_{i=1}^{N} D_i^2}$$

where $D_i$ is the Euclidean distance between the $i$th nondominant solution in Pareto solution set and the closest nondominant solution on the real Pareto front. $N$ is the number of Pareto optimal solutions. The smaller value of GD indicates that the Pareto optimal solution is closer to the real Pareto front.

$$SP = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1) \cdot \bar{d}}$$

where $d_f$ and $d_l$ are the Euclidean distances between the endpoint of Pareto optimal solution and real Pareto front. $N$ is the number of Pareto optimal solutions. The smaller value of SP indicates that Pareto optimal solution distribution is more homogenized.

4.4. Result Analysis. In order to eliminate the contingency, each algorithm is run 30 times independently. The statistical results of the evaluation indexes are shown in Tables 2 and 3. In order to reflect the advantages and disadvantages of the optimization results more directly, the boxplots of GD value and SP value of each algorithm are given as in Figures 5 and 6.

The best optimal results of the algorithms in repeated experiments are shown in Figures 7, 8, and 9. The comparison is more intuitive.

For UF3 test problem, the sMOGWO, NSGA-III, and SPEA2 are the first class in GD value which means the solutions of these methods are closer to the real Pareto front. The GD values of MOPSO, MOGWO, and MOEA/D are high and fluctuating. The sMOGWO method has a better SP value which means the solutions are well distributed. Although the NSGA-III and SPEA2 are closer to the real Pareto front, the SP values are high, which means the solutions of these methods tend to fall into local optimum. We can discover that the NSGA-III and SPEA2 methods fall into local optimum on UF3 test problem intuitively according to Figure 7. The solutions of the sMOGWO method are the best.

For UF4 test problem, the sMOGWO has the best GD value. The MOPSO, MOGWO, and MOEA/D are in the second class in GD value. The sMOGWO, MOGWO, and MOPSO are in the first class in SP value. From Figure 8, the best optimal results of sMOGWO, MOPSO, MOGWO, and MOEA/D are all close to the real Pareto front and are well distributed.

For UF7 test problem, the GD values of sMOGWO, MOGWO, NSGA-III, and SPEA2 are in the first class, but the SP value of sMOGWO is better than those of other methods. From Figure 9, the best optimal result of all the
methods is close to the real Pareto front, but the solutions of MOPSO, MOGWO, MOEA/D, NSGA-III, and SPEA2 are not well distributed. Some methods fall into local optimum. The solutions of the sMOGWO method are the best.

Accordingly, the proposed sMOGWO algorithm has good stability under all test functions, and the proposed algorithm performs well in repeated experiments with few poor results.

Figure 6: The boxplots of SP value of three algorithms.
**Figure 7:** The best optimal results of the different algorithms in repeated experiments on UF3.

**Figure 8:** The best optimal results of the different algorithms in repeated experiments on UF4.
Complexity

5. Case Study

In this section, a case study is presented to show the effect of the proposed algorithm on optimal control of HTGS under multiple operation conditions.

The no-load and on-load operation conditions are considered. The model of HTGS and its parameters are provided in Section 2. The multiobjective functions refer to (5) and (6). The simulation time is 20 seconds. The sMOGWO, MOGWO, MOPSO, MOEA/D, NSGA-III, and SPEA2 algorithms are utilized for optimal control. The maximum number of iterations is 100. The other parameters of the three algorithms are set as in Table 1. Each algorithm is run 30 times independently. The best optimal results of each algorithms in repeated experiments are shown in Figure 10.

The solutions of MOPSO method are far away from Pareto front. The solutions of sMOGWO, MOGWO, MOPSO, MOEA/D, NSGA-III, and SPEA2 are similar to the Pareto front, but the solutions of sMOGWO are more evenly distributed than those of other methods.

In order to analyze the detailed control transient process of the optimization results, some representative control strategies are chosen from the Pareto optimal solution set as shown in Table 4. It can be seen that the optimization of ITAE under no-load and ITAE under on-load are opposites. Transient process of some of the control strategies is shown in Figure 11 to exhibit control effects more intuitively.

All the three control strategies have good control stability under multiple operation conditions. Strategy 1 is most stable.
The Pareto front of sMOGWO, MOGWO and MOPSO

The Pareto front of sMOGWO and MOEA/D

The Pareto front of sMOGWO and NSGA-III

The Pareto front of sMOGWO and SPEA2

Figure 10: The Pareto front of different algorithms.

Figure 11: Transient process by different control strategies.
under on-load operation condition, but it has bigger overshoot than the other two strategies under no-load operation condition. Strategy 3 has the smallest overshoot among the three strategies under no-load operation condition, but its stability is the worst among the control strategies under on-load operation condition. Strategy 2 is a compromise control strategy.

Thus, a Pareto solution set can be got after one optimization which can be suitable for multiple operation conditions. And the solutions have different emphasis on objective functions. According to the specific requirements of the HTGS, the decision maker can select one or some satisfactory solutions from this solution set as the final solution, so that the HTGS can obtain better control quality.

6. Conclusion

The control strategies of hydraulic turbine governing system need to consider the multiple operation conditions. A multi-objective optimal function under different operation conditions is proposed in this paper to solve this control problems. In order to optimize the multiobjective problems more effectively, a novel MOGWO algorithm based on searching factor (SMOGWO) is proposed. The SMOGWO method is verified with several UF test problems compared to MOGWO, MOPSO, MOEA/D, NSGA-III, and SPEA2. The SMOGWO provided better solution compared with its competitors. And the proposed algorithm has good stability under all test functions, and the proposed algorithm performs well in repeated experiments with few poor results.

A case study has been designed to test the control quality of the control strategies which are got by the proposed method. The experimental results have confirmed that the control strategies perform well under multiple operation conditions.

Data Availability

The simulation data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

We declare that we have no conflicts of interest regarding the publication of this manuscript.

Acknowledgments

This paper is supported by the National Natural Science Foundation of China (no. 51709121, no. 51709122) and Six Talent Peaks Project of Jiangsu Province of China (no. RJFW-028).

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