Research Article

Finite-Time Nonfragile Dissipative Control for Discrete-Time Neural Networks with Markovian Jumps and Mixed Time-Delays

Ling Hou ¹, Dongyan Chen ^{1,2} and Chan He³

¹Department of Mathematics, Harbin University of Science and Technology, Harbin 150080, China

²Heilongjiang Provincial Key Laboratory of Optimization Control and Intelligent Analysis for Complex Systems, Harbin University of Science and Technology, Harbin 150080, China

³Department of Mathematics, North University of China, Taiyuan 030051, Shanxi, China

Correspondence should be addressed to Ling Hou; houlng0729@163.com

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This paper considers the stochastic finite-time dissipative (SFTD) control problem based on nonfragile controller for discrete-time neural networks (NNS) with Markovian jumps and mixed delays, in which the mode switching phenomenon, is described as Markov chain, and the mixed delays are composed of discrete time-varying delay and distributed delays. First, by selecting an appropriate Lyapunov-Krasovskii functional and applying stochastic analysis methods, some parameters-dependent sufficient conditions for solvability of stochastic finite-time boundedness are derived. Then, the main results are extended to SFTD control. Furthermore, existence condition of nonfragile controller is derived based on solution of linear matrix inequalities (LMIs). Finally, two numerical examples are employed to show the effectiveness of the obtained methods.

1. Introduction

NNS are a complex network system formed by a large number of simple processing units connected with each other. In the last decades, NNS have been widely used in many real-world problems, such as pattern recognition, smart antenna arrays, signal and image processing, target tracking, and combinatorial optimization; see [1–4]. As we all know, these practical applications rely heavily on the dynamic behavior of NNS. Thus, dynamic property of various types of NNS has become a hot research topic. The corresponding results can be found in [5–8] and the reference therein. In addition, in the electronic implementation of many biological and artificial NNS, time delays often occur in signals transmission between difference neurons, which may cause oscillation and instability of dynamic network behaviors. In the last decades, some important works on NNS with time delays have been presented, such as [9–13]. On the other hand, in NNS, information latching often occurs, which means that at different times the NNS may be limited to switching from one mode to another. Thus, in this case, the NNS can be

handled as a Markov jump NNS. Up to now, there have been some results for this kind of system, such as [14–18]. In [14], slow state variables feedback stabilization problem for Markov jump system was investigated. In [15], the passivity problem of Markov jump NNS was discussed by using linear matrix inequality (LMI) technique. In [16], the exponential stability of the NNS with Markov jump parameters was studied. In [17, 18], the stability and synthesis problem of Markov jump NNS with model-dependent mixed delays were discussed.

Dissipativity is a more general criterion compared with Lyapunov stability, H_∞ performance, and passive performance. Its essence is that there is a nonnegative energy function so that the energy loss inside for the system is always less than the supply rate of external energy. Since Willems put forward the concept of dissipativity in 1972 [19], dissipative theory has played an important role in systems, networks, control theory, and control engineering. During the past several decades, many results about dissipative theory have been obtained. For example, the dissipative problems of linear systems were studied in [20–22]. In [23], Shen et al. discussed

the dissipative performance of sampled-data Markov jump system.

It is worth noting that all above mentioned results are mainly concerned with the dynamic behavior in an infinite interval. However, in practice, we may be concerned with the dynamic performance in a specified time interval, that is, finite-time stability (FTS). The concept of FTS was first introduced by Dorato [24]. And in recent years, the FTS attracts more and more consideration. For example, Lv et al. studied the FTS for a class of nonlinear system in [25]. Yang et al. dealt with the FTS for singular system [26]. Subsequently, FTS was extended to finite-time boundedness (FTB), the finite-time H_∞ control, finite-time passive (FTP) control, and so on [27–31]. And recently, Ma et al. studied the finite-time dissipative (FTD) problem of singular discrete-time Markov jump systems [32]. In [33–35], Song, Mathiyalagan, and Chen studied the FTD problem for general network systems, stochastic interval systems, and network series control systems, respectively. However, in the abovementioned research, the designed controllers are all accurate. But in many industrial applications, controllers such as actuator damage and digital switching errors are inevitably inaccurate. Therefore, it is the more ideal method to design a controller that is insensitive to uncertainty coefficients and can provide sufficient tuning range. This leads to the study of nonfragile control. Many results about nonfragile state feedback controller design problem can be found in [36–38]. Recently, Ma et al. discussed the FTD problem based on nonfragile controller for uncertain T-S fuzzy systems in [39]. Furthermore, Gao and Xia studied the finite-time extended dissipative control for uncertain switched NNS based on nonfragile controller in [40, 41]. Summarizing the discussions made so far, stochastic finite-time nonfragile dissipative control problem for discrete-time NNS contain Markovian jumps and mixed delays has not yet been investigated, which remains a good challenge. Moreover, this kind of system model can be used to solve water pollution problems; see [32].

The objective of this paper is to study the problem of SFTD analysis based on nonfragile controller for a class of discrete time NNS with Markovian jumps and mixed delays. Different from the existing works, our objective is to design a finite-time nonfragile dissipative controller by fully taking into account the Markovian jumps and mixed delays. Through such a nonfragile controller, both stochastic finite-time boundedness and desired dissipative performance of a class of discrete-time NNS can be ensured. The main contributions of this paper can be highlighted. (1) A more comprehensive model of discrete-time NNS is proposed that contains Markovian jumps and mixed delays than existing literatures [30, 31], since the Markovian jumps and mixed delays were not considered in [30, 31]. (2) By applying stochastic analysis method and LMI tool, sufficient conditions for the stochastic finite-time boundedness and stochastic finite-time dissipativity of the discrete-time NNS with Markovian jumps and mixed delays are given. Our model and conditions are more general than those given in [27–30]. (3) A more general controller is designed, that is, the nonfragile state feedback controller, which can greatly reduce conservation of the system.

Notation. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ represent the n -dimension real Euclidean space and the set of $m \times n$ real matrices, respectively. $\mathbb{E}\{\cdot\}$ expresses the mathematical expectation of matrix X , and $\text{Prob}\{X\}$ is used to denote the occurrence probability of the event. The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semidefinite or positive definite, respectively, in which X and Y are symmetric matrices. I and 0 represent the identity and zero matrices with compatible dimensions respectively. $\text{Diag}\{\cdot\}$ denotes the block diagonal matrix. $*$ denotes symmetry terms in a symmetric matrix, and M^T denotes the transposition of matrix M .

2. Problem Formulation and Preliminaries

The discrete-time NNS considered in this paper is with Markovian jumps and mixed-delays as follows:

$$\begin{aligned} x(k+1) &= A(\theta_k)x(k) + A_d(\theta_k)x(k-d(k)) \\ &\quad + A_m(\theta_k) \sum_{m=1}^{+\infty} \mu_m x(k-m), \\ &\quad + B(\theta_k)u(k) + C(\theta_k)f(x(k)) + D(\theta_k)\omega(k), \\ z(k) &= F(\theta_k)x(k), \\ x(j) &= \varphi(j), \quad -\infty < j \leq 0. \end{aligned} \quad (1)$$

In system (1), $x(k) = [x_1(k), \dots, x_n(k)]^T \in \mathbb{R}^n$ is the state vector of the NNS associated with n neurons. $u(k) \in \mathbb{R}^p$ denotes the control input, $z(k) \in \mathbb{R}^t$ denotes the controlled output, and $\omega(k)$ is external disturbance input. $f(x(\cdot)) = [f(x_1(\cdot)), \dots, f(x_n(\cdot))]^T$ is the neuron activation function, and $\varphi(j)$ denotes the vector-value initial condition. $\sum_{m=1}^{+\infty} \mu_m x(k-m)$ is distributed delay, where the constants μ_m satisfy the following convergence conditions:

$$\bar{\mu} = \sum_{m=1}^{+\infty} \mu_m \leq \sum_{m=1}^{+\infty} m\mu_m < +\infty, \quad (2)$$

$A(\theta_k)$, $A_d(\theta_k)$, $A_m(\theta_k)$, $B(\theta_k)$, $C(\theta_k)$, $D(\theta_k)$, and $F(\theta_k)$ are known constant matrices with compatible dimensions, and θ_k is discrete-time Markov stochastic process and takes discrete values in a finite set $\Lambda = \{1, 2, \dots, s\}$ with transition matrix $\Pi = (\pi_{ij})_{s \times s}$, that is,

$$\pi_{ij} = \Pr\{\theta_{k+1} = j \mid \theta_k = i\}, \quad (3)$$

where $0 \leq \pi_{ij} \leq 1$ and $\sum_{j=1}^s \pi_{ij} = 1$.

To obtain the main results, the following assumptions are needed.

Assumption 1. $d(k)$ is time-varying delay satisfying

$$d_m \leq d(k) \leq d_M, \quad d_m, d_M > 0. \quad (4)$$

Assumption 2. The neuron activation function $f(x(\cdot))$ is continuous and satisfies the following two conditions:

$$(A1) \quad f(0) = 0,$$

$$(A2) \quad [f(x) - f(y) - U_1(x - y)]^T [f(x) - f(y) - U_2(x - y)] \leq 0, \\ \forall x, y \in \mathbb{R}^n,$$

where $U_1, U_2 \in \mathbb{R}^{n \times n}$ are real matrices of appropriate dimensions.

It should be pointed out that condition (A2) is more general than the Lipschitz condition.

Assumption 3. The external disturbance input $\omega(k)$ satisfies the constraint

$$\sum_{k=0}^N \omega^T(k) \omega(k) \leq d, \quad d \geq 0. \quad (5)$$

where d is a given positive number.

For simplifying the notation of this paper, we denote a matrix $G(\theta_k) = G_i$ for each possible $\theta_k = i$ ($i \in \Lambda$). For example, $A(\theta_k)$ and $B(\theta_k)$ will be presented by A_i and B_i , and so forth. Moreover, we denote $x(k - d(k)) = x_d$ and $x(k - m) = x_m$.

This paper adopts the following nonfragile controller for the discrete-time NNS (1):

$$u(k) = \bar{K}x(k), \quad (6)$$

where $\bar{K} = K_i + \Delta K_i$, K_i is the controller gain that will be determined later, and ΔK_i is an uncertain matrix with

$$\Delta K_i = M_i \Delta(k) N_i, \quad (7)$$

in which M_i and N_i are known real constant matrices with compatible dimensions and $\Delta(k)$ is an unknown perturbed matrix satisfying $\Delta^T(k) \Delta(k) \leq I$.

Remark 4. FTS means that the system state trajectories do not exceed a certain threshold during the specified finite-time interval under a given the initial condition. Up to now, most of the existing results concerning nonfragile controller design for NNS are considered in an infinite time interval, for example [37, 38], which cannot reflect the transient behavior of the system. In many practical applications, one may be interested in not only Lyapunov asymptotic stability on an infinite time interval but also bound of system trajectories over a fixed short period. Thus, it is necessary to study nonfragile controller design problem in a finite-time interval. Different from [37, 38], in this paper, nonfragile control problem for NNS is considered in a specified finite-time interval.

Then the resultant closed-loop system can be written as follows:

$$x(k+1) = \bar{A}_i x(k) + A_{di} x_d + A_{mi} \sum_{m=1}^{+\infty} \mu_m x_m \\ + C_i f(x(k)) + D_i \omega(k), \quad (8)$$

$$z(k) = F_i x(k),$$

$$x(j) = \varphi(j), \quad -\infty < j \leq 0$$

The aim of this article is to study nonfragile state feedback controller design problem which guarantees resultant closed-loop system is stochastic finite-time bounded (SFTB) and SFTD in spite of Markovian jumps and mixed delays. To study the stochastic finite-time boundedness and stochastic finite-time dissipativity of system (8), the some necessary definitions and lemmas are given below.

Definition 5. For given symmetric matrices $R_i > 0$, the system (8) is said to be SFTB with respect to (c_1, c_2, d, N, R_i) , where $0 < c_1 < c_2$, $d > 0$, and $N > 0$, if

$$\sup_{-\infty < l \leq 0} \mathbb{E} \{x(l)^T R_i x(l)\} \leq c_1 \implies \\ \mathbb{E} \{x(k)^T R_i x(k)\} \leq c_2, \quad (9) \\ k \in \{1, \dots, N\},$$

for all $\omega(k)$ satisfying Assumption 3.

Definition 6. For given symmetric matrices $R_i > 0$, the system (8) is said to be stochastic finite-time $(\mathcal{W}, \mathcal{S}, \mathcal{R}) - \gamma$ -dissipative with respect to (c_1, c_2, d, N, R_i) if the system (8) is SFTB with respect to (c_1, c_2, d, N, R_i) and, under the zero-initial condition, the output $z(k)$ satisfies

$$\sum_{k=0}^N \mathbb{E} [z^T(k) \mathcal{W} z(k) + 2z^T(k) \mathcal{S} \omega(k) \\ + \omega^T(k) \mathcal{R} \omega(k)] \geq \gamma \sum_{k=0}^N \omega^T(k) \omega(k), \quad (10)$$

for any nonzero $\omega(k)$ satisfying Assumption 3, $\mathcal{W}$, $\mathcal{S}$, and $\mathcal{R}$ are specified real matrices with $\mathcal{W}$ and $\mathcal{R}$ symmetric. Also, we assume that $\mathcal{W} \leq 0$ and denote $-\mathcal{W} = (\mathcal{W}_-)^T \mathcal{W}_-$ for some $\mathcal{W}_-$.

Remark 7. It is worth noting that the concept of stochastic finite-time dissipativity is different from the conventional dissipative results [20–23]. Conventional dissipative theory discussed the asymptotic behavior of the system over an infinite-time interval, while the proposed dissipative methods of this paper provide an input-output energy-related characterization to the analysis in a specified finite-time interval. On the other hand, the dissipative control considered in this paper is more general than most of existing finite-time stabilization, finite-time H_∞ control, and FTP control [25–31], which contains H_∞ performance and passive performance as two special cases by choosing appropriate parameters $\mathcal{W}$, $\mathcal{S}$, and $\mathcal{R}$ in inequality (10). When setting $\mathcal{W} = -I$, $\mathcal{S} = 0$, and $\mathcal{R} = (\gamma + \gamma^2)I$, the performance prescribed in Definition 6 becomes stochastic finite-time H_∞ performance [27–30]; if $\mathcal{W} = 0$, $\mathcal{S} = I$, and $\mathcal{R} = 2I$, then it corresponds to FTP performance [31].

Lemma 8 ([40] (schur complement)). *Given constant matrices $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$, where $\mathcal{S}_1 = \mathcal{S}_1^T$ and $\mathcal{S}_2 = \mathcal{S}_2^T > 0$. Then $\mathcal{S}_1 + \mathcal{S}_3^T \mathcal{S}_2^{-1} \mathcal{S}_3 < 0$ if and only if*

$$\begin{aligned} & \begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^T \\ * & -\mathcal{S}_2 \end{bmatrix} < 0 \\ \text{or} & \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ * & \mathcal{S}_1 \end{bmatrix} < 0. \end{aligned} \quad (11)$$

Lemma 9 (see [15]). Let $M \in \mathbb{R}^{n \times n}$ be a positive semidefinite matrix, $x_i \in \mathbb{R}^n$, and α_i ($i = 1, 2, \dots$) be a constant. If the series $\alpha_i \geq 1$ is convergent, then

$$\left(\sum_{i=1}^{+\infty} \alpha_i x_i \right)^T M \left(\sum_{i=1}^{+\infty} \alpha_i x_i \right) \leq \left(\sum_{i=1}^{+\infty} \alpha_i \right) \sum_{i=1}^{+\infty} \alpha_i x_i^T M x_i. \quad (12)$$

Lemma 10 ([40] (S-procedure)). Let $L = L^T$, M , and N be real matrices of appropriate dimensions with F satisfying $F^T F \leq I$, then $L + MFN + N^T F^T M^T < 0$ if and only if there exists a positive scalar ε such that $L + \varepsilon^{-1} M M^T + \varepsilon N^T N < 0$, or, equivalently,

$$\begin{bmatrix} L & M & \varepsilon N^T \\ M^T & -\varepsilon I & 0 \\ \varepsilon N & 0 & -\varepsilon I \end{bmatrix} < 0. \quad (13)$$

3. Main Result

This section will establish the criteria for stochastic finite-time boundedness and stochastic finite-time dissipativity. On this basis, the design method of nonfragile controller is provided.

In this subsection, we focus on the problem for stochastic finite-time boundedness of the system (8).

Theorem 11. For given a scalar $\delta \geq 1$, symmetric matrices $R_i > 0$, the system (8) is SFTB with respect to (c_1, c_2, d, N, R_i) , if there exist symmetric matrices $P_i > 0$, $Q > 0$, and $Z > 0$ and positive scalars ε_i and λ_j ($j = 1, 2, 3, 4$) such that the following conditions hold for all $i \in \Lambda$:

$$\Pi_i = \begin{bmatrix} \Pi_{i11} & \Pi_{i12} \\ * & \Pi_{i22} \end{bmatrix} < 0, \quad (14)$$

$$\lambda_1 R_i < P_i < \lambda_2 R_i,$$

$$0 < Q < \lambda_3 R_i, \quad (15)$$

$$0 < Z < \lambda_4 R_i,$$

$$\begin{bmatrix} \gamma d - c_2 \delta \lambda_1 & c_1 \delta^{N+1} \lambda_2 & \rho_1 \lambda_3 & \rho_2 \lambda_4 \\ * & -c_1 \delta^{N+1} \lambda_2 & 0 & 0 \\ * & * & -\rho_1 \lambda_3 & 0 \\ * & * & * & -\rho_2 \lambda_4 \end{bmatrix} < 0, \quad (16)$$

where

$$\Pi_{i11} = \begin{bmatrix} \Delta_{1i} & 0 & 0 & -\varepsilon_i H_2^T & 0 \\ * & -\delta^{d_m} Q & 0 & 0 & 0 \\ * & * & -\frac{1}{\mu} Z & 0 & 0 \\ * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & -\frac{\gamma}{\delta^N} I \end{bmatrix},$$

$$\Pi_{i12} = [A_i \ A_{di} \ A_{mi} \ C_i \ D_i]^T \cdot [\sqrt{\pi_{i1}} I \ \sqrt{\pi_{i2}} I \ \cdots \ \sqrt{\pi_{is}} I],$$

$$\Pi_{i22} = \text{diag} \{-P_1^{-1}, \dots, -P_s^{-1}\},$$

$$H_1 = \frac{U_1^T U_2 + U_2^T U_1}{2}, \quad (17)$$

$$H_2 = -\frac{U_1^T + U_2^T}{2},$$

$$\Delta_{1i} = -\delta P_i + \tau Q - \bar{\mu} Z - \varepsilon_i H_1,$$

$$\rho_1 = \delta^{N+1} [d_M \delta^{d_m-1} + (\tau_1 - 1) \tau_2 \delta^{d_m-2}],$$

$$\rho_2 = \delta^{N+1} \sigma,$$

$$\tau_1 = d_M - d_m + 1,$$

$$\tau_2 = d_M + d_m - 1,$$

$$\sigma = \mu_1 + (1 + \delta) \mu_2 + \cdots + (1 + \delta + \cdots + \delta^{d_M-1}) \mu_{d_M}$$

Proof. Choose Lyapunov-Krasovskii functional as follows:

$$V(\theta_k, x(k)) = \sum_{i=1}^3 V_i(k), \quad (18)$$

where

$$V_1(k) = x^T(k) P_i x(k),$$

$$\begin{aligned} V_2(k) = & \sum_{s=k-d(k)}^{k-1} \delta^{k-1-s} x^T(s) Q x(s) \\ & + \sum_{s=-d_M+1}^{-d_m+1} \sum_{t=k+s-1}^{k-1} \delta^{k-1-t} x^T(t) Q x(t), \end{aligned} \quad (19)$$

$$V_3(k) = \sum_{m=1}^{+\infty} \mu_m \sum_{l=k-m}^{k-1} \delta^{k-1-l} x^T(l) Z x(l).$$

Now, we assume that

$$\mathbb{E} \{x(l)^T R_i x(l)\} \leq c_1, \quad -\infty < l \leq 0. \quad (20)$$

Our purpose below is to prove that $\mathbb{E} \{x(k)^T R_i x(k)\} \leq c_2$, $k \in \{1, \dots, N\}$. First, by taking the difference variation of $V(k)$ along the trajectory of system (8), we can obtain that

$$\begin{aligned}
& \mathbb{E} \{V_1(\theta_{k+1}, x(k+1)) - \delta V_1(\theta_k, x(k))\} \\
&= \mathbb{E} \{x^T(k+1) P_i x(k+1) - \delta x^T(k) P_i x(k)\} \\
&= x^T(k) \left(\bar{A}_i^T \hat{P}_j \bar{A}_i - \delta P_i \right) x(k) + 2x^T(k) \\
&\quad \cdot \bar{A}_i^T \hat{P}_j \left[A_{di} x_d + A_{mi} \sum_{m=1}^{+\infty} \mu_m x_m + C_i f(x(k)) \right. \\
&\quad \left. + D_i \omega(k) \right] + x_d^T A_{di}^T \hat{P}_j A_{di} x_d \\
&\quad + 2x_d^T A_{di}^T \hat{P}_j \left[A_{mi} \sum_{m=1}^{+\infty} \mu_m x_m + C_i f(x(k)) \right. \\
&\quad \left. + D_i \omega(k) \right] + \left(\sum_{m=1}^{+\infty} \mu_m x_m \right)^T A_{mi}^T \hat{P}_j A_{mi} \sum_{m=1}^{+\infty} \mu_m x_m \\
&\quad + 2 \left(\sum_{m=1}^{+\infty} \mu_m x_m \right)^T A_{mi}^T \hat{P}_j [C_i f(x(k)) + D_i \omega(k)] \\
&\quad + f^T(x(k)) C_i^T \hat{P}_j C_i f(x(k)) + 2f^T(x(k)) \\
&\quad \cdot C_i^T \hat{P}_j D_i \omega(k) + \omega^T(k) D_i^T \hat{P}_j D_i \omega(k),
\end{aligned} \tag{21}$$

where $\hat{P}_j = \sum_{i=1}^s \pi_{ij} P_i$.

Similarly, we have

$$\begin{aligned}
& \mathbb{E} \{V_2(\theta_{k+1}, x(k+1)) - \delta V_2(\theta_k, x(k))\} \\
&= \sum_{s=k+1-d(k+1)}^k \delta^{k-s} x^T(s) Q x(s) \\
&\quad - \sum_{s=k-d(k)}^{k-1} \delta^{k-s} x^T(s) Q x(s) \\
&\quad + \sum_{s=-d_M+2}^{-d_M+1} \sum_{t=k+s}^k \delta^{k-t} x^T(t) Q x(t)
\end{aligned}$$

$$\begin{aligned}
& - \sum_{s=-d_M+2}^{-d_M+1} \sum_{t=k+s-1}^{k-1} \delta^{k-t} x^T(t) Q x(t) \leq (d_M - d_m + 1) \\
& \cdot x^T(k) Q x(k) - \delta^{d_m} x_d^T Q x_d,
\end{aligned} \tag{22}$$

$$\begin{aligned}
& \mathbb{E} \{V_3(\theta_{k+1}, x(k+1)) - \delta V_3(\theta_k, x(k))\} = \sum_{m=1}^{+\infty} \mu_m \\
& \cdot \left[\sum_{l=k+1-m}^k \delta^{k-l} x^T(l) Z x(l) \right. \\
& \quad \left. - \sum_{l=k-m}^{k-1} \delta^{k-l} x^T(l) Z x(l) \right] \\
&= \sum_{m=1}^{+\infty} \mu_m [x^T(k) Z x(k) - \delta^m x_m^T Z x_m].
\end{aligned} \tag{23}$$

Noticing that $\delta \geq 1$ and $\bar{\mu} = \sum_{m=1}^{+\infty} \mu_m$, we can get that

$$\begin{aligned}
& \mathbb{E} \{V_3(\theta_{k+1}, x(k+1)) - \delta V_3(\theta_k, x(k))\} \\
& \leq \bar{\mu} x^T(k) Z x(k) - \sum_{m=1}^{+\infty} \mu_m x_m^T Z x_m.
\end{aligned} \tag{24}$$

According to Lemma 9, we have

$$- \sum_{m=1}^{+\infty} \mu_m x_m^T Z x_m \leq -\frac{1}{\bar{\mu}} \left(\sum_{m=1}^{+\infty} \mu_m x_m \right)^T Z \sum_{m=1}^{+\infty} \mu_m x_m. \tag{25}$$

Then

$$\begin{aligned}
& \mathbb{E} \{V_3(\theta_{k+1}, x(k+1)) - \delta V_3(\theta_k, x(k))\} \\
& \leq \bar{\mu} x^T(k) Z x(k) - \frac{1}{\bar{\mu}} \left(\sum_{m=1}^{+\infty} \mu_m x_m \right)^T Z \sum_{m=1}^{+\infty} \mu_m x_m.
\end{aligned} \tag{26}$$

Therefore

$$\begin{aligned}
& \mathbb{E} \{V(\theta_{k+1}, x(k+1)) - \delta V(\theta_k, x(k))\} \\
& \leq \xi^T(k) \Sigma_i \xi(k),
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
& \xi(k) = \begin{bmatrix} x^T(k) & x_d^T & \left(\sum_{m=1}^{+\infty} \mu_m x_m \right)^T & f^T(x(k)) & \omega^T(k) \end{bmatrix}, \\
& \Sigma_i = \begin{bmatrix} \bar{A}_i^T \hat{P}_j \bar{A}_i - \delta P_i + \tau Q - \bar{\mu} Z & \bar{A}_i^T \hat{P}_j A_{di} & \bar{A}_i^T \hat{P}_j A_{mi} & \bar{A}_i^T \hat{P}_j C_i & \bar{A}_i^T \hat{P}_j D_i \\ * & A_{di}^T \hat{P}_j A_{di} - \delta^{d_m} Q & A_{di}^T \hat{P}_j A_{mi} & A_{di}^T \hat{P}_j C_i & A_{di}^T \hat{P}_j D_i \\ * & * & A_{mi}^T \hat{P}_j A_{mi} - \frac{1}{\bar{\mu}} Z & A_{mi}^T \hat{P}_j C_i & A_{mi}^T \hat{P}_j D_i \\ * & * & * & C_i^T \hat{P}_j C_i & C_i^T \hat{P}_j D_i \\ * & * & * & * & D_i^T \hat{P}_j D_i \end{bmatrix}.
\end{aligned} \tag{28}$$

In addition, from Assumption 2, we can obtain the following inequality:

$$\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} H_1 & H_2 \\ * & I \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \leq 0. \quad (29)$$

Let $\varepsilon_i > 0$, it follows from (27) and (29) that

$$\begin{aligned} & \mathbb{E} \{V(\theta_{k+1}, x(k+1)) - \delta V(\theta_k, x(k))\} \\ & \leq \xi^T(k) \bar{\Sigma}_i \xi(k) + \frac{\gamma}{\delta^N} \omega^T(k) \omega(k), \end{aligned} \quad (30)$$

where

$$\bar{\Sigma}_i = \begin{bmatrix} \Delta_{2i} & \bar{A}_i^T \hat{P}_j A_{di} & \bar{A}_i^T \hat{P}_j A_{mi} & \bar{A}_i^T \hat{P}_j C_i - \varepsilon_i H_2^T & \bar{A}_i^T \hat{P}_j D_{li} \\ * & A_{di}^T \hat{P}_j A_{di} - \delta^{d_m} Q & A_{di}^T \hat{P}_j A_{mi} & A_{di}^T \hat{P}_j C_i & A_{di}^T \hat{P}_j D_{li} \\ * & * & A_{mi}^T \hat{P}_j A_{mi} - \frac{1}{\mu} Z & A_{mi}^T \hat{P}_j C_i & A_{mi}^T \hat{P}_j D_{li} \\ * & * & * & C_i^T \hat{P}_j C_i - \varepsilon_i I & C_i^T \hat{P}_j D_{li} \\ * & * & * & * & D_i^T \hat{P}_j D_{li} - \frac{\gamma}{\delta^N} I \end{bmatrix}, \quad (31)$$

$$\Delta_{2i} = \bar{A}_i^T \hat{P}_j \bar{A}_i - \delta P_i + \tau Q - \bar{\mu} Z - \varepsilon_i H_1.$$

By Lemma 8 and inequality (14), we have $\bar{\Sigma}_i < 0$. Thus

$$\begin{aligned} & \mathbb{E} \{V(\theta_{k+1}, x(k+1)) - \delta V(\theta_k, x(k))\} \\ & < \frac{\gamma}{\delta^N} \omega^T(k) \omega(k), \quad \forall k \in \mathbb{Z}_+. \end{aligned} \quad (32)$$

It follows from (5) and (32) that

$$\begin{aligned} & \mathbb{E} \{V(\theta_k, x(k))\} < \delta^k \mathbb{E} \{V(0)\} \\ & + \frac{\gamma}{\delta^N} \sum_{s=0}^{k-1} \delta^{k-1-s} \omega^T(s) \omega(s) \\ & < \delta^N \mathbb{E} \{V(0)\} \\ & + \frac{\gamma}{\delta^N} \delta^{N-1} \sum_{s=0}^{N-1} \omega^T(s) \omega(s) \\ & < \delta^N \mathbb{E} \{V(0)\} + \frac{\gamma}{\delta} d. \end{aligned} \quad (33)$$

Noticing that condition (15), it yields that

$$\begin{aligned} \mathbb{E} \{V(0)\} &= \mathbb{E} \left\{ x^T(0) P_i x(0) \right. \\ &+ \sum_{s=-d(0)}^{-1} \delta^{-1-s} x^T(s) Q x(s) \\ &+ \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=s-1}^{-1} \delta^{-1-t} x^T(t) Q x(t) \\ &\left. + \sum_{m=1}^{+\infty} \mu_m \sum_{l=-m}^{-1} \delta^{-1-l} x^T(l) Z x(l) \right\} \end{aligned}$$

$$\begin{aligned} & < \lambda_2 \mathbb{E} \{x^T(0) R_i x(0)\} \\ & + \lambda_3 \delta^{d_M-1} \sum_{s=-d_M}^{-1} x^T(s) R_i x(s) \\ & + \lambda_3 \delta^{d_M-2} \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=-1+s}^{-1} x^T(t) R_i x(t) + \lambda_4 \sum_{m=1}^{+\infty} \mu_m \\ & \cdot \sum_{l=-m}^{-1} \delta^{-1-l} x^T(l) R_i x(l). \end{aligned} \quad (34)$$

Owing to

$$\begin{aligned} & \sum_{m=1}^{+\infty} \mu_m \sum_{l=-m}^{-1} \delta^{-1-l} x^T(l) R_i x(l) = \mu_1 x^T(-1) R_i x(-1) \\ & + \mu_2 [\delta x^T(-2) R_i x(-2) + x^T(-1) R_i x(-1)] \\ & + \mu_3 [\delta^2 x^T(-3) R_i x(-3) + \delta x^T(-2) R_i x(-2) \\ & + x^T(-1) R_i x(-1)] + \dots \\ & + \mu_m [\delta^{m-1} x^T(-m) R_i x(-m) \\ & + \delta^{m-2} x^T(-m+1) R_i x(-m+1) + \dots \\ & + x^T(-1) R_i x(-1)] + \dots = \mu_1 x^T(-1) R_i x(-1) \\ & + \mu_2 [\delta x^T(-2) R_i x(-2) + x^T(-1) R_i x(-1)] \\ & + \mu_3 [\delta^2 x^T(-3) R_i x(-3) + \delta x^T(-2) R_i x(-2) \\ & + x^T(-1) R_i x(-1)] + \dots \end{aligned}$$

$$\begin{aligned}
& + \mu_{d_M} \left[\delta^{d_M-1} x^T (-d_M) R_i x (-d_M) \right. \\
& + \delta^{d_M-2} x^T (-d_M+1) R_i x (-d_M+1) + \cdots \\
& \left. + x^T (-1) R_i x (-1) \right] \triangleq c_1 \sigma,
\end{aligned} \tag{35}$$

we have

$$\begin{aligned}
& \mathbb{E} \{V(0)\} \\
& < \left[\lambda_2 + \lambda_3 d_M \delta^{d_M-1} + \lambda_3 (\tau_1 - 1) \tau_2 \delta^{d_M-2} + \lambda_4 \sigma \right] c_1 \tag{36} \\
& \triangleq c_1.
\end{aligned}$$

Taking (33) and (36) into account, we have

$$\mathbb{E} \{V(\theta_k, x(k))\} < \delta^N c_1 + \frac{\gamma}{\delta} d, \quad \forall k \in Z_+. \tag{37}$$

On the other hand

$$\begin{aligned}
& \mathbb{E} \{V(\theta_k, x(k))\} \geq \mathbb{E} \{x^T(k) P_i x(k)\} \\
& > \lambda_1 \{ \mathbb{E} x^T(k) R_i x(k) \} \quad \forall k \in Z_+.
\end{aligned} \tag{38}$$

Consequently, we get from (37) and (38) that

$$\mathbb{E} \{x^T(k) R_i x(k)\} < \frac{\delta^{N+1} c_1 + \gamma d}{\delta \lambda_1}. \tag{39}$$

Note that

$$\begin{aligned}
& \delta^{N+1} c_1 + \gamma d - c_2 \delta \lambda_1 < 0 \iff \\
& \gamma d - c_2 \delta \lambda_1 + c_1 \delta^{N+1} \lambda_2 + c_1 \rho_1 \lambda_3 + c_1 \rho_2 \lambda_4 < 0 \iff \\
& \gamma d - c_2 \delta \lambda_1 \\
& + [c_1 \delta^{N+1} \lambda_2 \quad \rho_1 \lambda_3 \quad \rho_2 \lambda_4] \Phi_1 [c_1 \delta^{N+1} \lambda_2 \quad \rho_1 \lambda_3 \quad \rho_2 \lambda_4]^T \\
& < 0,
\end{aligned} \tag{40}$$

where

$$\Phi_1 = \begin{bmatrix} c_1 \delta^{N+1} \lambda_2 & 0 & 0 \\ 0 & \rho_1 \lambda_3 & 0 \\ 0 & 0 & \rho_2 \lambda_4 \end{bmatrix}^{-1}. \tag{41}$$

By Lemma 8, inequality (40) is equivalent to LMI (14). Then we have

$$\mathbb{E} \{x^T(k) R_i x(k)\} < c_2. \tag{42}$$

The proof is completed. $\square$

3.1. Stochastic Finite-Time $(\mathcal{W}, \mathcal{S}, \mathcal{R}) - \gamma$ Dissipative. In this subsection, we give the following SFTD conditions for system (8) based on Theorem 11.

Theorem 12. For given a scalar $\delta \geq 1$, symmetric matrices $R_i > 0$, and matrices $\mathcal{W}$, $\mathcal{S}$, and $\mathcal{R}$ satisfying that $\mathcal{W}$ and $\mathcal{R}$ are symmetric and $\mathcal{W} \leq 0$, the system (8) is stochastic finite-time $(\mathcal{W}, \mathcal{S}, \mathcal{R}) - \gamma$ dissipative if there exist symmetric matrices $P_i > 0$, $Q > 0$, and $Z > 0$ and positive scalars ε_i and λ_j ($j = 1, \dots, 7$) such that the following conditions hold for all $i \in \Lambda$:

$$\widehat{\Pi}_i = \begin{bmatrix} \widehat{\Pi}_{i11} & \widehat{\Pi}_{i12} & \widehat{\Pi}_{i13} \\ * & \widehat{\Pi}_{i22} & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{43}$$

$$\lambda_1 R_i < P_i < \lambda_2 R_i,$$

$$0 < Q < \lambda_3 R_i,$$

$$0 < Z < \lambda_4 R_i,$$

$$0 < F_i^T F_i < \lambda_5 R_i,$$

$$0 < \mathcal{S}^T \mathcal{S} < \lambda_6 I,$$

$$0 < \mathcal{R} < \lambda_7 I,$$

(44)

$$\begin{bmatrix} -c_2 \lambda_1 & c_1 \delta^{N+1} \lambda_2 & \rho_1 \lambda_3 & \rho_2 \lambda_4 & \rho_3 \lambda_5 & \rho_4 \lambda_6 & \rho_5 \lambda_7 \\ * & -c_1 \delta^{N+1} \lambda_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\rho_1 \lambda_3 & 0 & 0 & 0 & 0 \\ * & * & * & -\rho_2 \lambda_4 & 0 & 0 & 0 \\ * & * & * & * & -\rho_3 \lambda_5 & 0 & 0 \\ * & * & * & * & * & -\rho_4 \lambda_6 & 0 \\ * & * & * & * & * & * & -\rho_5 \lambda_7 \end{bmatrix} < 0, \tag{45}$$

where

$$\begin{aligned}
& \widehat{\Pi}_{i11} = \begin{bmatrix} \Delta_{1i} & 0 & 0 & -\varepsilon_i H_2^T & -F_i^T \mathcal{S} \\ * & -\delta^{d_m} Q & 0 & 0 & 0 \\ * & * & -\frac{1}{\mu} Z & 0 & 0 \\ * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & \gamma I - \mathcal{R} \end{bmatrix}, \\
& \widehat{\Pi}_{i12} = \Pi_{i12}, \\
& \widehat{\Pi}_{i13} = [\mathcal{W} - F_i \quad 0 \quad 0 \quad 0 \quad 0]^T, \\
& \widehat{\Pi}_{i22} = \Pi_{i22}, \\
& \rho_3 = \delta^{N-1} N, \\
& \rho_4 = \rho_5 = \delta^{N-1} d.
\end{aligned} \tag{46}$$

Other parameters are shown in Theorem 11.

Proof. Firstly, we will show that inequality (10) holds. Define the following function:

$$\begin{aligned}
J(k) &= z^T(k) \mathcal{W} z(k) + 2z^T(k) \mathcal{S} \omega(k) \\
&+ \omega^T(k) (\mathcal{R} - \gamma I) \omega(k)
\end{aligned} \tag{47}$$

Then by choosing the same Lyapunov-Krasovskii functional and using similar handling methods as Theorem 11, we can obtain that

$$\begin{aligned} & \mathbb{E} \{V(\theta_{k+1}, x(k+1)) - \delta V(\theta_k, x(k)) - J(k)\} \\ & < \xi^T(k) \Theta_i \xi(k), \end{aligned} \quad (48)$$

where

$$\Theta_i = \begin{bmatrix} \Delta_{1i} - F_i^T \mathcal{W} F_i & \bar{A}_i^T \hat{P}_j A_{di} & \bar{A}_i^T \hat{P}_j A_{mi} & \bar{A}_i^T \hat{P}_j C_i - \varepsilon_i H_2^T & \bar{A}_i^T \hat{P}_j D_i - F_i^T \mathcal{S} \\ * & A_{di}^T \hat{P}_j A_{di} - \delta^{d_m} Q & A_{di}^T \hat{P}_j A_{mi} & A_{di}^T \hat{P}_j C_i & A_{di}^T \hat{P}_j D_i \\ * & * & A_{mi}^T \hat{P}_j A_{mi} - \frac{1}{\mu} Z & A_{mi}^T \hat{P}_j C_i & A_{mi}^T \hat{P}_j D_i \\ * & * & * & C_i^T \hat{P}_j C_i - \varepsilon_i I & C_i^T \hat{P}_j D_i \\ * & * & * & * & D_i^T \hat{P}_j D_i - \mathcal{R} + \gamma I \end{bmatrix}. \quad (49)$$

By Lemma 8, we know that the condition (43) ensures that $\Theta_i < 0$. This leads to

$$\mathbb{E} \{V(\theta_{k+1}, x(k+1))\} < \mathbb{E} \{\delta V(\theta_k, x(k)) + J(k)\}, \quad (50)$$

and, by iteration, we get

$$\mathbb{E} \{V(\theta_k, x(k))\} < \mathbb{E} \left\{ \delta^k V(0) + \sum_{s=0}^{k-1} \delta^{k-1-s} J(s) \right\}. \quad (51)$$

Notice that $\mathbb{E}\{V(\theta_k, x(k))\} > 0$ and under the zero-initial condition, we can obtain that

$$\begin{aligned} & \sum_{s=0}^N \delta^{N-s} \mathbb{E} \{z^T(s) \mathcal{W} z(s) + 2z^T(s) \mathcal{S} \omega(s) \\ & + \omega^T(s) \mathcal{R} \omega(s) - \gamma \omega^T(s) \omega(s)\} > 0. \end{aligned} \quad (52)$$

Owing to $\delta \geq 1$, (55) immediately yields that

$$\begin{aligned} & \sum_{s=0}^N \mathbb{E} \{z^T(s) \mathcal{W} z(s) + 2z^T(s) \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s) \\ & - \gamma \omega^T(s) \omega(s)\} > 0, \end{aligned} \quad (53)$$

that is,

$$\begin{aligned} & \sum_{s=0}^N \mathbb{E} \{z^T(s) \mathcal{W} z(s) + 2z^T(s) \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s)\} \\ & > \gamma \sum_{s=0}^N \omega^T(s) \omega(s). \end{aligned} \quad (54)$$

Thus inequality (10) holds.

In the following, we prove that the system (8) is SFTB. From the above proof and Theorem 11, we can obtain that

$$\begin{aligned} & \mathbb{E} \{V(\theta_k, x(k))\} < \delta^k c_1 + \sum_{s=0}^{k-1} \delta^{k-1-s} \mathbb{E} \{J(s)\} \\ & < \delta^N c_1 + \delta^{N-1} \sum_{s=0}^{N-1} \mathbb{E} \{J(s)\}. \end{aligned} \quad (55)$$

Note that $\mathcal{W} < 0$, we derive that

$$\begin{aligned} & \sum_{s=0}^{N-1} \mathbb{E} \{J(s)\} \leq \sum_{s=0}^{N-1} \{2z^T(s) \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s)\}, \\ & \mathbb{E} \{V(\theta_k, x(k))\} \\ & < \delta^N c_1 + \sum_{s=0}^{N-1} \mathbb{E} \{2z^T(s) \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s)\}. \end{aligned} \quad (56)$$

Thus we have

$$\begin{aligned} & \mathbb{E} \{x^T(k) R_i x(k)\} < \frac{\mathbb{E} \{V(r_k, x(k))\}}{\lambda_1} \\ & < \frac{\delta^N c_1 + \sum_{s=0}^{N-1} \mathbb{E} \{2z^T(s) \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s)\}}{\lambda_1}. \end{aligned} \quad (57)$$

Since

$$\begin{aligned} & \sum_{s=0}^{N-1} \{2z^T(s) \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s)\} \\ & = \sum_{s=0}^{N-1} \{2x^T(s) F_i^T \mathcal{S} \omega(s) + \omega^T(s) \mathcal{R} \omega(s)\}, \end{aligned} \quad (58)$$

by nature of the vector, we have

$$\begin{aligned} & 2x^T(s) F_i^T \mathcal{S} \omega(s) \leq x^T(s) F_i^T F_i x(s) \\ & + \omega^T(s) \mathcal{S}^T \mathcal{S} \omega(s). \end{aligned} \quad (59)$$

It follows that

$$\mathbb{E} \{x^T(k) R_i x(k)\} < \frac{\delta^{N-1} [\zeta_2 + (\lambda_6 + \lambda_7) d]}{\lambda_1}, \quad (60)$$

where $\zeta_2 = \delta \lambda_2 + \lambda_3 [d_M \delta^{d_M} + (\tau_1 - 1) \tau_2 \delta^{d_M-1} + \delta \lambda_4 \sigma + N \lambda_5]$.

Therefore, it can be easily verified from inequality (60) that

$$\mathbb{E} \{x^T(k) R_i x(k)\} < c_2. \quad (61)$$

Then, we conclude that the system (8) is stochastic finite-time $(\mathcal{W}, \mathcal{S}, \mathcal{R}) - \gamma$ -dissipative with respect to (c_1, c_2, d, N, R_i) from Definition 6. The proof is now completed. $\square$

3.2. Nonfragile Feedback Controller Design. Now we will provide the design scheme of nonfragile controller for NNS (1).

Theorem 13. For given scalar $\delta \geq 1$ and symmetric matrices $R_i > 0$ and matrices $\mathcal{W}$, $\mathcal{S}$, and $\mathcal{R}$ satisfying that $\mathcal{W}$ and $\mathcal{R}$

are symmetric and $\mathcal{W} \leq 0$, the system (1) is stochastic finite-time $(\mathcal{W}, \overline{\mathcal{S}}, \overline{\mathcal{R}})$ - γ dissipative if there exist symmetric matrices $T_i > 0$, $X_i > 0$, $\overline{Q}_i > 0$, $\overline{Z}_i > 0$, and $W_{ji} > 0$ ($j = 1, \dots, 7$), a nonsingular matrix L_i , and positive scalars ρ_l ($l = 1, 2, 3, 4, 5$) and α_i such that the following conditions hold for all $i \in \Lambda$:

$$\check{\Pi}_i = \begin{bmatrix} \check{\Pi}_{i11} & \widetilde{M}_i & \alpha_i \widetilde{N}_i^T \\ * & -\alpha_i I & 0 \\ * & * & -\alpha_i I \end{bmatrix} < 0, \quad (62)$$

$$\begin{aligned} T_i &< W_{2i}, \\ \overline{Q}_i &< W_{3i}, \\ \overline{Z}_i &< W_{4i}, \\ \overline{F}_i^T \overline{F}_i &< W_{5i}, \\ \overline{\mathcal{S}}^T \overline{\mathcal{S}} &< W_{6i}, \\ \overline{\mathcal{R}} &< W_{7i}, \end{aligned} \quad (63)$$

$$\begin{bmatrix} -W_{1i} & c_1 \delta^N W_{2i} & \rho_1 W_{3i} & \rho_2 W_{4i} & \rho_3 W_{5i} & \rho_4 W_{6i} & \rho_5 W_{7i} \\ * & -c_1 \delta^N W_{2i} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\rho_1 W_{3i} & 0 & 0 & 0 & 0 \\ * & * & * & -\rho_2 W_{4i} & 0 & 0 & 0 \\ * & * & * & * & -\rho_3 W_{5i} & 0 & 0 \\ * & * & * & * & * & -\rho_4 W_{6i} & 0 \\ * & * & * & * & * & * & -\rho_5 W_{7i} \end{bmatrix} < 0, \quad (64)$$

The desired state feedback controller gain matrix is given by

$$K_i = L_i T_i^{-1}, \quad (65)$$

where

$$\check{\Pi}_{i11} = \begin{bmatrix} \Delta_{3i} & 0 & 0 & -\varepsilon_i H_2^T & -F_i^T \mathcal{S} & \sqrt{\pi_{1i}} (T_i A_i^T + L_i^T B_i^T) & \cdots & \sqrt{\pi_{si}} (T_i A_i^T + L_i^T B_i^T) & F_i^T \mathcal{W}^- \\ * & -\delta^{d_m} Q_i & 0 & 0 & 0 & \sqrt{\pi_{1i}} A_{di}^T & \cdots & \sqrt{\pi_{si}} A_{di}^T & 0 \\ * & * & -\frac{1}{\mu} Z_i & 0 & 0 & \sqrt{\pi_{1i}} A_{mi}^T & \cdots & \sqrt{\pi_{si}} A_{mi}^T & 0 \\ * & * & * & -\varepsilon_i I & 0 & \sqrt{\pi_{1i}} C_i^T & \cdots & \sqrt{\pi_{si}} C_i^T & 0 \\ * & * & * & * & \gamma I - \mathcal{R} & \sqrt{\pi_{1i}} D_i^T & \cdots & \sqrt{\pi_{si}} D_i^T & 0 \\ * & * & * & * & * & -T_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 \\ * & * & * & * & * & * & * & -T_s & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\Delta_{3i} = -\delta U_i + \tau Q_i - \bar{\mu} Z_i - \varepsilon_i X_i,$$

$$\begin{aligned}\widetilde{M}_i &= [0, 0, 0, 0, 0, \sqrt{\pi_{1i}}B_iM_i, \dots, 0]^T, \\ \widetilde{N}_i &= [N_i, 0, 0, 0, 0, 0, \dots, 0].\end{aligned}$$

(66)

Proof. From Theorem 12, it is derived that

$$\widehat{\Pi}_i < 0. \quad (67)$$

Noting that

$$\Delta K_i = M_i \Delta(k) N_i, \quad (68)$$

we have

$$\widehat{\Pi}_i = \Xi_i + \widetilde{M}_i \Delta(k) \widetilde{N}_i + \widetilde{N}_i^T \Delta(k) \widetilde{M}_i^T < 0. \quad (69)$$

Moreover, inequality (69) is equivalent to

$$\Xi_i + \alpha_i^{-1} \widetilde{M}_i \widetilde{M}_i^T + \alpha_i^{-1} (\alpha_i \widetilde{N}_i)^T \Delta(k) (\alpha_i \widetilde{N}_i) < 0. \quad (70)$$

By using Lemma 10, the following inequality holds:

$$\begin{bmatrix} \Xi_i & \widetilde{M}_i & \alpha_i \widetilde{N}_i^T \\ * & -\alpha_i I & 0 \\ * & * & -\alpha_i I \end{bmatrix} < 0, \quad (71)$$

where

$$\Xi_i = \begin{bmatrix} \Xi_{i11} & \Xi_{i12} & \Xi_{i13} \\ * & \Xi_{i22} & 0 \\ * & * & \Xi_{i33} \end{bmatrix},$$

$$\Xi_{i11} = \begin{bmatrix} \Delta_{1i} & 0 & 0 & -\varepsilon_i H_2^T & -F_i^T \mathcal{S} \\ * & -\delta^{d_m} Q_i & 0 & 0 & 0 \\ * & * & -\frac{1}{\mu} Z_i & 0 & 0 \\ * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & \gamma I - \mathcal{R} \end{bmatrix},$$

$$\Xi_{i12}$$

$$= \begin{bmatrix} \sqrt{\pi_{1i}}(A_i + B_i K_i)^T & \dots & \sqrt{\pi_{si}}(A_i + B_i K_i)^T \\ \sqrt{\pi_{1i}}A_{di}^T & \dots & \sqrt{\pi_{si}}A_{di}^T \\ \sqrt{\pi_{1i}}A_{mi}^T & \dots & \sqrt{\pi_{si}}A_{mi}^T \\ \sqrt{\pi_{1i}}C_i^T & \dots & \sqrt{\pi_{si}}C_i^T \\ \sqrt{\pi_{1i}}D_i^T & \dots & \sqrt{\pi_{si}}D_i^T \end{bmatrix},$$

$$\Xi_{i13} = [\mathcal{W}_i F_i \ 0 \ 0 \ 0 \ 0]^T,$$

$$\Xi_{i22} = \text{diag}\{-U_1, \dots, -U_s\},$$

$$\Xi_{i33} = -I.$$

(72)

Letting $J = \text{diag}\{P_i^{-1}, P_i^{-1}, P_i^{-1}, I, I, I, I, \dots, I\}$ and then pre- and postmultiplying inequality (71) with J and letting $P_i^{-1}K_i = L_i$, $T_i = P_i^{-1}$, $Q_i = P_i^{-1}Q_i P_i^{-1}$, $Z_i = P_i^{-1}Z_i P_i^{-1}$, and $\overline{F}_i = F_i^T F_i$ ($i \in \Lambda$), it is easy to see that inequality (71) is equivalent to inequality (62).

For getting LMI (64), we note that the inequality (45) can be regarded as

$$\begin{bmatrix} -c_2 \lambda_1 I & (c_1 \delta^{N+1} \lambda_2) I & (\rho_1 \lambda_3) I & (\rho_2 \lambda_4) I & (\rho_3 \lambda_5) I & (\rho_4 \lambda_6) I & (\rho_5 \lambda_7) I \\ * & (-c_1 \delta^{N+1} \lambda_2) I & 0 & 0 & 0 & 0 & 0 \\ * & * & (-\rho_1 \lambda_3) I & 0 & 0 & 0 & 0 \\ * & * & * & (-\rho_2 \lambda_4) I & 0 & 0 & 0 \\ * & * & * & * & (-\rho_3 \lambda_5) I & 0 & 0 \\ * & * & * & * & * & (-\rho_4 \lambda_6) I & 0 \\ * & * & * & * & * & * & (-\rho_5 \lambda_7) I \end{bmatrix} < 0. \quad (73)$$

Postmultiplying inequality (73) by the matrix $\text{diag}\{R_i, R_i, R_i, R_i, R_i, I, I\}$, we can get that

$$\begin{bmatrix}
-c_2\lambda_1 R_i & (c_1\delta^{N+1}\lambda_2) R_i & (\rho_1\lambda_3) R_i & (\rho_2\lambda_4) R_i & (\rho_3\lambda_5) R_i & (\rho_4\lambda_6) I & (\rho_5\lambda_7) I \\
* & (-c_1\delta^{N+1}\lambda_2) R_i & 0 & 0 & 0 & 0 & 0 \\
* & * & (-\rho_1\lambda_3) R_i & 0 & 0 & 0 & 0 \\
* & * & * & (-\rho_2\lambda_4) R_i & 0 & 0 & 0 \\
* & * & * & * & (-\rho_3\lambda_5) R_i & 0 & 0 \\
* & * & * & * & * & (-\rho_4\lambda_6) I & 0 \\
* & * & * & * & * & * & (-\rho_5\lambda_7) I
\end{bmatrix} < 0. \quad (74)$$

Denoting $J_1 = \text{diag}\{P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}\}$. By pre- and postmultiplying (74) by J_1 and letting $W_{1i} = c_2\delta P_i^{-1}(\lambda_1 R_i)P_i^{-1}$, $W_{2i} = P_i^{-1}(\lambda_2 R_i)P_i^{-1}$, $W_{3i} = P_i^{-1}(\lambda_3 R_i)P_i^{-1}$, $W_{4i} = P_i^{-1}(\lambda_4 R_i)P_i^{-1}$, $W_{5i} = P_i^{-1}(\lambda_5 R_i)P_i^{-1}$, $W_{6i} = P_i^{-1}(\lambda_6 I)P_i^{-1}$, and $W_{7i} = P_i^{-1}(\lambda_7 I)P_i^{-1}$, we can obtain inequality (65). This completes the proof. $\square$

Remark 14. In Theorem 13, sufficient conditions for guaranteeing stochastic finite-time dissipativity are proposed. It is easy to see that the obtained results of this paper are influenced by all system parameters, such as the bounds on the time-delay and the jumping transition probabilities. In addition, the design problem of the nonfragile state gain can be transformed into the following optimization problem:

$$\begin{aligned}
\min \quad & (c_2 + \gamma) \\
\text{s.t.} \quad & (64), (65) \text{ and } (69).
\end{aligned} \quad (75)$$

4. Numerical Examples

In this section, two numerical examples are provided to illustrate the effectiveness of the approach addressed.

Example 1. We consider the same example in [28]; the system parameters are as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} -1.5 & 1 \\ -1.03 & -5.2 \end{bmatrix}, \\
A_{d1} &= \begin{bmatrix} 0.02 & 0 \\ -0.01 & 0.5 \end{bmatrix}, \\
A_{m1} &= \begin{bmatrix} 0.04 & 0.02 \\ 0.05 & 0.04 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -0.5 & 6 \\ 0 & -1 \end{bmatrix}, \\
A_{d2} &= \begin{bmatrix} 0 & 0.12 \\ 0 & 0.2 \end{bmatrix}, \\
A_{m2} &= \begin{bmatrix} -0.08 & 0.06 \\ 0 & -0.1 \end{bmatrix},
\end{aligned}$$

$$B_1 = \begin{bmatrix} -2.5 & 1 \\ 0 & -5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.04 & 0 \\ 0.05 & -0.14 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.18 & 0 \\ 0 & 0.02 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.02 & 0 \\ -0.04 & 0.1 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.1 & 0.02 \\ 0.01 & 0.3 \end{bmatrix},$$

$$F_1 = [-0.12 \quad -0.02],$$

$$F_2 = [0.04 \quad 0.5],$$

(76)

Assume that $N = 15$, $c_1 = 0.9$, $\delta = 1.5$. We choose $\mathcal{W} = 0$, $\mathcal{S} = I$, $\mathcal{R} = 2I$, and $R_1 = R_2 = I$; the time-varying delay $d(k) = 1 + 0.03 \sin^2(0.5k\pi)$ satisfies $0.97 < d(k) < 4$. The transition matrix $\Pi = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$. Moreover, we can get $\omega^T(k)\omega(k) \leq 0.7 = d$ when the external disturbance $\omega(k)$ is chosen as $\omega(k) = [0.21/(1+k^2), 0.21/(1+k^2)]^T$.

In addition, we take the following activation function:

$$f(x(k)) = [0, 0]^T \quad (77)$$

and we can conclude that

$$\begin{aligned}
U_1 &= V_1 = \text{diag}\{0, 0\}, \\
U_2 &= V_2 = \text{diag}\{0, 0\}
\end{aligned} \quad (78)$$

According to the methods in [28], we can get the optimal values $\gamma_{\min} = 5.9474$ and $c_{2\min} = 11.5133$, respectively. While using methods of this paper, we can obtain the optimal values

FIGURE 1: The trajectory of state $x(k)$. FIGURE 2: The curve of $\mathbb{E}\{x^T(k)R_i x(k)\}$.

$\gamma_{min} = 4.3640$ and $c_{2min} = 9.2560$, respectively, and the corresponding controller gain matrices are given as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.5959 & -0.0143 \\ -0.2016 & -1.0353 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.2770 & 3.7916 \\ -0.1020 & 2.2540 \end{bmatrix}. \end{aligned} \quad (79)$$

Since the smaller γ and c_2 , the better system performance is. Thus, our results are better than that in [28]. In addition, we can get that the corresponding system is SFTB and satisfies the desired finite-time H_∞ performance by simulation results in Figures 1 and 2. This shows that the results of this paper can be applied to the stochastic finite-time H_∞ control problem, which further verifies that the proposed results in this paper are more general than that in [28].

Example 2. Consider the discrete time NNS (1). Its parameters are shown below:

$$A_1 = \begin{bmatrix} 0.02 & -0.05 & -0.12 \\ 0.03 & 0.06 & -0.1 \\ 0.06 & 0.08 & -0.08 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.02 & 0.06 & 0.04 \\ 0.04 & 0.01 & -0.03 \\ 0.01 & 0.02 & 0.05 \end{bmatrix},$$

$$A_{m1} = \begin{bmatrix} 0.04 & 0.02 & 0 \\ 0.05 & 0.04 & 0 \\ 0.02 & 0.06 & 0.03 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.02 & -0.06 & 0.07 \\ 0.06 & 0.04 & -0.04 \\ 0.01 & 0.02 & 0.014 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0.08 & 0.1 \\ 0 & 0.05 & -0.06 \\ -0.03 & 0.04 & -0.05 \end{bmatrix},$$

$$A_{m2} = \begin{bmatrix} -0.08 & 0.06 & 0 \\ 0 & -0.1 & 0.04 \\ 0.02 & 0.04 & 0.02 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -1.0 & 0.23 \\ 0.5 & 0.05 \\ 0.4 & 0.3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.8 & -0.17 \\ 1.0 & 0.4 \\ 0.3 & 0.26 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ 0 & 0 & 0.001 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2.6 & 2 \\ 3 & 3 & 2 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0 & 0.001 & 0.01 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.001 \end{bmatrix},$$

FIGURE 3: The trajectory of state $x(k)$.

$$F_2 = \begin{bmatrix} 1.2 & 1.3 & 0.1 \\ 2 & 2.5 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \quad (80)$$

Assume that $N = 40$, $c_1 = 1$, and $\delta = 1.0027$ and M_1 , M_2 , N_1 , and N_2 and $\Delta(k)$ are selected as follows:

$$\begin{aligned} M_1 &= \begin{bmatrix} 0.2 & 0.4 \\ -0.1 & 0.2 \\ 0.2 & -0.3 \end{bmatrix}^T, \\ M_2 &= \begin{bmatrix} 0.2 & 0.4 \\ -0.1 & 0.2 \\ 0.4 & -0.6 \end{bmatrix}^T, \\ N_1 &= \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.3 \end{bmatrix}, \\ N_2 &= \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.15 \end{bmatrix}, \\ \Delta(k) &= \begin{bmatrix} \sin(0.2k) & 0 & 0 \\ 0 & \cos(0.2k) & 0 \\ 0 & 0 & \sin(0.2k) \end{bmatrix}. \end{aligned} \quad (81)$$

In addition, in this example, we take dissipativity matrices $\mathcal{W} = -0.3$, $\mathcal{S} = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}^T$, $\mathcal{R} = \begin{bmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 1 & 0.1 \\ 0.1 & 0.1 & 1 \end{bmatrix}$, $R_1 = R_2 =$

$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, and transition matrix $\Pi = \begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}$. The constant sequence $\mu_m = 3^{-(3+m)}$. It can be checked that $\bar{\mu} = \sum_{m=1}^{+\infty} \mu_m = 1/54 < \sum_{m=1}^{+\infty} m\mu_m = 1/36 < +\infty$, which satisfies the convergence condition (2). Moreover, we can get $\omega^T(k)\omega(k) \leq 0.62 = d$ when the external disturbance $\omega(k)$ is chosen as $\omega(k) = [0.21/(1+k^2), 0.21/(1+k^2), 0.21/(1+k^2)]^T$.

Activation function is given as

$$\begin{aligned} f(x(k)) &= [\tanh(-1.2x_1(k)), \tanh(0.8x_1(k)), \tanh(x_1(k))]^T \end{aligned} \quad (82)$$

and we can conclude that

$$\begin{aligned} U_1 &= V_1 = \text{diag}\{0, 0, 0\}, \\ U_2 &= V_2 = \text{diag}\{1.2, 0.8, 1\} \end{aligned} \quad (83)$$

Other parameters are the same as Example 1. By solving the LMIs (65), (69), (70), and (71), we can obtain the optimal value $\gamma_{\min} = 1.340e^{-12}$, $c_{2\min} = 6.4624$, and state feedback gain matrices as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.8625 & -0.4365 & -0.0090 \\ 0.2306 & -0.3940 & -0.0240 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 0.2016 & 0.1725 & 0.0041 \\ -1.2945 & -0.5934 & -0.0324 \end{bmatrix} \end{aligned} \quad (84)$$

The effectiveness of the proposed methods will be shown in Figures 3 and 4, in which Figure 3 shows the state trajectories of the closed-loop system and Figure 4 denotes the time history of $\mathbb{E}\{x(k)R_i x(k)\}$. From Figures 3 and 4, we can obtain that the system (8) is SFTD; this shows that designed nonfragile state feedback controller is effective.

FIGURE 4: The curve of $\mathbb{E}\{x^T(k)R_i x(k)\}$.

TABLE 1: Optimized variable for each case.

Case	c_{2min}	γ_{min}
H_∞ control	7.2134	$3.4302e^{-12}$
Passivity control	8.1320	$1.8893e^{-12}$

FIGURE 5: The trajectory of state $x(k)$.

Furthermore, in order to show that the dissipative results in this paper are more general, we choose matrices on Remark 14 for each case. By using LMI toolbox in Matlab, the optimal value and the corresponding controller gain matrices can be obtained for each case in Tables 1 and 2.

The simulation results are shown in Figures 5–8. Figures 5 and 7 show the state responses of the closed-loop system under two difference cases, respectively. Figures 6 and 8 show the time history of $\mathbb{E}\{x(k)R_i x(k)\}$ under two difference cases, respectively. It is not difficult to find that the system is SFTB and satisfies corresponding performance under two

FIGURE 6: The curve of $\mathbb{E}\{x^T(k)R_i x(k)\}$.

difference cases from Figures 5–8. This shows that the dissipative control method obtained in this paper can also solve the H_∞ control and passive control problem, which proves that dissipative performance is a more general result than H_∞ performance and passive performance. These results show that different control goals can be achieved in the process of dissipative control. Therefore, in the actual system dissipative control can save costs and time.

5. Conclusions

In this paper, stochastic finite-time nonfragile dissipative control issue for a class of discrete-time NNS in the presence of Markovian jumps and mixed delays has been presented. By employing stochastic analysis method and LMI technique, some sufficient conditions of stochastic finite-time dissipativity for the system (8) are obtained. Through the feasible solution of LMIs, the analytic expression of the expected nonfragile controller gain is provided. Two numerical examples have been used to verify the effectiveness of the proposed theoretical results. It is interesting for us to extended derived results to SFTD filtering problems. On the

TABLE 2: Controller gain for each case.

Case	K_1	K_2
H_∞ control	$\begin{bmatrix} -0.8688 & -0.4377 & -0.0090 \\ 0.2273 & -0.3513 & -0.0205 \end{bmatrix}$	$\begin{bmatrix} 0.2038 & 0.1707 & 0.0038 \\ -1.2731 & -0.5455 & -0.0295 \end{bmatrix}$
Passivity control	$\begin{bmatrix} -0.8684 & -0.4398 & -0.0089 \\ 0.2325 & -0.4040 & -0.0241 \end{bmatrix}$	$\begin{bmatrix} 0.2023 & 0.1740 & 0.0041 \\ -1.3098 & -0.6022 & -0.0323 \end{bmatrix}$

FIGURE 7: The trajectory of state $x(k)$. FIGURE 8: The curve of $\mathbb{E}\{x^T(k)R_i x(k)\}$.

other hand, the proposed approach in this paper can also further extended to more complicated discrete-time NNS such as actuator saturation [32] and fading measurements [33].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors confirm that none of the authors have any conflicts of interest in the manuscript.

Authors' Contributions

Ling Hou contributed significantly to analysis and manuscript preparation, and she performed the data analysis and wrote the manuscript. Dongyan Chen helped to perform the analysis with constructive discussions. Chan He helped to complete the simulation of the examples. All authors read and approved the final manuscript.

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