

## Editorial

# Memory Circuit Elements: Complexity, Complex Systems, and Applications

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Received 6 November 2018; Accepted 6 November 2018; Published 15 January 2019

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Mem-systems, based on the Memory Circuit Elements (memristor, memcapacitor, and meminductor), have received significant attention after the realization of a solid-state memristor in the Hewlett-Packard laboratories in 2008. Various mem-systems have been reported in different fields, ranging from physics and biological models to engineering. In particular, mem-systems find potential applications in switching devices, bioinspired electronics, neural networks, memory elements, and so forth.

In the last few years, we have witnessed the rapid developments in investigating mem-systems such as theoretical models, complexity, chaos, fundamental fingerprints, numerical simulations, nonlinear properties, fabrication aspects, and experimentations. However, there are still different questions, which invite more discoveries in such systems. The special issue aims at presenting and discussing advanced topics of mem-systems with complex dynamic behavior. We had received a total of 38 submissions. After the review process, this special issue contains 17 articles, the contents of which are summarized as follows.

In the article “Fully Integrated Memristor and Its Application on Scroll-Controllable Hyperchaotic System” by J. Jin and C. Li, a fully integrated memristor emulator using operational amplifiers (OA) and analog multipliers is simulated.

Based on the fully integrated memristor, a scroll-controllable hyperchaotic system is presented. By controlling the nonlinear function with programmable switches, the memristor-based hyperchaotic system achieves scroll numbers controllably. Moreover, the memristor-based hyperchaotic system is fully integrated in one single chip, and it achieves lower supply voltage, lower power dissipation, and smaller chip area. The fully integrated memristor and memristor-based hyperchaotic system are verified with GlobalFoundries’ 0.18 $\mu$ m CMOS process using Cadence IC Design Tools. The postlayout simulation results demonstrate that the memristor-based fully integrated hyperchaotic system consumes 90.5mW from  $\pm 2.5$ V supply voltage, and it takes a compact chip area of 1.8mm<sup>2</sup>.

In the article “A New Memristor-Based 5D Chaotic System and Circuit Implementation” by R. Wang et al., a 5D chaotic system with the flux-controlled memristor is proposed. The dynamics analysis of the new system can also demonstrate the hyperchaotic characteristics. The design and analysis of adaptive synchronization for the new memristor-based chaotic system and its slave system are carried out. Furthermore, the modularized circuit designs method is used in the new chaotic system circuit implementation. The Multisim simulation and the physical experiments are

conducted and compare and match with each other which can demonstrate the existence of the attractor for the new system.

In the article “A Novel Memductor-Based Chaotic System and its Applications in Circuit Design and Experimental Validation” by L. Xiong et al., a novel memductor-based chaotic system is introduced. The local dynamical entities, such as the basic dynamical behavior, the divergence, the stability of equilibrium set, and the Lyapunov exponent, are all investigated analytically and numerically to reveal the dynamic characteristics of the new memductor-based chaotic system as the system parameters and the initial state of memristor change. Subsequently, an active control method is derived to study the synchronous stability of the novel memductor-based chaotic system through making the synchronization error system asymptotically stable at the origin. Further to these, a memductor-based chaotic circuit is designed, realized, and applied to construct a new memductor-based secure communication circuit by employing the basic electronic components and memristor. Furthermore, the design principle of the memductor-based chaotic circuit is thoroughly analyzed and the concept of “the memductor-based chaotic circuit defect quantification index” is proposed for the first time to verify whether the chaotic output is consistent with the mathematical model. A good qualitative agreement is shown between the simulations and the experimental validation results.

In the article “New Results on Fuzzy Synchronization for a Kind of Disturbed Memristive Chaotic System” by B. Wang and L. L. Chen, the problem on the fuzzy synchronization for a kind of disturbed memristive chaotic system is studied. First, based on fuzzy theory, the fuzzy model for a memristive chaotic system is presented; next, based on H-infinity technique, a multidimensional fuzzy controller and a single-dimensional fuzzy controller are designed to realize the synchronization of master-slave chaotic systems with disturbances. Finally, some typical examples are included to illuminate the correctness of the given control method.

In the article “Exact Analysis and Physical Realization of the 6-Lobe Chua Corsage Memristor” by Z. I. Mannan et al., a novel generic memristor, dubbed the 6-lobe Chua corsage memristor, is proposed with its nonlinear dynamical analysis and physical realization. The proposed corsage memristor contains four asymptotically stable equilibrium points on its complex and diversified dynamic routes which reveals a 4-state nonlinear memory device. The higher degree of versatility of its dynamic routes reveals that the proposed memristor has a variety of dynamic paths in response to different initial conditions and exhibits a highly nonlinear contiguous DC  $V$ - $I$  curve. The DC  $V$ - $I$  curve of the proposed memristor is endowed with an explicit analytical parametric representation. Moreover, the derived three formulas, exponential trajectories of state  $x_n(t)$ , time period  $t_{fn}$ , and minimum pulse amplitude  $V_A$ , are required to analyze the movement of the state trajectories on the piecewise linear (PWL) dynamic route map (DRM) of the corsage memristor. These formulas are universal, that is, applicable to any PWL DRM curves for any DC or pulse input and with any number of segments. Nonlinear dynamics and circuit and system

theoretic approach are employed to explain the asymptotic quad-stable behavior of the proposed corsage memristor and to design a novel real memristor emulator using off-the-shelf circuit components.

In the article “On Designing Feedback Controllers for Master-Slave Synchronization of Memristor-Based Chua’s Circuits” by K. Ding, designing feedback controllers for master-slave synchronization of two chaotic memristor-based Chua’s circuits is investigated. The memductance function of memristor-based Chua’s circuits is a bounded function with a bounded derivative which is more generalized than those piecewise constant-valued functions or quadratic functions in some existing papers. The main contributions are that one master-slave synchronization criterion is established for two chaotic memristor-based Chua’s circuits, and the feedback controller gain is easily obtained by solving a set of linear matrix inequalities. One numerical example is given to illustrate the effectiveness of the design method.

In the article “Family of Bistable Attractors Contained in an Unstable Dissipative Switching System Associated to a SNLF” by J. L. Echenausia-Monroy et al., a multiscroll generator system is presented, which addresses the issue by the implementation of 9-level saturated nonlinear function, SNLF, being modified with a new control parameter that acts as a bifurcation parameter. By means of the modification of the newly introduced parameter, it is possible to control the number of scrolls to generate. The proposed system has richer dynamics than the original, not only presenting the generation of a global attractor; it is capable of generating monostable and bistable multiscrolls. The study of the basin of attraction for the natural attractor generation (9-scroll SNLF) shows the restrictions in the initial conditions space where the system is capable of presenting dynamical responses, limiting its possible electronic implementations.

In the article “Dynamical Behavior of a 3D Jerk System with a Generalized Memristive Device” by W. Feng et al., a 3D jerk system is proposed by introducing a generalized memristive device. It is found that the dynamical behavior of the system is sensitive to the initial conditions even the system parameters are fixed, which results in the coexistence of multiple attractors. And there exists different transition behavior depending on the selection of the parameters and initial values. Thereby, it is one important type of the candidate system for secure communication since the reconstruction of accurate state space becomes more difficult. Moreover, authors build a hardware circuit and the experimental results effectively confirm the theoretical analyses.

In the article “Chaos and Symbol Complexity in a Conformable Fractional-Order Memcapacitor System” by S. He et al., numerical solution of a conformable fractional nonlinear system is obtained based on the conformable differential transform method. Dynamics of a conformable fractional memcapacitor (CFM) system is analyzed by means of bifurcation diagram and Lyapunov characteristic exponents (LCEs). Rich dynamics is found, and coexisting attractors and transient state are observed. Symbol complexity of the CFM system is estimated by employing the symbolic entropy (SybEn) algorithm, symbolic spectral entropy (SybSEn) algorithm, and symbolic  $C_0$  (Syb $C_0$ ) algorithm. It shows

that pseudorandom sequences generated by the system have high complexity and pass the rigorous NIST test. Results demonstrate that the conformable memcapacitor nonlinear system can also be a good model for real applications.

In the article “An Integer-Order Memristive System with Two- to Four-Scroll Chaotic Attractors and Its Fractional-Order Version with a Coexisting Chaotic Attractor” by P. Zhou and M. Ke, based on a linear passive capacitor  $C$ , a linear passive inductor  $L$ , an active-charge-controlled memristor, and a fourth-degree polynomial function determined by charge, an integer-order memristive system is suggested. The proposed integer-order memristive system can generate two-scroll, three-scroll, and four-scroll chaotic attractors. The complex dynamics behaviors are investigated numerically. The Lyapunov exponent spectrum with respect to linear passive inductor  $L$  and the two-scroll, three-scroll, and four-scroll chaotic attractors are yielded by numerical calculation. Second, based on the integer-order memristive chaotic system with a four-scroll attractor, a fractional-order version memristive system is suggested. The complex dynamics behaviors of its fractional-order version are studied numerically. The largest Lyapunov exponent spectrum with respect to fractional-order  $p$  is yielded. The coexisting two kinds of three-scroll chaotic attractors and the coexisting three-scroll and four-scroll chaotic attractors can be found in its fractional-order version.

In the article “Dynamic Behaviors in Coupled Neuron System with the Excitatory and Inhibitory Autapse under Electromagnetic Induction” by Y. Xu et al., numerical simulation method is adopted with the aim of investigating the synchronous behavior in the neuronal system that is coupled by chemical and electrical synapses under electromagnetic induction. Within the improved model, the effects of electromagnetic induction on neurons are described with additive memristive current on the membrane variable, and the memristive current is dependent on the variation of magnetic flow. The simulation results show that the two coupling modes play an important role in the synchronization of the system. By increasing the chemical synaptic feedback gain, authors observe a transition from mixed oscillatory to periodic state at a critical value. In addition, two Hopf bifurcation points are found with the change of the external stimuli, and the state of neuron discharge is influenced by initial values. Furthermore, there is a domain of coupling strength and feedback gain values, in which the two-coupled neuron system is synchronized and longer time lag is not conducive to the system synchronization.

In the article “Evidence of Exponential Speed-Up in the Solution of Hard Optimization Problems” by F. L. Traversa et al., a noncombinatorial approach is applied to hard optimization problems that achieves an exponential speed-up and finds better approximations than the current state of the art. First, authors map the optimization problem into a Boolean circuit made of specially designed, self-organizing logic gates, which can be built with (nonquantum) electronic elements with memory. The equilibrium points of the circuit represent the approximation to the problem at hand. Then, authors solve its associated nonlinear ordinary differential equations numerically, towards the equilibrium

points. Authors demonstrate this exponential gain by comparing a sequential MATLAB implementation of authors' solver with the winners of the 2016 Max-SAT competition on a variety of hard optimization instances. Authors show empirical evidence that authors' solver scales linearly with the size of the problem, both in time and in memory, and argue that this property derives from the collective behavior of the simulated physical circuit. Authors' approach can be applied to other types of optimization problems, and the results presented here have far-reaching consequences in many fields.

In the article “Analysis and Implementation of a New Switching Memristor Scroll Hyperchaotic System and Application in Secure Communication” by P. Liu et al., a novel switching scroll hyperchaotic system based on a memristor device is proposed and applied to secure communication. The new system could be switched between the double-scroll chaotic system and the multiscroll one by switch  $S1$  and switch  $S2$ . Authors gave the construction process of the novel system, its numerical simulations, and dynamical properties, firstly. Moreover, the memristive circuit implementation of the new switching system was presented and the results were also in agreement with those of numerical simulation. Finally, the new switching memristive system was applied to secure communication by means of the drive-response synchronization with chaotic masking. When the voice signal is a rising waveform, it is encrypted by the double-scroll memristive system. When the voice signal is a falling waveform, the multiscroll memristive system works. The voice signal is completely submerged in the chaotic signal and could not be distinguished at all. Security analyses show that it is a successful application to secure communication.

In the article “Investigation of Cortical Signal Propagation and the Resulting Spatiotemporal Patterns in Memristor-Based Neuronal Network” by K. Ding et al., it is shown that memristive neuronal network can represent plasticity phenomena observed in biological cortical synapses. A network of neuronal units as a two-dimensional excitable tissue is designed with 3-neuron Hopfield neuronal model for the local dynamics of each unit. The results show that the lattice supports spatiotemporal pattern formation without supervision. It is found that memristor-type coupling is more noticeable against resistor-type coupling, while determining the excitable tissue switch over different complex behaviors. The stability of the resulting spatiotemporal patterns against noise is studied as well. Finally, the bifurcation analysis is carried out for variation of memristor effect. Authors' study reveals that the spatiotemporal electrical activity of the tissue concurs with the bifurcation analysis. It is shown that the memristor coupling intensities, by which the system undergoes periodic behavior, prevent the tissue from holding wave propagation. Besides, the chaotic behavior in bifurcation diagram corresponds to turbulent spatiotemporal behavior of the tissue. Moreover, authors found that the excitable media are very sensitive to noise impact when the neurons are set close to their bifurcation point, so that the respective spatiotemporal pattern is not stable.

In the article “Fractional-Order Memristor Emulator Circuits” by C. Sánchez-López et al., the synthesis of

fractional-order memristor (FOM) emulator circuits is studied. To do so, a novel fractional-order integrator (FOI) topology based on current-feedback operational amplifier and integer-order capacitors is proposed. Then, the FOI is substituting the integer-order integrator inside flux- or charge-controlled memristor emulator circuits previously reported in the literature and in both versions: floating and grounded. This demonstrates that FOM emulator circuits can also be configured at incremental or decremental mode and the main fingerprints of an integer-order memristor are also holding up for FOMs. Theoretical results are validated through HSPICE simulations and the synthesized FOM emulator circuits can easily be reproducible. Moreover, the FOM emulator circuits can be used for improving future applications such as cellular neural networks, modulators, sensors, chaotic systems, relaxation oscillators, nonvolatile memory devices, and programmable analog circuits.

In the article “Memristor-Based Canonical Chua’s Circuit: Extreme Multistability in Voltage-Current Domain and Its Controllability in Flux-Charge Domain” by H. Bao et al., authors investigate extreme multistability and its controllability for an ideal voltage-controlled memristor emulator-based canonical Chua’s circuit. With the voltage-current model, the initial condition-dependent extreme multistability is explored through analyzing the stability distribution of line equilibrium point and then the coexisting infinitely many attractors are numerically uncovered in such a memristive circuit by the attraction basin and phase portraits. Furthermore, based on the accurate constitutive relation of the memristor emulator, a set of incremental flux-charge describing equations for the memristor-based canonical Chua’s circuit is formulated and a dimensionality reduction model is thus established. As a result, the initial condition-dependent dynamics in the voltage-current domain is converted into the system parameter-associated dynamics in the flux-charge domain, which is confirmed by numerical simulations and circuit simulations. Therefore, a controllable strategy for extreme multistability can be expediently implemented, which is greatly significant for seeking chaos-based engineering applications of multistable memristive circuits.

In the article “Three-Dimensional Memristive Hindmarsh–Rose Neuron Model with Hidden Coexisting Asymmetric Behaviors” by B. Bao et al., a novel three-dimensional memristive Hindmarsh–Rose (HR) neuron model is presented to describe complex dynamics of neuronal activities with electromagnetic induction. The proposed memristive HR neuron model has no equilibrium point but can show hidden dynamical behaviors of coexisting asymmetric attractors, which has not been reported in the previous references for the HR neuron model. Mathematical model based numerical simulations for hidden coexisting asymmetric attractors are performed by bifurcation analyses, phase portraits, attraction basins, and dynamical maps, which just demonstrate the occurrence of complex dynamical behaviors of electrical activities in neuron with electromagnetic induction. Additionally, circuit breadboard based experimental results well confirm the numerical simulations.

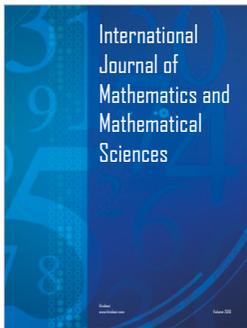
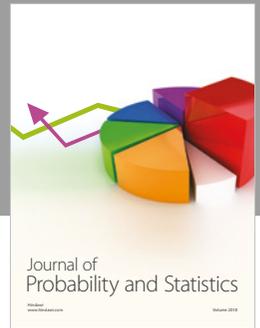
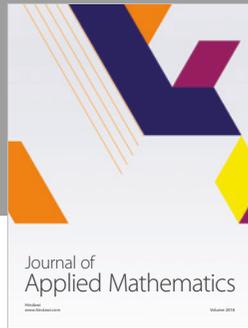
## Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

## Acknowledgments

The editorial team would like to express appreciation to all authors for their valuable contributions and to all reviewers for their valuable comments. In addition, the editors would like to thank the Complexity journal’s Editorial Board for their valuable help and support regarding this special issue.

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