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Research Article

A Grey CES Production Function Model and Its Application in Calculating the Contribution Rate of Economic Growth Factors

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In analyses of economic growth factors, people generally use the CES (Constant Elasticity of Substitution) production function model to calculate the contribution rates of the factors that influence economic growth. However, the traditional CES function model that is built directly from economic data often shows apparent errors in parameter estimation due to data fluctuations. Such a model also may cause a negative calculation of the contribution rates of economic growth factors, or it may create abnormal fluctuations for some periods, and thus it fails to meet economic growth laws. In this paper, we propose a grey CES production function that can eliminate the random fluctuations of data and make the estimated parameters more reasonable, and this model can reflect the relationship between inputs and outputs more accurately. With regard to model application, the paper puts forward a scientific calculation method to avoid the calculation deviations caused by the substitution of difference equation for a differential equation with Solow's formula. With the grey two-level nested CES production function model and the calculation method proposed, the paper makes an empirical analysis of the contribution rates of factors that influence China's economic growth.

1. Introduction

In economics, the study of the production function is very important. In 1928, Cobb and Douglas verified the relationship between the output and the input factors of labor and capital with a statistical approach in their study of American economic growth, and they first proposed the famous Cobb-Douglas production function (C-D production function), which has been applied widely [1–3]. However, the C-D production function's elasticity of substitution of factors is 1, which is inconsistent with reality. Thus, scholars Arrow, Chenery, Mihas, and Solow proposed the CES (Constant Elasticity of Substitution) production function. Its basic form [4–6] is $Y = A(\delta_K K^{-\rho} + \delta_L L^{-\rho})^{-\mu/\rho}$ in which Y is the output, K is the capital input, L is the labor input, A is the coefficient of efficiency, $A > 0$, δ_K, δ_L represents the intensive degree of factors technically, μ represents the homogeneous order of the function or the return to scale, $\mu > 0$, ρ is the substitution parameter, and $\rho \geq -1$. The CES production function's elasticity of the substitution of factors is $\sigma = 1/(1+\rho)$, changing the C-D production function's limit that the elasticity of the substitution of factors is 1; therefore, the CES production function is closer to reality as compared with the C-D production function. In the previous research on the production function noted above, we find that a key hypothesis is that output is only affected by capital and labor. However, the energy input is also a very important factor that affects the growth of a national economy. Therefore, since the work of the four scholars noted above, many researchers have tried to generalize the CES production function on the basis of a two-factor CES production function by adding the input factor of energy. A basic form of this function is $Y = A(\delta_1 K^{-\rho}+\delta_2 L^{-\rho}+\delta_3 E^{-\rho})^{-\mu/\rho}$ in which Y is the output, K is the capital input, L is the labor input, and E is the energy input. However, in the production function, factors have the same elasticity of substitution, which is inconsistent with reality. Therefore, some scholars have proposed a two-level nested CES production function [7–12]. Its basic form is

$$
\begin{aligned}
Y_{KL} &= \left(aK^{-\alpha} + (1-a)\, L^{-\alpha}\right)^{-1/\alpha}, \\
Y_{KLE} &= A\left(bY_{KL}^{-\beta} + (1-b)\, E^{-\beta}\right)^{-\gamma/\beta}.
\end{aligned} \tag{1}
$$

This paper generalizes it as

$$Y_{KL} = \left[a\left(\lambda_1 K\right)^{-\alpha} + (1-a)\left(\lambda_2 L\right)^{-\alpha}\right]^{-1/\alpha},$$
$$Y_{KLE} = A\left[bY_{KL}^{-\beta} + (1-b)\left(\lambda_3 E\right)^{-\beta}\right]^{-\gamma/\beta}. \tag{2}$$

In these equations, the newly added λ_i is the gain component of the input factor, and $A = A_0 e^{\sigma t}$ is the technological progress level. $a, b, A_0, \sigma, \alpha, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3$ are the parameters to be estimated, and Y, K, L, and E are the economic data of the time sequence. In practice, the idea of CES has a variety of applications. Ravelojaona [13] presents various nonlinear CES (Constant Elasticity of Substitution)–CET (Constant Elasticity of Transformation) Directional Distance Functions. These measures inherit the structure of the standard Directional Distance Functions as well as that of the CES–CET technology. These functions allow for the nonparametric estimation of efficiency scores through a linear programming method. Carrara and Marangoni [14] discuss how different modeling mechanisms can profoundly impact the evolution of the electricity mix, and specifically of renewable penetration. In particular, these scholars focus on the effects of introducing a set of explicit system integration constraints into a model, known as WITCH, that is characterized by a Constant Elasticity of Substitution (CES) framework. The role of the implementation of storage into this system is discussed as well. This application of CES into the production function was a success.

The traditional CES production function model has many applications. Zha et al. [15] developed a theoretical framework to study energy-based technical change that considers capital, labor, and energy as inputs. Their framework involves a first-order condition estimation of elasticity and technical change parameters for a three factor-nested Constant Elasticity of Substitution (CES) function. Olutayo et al. [16] made use of classical and Bayesian approaches to estimate the Constant Elasticity of Substitution (CES) production function. The Metropolis within Gibbs Algorithm was used to carry out their analysis, as shown by empirical illustrations; the result showed that the Numerical Standard Error (NSE) was minimal, while the posterior estimates converged to the region of true values, making the Bayesian approach more preferred. Kiselev et al. [17] considered the resource allocation problem in a two-sector economy using a Constant Elasticity of Substitution (CES) production function over a sufficiently long and finite planning horizon. In the application of the CES production function model, an important aspect is calculating the contribution rates of influencing factors on economic growth. He et al. [18] set out to develop an intelligent computing method for the evaluation of the "economic contribution rate of talent" (ECRT). Liu and Lai [19] used the modified CES production function model to (1) construct the production function of China's civil aviation transportation, (2) calculate the contribution rate of the key factors that affect the production capacity of China's civil aviation transportation, and (3) analyze and put forward relevant theoretical suggestions. However, the contribution rates obtained by calculating with the traditional CES production function changed greatly from year to year, which was inconsistent with the reality. The main reason for this is that, when building a traditional CES production function model directly from the data, the parameter estimates often have apparent errors due to data fluctuations. This can cause a negative calculation of the contribution rates of economic growth factors, create abnormal fluctuations for some periods, or cause the elasticity estimate to fall outside of the reasonable range; none of these results accord with economic growth laws. In this paper, we first build the grey model GM (1, 1) of Y, K, L, and E [20–22] by using their simulation values as the initial data, and we then estimate the parameters with the least square method. In this way, we eliminate random fluctuations and obtain more reasonable parameter estimates. Our model can reflect the input-output relationship more accurately, and the contribution rates of economic growth factors that we obtain though our calculations are more reliable.

Currently, the grey model has been widely applied in many fields, and many scholars have conducted relevant studies with rich achievements. To improve the simulation and prediction accuracy, Wu and Zhang [23] put forward the GMC (1, N) model with the accumulation of new information. A parameter is added to adjust the weight of data. By giving a large weight to the new information, the accuracy of the prediction is improved in theory. The priority of the new GMC (1, N) model is verified using case studies. Zeng and Liu [24] proposed a new SAIGM model with the fractional order accumulating operator (SAIGM_FO). Specifically, the final restored expression of the SAIGM_FO model was deduced in detail, and the parameter estimation method of SAIGM_FO was studied. In later studies, the Particle Swarm Optimization algorithm was used to optimize the order number of the SAIGM_FO model, and some steps were provided. On the basis of the grey GM (1,1) model, Tian and Liu [25] reconstructed the background value of the gray GM (1,1) model by Gauss orthogonal interpolation to eliminate the prediction bias caused by the traditional grey prediction model. Zhao et al. [26] built the dynamic equal dimension GM (1,1) model to predict the use of agricultural irrigation water in China based on the application of grey theory.

The paper uses the grey model GM (1,1) in the application of a production function model. The grey model GM (1, 1) is a smooth and continuous curve. Building a production function using the grey model to substitute for data with random fluctuations eliminates abnormal fluctuations or negative calculations of the contribution rates of the economic growth factors. Thus, the grey production function built in this paper has more reliable applications. The combination of grey theory with many models has many successful applications. We predict that the grey theory will have increasingly wide applications in various fields, and it is especially suitable for the modeling of small samples, poor information systems, and uncertain systems [27, 28].

Suppose $\widehat{Y} = (\widehat{y}(1), \widehat{y}(2), \cdots, \widehat{y}(n))$, $\widehat{K} = (\widehat{k}(1), \widehat{k}(2), \cdots, \widehat{k}(n))$, $\widehat{L} = (\widehat{l}(1), \widehat{l}(2), \cdots, \widehat{l}(n))$, and $\widehat{E} = (\widehat{e}(1), \widehat{e}(2), \cdots, \widehat{e}(n))$ are the simulative sequences in GM (1, 1) of Y, K, L, and E, respectively, and then the grey two-level nested CES production function is

$$\widehat{Y}_{KL} = \left[a\left(\lambda_1\widehat{K}\right)^{-\alpha} + (1-a)\left(\lambda_2\widehat{L}\right)^{-\alpha}\right]^{-1/\alpha},$$
$$\widehat{Y}_{KLE} = A\left[b\widehat{Y}_{KL}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta}. \quad (3)$$

The grey two-level nested CES production function model is essentially a nonlinear model that cannot be linearized. The paper uses the nonlinear least square method to estimate parameters [29–31]. With regard to model application, many scholars use the formula proposed by Solow to calculate the contribution rates of the factors, but Solow's formula substitutes the difference equation for the differential equation in the calculation and thus causes deviations. Therefore, this paper proposes a scientific calculation method [32–35]. The paper uses a grey two-level nested CES production function model and calculates the contribution rates of influencing factors on Chinese economic growth. Besides calculating the contribution rates of economic growth factors, the grey two-level nested CES production function model can also be applied in other fields, such as analyzing production factors' output elasticity and substitution elasticity, calculating the potential growth of the economy, predicting economic growth, and so on.

2. Parameter Estimation of Grey Two-Level Nested CES Production Function Model

Suppose the original sequence of Y is

$$Y = \left(y(1), y(2), \cdots, y(n)\right), \quad (4)$$

and the column generated after the accumulated generating operation is

$$Y^{(1)} = \left(y^{(1)}(1), y^{(1)}(2), \cdots, y^{(1)}(n)\right). \quad (5)$$

Suppose $Y^{(1)}$ satisfies

$$\frac{dY^{(1)}}{dt} + a_0Y^{(1)} = b_0, \quad (6)$$

and the solution of the model is

$$Y^{(1)}(t+1) = \left[y(1) - \frac{b_0}{a_0}\right]e^{-a_0t} + \frac{b_0}{a_0}. \quad (7)$$

From $\widehat{Y}(t+1) = Y^{(1)}(t+1) - Y^{(1)}(t)$, we obtain Y's simulated value of GM (1, 1):

$$\widehat{Y}(t+1) = \left(1-e^{a_0}\right)\left(y(1) - \frac{b_0}{a_0}\right)e^{-a_0t} \quad (t = 0, 1, 2, \cdots, n-1). \quad (8)$$

i.e.,

$$\widehat{Y}(t) = \left(1-e^{a_0}\right)\left(y(1) - \frac{b_0}{a_0}\right)e^{-a_0(t-1)} = \left(e^{a_0} - e^{2a_0}\right)\left(y(1) - \frac{b_0}{a_0}\right)e^{-a_0t}, \quad (t = 1, 2, \cdots, n). \quad (9)$$

Let $h_0 = (e^{a_0} - e^{2a_0})(y(1) - b_0/a_0)$, $c_0 = -a_0$, and then the simulated value of Y is

$$\widehat{Y}(t) = h_0e^{c_0t}, \quad (t = 1, 2, \cdots, n). \quad (10)$$

Similarly, we obtain

K's simulated value $\widehat{K}(t) = h_1e^{c_1t}$, $(t = 1, 2, \cdots, n)$.

L's simulated value $\widehat{L}(t) = h_2e^{c_2t}$, $(t = 1, 2, \cdots, n)$.

E's simulated value $\widehat{E}(t) = h_3e^{c_3t}$, $(t = 1, 2, \cdots, n)$.

For the parameters in the grey two-level nested CES production function model, we use the nonlinear least square method to make the estimations.

We write the nonlinear production function as

$$\widehat{Y} = f\left(\widehat{K}, \widehat{L}, \widehat{E}, a, b, A_0, \sigma, \alpha, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3\right) + \varepsilon, \quad (11)$$

where f is a theoretical model; $\widehat{Y}, \widehat{K}, \widehat{L}, \widehat{E}$ are the simulated values of grey model GM (1,1); $a, b, A_0, \sigma, \alpha, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3$ are the parameters; and ε is the fitting error of the model. The standard used to estimate parameters $a, b, A_0, \sigma, \alpha, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3$ is the same as that of linear regression, i.e., the minimization of the sum of the squared errors. That is, let

$$\begin{aligned}\sum_{t=1}^{n}\varepsilon_t^2 &= \sum_{t=1}^{n}\left[\widehat{Y}_t - f\left(\widehat{K}_t, \widehat{L}_t, \widehat{E}_t, a, b, A_0, \sigma, \alpha, \beta, \gamma, \lambda_1, \lambda_2, \lambda_3\right)\right]^2, \\ &= \sum_{t=1}^{n}\left\{\widehat{Y}_t - A\left[b\widehat{Y}_{KL}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}_t\right)^{-\beta}\right]^{-\gamma/\beta}\right\}^2 \\ &= \sum_{t=1}^{n}\left\{\widehat{Y}_t - A_0e^{\sigma t}\left[b\left(a\left(\lambda_1\widehat{K}_t\right)^{-\alpha} + (1-a)\left(\lambda_2\widehat{L}_t\right)^{-\alpha}\right)^{-\beta/\alpha} + (1-b)\left(\lambda_3\widehat{E}_t\right)^{-\beta}\right]^{-\gamma/\beta}\right\}^2\end{aligned} \quad (12)$$

and calculate the minimum and estimated unknown parameters with the nonlinear least square method. However, the calculation of nonlinear estimation is relatively complex and is typically conducted using software, so this paper uses the software MATLAB to estimate the parameters.

3. The Calculation Method for the Contribution Rates of Economic Growth Factors

Suppose the grey two-level nested CES production function is

$$\begin{aligned} \widehat{Y}_{KL} &= \left[a\left(\lambda_1\widehat{K}\right)^{-\alpha} + (1-a)\left(\lambda_2\widehat{L}\right)^{-\alpha}\right]^{-1/\alpha}, \\ \widehat{Y}_{KLE} &= A\left[b\widehat{Y}_{KL}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta}, \end{aligned} \tag{13}$$

where $A(t) = A_0 e^{\sigma t}$ is the technological progress level, $\widehat{Y}$ is the simulated value of output, $\widehat{K}$ is the simulated value of capital input, $\widehat{L}$ is the simulated value of labor input, and $\widehat{E}$ is the simulated value of energy input.

The equations above can be written briefly as

$$\begin{aligned} \widehat{X} &= \left[a\left(\lambda_1\widehat{K}\right)^{-\alpha} + (1-a)\left(\lambda_2\widehat{L}\right)^{-\alpha}\right]^{-1/\alpha}, \\ \widehat{Y} &= A_0 e^{\sigma t}\left[b\widehat{X}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta}, \end{aligned} \tag{14}$$

where $\widehat{X} = \widehat{Y}_{KL}$, $\widehat{Y} = \widehat{Y}_{KLE}$.

We calculate and obtain $\widehat{X}$'s simulated value of GM (1, 1) with the parameter estimation results from the least square method and write the simulated value as

$$\widehat{X}(t) = u_0 e^{v_0 t}, \quad (t = 1, 2, \cdots, n). \tag{15}$$

We conduct the differential operation of the two-level nested CES production function, and we get

$$\begin{aligned}
\frac{\partial \widehat{Y}}{\partial A} &= \left[b\widehat{X}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta} = \frac{\widehat{Y}}{A_0 e^{\sigma t}}, \\
\frac{\partial \widehat{Y}}{\partial \widehat{X}} &= -\frac{\gamma}{\beta}A(t)\left[b\widehat{X}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta-1} \\
&\quad \cdot b(-\beta)\widehat{X}^{-\beta-1} = -\frac{\gamma}{\beta}A(t)\left(\frac{\widehat{Y}}{A(t)}\right)^{-(\beta/\gamma)(-\gamma/\beta-1)} \\
&\quad \cdot b(-\beta)\widehat{X}^{-\beta-1} = b\gamma A_0^{-\beta/\gamma}\widehat{X}^{-\beta-1}\widehat{Y}^{1+\beta/\gamma}e^{-(\sigma\beta/\gamma)t}, \\
\frac{\partial \widehat{X}}{\partial \widehat{K}} &= -\frac{1}{\alpha}\left[a\left(\lambda_1\widehat{K}\right)^{-\alpha} + (1-a)\left(\lambda_2\widehat{L}\right)^{-\alpha}\right]^{-1/\alpha-1} \\
&\quad \cdot a\lambda_1^{-\alpha}(-\alpha)\widehat{K}^{-\alpha-1} = -\frac{1}{\alpha}\widehat{X}^{-\alpha(-1/\alpha-1)}a\lambda_1^{-\alpha}(-\alpha) \\
&\quad \cdot \widehat{K}^{-\alpha-1} = a\lambda_1^{-\alpha}\widehat{X}^{1+\alpha}\widehat{K}^{-\alpha-1}, \\
\frac{\partial \widehat{X}}{\partial \widehat{L}} &= -\frac{1}{\alpha}\left[a\left(\lambda_1\widehat{K}\right)^{-\alpha} + (1-a)\left(\lambda_2\widehat{L}\right)^{-\alpha}\right]^{-1/\alpha-1} \\
&\quad \cdot (1-a)\lambda_2^{-\alpha}(-\alpha)\widehat{L}^{-\alpha-1} = -\frac{1}{\alpha}\widehat{X}^{-\alpha(-1/\alpha-1)}(1-a) \\
&\quad \cdot \lambda_2^{-\alpha}(-\alpha)\widehat{L}^{-\alpha-1} = (1-a)\lambda_2^{-\alpha}\widehat{X}^{1+\alpha}\widehat{L}^{-\alpha-1}, \\
\frac{\partial \widehat{Y}}{\partial \widehat{E}} &= A(t)\left(-\frac{\gamma}{\beta}\right)\left[b\widehat{X}^{-\beta} + (1-b)\left(\lambda_3\widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta-1} \\
&\quad \cdot (1-b)\lambda_3^{-\beta}(-\beta)\widehat{E}^{-\beta-1} = A(t)\left(-\frac{\gamma}{\beta}\right) \\
&\quad \cdot \left(\frac{\widehat{Y}}{A(t)}\right)^{-(\beta/\gamma)(-\gamma/\beta-1)}(1-b)\lambda_3^{-\beta}(-\beta)\widehat{E}^{-\beta-1} \\
&= (1-b)\gamma A_0^{-\beta/\gamma}\widehat{Y}^{1+\beta/\gamma}\lambda_3^{-\beta}e^{-(\sigma\beta/\gamma)t}\widehat{E}^{-\beta-1}.
\end{aligned} \tag{16}$$

In this case, technological progress A's influence value on economic growth from period 1 to period n is

$$\begin{aligned}
\Delta Y_A &= \int_1^n \frac{\partial \widehat{Y}}{\partial A}dA = \int_1^n \frac{\widehat{Y}}{A_0 e^{\sigma t}}dA \\
&= \int_1^n \frac{h_0 e^{c_0 t}}{A_0 e^{\sigma t}}d\left(A_0 e^{\sigma t}\right) = \int_1^n \sigma h_0 e^{c_0 t}dt \\
&= \frac{\sigma h_0}{c_0}\left[e^{c_0 n} - e^{c_0}\right].
\end{aligned} \tag{17}$$

Factor K's influence value on economic growth from period 1 to period n is

$$\begin{aligned}
\Delta Y_K &= \int_1^n \frac{\partial \widehat{Y}}{\partial \widehat{K}}d\widehat{K} = \int_1^n \frac{\partial \widehat{Y}}{\partial \widehat{X}} \cdot \frac{\partial \widehat{X}}{\partial \widehat{K}}d\widehat{K} = \int_1^n ab\gamma A_0^{-\beta/\gamma}\widehat{X}^{\alpha-\beta}\widehat{Y}^{1+\beta/\gamma}\lambda_1^{-\alpha}e^{-(\sigma\beta/\gamma)t}\widehat{K}^{-\alpha-1}d\widehat{K} \\
&= \int_1^n ab\gamma A_0^{-\beta/\gamma}\left(u_0 e^{v_0 t}\right)^{\alpha-\beta}\left(h_0 e^{c_0 t}\right)^{1+\beta/\gamma}\lambda_1^{-\alpha}e^{-(\sigma\beta/\gamma)t}\left(h_1 e^{c_1 t}\right)^{-\alpha-1}d\left(h_1 e^{c_1 t}\right) \\
&= \int_1^n ab\gamma A_0^{-\beta/\gamma}\lambda_1^{-\alpha}u_0^{\alpha-\beta}h_0^{1+\beta/\gamma}h_1^{-\alpha}c_1 e^{[(\alpha-\beta)v_0+(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\alpha c_1]t}dt \\
&= \frac{ab\gamma A_0^{-\beta/\gamma}\lambda_1^{-\alpha}u_0^{\alpha-\beta}h_0^{1+\beta/\gamma}h_1^{-\alpha}c_1}{(\alpha-\beta)v_0 + (1+\beta/\gamma)c_0 - \sigma\beta/\gamma - \alpha c_1}\left[e^{[(\alpha-\beta)v_0+(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\alpha c_1]n} - e^{[(\alpha-\beta)v_0+(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\alpha c_1]}\right].
\end{aligned} \tag{18}$$

Factor L's influence value on economic growth from period 1 to period n is

$$\begin{aligned}
\Delta Y_L &= \int_1^n \frac{\partial \widehat{Y}}{\partial \widehat{L}} d\widehat{L} = \int_1^n \frac{\partial \widehat{Y}}{\partial \widehat{X}} \cdot \frac{\partial \widehat{X}}{\partial \widehat{L}} d\widehat{L} \\
&= \int_1^n (1-a)\, b\gamma A_0^{-\beta/\gamma} \widehat{X}^{\alpha-\beta} \widehat{Y}^{1+\beta/\gamma} \lambda_2^{-\alpha} e^{-(\sigma\beta/\gamma)t} \widehat{L}^{-\alpha-1} d\widehat{L} \\
&= \int_1^n (1-a)\, b\gamma A_0^{-\beta/\gamma} \left(u_0 e^{v_0 t}\right)^{\alpha-\beta} \left(h_0 e^{c_0 t}\right)^{1+\beta/\gamma} \\
&\quad \cdot \lambda_2^{-\alpha} e^{-(\sigma\beta/\gamma)t} \left(h_2 e^{c_2 t}\right)^{-\alpha-1} d\left(h_2 e^{c_2 t}\right) \\
&= \int_1^n (1-a) \\
&\quad \cdot b\gamma A_0^{-\beta/\gamma} \lambda_2^{-\alpha} u_0^{\alpha-\beta} h_0^{1+\beta/\gamma} h_2^{-\alpha} c_2 e^{[(\alpha-\beta)v_0+(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\alpha c_2]t} dt \\
&= \frac{(1-a)\, b\gamma A_0^{-\beta/\gamma} \lambda_2^{-\alpha} u_0^{\alpha-\beta} h_0^{1+\beta/\gamma} h_2^{-\alpha} c_2}{(\alpha-\beta) v_0 + (1+\beta/\gamma) c_0 - \sigma\beta/\gamma - \alpha c_2} \left[e^{[(\alpha-\beta)v_0+(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\alpha c_2]n}\right. \\
&\quad \left. - e^{[(\alpha-\beta)v_0+(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\alpha c_2]}\right].
\end{aligned} \tag{19}$$

Factor E's influence value on economic growth from period 1 to period n is

$$\begin{aligned}
\Delta Y_E &= \int_1^n \frac{\partial \widehat{Y}}{\partial \widehat{E}} d\widehat{E} \\
&= \int_1^n (1-b)\, \gamma A_0^{-\beta/\gamma} Y^{1+\beta/\gamma} \lambda_3^{-\beta} e^{-(\sigma\beta/\gamma)t} \widehat{E}^{-\beta-1} d\widehat{E} \\
&= \int_1^n (1-b)\, \gamma A_0^{-\beta/\gamma} \left(h_0 e^{c_0 t}\right)^{1+\beta/\gamma} \\
&\quad \cdot \lambda_3^{-\beta} e^{-(\sigma\beta/\gamma)t} \left(h_3 e^{c_3 t}\right)^{-\beta-1} d\left(h_3 e^{c_3 t}\right) \\
&= \int_1^n (1-b)\, \gamma A_0^{-\beta/\gamma} \lambda_3^{-\beta} h_0^{1+\beta/\gamma} h_3^{-\beta} c_3 e^{[(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\beta c_3]t} dt \\
&= \frac{(1-b)\, \gamma A_0^{-\beta/\gamma} \lambda_3^{-\beta} h_0^{1+\beta/\gamma} h_3^{-\beta} c_3}{(1+\beta/\gamma) c_0 - \sigma\beta/\gamma - \beta c_3} \left[e^{[(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\beta c_3]n}\right. \\
&\quad \left. - e^{[(1+\beta/\gamma)c_0-\sigma\beta/\gamma-\beta c_3]}\right].
\end{aligned} \tag{20}$$

Then, the contribution rate of technological progress to economic growth from period 1 to period n is

$$\frac{\Delta Y_A}{\Delta Y} = \frac{\Delta Y_A}{\Delta Y_A + \Delta Y_K + \Delta Y_L + \Delta Y_E}. \tag{21}$$

Capital's contribution rate to economic growth from period 1 to period n is

$$\frac{\Delta Y_K}{\Delta Y} = \frac{\Delta Y_K}{\Delta Y_A + \Delta Y_K + \Delta Y_L + \Delta Y_E}. \tag{22}$$

Labor's contribution rate to economic growth from period 1 to period n is

$$\frac{\Delta Y_L}{\Delta Y} = \frac{\Delta Y_L}{\Delta Y_A + \Delta Y_K + \Delta Y_L + \Delta Y_E}. \tag{23}$$

Energy's contribution rate to economic growth from period 1 to period n is

$$\frac{\Delta Y_E}{\Delta Y} = \frac{\Delta Y_E}{\Delta Y_A + \Delta Y_K + \Delta Y_L + \Delta Y_E}. \tag{24}$$

4. Empirical Analysis of the Contribution Rates of Influencing Factors to China's Economic Growth

To research Chinese economic growth, and to explore the growth mechanisms and calculate the contribution rates of influencing factors to economic growth, this paper selects GDP Y (¥0.1 billion) as the output, and it considers the following to be economic influencing factors: fixed-asset investment K (¥0.1 billion), the number of employees L (10,000 people), and the total energy consumed E (10,000 tons of standard coal). See Table 1 for the data. The data comes from the *China Statistical Yearbook 2017*.

We build the grey two-level nested CES production function as follows:

$$\begin{aligned}
\widehat{Y}_{KL} &= \left[a\left(\lambda_1 \widehat{K}\right)^{-\alpha} + (1-a)\left(\lambda_2 \widehat{L}\right)^{-\alpha}\right]^{-1/\alpha}, \\
\widehat{Y}_{KLE} &= A\left[b\widehat{Y}_{KL}^{-\beta} + (1-b)\left(\lambda_3 \widehat{E}\right)^{-\beta}\right]^{-\gamma/\beta},
\end{aligned} \tag{25}$$

where $A(t) = A_0 e^{\sigma t}$ is the technological progress level, $\widehat{Y}$ is the simulated value of output, $\widehat{K}$ is the simulated value of capital input, $\widehat{L}$ is the simulated value of labor input, and $\widehat{E}$ is the simulated value of energy input.

First, we get the simulated values of Y, K, L, E with GM (1, 1):

$$\begin{aligned}
\widehat{Y}(t) &= h_0 e^{c_0 t} = 6.1783e+04 e^{0.1239t}, \\
&\qquad (t = 1, 2, \cdots, 21); \\
\widehat{K}(t) &= h_1 e^{c_1 t} = 2.5299e+04 e^{0.1687t}, \\
&\qquad (t = 1, 2, \cdots, 21); \\
\widehat{L}(t) &= h_2 e^{c_2 t} = 7.0322e+04 e^{0.0051t}, \\
&\qquad (t = 1, 2, \cdots, 21); \\
\widehat{E}(t) &= h_3 e^{c_3 t} = 1.1677e+05 e^{0.0664t}, \\
&\qquad (t = 1, 2, \cdots, 21).
\end{aligned} \tag{26}$$

With the least square method, we calculate and get

$$\begin{aligned}
a &= 0.6667, \\
b &= 0.5021, \\
A_0 &= 1.5800, \\
\sigma &= 0.0694, \\
\alpha &= 5.8431, \\
\beta &= 0.3500, \\
\gamma &= 0.9996, \\
\lambda_1 &= 0.7912, \\
\lambda_2 &= 0.9754, \\
\lambda_3 &= 0.2999.
\end{aligned} \tag{27}$$

Table 1: Related data of Chinese economic growth.

Year	t	Y (RMB 0.1 billion yuan)	L (10,000 people)	K (RMB 0.1 billion yuan)	E (10,000 tons of standard coal)
1996	1	71176.6	68950	22913.5	135192
1997	2	78973.0	69820	24941.1	135909
1998	3	84402.3	70637	28406.2	136184
1999	4	89677.1	71394	29854.7	140569
2000	5	99214.6	72085	32917.7	145531
2001	6	109655.2	72797	37213.5	150406
2002	7	120332.7	73280	43499.9	159431
2003	8	135822.8	73736	55566.6	183792
2004	9	159878.3	74264	70477.4	213456
2005	10	183217.5	74647	88773.6	235997
2006	11	211923.5	74978	109998.2	258676
2007	12	249529.9	75321	137239.0	280508
2008	13	316228.8	75564	172828.4	291448
2009	14	343464.7	75828	224598.8	306647
2010	15	401512.8	76105	251683.8	324939
2011	16	473104.0	76420	311485.1	348002
2012	17	519470.1	76704	374694.7	361732
2013	18	568845.0	76977	447074.0	375252
2014	19	636462.7	77253	512760.7	426000
2015	20	676780.0	77451	562000.0	430000
2016	21	744127.0	77603	606466.0	436000

i.e.,

$$\begin{aligned}\widehat{Y}_{KL} &= \left[0.6667\left(0.7912\widehat{K}\right)^{-5.8431} + 0.3333\left(0.9754\widehat{L}\right)^{-5.8431}\right]^{-1/5.8431}, \\ \widehat{Y}_{KLE} &= 1.5800e^{0.0694t}\left[0.5021\widehat{Y}_{KL}^{-0.3500} + 0.4979\left(0.2999\widehat{E}\right)^{-0.3500}\right]^{-0.9996/0.3500}.\end{aligned} \tag{28}$$

The model's coefficient of determination is $R^2 = 1 - \sum(Y_t - \widehat{Y}_t)^2 / \sum(Y_t - \overline{Y})^2 = 0.9879$, which shows the high fitting precision.

$\widehat{X}$'s simulated value with GM (1, 1) is

$$\widehat{X}(t) = u_0 e^{v_0 t} = 4.8811e+04e^{0.0366t}, \quad (t = 1, 2, \cdots, 21). \tag{29}$$

And then, we get

$$\begin{aligned}\Delta Y_A &= 4.2783e+05, \\ \Delta Y_K &= 4.8495e+05, \\ \Delta Y_L &= 0.1568e+05, \\ \Delta Y_E &= 1.9935e+05.\end{aligned} \tag{30}$$

In this case, from 1996 to 2016, the factors' contribution rates to economic growth are as follows:

Technological progress A's contribution rate to economic growth is

$$\frac{\Delta Y_A}{\Delta Y} = 37.93\%; \tag{31}$$

Factor K's contribution rate to economic growth is

$$\frac{\Delta Y_K}{\Delta Y} = 43.00\%; \tag{32}$$

Factor L's contribution rate to economic growth is

$$\frac{\Delta Y_L}{\Delta Y} = 1.39\%; \tag{33}$$

Factor E's contribution rate to economic growth is

$$\frac{\Delta Y_E}{\Delta Y} = 17.68\%. \tag{34}$$

Figure 1 shows a pie chart of contribution rates of China's economic growth factors. The calculation results show that economic growth in China mainly depends on capital input, and then on technological progress and energy input. The labor input contributes less to economic growth. These results are consistent with the reality in China. In fact, the Chinese economy has been growing rapidly since the implementation

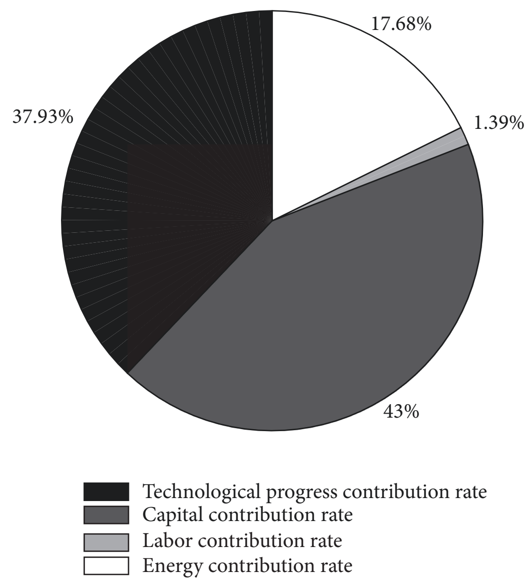

Figure 1: Pie chart of contribution rates of China's economic growth factors.

of various reforms and the opening-up policy, and it is mainly dependent on the rapid growth of capital input. Therefore, capital has the largest contribution rate to economic growth at close to 45%. China has also continuously increased investment in technological progress, which has a contribution rate of approximately 38%. The energy input is another important factor for economic growth in China and has been steadily increasing, with a contribution rate of approximately 18%. Labor has the lowest contribution rate to economic growth, because the agricultural labor force is of low quality, and, for the most part, it has increased in quantity and quality only slightly, and very slowly.

5. Conclusion

The paper proposes a grey CES production function model. With regard to the model's parameter estimation, we use the simulated values of grey model GM (1, 1) with economic data, and we employ the least square method to eliminate apparent errors in parameter estimates caused by data fluctuations. Our method also avoids the problems of negative values and abnormal fluctuations in the calculated economic growth factors' contribution rates. Our model provides a method to scientifically calculate the contribution rates of influencing factors to economic growth. The method eliminates the calculation deviations caused by difference equation substitution for differential equations that are created by Solow's formula. The empirical analysis results show that the contribution rates of economic growth factors obtained with the model and method we propose are consistent with the reality in China. The research in this paper is applicable for intensive research, for the popularization and application of production function models, and as a reference for macroeconomic departments to analyze economic growth processes, explore economic growth mechanisms, and make economic policies.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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