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Research Article

MI-Based Robust Waveform Design in Radar and Jammer Games

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Due to the uncertainties of the radar target prior information in the actual scene, the waveform designed based on the radar target prior information cannot meet the needs of parameter estimation. To improve the performance of parameter estimation, a novel transmitted waveform design method under the hierarchical game model of radar and jammer, which maximizes the mutual information (MI) between the radar target echo and the random target spectrum response, is proposed. In the hierarchical game model of radar and jammer, the radar is in a leading position while the jammer is in a following position. The strategy of the jammer is optimized based on the radar transmitted waveform of previous moment, then the radar selects its own strategy based on the strategy of the jammer. It is generally assumed that the radar and the jammer have intercepted the real target spectrum and then the optimal jamming and the optimal transmitted waveform spectrum are obtained. However, the exact characteristic of the real target spectrum is hard to capture accurately in actual scenes. To simulate this, the real target spectrum is considered to be within an uncertainty range which is confined by known upper and lower bounds. Then, the minimax robust jamming and the maximin robust transmitted waveform are designed successively based on the MI criteria, which optimizes the performance under the most unfavorable condition of the radar and the jammer, respectively. Simulation results demonstrate that the robust transmitted waveform design method guarantees the parameter estimation performance effectively and provides useful guidance for waveform energy allocation.

1. Introduction

Cognitive radar (CR) is a new radar system concept proposed in recent years. This system is inspired by the bat echolocation, which improves the system performance of the radar through using the feedback structure from the receiver to the transmitter to optimize the transmitted waveform based on the recognition of the target and the scene [1]. The transmitted waveform of the traditional radar is independent of the environment and each transmission repeats the same waveform. Therefore the research of the traditional radar is devoted to optimizing the receiver design through radar signal processing [2]. Different from the traditional radar, CR transmitter can adjust the transmitted waveform adaptively to achieve optimal matching with the environment according to the acquired information [3]. During the past decades, many adaptive waveform design methods for radar target parameter estimation have been developed by a lot of experts and scholars. MI is a useful information metric in information theory [4], which has been widely adopted in cognitive radar and other engineering fields [5, 6]. One important method for radar waveform design is to use information theory. Researchers such as Vaidyanathan applied information theory to the radar of the MIMO system [7]. Many radar experts are devoted to improving the parameter estimation performance by boosting MI [8–10]. The innovative study in [9] optimizes the transmitted waveform through maximizing the conditional MI between the radar target echo and the random target spectrum response. Kwon et al. studied multitarget detection at low SNR, using the maximum eigenvalue of the sample covariance matrix and the correlation coefficient between the transmitted signal and the echo signal to obtain a modified full correlation detector from the perspective of average mutual information [11]. Under certain assumptions, such as fully known noise PSD and white noise, better estimation performance can be obtained by maximizing MI, as shown in [12].

However, the designed optimal waveforms under the environment of complex target model are not well known and do not consider the complex battlefield game environment, while in practice precise estimation of the real target spectrum is impossible and the game between radar and jammer is real existence in many cases. The mismatch of prior information of the real target and the ignorance of battlefield game environment might reduce the waveform performance transmitted by the radar transmitter.

In this paper, a novel transmitted waveform design technique based on MI under the environment of complex battlefield game and complicated target model is presented. Minimax robust jamming and maximin robust waveform design methods are proposed successively to reduce the impact of insufficient prior information on the performance of the designed waveform. Our main contribution is that the imprecise estimation of target spectrum [12] is considered in the optimal jamming and the optimal transmitted waveform design strategies. In addition, we also establish a hierarchical game model of radar and jammer, which regards radar as the leader and jammer as the follower. The minimax robust jamming and maximin robust transmitted waveform techniques under the established hierarchical game model above based on the MI are developed successively. In summary, first of all, given that the real target spectrum is known, the optimal jamming and optimal transmitted waveform design methods for random target based on MI are proposed successively. Secondly, by considering the uncertainty of the target spectrum, the MI-based minimax robust jamming and maximin robust transmitted waveform techniques are proposed successively. In this paper, we consider the single target model and multitarget model, respectively; then the minimax robust jamming and maximin robust transmitted waveform techniques under the two different target models above based on MI are proposed, respectively. The minimax robust jamming and maximin robust transmitted waveform design methods optimize the performance under the most unfavorable condition of the jammer and the radar transmitter, respectively. Their behaviour with regard to the uncertainty of target spectrum is also analyzed. The MI-based robust jamming and robust waveform provide useful guidance for waveform energy allocation strategy. These two waveform design techniques are easy to implement in an intelligent jammer and a cognitive radar and will have important applications in electronic warfare. In the actual situation, the minimax and maximin robust waveform design methods proposed in this paper can provide the most favorable strategies for intelligent jammer and radar, respectively, in complex target environments. And the waveform design method proposed in this paper can be well applied to the battlefield game environment which regards radar as the leader and the intelligent jammer as the follower. In electronic warfare, radar and jammer can design their own waveform, respectively, according to the strategy of their opponent, which can improve their own performance and weaken the performance of their opponent. As the method proposed in this paper assumes that the radar is more powerful than the jammer, the radar finally won this electronic warfare, and the transmitted waveform designed by radar maximizes the estimation performance of radar.

2. Signal Model and Problem Formulation

For the jammer, to impair the estimation performance, the minimization of MI means that the radar target echo contains little information on the target, which will result in poor performance of the radar transmitter. But for the radar transmitter, the maximization of the MI will improve the estimation performance.

2.1. Model of Random Target and Optimal Jamming Design Based on MI. In this subsection, to minimize the estimation performance of a general radar system, the optimal jamming design method based on MI is proposed. The model of the random target is given in Figure 1 [9, 13], where Figure 1(a) illustrates that the duration of the random target is finite. In this model, $a(t)$ denotes a window function with duration T_h and $g(t)$ represents a generalized stationary random process. Thus, the product $h(t) = a(t)g(t)$ is a generalized stationary random process within $[0, T_h]$. The random target model is shown in radar signal processing system in Figure 1(b), where $x(t)$ denotes the transmitted waveform signal and $h(t)$ represents the signal model of random target. The spectrum response of $x(t)$ can be denoted by $X(f)$, and similarly $H(f)$ is the spectrum response of $h(t)$. $r(t)$ denotes the signal model of receiver filter and $n(t)$ is a noise process with the power spectrum density (PSD) $S_{nn}(f)$. Likewise, $c(t)$ represents a jamming component which is a Gaussian random process and the PSD of $c(t)$ can be denoted by $J(f)$.

The energy spectrum variance (ESV) of $h(t)$ is represented as follows [9, 13].

$$\sigma_H{}^2(f) = \mathrm{E}\left[\left|H(f) - \mu_H(f)\right|^2\right] \tag{1}$$

In the expression of (1), $\mathrm{E}[\cdot]$ represents the expectation of an input entity, and $\mu_H(f)$ denotes the mean value of $H(f)$ which is assumed to be 0.

The signal model depicted in Figure 1(b) can be applied for MI-based jamming and transmitted waveform design. Therefore, the expression of MI for the model of random target can be denoted by [13]

$$MI = T_y \int_{BW} \ln\left[1 + \frac{\sigma_H{}^2(f)\left|X(f)\right|^2}{T_y\left(J(f)\left|X(f)\right|^2 + S_{nn}(f)\right)}\right] df \tag{2}$$

where T_y represents the duration of the radar target echo $y(t)$. The energy constraint of the jamming is set to be E_X. Therefore, for the jammer to impair the performance of the radar transmitter, the optimization problem can be expressed as follows.

$$\min_{J(f)} \quad \mathrm{MI}(J(f)) \tag{3}$$

$$\text{s.t.} \quad \int_{BW} J(f)\, df \le E_X \tag{4}$$

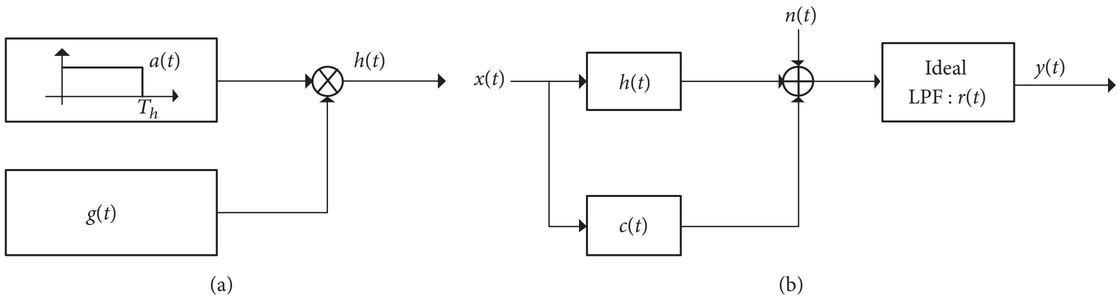

FIGURE 1: Random target model for waveform design based on MI. (a) Signal model for random target with finite duration T_h. (b) Signal model for waveform design based on MI.

In the expression of (4), BW is the bandwidth that the spectrum response of transmitted waveform and jamming are virtually limited to. The minimization of MI means that the radar target echo contains little information on the real target, which will result in poor performance for the radar transmitter. The equation of MI is expressed by the transmitted waveform of previous moment, the noise PSD, the target ESV, and the jamming PSD.

The optimal jamming solution obtained by Lagrange multiplier method that minimizes the MI (2) under the energy constraint (4) can be denoted by [14]

$$J(f) = \max\left[0, B(f)\left(A + D(f)\right)\right] \tag{5}$$

where

$$B(f) = -\frac{\sigma_H^2(f)\left|X(f)\right|^2}{2T_y \cdot S_{nn}(f) + \sigma_H^2(f)\left|X(f)\right|^2}, \tag{6}$$

$$D(f) = \frac{T_y S_{nn}^2(f) + \sigma_H^2(f)\left|X(f)\right|^2 S_{nn}(f)}{\sigma_H^2(f)\left|X(f)\right|^4}, \tag{7}$$

and A is a constant that can be derived from the energy constraint of the jamming.

$$\int_{BW} \max\left[0, B(f)\left(A + D(f)\right)\right] df \le E_X \tag{8}$$

The results show that the optimal jamming solution based on MI can be obtained by water-filling operation which assumes that the target spectrum $H(f)$ and transmitted waveform of previous moment $X(f)$ are greater than zero at each sampling frequency.

2.2. MI-Based Transmitted Waveform Design. According to the jamming solved above, for radar transmitter to improve the parameter estimation performance, MI is also adopted as the criterion. The energy constraint of the transmitted waveform is also set to be E_X. In (9), the MI is expressed by the jamming designed by jammer, the noise PSD, the target ESV, and the transmitted waveform spectrum. The designed optimal transmitted waveform should satisfy the following.

$$\max_{|X(f)|^2} \quad \mathrm{MI}\left(\left|X(f)\right|^2\right) \tag{9}$$

$$\text{s.t.} \quad \int_{BW} \left|X(f)\right|^2 df \le E_X \tag{10}$$

The maximization of MI means that the radar target echo contains much target information, which will result in rich parameter estimation performance for the radar.

The optimal transmitted waveform obtained by Lagrange multiplier method which maximizes the MI (9) under the energy constraint (10) can be written as

$$\left|\overline{X}(f)\right|^2 = \max\left[0, \overline{B}(f)\left(\overline{A} - \overline{D}(f)\right)\right] \tag{11}$$

where

$$\overline{B}(f) = \frac{\sigma_H^2(f)}{2T_y \cdot J(f) + \sigma_H^2(f)}, \tag{12}$$

$$\overline{D}(f) = \frac{T_y S_{nn}(f)}{\sigma_H^2(f)}, \tag{13}$$

and $\overline{A}$ is a constant that can be derived from the energy constraint of the transmitted waveform.

$$\int_{BW} \max\left[0, \overline{B}(f)\left(\overline{A} - \overline{D}(f)\right)\right] df \le E_X \tag{14}$$

The optimal transmitted waveform based on MI is obtained by performing a water-filling operation when the jamming spectrum, the target ESV, and the noise PSD are known. By using the designed transmitted waveform, the available energy can be used more efficiently in battlefield game environment, which will achieve better performance of the transmitted waveform. Simulation results show that the optimal jamming and optimal transmitted waveform actually lead to opposite energy allocation strategies.

Note that in the designed jamming and transmitted waveform above, the real target spectrum response is assumed to be fully known, while in practice the real target spectrum is difficult to capture. When the target model is blurred, the designed jamming and transmitted waveform based on target

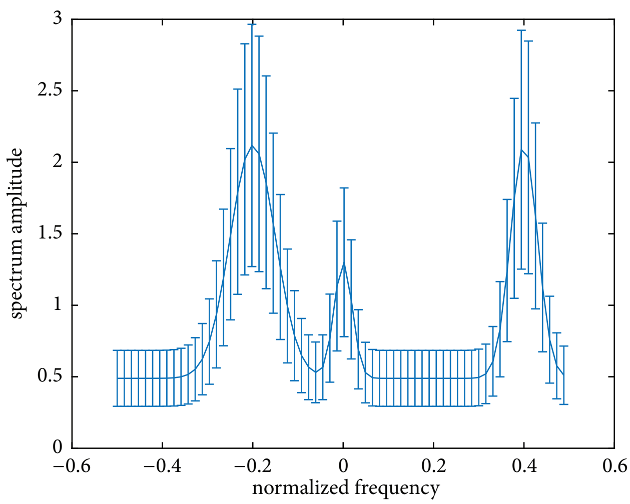

Figure 2: Model of the uncertainty range of single target spectrum.

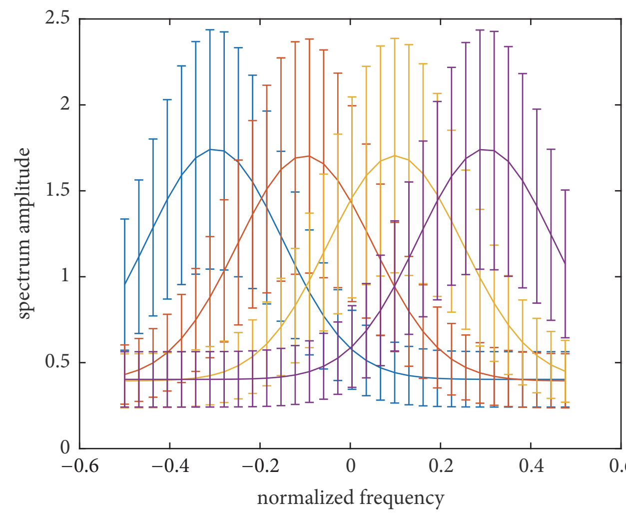

Figure 3: Model of the uncertainty range of multitarget spectrum.

prior information will not guarantee the performance of the radar or the jammer effectively, so it is critical to minimize the loss of the performance. Therefore, the robust jamming technique and robust transmitted waveform technique are considered next.

3. Maximin Robust Waveform Design

Taking the target spectrum uncertainty into account, the band model in [15] is adopted. For single target, assume that the real target spectrum exists in an uncertainty range ε where both the upper and the lower bound are known, that is,

$$H(f) \in \varepsilon = \{l_k \le H(f_k) \le u_k, k = 1, 2 \ldots, K\} \tag{15}$$

where f_k denotes the sampling frequency. The model of uncertainty single target is shown as Figure 2.

For multiple targets, assume that each real target spectrum of the multiple targets exists within an uncertainty range ε_i which is confined also by known upper and lower bounds, that is,

$$H_i(f) \in \varepsilon_i = \{l_{ik} \le H_i(f_k) \le u_{ik}, k = 1, 2 \ldots, K\} \tag{16}$$

where $i = 1, 2, 3, 4 \ldots$, which is used to distinguish different targets. The uncertainty class ε_i for each target is different. The model of uncertainty multiple targets is shown as Figure 3.

In practice, uncertainty target model is widely adopted in robust waveform design because the uncertainty range can be captured through spectrum estimation [15]. The larger the difference between the upper and the lower bound, the greater the uncertainty of the target spectrum. Moreover, pay attention to the fact that the difference in amplitude between the upper and the lower bound of the blurred target spectrum could be different at each sampling frequency.

Now the game model of radar and jammer is built: assume that the radar is in a leading position and the jammer is in a following position. The information of the leader and follower is not equal. Firstly, the strategy of the jammer is optimized according to the radar transmitted waveform of the previous moment. Then the radar selects its own strategy according to the jammer's strategy.

For each particular target spectrum, there exist an optimal jamming and an optimal transmitted waveform, respectively, for jammer and radar. However, the real target spectrum may vary in this uncertainty range, so the minimax robust technique for jammer and the maximin robust technique for radar are good approaches which guarantee the performance under the most unfavorable case. In this section, the minimax robust jamming technique and the maximin robust transmitted waveform technique for MI are proposed successively.

3.1. Minimax Robust Jamming Technique Based on MI. Let $\xi(J(f), {\sigma_H}^2(f))$ denote the optimization criterion MI. The expression of MI can be expressed by the jamming spectrum $J(f)$ and the target ESV ${\sigma_H}^2(f)$ or target spectrum $H(f)$. Note that the expressions of ${\sigma_H}^2(f)$ for single target and multiple targets are different, which will be given in Sections 3.1.1 and 3.1.2, respectively. The minimax robust jamming design method should satisfy the following [12, 15, 16].

$$\min_{J(f)} \left\{ \max_{|H(f)| \in \varepsilon} \xi\left(J(f), {\sigma_H}^2(f)\right) \Big|_{\int_{BW} J(f) df \le E_X} \right\} \tag{17}$$

Based on the theory of robust signal processing in [16], the solution of this minimax optimization problem can be denoted as follows.

$$\begin{aligned}
&\xi\left(J^{\min\max}(f),\sigma_H^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X}\\
&\quad\le \xi\left(J^{\min\max}(f),\sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X}\\
&\quad\le \xi\left(J(f),\sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J(f)df\le E_X}
\end{aligned}\tag{18}$$

From the right side of the inequality above, the minimax optimal jamming is optimal for the jammer when $\sigma_H^{\,2}(f) = \sigma_{H_{worst}}^{\,2}(f)$. It minimizes the performance of radar transmitter. If other jamming spectrum is adopted, the performance of the jammer will be degraded. Meanwhile, the left side of the inequality indicates that $\sigma_{H_{worst}}^{\,2}(f)$ is the most unfavorable target ESV for the minimax optimal jamming. If the minimax optimal jamming spectrum $J^{\min\max}(f)$ is adopted, for all target ESV in the uncertainty range ε or ε_i, the MI performance will be better than the unfavorable case, at least as good as the case of $\sigma_H^{\,2}(f) = \sigma_{H_{worst}}^{\,2}(f)$. Therefore the minimax optimal jamming for the most unfavorable target ESV within the uncertainty range is optimal. By ensuring the performance under the most unfavorable condition, the performance for all target spectra within the uncertainty range will not be worse than this case.

3.1.1. Minimax Robust Jamming Technique for Single Target Based on MI. For the jammer, the upper bound of the uncertainty range is taken as the most unfavorable target spectrum. Therefore the minimax robust jamming technique for single target based on MI should satisfy the following.

$$\min_{J(f)}\left\{\max_{|H(f)|\in\varepsilon}\mathrm{MI}\left(J(f),\sigma_H^{\,2}(f)\right)\Big|_{\int_{BW}J(f)df\le E_X}\right\}\tag{19}$$

Theorem 1. *The solution to the minimax optimum problem described in (19) is*

$$\widetilde{J}^{\min\max}(f) = \max\left[0,\widetilde{B}(f)\left(\widetilde{A}+\widetilde{D}(f)\right)\right]\tag{20}$$

where

$$\widetilde{B}(f) = -\frac{\sigma_U^{\,2}(f)\,|X(f)|^2}{2T_y\cdot S_{nn}(f)+\sigma_U^{\,2}(f)\,|X(f)|^2}\tag{21}$$

and

$$\widetilde{D}(f) = \frac{T_yS_{nn}^{\,2}(f)+\sigma_U^{\,2}(f)\,|X(f)|^2S_{nn}(f)}{\sigma_U^{\,2}(f)\,|X(f)|^4}.\tag{22}$$

$\sigma_U^{\,2}(f) = |U(f)|^2$ denotes the unfavorable target ESV for jammer, where $U(f) = \{u_k, k = 1,2,\dots,K\}$ represents the upper bound of the target uncertainty range, and $\widetilde{A}$ is a constant which can be derived by the following.

$$\int_{BW}\max\left[0,\widetilde{B}(f)\left(\widetilde{A}+\widetilde{D}(f)\right)\right]df\le E_X\tag{23}$$

3.1.2. Minimax Robust Jamming Technique for Multiple Targets Based on MI. The minimax robust jamming technique for multiple targets based on MI should satisfy the following.

$$\min_{J(f)}\left\{\max_{|H_i(f)|\in\varepsilon_i}\mathrm{MI}\left(J(f),\sigma_H^{\,2}(f)\right)\Big|_{\int_{BW}J(f)df\le E_X}\right\}\tag{24}$$

Theorem 2. *The solution to the minimax optimum problem described in (24) is*

$$\widetilde{\widetilde{J}}^{\min\max}(f) = \max\left[0,\widetilde{\widetilde{B}}(f)\left(\widetilde{\widetilde{A}}+\widetilde{\widetilde{D}}(f)\right)\right]\tag{25}$$

where

$$\widetilde{\widetilde{B}}(f) = -\frac{\sigma_U^{\,2}(f)\,|X(f)|^2}{2T_y\cdot S_{nn}(f)+\sigma_U^{\,2}(f)\,|X(f)|^2}\tag{26}$$

and

$$\widetilde{\widetilde{D}}(f) = \frac{T_yS_{nn}^{\,2}(f)+\sigma_U^{\,2}(f)\,|X(f)|^2S_{nn}(f)}{\sigma_U^{\,2}(f)\,|X(f)|^4}.\tag{27}$$

$|U_i(f)| = \{u_{ik}, k = 1,2,\dots,K\}$ represents the upper bound of $i-th$ target uncertainty range, where $\sigma_U^{\,2}(f) = \sum_{i=1}^{M}P_i|U_i(f)|^2 - |\sum_{i=1}^{M}P_iU_i(f)|^2$ [13] in (26) and (27), M denotes the number of targets, P_i denotes the occurrence probability of $i-th$ target, and $\widetilde{\widetilde{A}}$ is a constant which can be derived by the following.

$$\int_{BW}\max\left[0,\widetilde{\widetilde{B}}(f)\left(\widetilde{\widetilde{A}}+\widetilde{\widetilde{D}}(f)\right)\right]df\le E_X\tag{28}$$

Note that the optimization problem in (24) which minimizes the MI for the multitarget model is similar to the problem described in (19). The difference is that the expression of $\sigma_H^{\,2}(f)$ is varied from $\sigma_H^{\,2}(f) = |H(f)|^2$ to $\sigma_H^{\,2}(f) = \sum_{i=1}^{M}P_i|H_i(f)|^2 - |\sum_{i=1}^{M}P_iH_i(f)|^2$.

Proof of Theorems 1 and 2. To prove the conclusion above, the optimal problem should satisfy the following.

$$\begin{aligned}
&\xi\left(J^{\min\max}(f),\sigma_H^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X}\\
&\quad\le \xi\left(J^{\min\max}(f),\sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X}\\
&\quad\le \xi\left(J(f),\sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J(f)df\le E_X}
\end{aligned}\tag{29}$$

Firstly, we prove the right side of inequality (29). The expression of MI can be denoted as follows.

$$\begin{aligned}
&MI(J(f))\\
&\quad= T_y\int_{BW}\ln\left[1+\frac{\sigma_H^{\,2}(f)\,|X(f)|^2}{T_y\left(J(f)\,|X(f)|^2+S_{nn}(f)\right)}\right]df
\end{aligned}\tag{30}$$

The expression of $\sigma_H^{\,2}(f)$ in (30) is varied from single target to multiple targets, assuming that the most unfavorable target spectrum can be captured, which is $H_{worst}(f) =$

$|U(f)|$. Therefore the most unfavorable target ESV is $\sigma_U^{\,2}(f)$. Similarly, $\sigma_U^{\,2}(f)$ is the upper bound of $\sigma_H^{\,2}(f)$. The optimal problem is equivalent to designing the optimal jamming that minimizes the MI when the real target spectrum is $U(f)$.

We determine an objective function by using the Lagrangian multiplier technique.

$$\begin{aligned} &L(J(f),\lambda) \\ &= T_y \int_{BW} \ln\left[1+\frac{|X(f)|^2\sigma_U^{\,2}(f)}{T_y\left(S_{nn}(f)+|X(f)|^2 J(f)\right)}\right]df \\ &\quad + \lambda\left[E_X - \int_{BW} J(f)\,df\right] \end{aligned} \tag{31}$$

This is equivalent to minimizing $L(J(f))$ with respect to $J(f)$; the expression of (31) can be converted into

$$\begin{aligned} &L(J(f),\lambda) \\ &= T_y \int_{BW} \ln\left[1+\frac{|X(f)|^2\sigma_U^{\,2}(f)}{T_y\left(S_{nn}(f)+|X(f)|^2 J(f)\right)}\right]df \\ &\quad - \lambda\int_{BW} J(f)\,df \end{aligned} \tag{32}$$

where $L(J(f))$ can be denoted as follows.

$$\begin{aligned} L(J(f)) &= T_y \\ &\quad\cdot \ln\left[1+\frac{|X(f)|^2\sigma_U^{\,2}(f)}{T_y\left(S_{nn}(f)+|X(f)|^2 J(f)\right)}\right] \\ &\quad - \lambda J(f) \end{aligned} \tag{33}$$

The second order derivation of $L(J(f))$ with regard to $J(f)$ is greater than zero. Therefore, deriving $L(J(f))$ to $J(f)$ and setting it to zero yield the optimal jamming $J^{\min\max}(f)$, that is,

$$\begin{aligned} &J^{\min\max}(f) \\ &= \max\left[0, -\widetilde{R}(f)+\sqrt{\widetilde{R}^2(f)-\widetilde{S}(f)\left(\widetilde{A}+\widetilde{D}(f)\right)}\right] \end{aligned} \tag{34}$$

where $\widetilde{A}$ is a constant which can be derived by

$$\begin{aligned} &\int_{BW} \max\left[0, -\widetilde{R}(f)\right. \\ &\quad \left.+\sqrt{\widetilde{R}^2(f)-\widetilde{S}(f)\left(\widetilde{A}+\widetilde{D}(f)\right)}\right]df \le E_X \end{aligned} \tag{35}$$

where

$$\widetilde{R}(f) = \frac{S_{nn}(f)}{|X(f)|^2} + \frac{\sigma_U^{\,2}(f)}{2T_y} \tag{36}$$

$$\widetilde{S}(f) = \frac{\sigma_U^{\,2}(f)}{T_y} \tag{37}$$

$$\widetilde{D}(f) = \frac{T_y S_{nn}^{\,2}(f) + \sigma_U^{\,2}(f)|X(f)|^2 S_{nn}(f)}{\sigma_U^2(f)|X(f)|^4} \tag{38}$$

respectively.

We define the following

$$\widetilde{Q}(f) = -\widetilde{R}(f)+\sqrt{\widetilde{R}^2(f)-\widetilde{S}(f)\left(\widetilde{A}+\widetilde{D}(f)\right)} \tag{39}$$

and use the first order Taylor approximation to (39) to yield

$$Q(f) = \widetilde{B}(f)\left(\widetilde{A}+\widetilde{D}(f)\right) \tag{40}$$

where

$$\widetilde{B}(f) = -\frac{\sigma_U^{\,2}(f)|X(f)|^2}{2T_y\cdot S_{nn}(f)+\sigma_U^{\,2}(f)|X(f)|^2}. \tag{41}$$

Thus the designed jamming can be denoted as follows.

$$J^{\min\max}(f) = \max\left[0, \widetilde{B}(f)\left(\widetilde{A}+\widetilde{D}(f)\right)\right] \tag{42}$$

Therefore we obtain the following.

$$\begin{aligned} &\xi\left(J^{\min\max}(f), \sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X} \\ &\quad \le \xi\left(J(f), \sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J(f)df\le E_X} \end{aligned} \tag{43}$$

Then we continue to prove that $H_{worst}(f) = |U(f)|$ is the most unfavorable target spectrum which means that $\sigma_{Hworst}^{\,2}(f) = \sigma_U^{\,2}(f)$ is the most unfavorable target ESV. Substituting the designed jamming into the expression of MI in (30) for any $H(f) \in \varepsilon$ or $H_i(f) \in \varepsilon_i$, the integral is approximated by summation, which is

$$\begin{aligned} &\xi\left(J^{\min\max}(f), \sigma_H^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X} = T_y\cdot\sum_{k=1}^{K}\Delta f \\ &\quad\cdot \ln\left[1+\frac{|X(f_k)|^2\sigma_H^{\,2}(f_k)}{T_y\left(S_{nn}(f_k)+|X(f_k)|^2 J^{\min\max}(f_k)\right)}\right] \\ &\quad = T_y\cdot\sum_{k=1}^{K}\Delta f\cdot\ln\left[1\right. \\ &\quad \left.+\frac{|X(f_k)|^2\sigma_H^{\,2}(f_k)}{T_y\left(S_{nn}(f_k)+|X(f_k)|^2\cdot\max(0,Q(f_k))\right)}\right] = T_y \\ &\quad\cdot\sum_{k=1}^{K}\Delta f\cdot\ln\left[1\right. \\ &\quad \left.+\frac{|X(f_k)|^2\sigma_H^{\,2}(f_k)}{T_y\cdot\max\left(S_{nn}(f_k),|X(f_k)|^2\cdot Q(f_k)+S_{nn}(f_k)\right)}\right] \\ &\quad \le T_y\cdot\sum_{k=1}^{K}\Delta f\cdot\ln\left[1\right. \\ &\quad \left.+\frac{|X(f_k)|^2\sigma_U^{\,2}(f_k)}{T_y\cdot\max\left(S_{nn}(f_k),|X(f_k)|^2\cdot Q(f_k)+S_{nn}(f_k)\right)}\right] \\ &\quad = \xi\left(J^{\min\max}(f), \sigma_{H_{worst}}^{\,2}(f)\right)\Big|_{\int_{BW}J^{\min\max}(f)df\le E_X} \end{aligned} \tag{44}$$

where Δf denotes the sampling frequency interval. Therefore, the most unfavorable target spectrum which maximizes the MI is $H_{worst}(f) = |U(f)|$, and similarly the most unfavorable target ESV is $\sigma_{Hworst}{}^2(f) = \sigma_U{}^2(f)$; the proof is complete. □

3.2. Robust Transmitted Waveform Design Based on MI. According to the minimax robust jamming solved above, which is the strategy of the jammer, in order to improve the estimation performance, it is time for the radar to select its own strategy.

Let $\xi(|X(f)|^2, \sigma_H{}^2(f), J^{\min\max}(f))$ represent the optimization criterion MI. The MI criteria can be expressed by the transmitted waveform spectrum $X(f)$, minimax robust jamming $J^{\min\max}(f)$, and the target ESV $\sigma_H{}^2(f)$ or target spectrum $H(f)$. Note that the expressions of $\sigma_H{}^2(f)$ for single target and multiple targets are different, which will be given in Sections 3.2.1 and 3.2.2, respectively. The maximin robust transmitted waveform design method should satisfy the following [15, 16].

$$\max_{|X(f)|^2} \left\{ \min_{|H(f)|\in\varepsilon} \xi\left(|X(f)|^2, \sigma_H{}^2(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X(f)|^2 df \le E_X} \right\} \tag{45}$$

Based on the theory of robust signal processing in [16], the solution of this maximin optimization problem can be denoted as follows.

$$\begin{aligned} &\xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_H{}^2(f), \right.\\ &\quad \left. J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X} \\ &\quad \ge \xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_{H_{worst}}{}^2(f), \right.\\ &\quad \left. J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X} \ge \xi\left(|X(f)|^2, \right.\\ &\quad \left. \sigma_{H_{worst}}{}^2(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X(f)|^2 df \le E_X} \end{aligned} \tag{46}$$

From the right side of the inequality above, the maximin optimal transmitted waveform is optimal for the radar transmitter when $\sigma_H{}^2(f) = \sigma_{H_{worst}}{}^2(f)$. It maximizes the performance of radar transmitter. If another waveform spectrum is adopted, the performance of the radar will be degraded. Meanwhile, the left side of the inequality indicates that $\sigma_{H_{worst}}{}^2(f)$ is the most unfavorable target ESV for the maximin optimal transmitted waveform. If the maximin optimal transmitted waveform spectrum $|X^{\max\min}(f)|^2$ is adopted, for all target ESV in the uncertainty range ε or ε_i, the MI performance will be better than the unfavorable case, at least as good as the case of $\sigma_H{}^2(f) = \sigma_{H_{worst}}{}^2(f)$. Therefore the maximin optimal transmitted waveform for the most unfavorable target ESV within the uncertainty range is optimal. By ensuring the performance under the most unfavorable condition, the performance for all target spectra within the uncertainty range will not be worse than this case.

3.2.1. Robust Transmitted Waveform Design for Single Target Based on MI. For the radar, the lower bound of the uncertainty range is taken as the most unfavorable target spectrum. Therefore the maximin robust transmitted waveform technique for single target based on MI should satisfy the following.

$$\max_{|X(f)|^2} \left\{ \min_{|H(f)|\in\varepsilon} \xi\left(|X(f)|^2, \sigma_H{}^2(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X(f)|^2 df \le E_X} \right\} \tag{47}$$

Theorem 3. *The solution to the maximin optimum problem described in (47) is*

$$\left|\widehat{X}^{\max\min}(f)\right|^2 = \max\left[0, \widehat{B}(f)\left(\widehat{A} - \widehat{D}(f)\right)\right] \tag{48}$$

where

$$\widehat{B}(f) = \frac{\sigma_L{}^2(f)}{2T_y \cdot J^{\min\max}(f) + \sigma_L{}^2(f)} \tag{49}$$

and

$$\widehat{D}(f) = \frac{T_y S_{nn}(f)}{\sigma_L{}^2(f)}. \tag{50}$$

$\sigma_L{}^2(f) = |L(f)|^2$ denotes the unfavorable target ESV for radar, where $|L(f)| = \{l_k, k = 1, 2, \ldots, K\}$ represents the lower bound of the single target spectrum uncertainty range, and $\widehat{A}$ is a constant which can be derived by the following.

$$\int_{BW} \max\left[0, \widehat{B}(f)\left(\widehat{A} - \widehat{D}(f)\right)\right] df \le E_X \tag{51}$$

3.2.2. Robust Transmitted Waveform Design for Multiple Targets Based on MI. The maximin robust transmitted waveform technique for multiple targets based on MI should satisfy the following.

$$\max_{|X(f)|^2}\left\{\min_{|H_i(f)|\in\varepsilon_i}\xi\left(\left|X(f)\right|^2,\sigma_H{}^2(f),J^{\min\max}(f)\right)\Big|_{\int_{BW}|X(f)|^2df\le E_X}\right\} \tag{52}$$

Theorem 4. *The solution to the maximin optimum problem described in (52) is*

$$\left|\widehat{\widehat{X}}^{\max\min}(f)\right|^2=\max\left[0,\widehat{\widehat{B}}(f)\left(\widehat{\widehat{A}}-\widehat{\widehat{D}}(f)\right)\right] \tag{53}$$

where

$$\widehat{\widehat{B}}(f)=\frac{\sigma_L{}^2(f)}{2T_y\cdot J^{\min\max}(f)+\sigma_L{}^2(f)} \tag{54}$$

and

$$\widehat{\widehat{D}}(f)=\frac{T_yS_{nn}(f)}{\sigma_L{}^2(f)}. \tag{55}$$

$|L_i(f)|=\{l_{ik},k=1,2,\dots,K\}$ *denotes the lower bound of* $i-th$ *target spectrum uncertainty range, where* $\sigma_L{}^2(f)=\sum_{i=1}^M P_i|L_i(f)|^2-|\sum_{i=1}^M P_iL_i(f)|^2$ *in (54) and (55), and* $\widehat{\widehat{A}}$ *is a constant which can be derived by the following.*

$$\int_{BW}\max\left[0,\overline{B}(f)\left(A-\overline{D}(f)\right)\right]df\le E_X \tag{56}$$

Note that the optimization problem in (52) which maximizes the MI for the multitarget model is similar to the problem described in (47). The difference is that the expression of $\sigma_H{}^2(f)$ *is varied from* $\sigma_H{}^2(f)=|H(f)|^2$ *to* $\sigma_H{}^2(f)=\sum_{i=1}^M P_i|H_i(f)|^2-|\sum_{i=1}^M P_iH_i(f)|^2$.

Proof of Theorems 3 and 4. To prove the conclusion above, the optimal problem should satisfy the following.

$$\begin{aligned}&\xi\left(\left|X^{\max\min}(f)\right|^2,\sigma_H{}^2(f),\right.\\&\left.J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2df\le E_X}\\&\ge\xi\left(\left|X^{\max\min}(f)\right|^2,\sigma_{H_{worst}}{}^2(f),\right.\\&\left.J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2df\le E_X}\ge\xi\left(\left|X(f)\right|^2,\right.\\&\left.\sigma_{H_{worst}}{}^2(f),J^{\min\max}(f)\right)\Big|_{\int_{BW}|X(f)|^2df\le E_X}\end{aligned} \tag{57}$$

Firstly, we prove the right side of inequality (57). The expression of MI can be denoted as follows.

$$\begin{aligned}MI\left(|X(f)|^2\right)&=T_y\int_{BW}\ln\left[1\right.\\&\left.+\frac{\sigma_H{}^2(f)\left|X(f)\right|^2}{T_y\left(J^{\min\max}(f)\left|X(f)\right|^2+S_{nn}(f)\right)}\right]df\end{aligned} \tag{58}$$

The expression of $\sigma_H{}^2(f)$ in (58) is still different from single target to multiple targets, supposing that the most unfavorable target spectrum can be captured, which is $H_{worst}(f)=|L(f)|$. Therefore the most unfavorable target ESV is $\sigma_L{}^2(f)$. Similarly, $\sigma_L{}^2(f)$ is the lower bound of $\sigma_H{}^2(f)$. The optimal problem is equivalent to designing the optimal transmitted waveform that minimizes the MI when the real target spectrum is $L(f)$.

We determine an objective function by using the Lagrangian multiplier technique.

$$\begin{aligned}L\left(|X(f)|^2,\lambda\right)&=T_y\int_{BW}\ln\left[1\right.\\&\left.+\frac{\left|X(f)\right|^2\sigma_L{}^2(f)}{T_y\left(S_{nn}(f)+\left|X(f)\right|^2J^{\min\max}(f)\right)}\right]df\\&+\lambda\left[E_X-\int_{BW}\left|X(f)\right|^2df\right]\end{aligned} \tag{59}$$

This is equivalent to maximizing $L(|X(f)|^2)$ with respect to $|X(f)|^2$; the expression of (59) can be converted into

$$\begin{aligned}L\left(|X(f)|^2,\lambda\right)&=T_y\int_{BW}\ln\left[1\right.\\&\left.+\frac{\left|X(f)\right|^2\sigma_L{}^2(f)}{T_y\left(S_{nn}(f)+\left|X(f)\right|^2J^{\min\max}(f)\right)}\right]df\\&-\lambda\int_{BW}\left|X(f)\right|^2df\end{aligned} \tag{60}$$

where $L(|X(f)|^2)$ can be denoted as follows.

$$L\left(|X(f)|^2\right) = T_y \cdot \ln\left[1 + \frac{|X(f)|^2 \sigma_L^{\,2}(f)}{T_y\left(S_{nn}(f) + |X(f)|^2 J^{\min\max}(f)\right)}\right] - \lambda |X(f)|^2 \quad (61)$$

The second order derivation of $L(|X(f)|^2)$ with regard to $|X(f)|^2$ is greater than zero. Therefore, deriving $L(|X(f)|^2)$ to $|X(f)|^2$ and setting it to zero yield the optimal transmitted waveform $|X^{\max\min}(f)|^2$, that is,

$$\left|X^{\max\min}(f)\right|^2 = \max\left[0, -\widehat{R}(f) + \sqrt{\widehat{R}^2(f) + \widehat{S}(f)\left(\widehat{A} - \widehat{D}(f)\right)}\right]. \quad (62)$$

$\widehat{A}$ is a constant which can be derived by

$$\int_{BW} \max\left[0, -\widehat{R}(f) + \sqrt{\widehat{R}^2(f) + \widehat{S}(f)\left(\widehat{A} - \widehat{D}(f)\right)}\right] df \le E_X \quad (63)$$

where

$$\widehat{R}(f) = \frac{S_{nn}(f)\left(2T_y \cdot J^{\min\max}(f) + \sigma_L^{\,2}(f)\right)}{2J^{\min\max}(f)\left(T_y \cdot J^{\min\max}(f) + \sigma_L^{\,2}(f)\right)} \quad (64)$$

$$\widehat{S}(f) = \frac{S_{nn}(f)\sigma_L^{\,2}(f)}{J^{\min\max}(f)\left(T_y \cdot J^{\min\max}(f) + \sigma_L^{\,2}(f)\right)} \quad (65)$$

$$\widehat{D}(f) = \frac{S_{nn}(f)}{\sigma_L^{\,2}(f)\,T_y} \quad (66)$$

respectively.

We define the following

$$\widehat{N}(f) = -\widehat{R}(f) + \sqrt{\widehat{R}^2(f) + \widehat{S}(f)\left(\widehat{A} - \widehat{D}(f)\right)} \quad (67)$$

and use the first order Taylor approximation to (67) to yield

$$N(f) = \widehat{B}(f)\left(\widehat{A} - \widehat{D}(f)\right) \quad (68)$$

where

$$\widehat{B}(f) = \frac{\sigma_L^{\,2}(f)}{2T_y \cdot J^{\min\max}(f) + \sigma_L^{\,2}(f)}. \quad (69)$$

Thus the designed transmitted waveform can be denoted as follows.

$$\left|X^{\max\min}(f)\right|^2 = \max\left[0, \widehat{B}(f)\left(\widehat{A} - \widehat{D}(f)\right)\right] \quad (70)$$

Therefore we obtain the following.

$$\xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_{H_{worst}}^{\;\;2}(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X} \ge \xi\left(|X(f)|^2, \sigma_{H_{worst}}^{\;\;2}(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X(f)|^2 df \le E_X} \quad (71)$$

Then we continue to prove that $H_{worst}(f) = |L(f)|$ is the most unfavorable target spectrum which means that $\sigma_{Hworst}^{\;\;2}(f) = \sigma_L^{\,2}(f)$ is the most unfavorable target ESV. Substituting the designed transmitted waveform into the expression of MI in (58) for any $H(f) \in \varepsilon$ or $H_i(f) \in \varepsilon_i$, the integral is approximated by summation, which is as follows.

$$\begin{aligned}
&\xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_H^{\,2}(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X} = T_y \cdot \sum_{k=1}^{K}\Delta f \\
&\cdot \ln\left[1 + \frac{\left|X^{\max\min}(f_k)\right|^2 \sigma_H^{\,2}(f_k)}{T_y\left(S_{nn}(f_k) + \left|X^{\max\min}(f_k)\right|^2 J^{\min\max}(f_k)\right)}\right]\Bigg|_{\int_{BW}|X^{\max\min}(f_k)|^2 df \le E_X} = T_y \cdot \sum_{k=1}^{K}\Delta f \cdot \ln\Bigg[1 \\
&+ \frac{\max\left(0, N(f_k)\right) \cdot \sigma_H^{\,2}(f_k)}{T_y\left(S_{nn}(f_k) + \max\left(0, N(f_k)\right) \cdot J^{\min\max}(f_k)\right)}\Bigg] = T_y \cdot \sum_{k=1}^{K}\Delta f \cdot \ln\Bigg[1 \\
&+ \frac{\max\left(0, N(f_k) \cdot \sigma_H^{\,2}(f_k)\right)}{T_y \cdot \max\left(S_{nn}(f_k), N(f_k) \cdot J^{\min\max}(f_k) + S_{nn}(f_k)\right)}\Bigg] \ge T_y \cdot \sum_{k=1}^{K}\Delta f \cdot \ln\Bigg[1 \\
&+ \frac{\max\left(0, N(f_k) \cdot \sigma_L^{\,2}(f_k)\right)}{T_y \cdot \max\left(S_{nn}(f_k), N(f_k) \cdot J^{\min\max}(f_k) + S_{nn}(f_k)\right)}\Bigg] = \xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_{H_{worst}}^{\;\;2}(f), J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X}
\end{aligned} \quad (72)$$

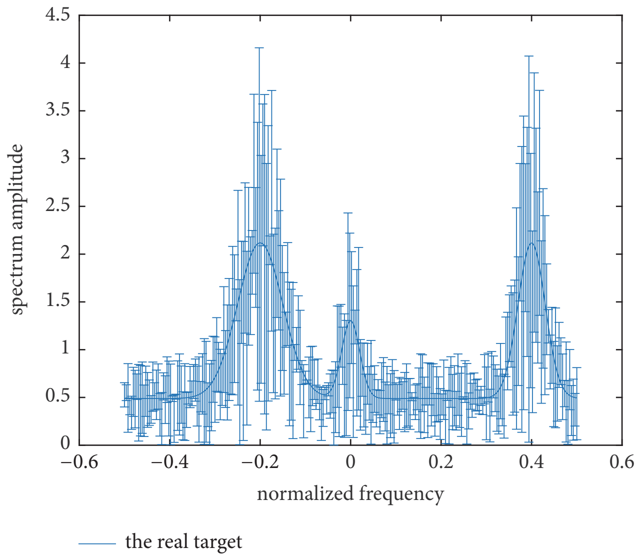

Figure 4: Bounded single target spectrum samples.

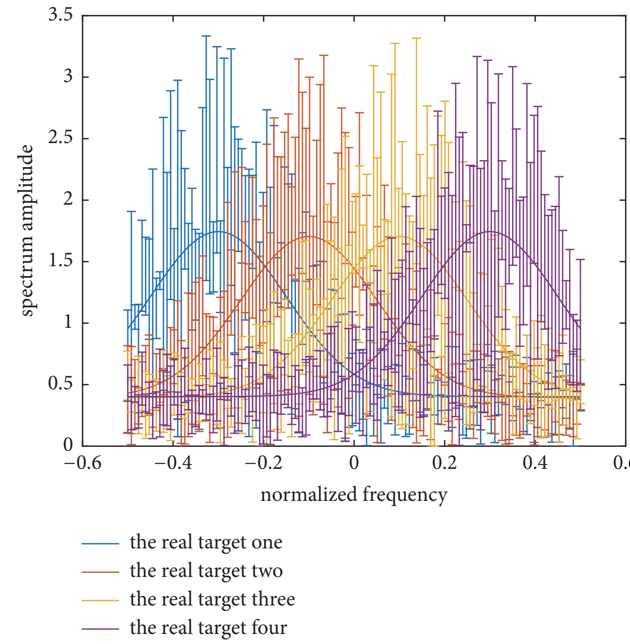

Figure 5: Bounded multitarget spectrum samples.

Therefore, the most unfavorable target spectrum which minimizes the MI is $H_{worst}(f) = |L(f)|$, and similarly the most unfavorable target ESV is $\sigma_{Hworst}{}^2(f) = \sigma_L{}^2(f)$; this guarantees that

$$\begin{aligned} &\xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_H{}^2(f), \right. \\ &\quad \left. J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X} \\ &\ge \xi\left(\left|X^{\max\min}(f)\right|^2, \sigma_{H_{worst}}{}^2(f), \right. \\ &\quad \left. J^{\min\max}(f)\right)\Big|_{\int_{BW}|X^{\max\min}(f)|^2 df \le E_X}, \end{aligned} \tag{73}$$

which is the left side of (57). Thus, the proof of Theorems 3 and 4 is complete. □

For the minimax robust jamming, the most unfavorable case is the upper bound of the target uncertainty range, and the lower bound of the uncertainty range is the most unfavorable case for the maximin robust transmitted waveform. The MI performance of the jammer and the radar for other target ESV within this uncertainty range will be better than the performance of these two unfavorable cases. Therefore, considering the uncertainty range of the target spectrum can optimize the system performance of the jammer and the radar. Through considering the hierarchical game model of radar and jammer, the maximin robust transmitted waveform is designed based on the minimax robust jamming. Although the jamming designed by the jammer can greatly impair the performance of the radar system, the radar is in a leading position and the jamming is intercepted by the radar system. Therefore, the transmitted waveform designed by the radar transmitter can finally guarantee the performance of the radar system. Furthermore, the robust jamming and robust transmitted waveform techniques based on MI provide useful guidance for waveform energy allocation.

4. Simulation and Results

To demonstrate the validity of the MI-based robust jamming and robust transmitted waveform techniques proposed in this paper, a lot of simulation analyses are performed. The uncertainty ranges of the single target and multitarget spectrum are presented in Figures 4 and 5, respectively, where the real single target and multitarget spectrum are denoted by the solid line. The main energy of the real single target is allocated near the normalized frequency -0.2, 0, and 0.4. For each target of the nominal multiple targets, the main energy is allocated near the normalized frequency 0.2, 0.4, 0.6, and 0.8, respectively, with the occurrence probability 0.1, 0.2, 0.3, and 0.4. The upper and the lower bound at each sampling frequency are represented by the deviation bounds. The amplitude of the upper bound is the real amplitude that added a random value, and similarly the lower bound is the real amplitude that subtracted a random value.

In Figures 6 and 7, the total energy of the transmitted waveform of previous moment is 1W, and the main energy is allocated near the normalized frequency of 0.3; the target spectrum response $H(f)$ in Figure 6 is the same as the solid line in Figure 4. As $\sigma_H{}^2(f)$ and $H(f)$ have similar shapes, $\sigma_H{}^2(f)$ is not presented in Figure 6. The real ESV of multiple targets is illustrated in Figure 8. The duration of the target echo $y(t)$ is supposed to be $T_y = 1.5s$. The energy constraint of the noise signal is 1W. The optimal jamming under the real target spectrum and the robust jamming under the most unfavorable target spectrum are also illustrated in Figures 6 and 7. Both the two jamming energy constraints are assumed to be 1W. In Figure 6, both the two jamming techniques distribute the finite energy in frequency bands where both the real target spectrum response and the transmitted waveform of previous moment are relatively strong. But in Figure 7,

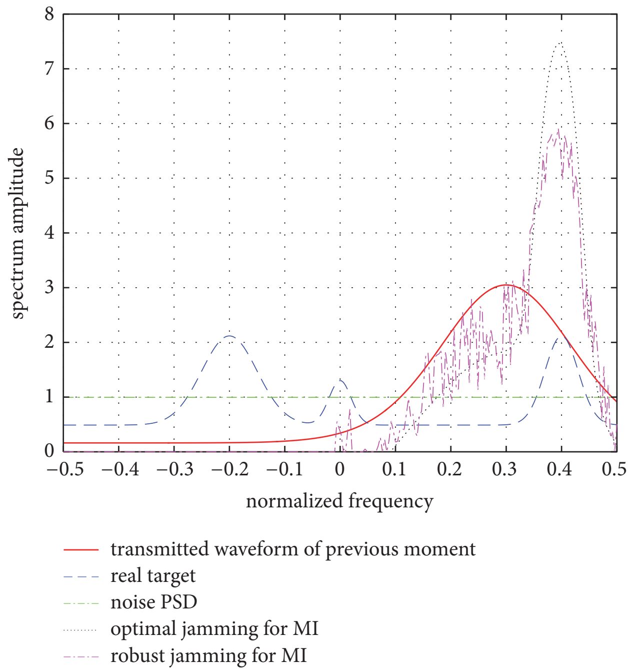

FIGURE 6: Jamming results for single target.

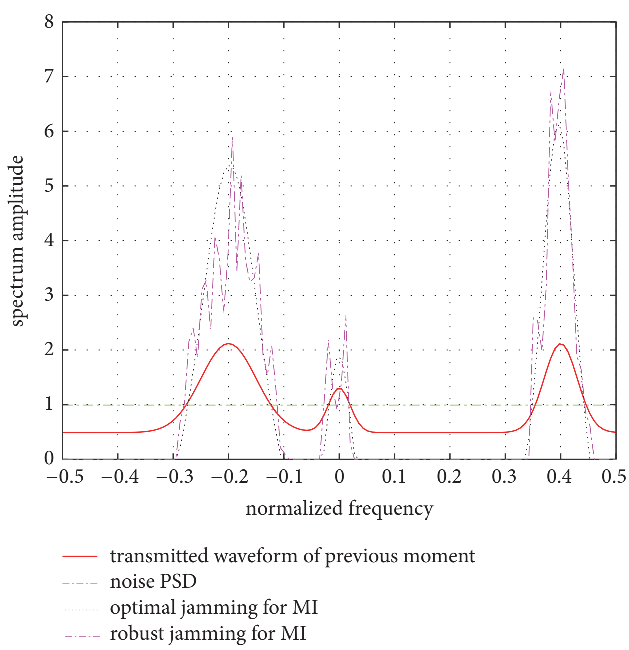

FIGURE 7: Jamming results for multiple targets.

both the two jamming techniques distribute the finite energy only in frequency bands where the transmitted waveform of previous moment is relatively strong, because the value of the nominal ESV of multiple targets is too small compared to the value of the waveform.

Assume that the total energy of the jamming varies from 1 to 7 W, the MI units (MIs) corresponding to the optimal jamming for real target spectrum, the optimal jamming for the most unfavorable target spectrum, the robust jamming

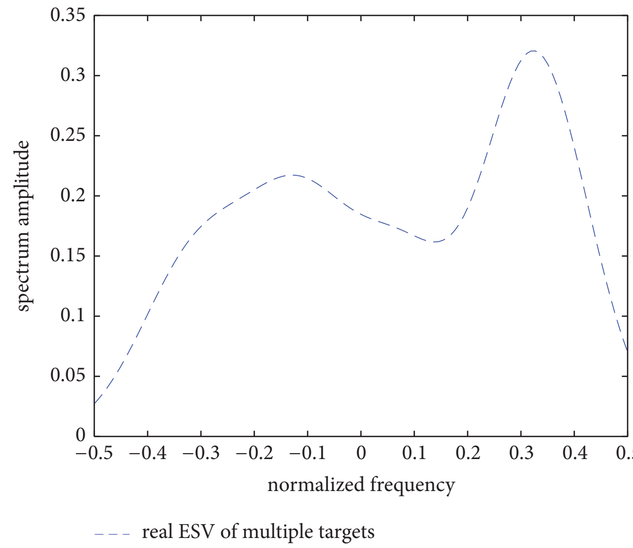

FIGURE 8: Nominal ESV for multiple targets.

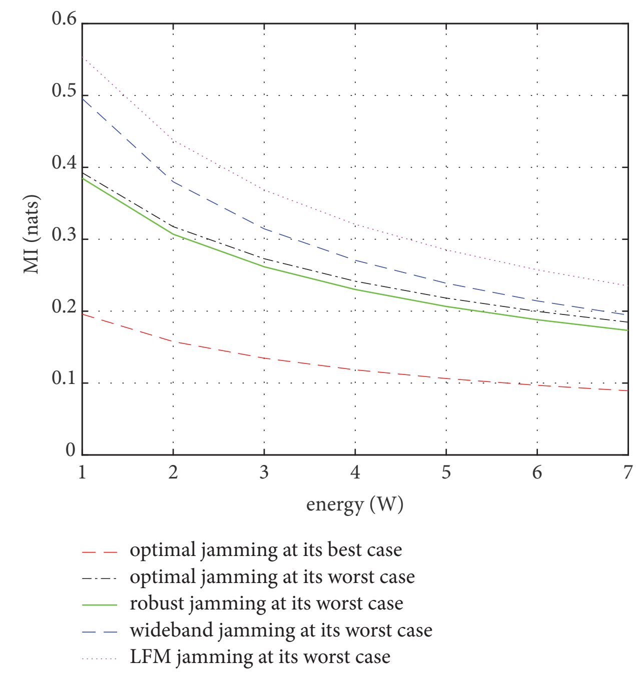

FIGURE 9: MI performance of the jamming results for single target.

for the most unfavorable target spectrum, the wide-band jamming for the most unfavorable target spectrum, and the LFM (linear frequency modulation) jamming for the most unfavorable target spectrum are compared in Figures 9 and 10 for single target and multiple targets, respectively. Simulation results show that the MI of the optimal jamming for real target spectrum is the smallest, which reaches its best performance for the jammer; the reason is that the real target spectrum is adopted and the optimal jamming is designed based on the real target spectrum. When using the most unfavorable target spectrum for the jammer, that is, the upper bound of the uncertainty range, we can get the MI corresponding to the

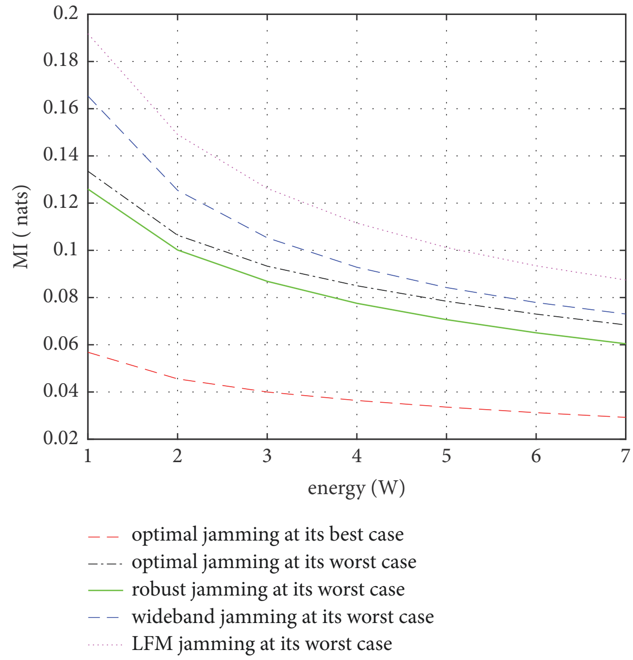

Figure 10: MI performance of the jamming results for multiple targets.

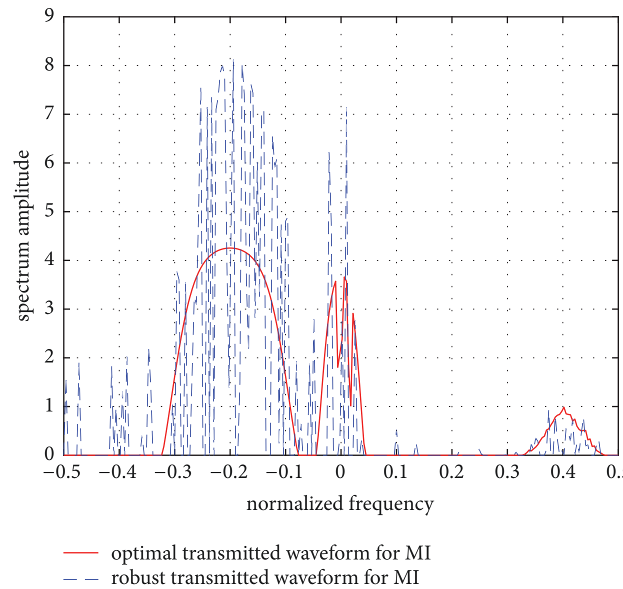

Figure 11: Waveform results for single target.

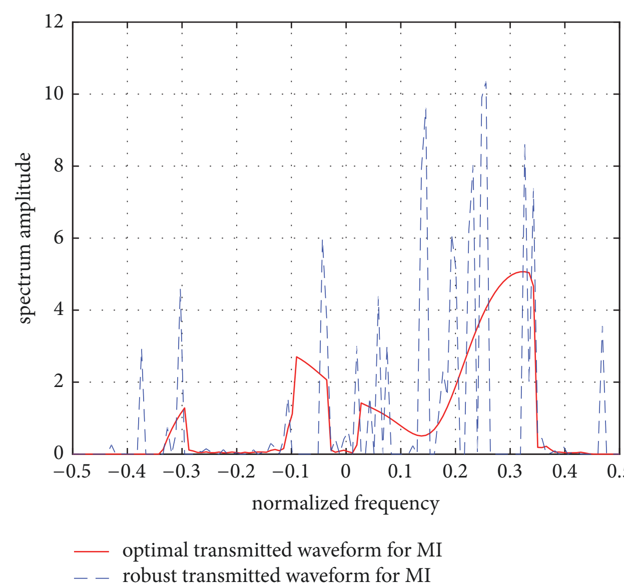

Figure 12: Waveform results for multiple targets.

optimal jamming for the most unfavorable target spectrum. As expected, the MI corresponding to the robust jamming for the most unfavorable target spectrum is between the above two. That is because the prior knowledge of the real target for the designed robust jamming is less. However, it is better than the MI corresponding to the optimal jamming for the most unfavorable target spectrum because the minimax robust technique improves the most unfavorable performance of the jammer. The wide-band jamming indicates that the jamming spectrum is a straight line over the entire frequency band, and the LFM jamming means that the instantaneous frequency of the jamming signal changes linearly with time. Both of these two jamming techniques do not contain the information about the target, noise, and transmitted waveform of previous moment. Therefore the MIs corresponding to the wide-band jamming and the LFM jamming for the most unfavorable target spectrum are larger than the above-mentioned three cases.

According to the prior information of the robust jamming shown in Figures 6 and 7, which denote the strategies of the jammer, the optimal waveform for real target spectrum and the robust waveform for the worst case are shown in Figure 11 for single target and Figure 12 for multiple targets. Both the waveform energy constraints are assumed to be 1W and distribute the finite energy in frequency bands where the target spectrum response is strong and the jamming spectrum response is weak.

Assume that the total energy of the transmitted waveform varies from 1 to 7 W, the MIs corresponding to the optimal transmitted waveform for real target spectrum, the optimal transmitted waveform for the most unfavorable target spectrum, the robust transmitted waveform for the most unfavorable target spectrum, the wide-band transmitted waveform for the most unfavorable target spectrum, and the LFM transmitted waveform for the most unfavorable target spectrum are compared in Figure 13 for single target and Figure 14 for multiple targets. Simulation results show that the MI of the optimal transmitted waveform for real target spectrum is the largest, which reaches its best performance for the radar; the reason is that the real target spectrum is adopted and the optimal transmitted waveform is designed based on the real target spectrum. When using the most unfavorable target spectrum for the radar, that is, the lower bound of the uncertainty range, we can get the MI corresponding to the optimal transmitted waveform for the most unfavorable target spectrum. As expected, the MI corresponding to the

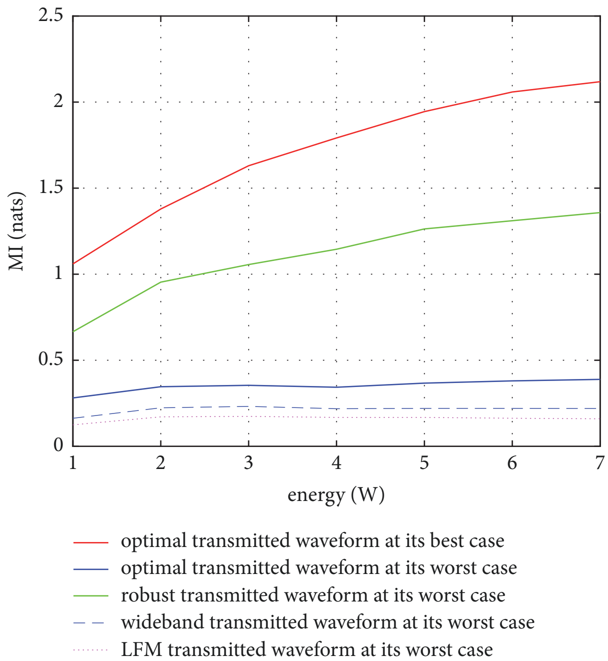

Figure 13: MI performance of the transmitted waveform results for single target.

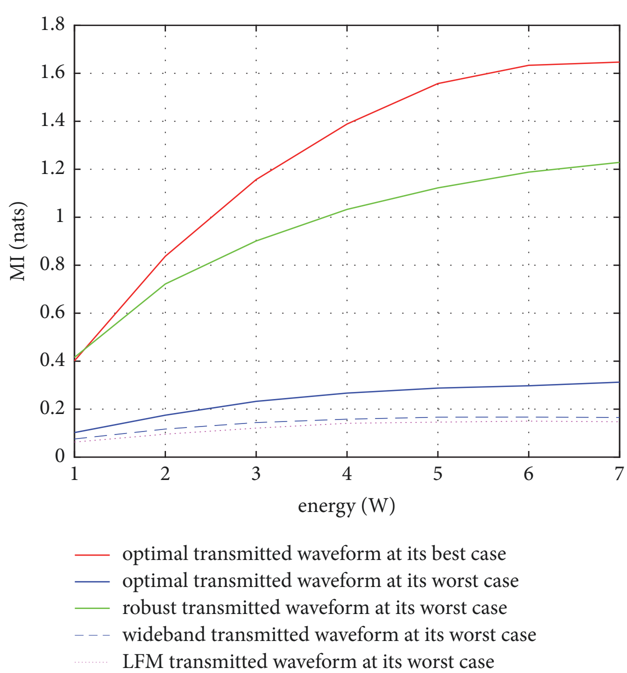

Figure 14: MI performance of the transmitted waveform results for multiple targets.

robust transmitted waveform for the most unfavorable target spectrum is between the above two. That is because the prior knowledge of the real target for the designed robust transmitted waveform is less. However, it is better than the MI corresponding to the optimal transmitted waveform for the most unfavorable target spectrum because the maximin robust technique improves the most unfavorable performance of the radar. The wide-band transmitted waveform indicates that the transmitted waveform spectrum is a straight line over the entire frequency band, and the LFM transmitted waveform means that the instantaneous frequency of the transmitted waveform signal changes linearly with time. Both of these two transmitted waveforms do not contain the information about the target, noise, and jamming, which is similar to the wide-band jamming and the LFM jamming. Therefore the MIs corresponding to the wide-band transmitted waveform and the LFM transmitted waveform for the most unfavorable target spectrum are smaller than the above-mentioned three cases.

5. Conclusion

In this paper, through assuming that the real target spectrum is known, the MI-based jamming and radar transmitted waveform techniques are proposed firstly. The designed jamming and transmitted waveform under the hierarchical game model of radar and jammer are proper for restricted energy condition. Then, the uncertainty range of the target spectrum is taken into account. The target model has been adopted, which assumes that the real target spectrum exists in an uncertainty range defined by the known upper and lower bounds. According to the uncertainty range above, the minimax robust jamming and maximin robust transmitted waveform have been designed successively. Although the jamming designed by the jammer can greatly impair the performance of the radar system, the radar is in a leading position and the jamming is intercepted by the radar system. Simulation results show that the transmitted waveform designed by the radar transmitter can finally guarantee the estimation performance of the radar system and provide useful guidance for waveform energy allocation. Although the data used in this paper does not come from the real world experimentation, the data has a certain representativeness, and it can provide reference for future hardware or actual device implementation.

Data Availability

All data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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