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Research Article

Interval State Estimation of Linear Multicellular Systems

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Linear multicellular system is a type of differential inclusion system, which can be deemed as an extension of linear control system with set-valued mapping. As an important issue in existing control systems, interval state estimation has been widely applied in engineering practices. Over the years, the objects of the studies on interval state estimation have been extended from the initial linear time-invariant systems to linear time-varying systems, chaotic systems, feedback linearization systems, and nonlinear Lipschitz systems. However, there is no report on the design of interval observer for linear multicellular system. To make up for this gap, this chapter attempts to explore the design of an interval observer for linear multicellular systems.

1. Introduction

With the rapid development of science and technology, the traditional state estimation theory can no longer accurately estimate the system state, due to the heavy presence of uncertainties in actual systems. The interval observer came into being under this background. In recent years, scholars have paid much attention to the interval observer, especially its applicability.

The earliest interval observers were designed for the state monitoring of such biochemical systems as biochemical reactions, wastewater treatment systems, and vehicle control systems. It is impossible to make an accurate model for any of these systems, as they receive uncertain inputs and suffer from external disturbances with unknown statistical laws. In this case, the observation error rarely converges to zero if the observer is constructed according to the traditional method. By contrast, the interval observer can measure the real-time upper and lower bounds of the state change at any time, thus fulfilling the control requirements.

The design of interval observer, a core issue in control system design, has been widely applied in engineering practice and intensively discussed in the academia [1–10]. It is generally agreed that the main obstacle in the observer design lies in the wrong assumption that the error observation system is cooperative. In fact, most systems are not cooperative, except for a few monotonous systems. Important progress was made in the study of interval observers around 2010. It is found that system coordination is related to the selected coordinates; under certain conditions, noncooperative systems can be transformed into cooperative ones. The existing designs of interval observers are based on one of the following two theories: the monotonous theory and the positive system theory. The research objects have been extended from the initial linear time-invariant systems to linear time-varying systems, chaotic systems, feedback linearization systems, and nonlinear Lipschitz systems.

Linear multicellular system is a type of differential inclusion system, which can be deemed as an extension of linear control system with set-valued mapping. However, there is no report on the design of interval observer for linear multicellular system. Thus, it is very meaningful to make up for this research gap.

Considering the uncertain parameters of multicellular system, this chapter gives a rational definition to the interval observer of multicellular system, converts the system into the linear parameter varying (LPV) form, and designs an interval observer for the LPV system. The proposed design algorithm

can effectively estimate the system state through the control of the deviation and the adjustment of control gain.

2. Description of the Problem and Related Concepts

Vector Comparison

$$x(t) \geq z(t)\,(x(t) > z(t)) \tag{1}$$

where $x(t), z(t) \in R^n$ is the comparison of all corresponding elements in vectors $x(t)$ and $z(t)$. Through the comparison, we have

$$x_i(t) \geq z_i(t)\left(x_i(t) > z_i(t)\right), \quad i = 1, 2, 3, \ldots, n \tag{2}$$

Matrix Comparison

$$A \geq B\,(A > B), \quad A, B \in R^{n\times n} \tag{3}$$

Comparing all corresponding elements in matrices A and B, we have

$$a_{ij} \geq b_{ij}\left(a_{ij} > b_{ij}\right), \quad i = 1, 2, \ldots, n, \; j = 1, 2, \ldots, n. \tag{4}$$

Let $A_i \in R^{n\times n}$, $i = 1, 2, \ldots, s$ be s matrices. Then, the convex hull formed by A_i can be expressed as

$$\begin{aligned} &co\left(A_i,\, i = 1, 2\cdots s\right) \\ &\quad = \left\{A = \sum_{i=1}^{s}\theta_i A_i;\, \left(\theta_1, \theta_2 \cdots \theta_s\right)^T \in \aleph\right\} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \aleph = \Big\{&\theta;\, \theta = \left(\theta_1, \theta_2 \cdots \theta_s\right);\, 0 \leq \theta_i \leq 1,\, i \\ &= 1, 2\cdots s,\, \sum_{i=1}^{s}\theta_i = 1\Big\}. \end{aligned} \tag{6}$$

If s is a finite integer, $co(A_i)$ can be called a multicellular body composed of $A_i, i = 1, 2, \ldots, s$. Thus, a multicellular body can be considered as the convex set of a finite number of linear matrices.

Consider the following linear multicellular system:

$$\dot{x}(t) \in co\left\{A_i x(t) + B_i u(t),\, i = 1, 2\cdots s\right\} \tag{7}$$

$$y(t) = Cx(t) \tag{8}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the state and input of the system, respectively; $A_i \in R^{n\times n}$, $B_i \in R^{n\times l}$, $i = 1, 2\cdots, s$; $y(t) \in R^m$ is the output of the system; $C \in R^{m\times n}$; $co\{\bullet\}$ is the convex hull of a set.

This chapter aims to solve the following problem: based on the measurable information $y(t)$ of the original system, two new systems should be established to estimate the upper and lower bounds $x^+(t)$ and $x^-(t)$ of the state of the original system. In other words, for any $t > 0$, there is

$$x^-(t) \leq x(t) \leq x^+(t) \tag{9}$$

The two new systems are called the upper and lower bound observers of system (7). Together, they constitute the interval observer of this system.

Definition 1. Matrix A is a Hurwitz matrix if all eigenvalues of the matrix have a negative real part, a Schur matrix if all eigenvalue norms of the matrix are smaller than one, a non-negative matrix if all elements of the matrix are non-negative, and a Metzler matrix if all non-diagonal elements of the matrix are non-negative.

3. Interval Observer Design of Linear Multicellular System

According to the convex analysis theory [11], system (7) is equivalent to

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + B_0 u(t) + \sum_{i=1}^{s}\theta_i\left[A_i x(t) + B_i u(t)\right] \\ &= A(\theta)\,x(t) + B(\theta)\,u(t) \end{aligned} \tag{10}$$

where θ_i, $i = 1, 2, \ldots, s$ is an uncertain parameter that satisfies

$$\theta_i\,[0, 1], \quad \sum_{i=1}^{s}\theta_i = 1 \tag{11}$$

and

$$\begin{aligned} A(\theta) &= A_0 + \sum_{i=1}^{s}\theta_i A_i, \\ B(\theta) &= B_0 + \sum_{i=1}^{s}\theta_i B_i \end{aligned} \tag{12}$$

Thus, systems (7) and (8) can be transformed as

$$\dot{x}(t) = A(\theta)\,x(t) + B(\theta)\,u(t) \tag{13}$$

$$y(t) = Cx(t) \tag{14}$$

where $\theta = (\theta_1, \theta_2 \cdots \theta_s)^T$ a time-varying parameter variable included in the multicellular body $\aleph$. The vertex of the variable can be expressed as

$$\theta^{(j)}, \quad j = 1, \ldots, h \tag{15}$$

Hypothesis 2. $(A(\theta), C)$ is observable for any $\theta \in \aleph$.

Hypothesis 3. The input $u(t)$ and output $y(t)$ in systems (13) and (14) are bounded.

Here, it is assumed that

$$\begin{aligned} A(\theta) &= A_0 + \theta_1 A_1 + \cdots \theta_s A_s, \\ B(\theta) &= B_0 + \theta_1 B_1 + \cdots \theta_s B_s \end{aligned} \tag{16}$$

where

$$A_i = A_{i,0} + A_{i,1} + \cdots A_{i,s},$$
$$B_i = B_{i,0} + B_{i,1} + \cdots B_{i,s} \tag{17}$$

Next, $w^+(x^+, \theta_e^+, u)$ and $w^-(x^-, \theta_e^-, u)$ are defined as

$$w^+(x^+, \theta_e^+, u) = \max_{\theta_e}\left(\left[\overline{A}(\theta_e) - A(\theta_0)\right]x^+ + \left[\overline{B}(\theta_e) - B(\theta_0)\right]u\right) \tag{18}$$

$$w^-(x^-, \theta_e^-, u) = \min_{\theta_e}\left(\left[\overline{A}(\theta_e) - A(\theta_0)\right]x^- + \left[\overline{B}(\theta_e) - B(\theta_0)\right]u\right) \tag{19}$$

where $\theta_e \in R^{s\cdot n\times 1}$ is defined as

$$\theta_e = \theta \otimes 1_n = [\theta_1 \cdots \theta_1, \cdots, \theta_s \cdots \theta_s] \tag{20}$$

where all elements of $1_n \in R^{n\times 1}$ are composed of 1_s; $\otimes$ is the Kronecker product. Let $\aleph_e$ be a set of multicellular bodies with h_n as the vertex. Then, $\overline{A}(\theta_e)$ and $\overline{B}(\theta_e)$ in (18) and (19) can be expressed as

$$\overline{A}(\theta_e) = A_0 + \sum_{i=1}^{s}\sum_{j=1}^{n}\theta_e((i-1)\cdot n + j)A_{ij}$$
$$\overline{B}(\theta_e) = B_0 + \sum_{i=1}^{s}\sum_{j=1}^{n}\theta_e((i-1)\cdot n + j)B_{ij} \tag{21}$$

where $A_{i,j}$, $B_{i,j}$ can be expressed as

$$A_{i,j} = D_j A_i,$$
$$B_{i,j} = D_j B_i \tag{22}$$

where D_j is a matrix in which all elements are zero except for the diagonal elements (whose value is one).

For system (13), an interval observer can be designed as

$$\dot{x}^+ = A(\theta_0)x^+ + B(\theta_0)u + w^+(x^+, \theta_e^+, u) + L(y - Cx^+) \tag{23}$$

$$\dot{x}^- = A(\theta_0)x^- + B(\theta_0)u + w^-(x^-, \theta_e^-, u) + L(y - Cx^-) \tag{24}$$

where θ_0 is the nominal value of the uncertain parameter θ ($\theta \in \aleph$).

Lemma 4. *If x^+, x^-, u are bounded, then $w^+(x^+, \theta_e^+, u)$ and $w^-(x^-, \theta_e^-, u)$ are bounded, and the optimal decision variables θ_e^+ and θ_e^- are at the vertex of the set of multicellular bodies $\aleph_e$.*

Proof. From the basic properties of the linear programming problem, it is learned that the optimal decision variables θ_e^+ and θ_e^- in (18) and (19) are located at a vertex of the set of multicellular bodies $\aleph_e$. Since all vertices are finite and the coefficients of θ_e^+ and θ_e^- are the affine functions of bounded variables x^+, x^-, and u, then $w^+(x^+, \theta_e^+, u)$ and $w^-(x^-, \theta_e^-, u)$ must be bounded. □

Lemma 5. *For a given matrix $T \in R^{m\times n}$, if $T^+ = \max\{0, T\}$ and $T^- = T^+ - T$, and if $v \in R^n$ is a variable vector satisfying $v^- \le v \le v^+$ where $v^-, v^+ \in R^n$, then*

$$T^+v^- - T^-v^+ \le T^+v^+ - T^-v^- \tag{25}$$

In (18) and (19),

$$\left[\overline{A}(\theta_e) - A(\theta_0)\right]x^+ + \left[\overline{B}(\theta_e) - B(\theta_0)\right]u = \left[\overline{A}(\theta_e)x^+ + \overline{B}(\theta_e)u\right] - \left[A(\theta_0)x^+ + B(\theta_0)\right]u \tag{26}$$

where θ_e is an uncertain parameter; $\overline{A}(\theta_e)x^+ + \overline{B}(\theta_e)u$ can be written as follows through linear transform:

$$\overline{A}(\theta_e)x^+ + \overline{B}(\theta_e)u = A_0x^+ + B_0u + \left[\underbrace{A_{11}x^+ + B_{11}u \cdots A_{sn}x^+ + B_{sn}u}_{T_1}\right]\underbrace{\theta_e}_{v} \tag{27}$$

Similarly, $[\overline{A}(\theta_e) - A(\theta_0)]x^- + [\overline{B}(\theta_e) - B(\theta_0)]u$ can be expressed as

$$\left[\overline{A}(\theta_e) - A(\theta_0)\right]x^+ + \left[\overline{B}(\theta_e) - B(\theta_0)\right]u = \left[A_0x^- + B_0u\right] - \left[A(\theta_0)x^- + B(\theta_0)u\right] + \left[\underbrace{A_{11}x^- + B_{11}u \cdots A_{sn}x^- + B_{sn}u}_{T_2}\right]\underbrace{\theta_e}_{v} \tag{28}$$

According to Lemma 5, $w^+(x^+, \theta_e^+, u)$ and $w^-(x^-, \theta_e^-, u)$ can be expressed as

$$w^+(x^+, \theta_e^+, u) = \left[A_0x^- + B_0u\right] - \left[A(\theta_0)x^+ + B(\theta_0)u\right] + \left[T_1{}^+v^+ - T_1{}^-v^-\right]$$
$$w^-(x^-, \theta_e^-, u) = \left[A_0x^- + B_0u\right] - \left[A(\theta_0)x^- + B(\theta_0)u\right] + \left[T_2{}^+v^+ - T_2{}^-v^-\right] \tag{29}$$

Among the matrices T_1, T_2 and v defined in (27) and (28) above, $T_1{}^+$, T_1, $T_2{}^+$, and T_2 are all time-varying matrices depending on the state estimates of x^+ and x^-. The adjustable inputs u, $v^+$and v^- are, respectively, the upper and lower bounds defined on the vector parameter θ_e.

Lemma 6. *If $\overline{A}(\theta_e) - LC$ is a Hurwitz matrix for any $\theta_e \in \aleph$, then $A(\theta) - LC$ is also a Hurwitz matrix for any $\theta \in \aleph$.*

Proof. According to the definition of $\overline{A}(\theta_e)$ in (21), $A_i = \sum_{j=1}^{n} A_{ij}$ is equal for any θ_i, $i = 1, 2 \cdots s$ in (17). Thus, we have

$$\theta_i = \theta_e((i-1)n + j), \quad j = 1, \ldots, n \tag{30}$$

Then, $A(\theta)$ can be expressed as

$$A(\theta) = A_0 + \sum_{i=1}^{s} A_i = A_0 + \sum_{i=1}^{s}\sum_{j=}^{n} A_{ij} = \overline{A}(\theta_e) \quad (31)$$

It is obvious that $A(\theta) - LC$ is a special case of $\overline{A}(\theta_e) - LC$. □

Lemma 7. *Suppose matrix P is a diagonal positive definite matrix; then matrix R is a Metzler matrix if and only if PR is a Metzler matrix.*

Proof. Considering that both matrices P and P^{-1} are diagonal positive definite matrices and that each element in matrix PR is identical to each element in matrix R, both matrices PR and R must be Metzler matrices. □

Theorem 8. *In system (13), both $A(\theta)$ and $B(\theta)$ are matrix functions relative to θ, if there exist a matrix P and a matrix $Q_0, Q_1 \ldots Q_r$ that satisfy the following three conditions:*

(1) $P \in R^{n\times n}$ is a diagonal positive definite matrix;

(2) For any $\theta_e^{(j)} \in \aleph_e$, there is

$$\overline{A}^T\left(\theta_e^{(j)}\right)P + P\overline{A}\left(\theta_e^{(j)}\right) - \left(C^T Q^T + QC\right) \prec 0 \quad (32)$$

where $Q \in R^{n\times n}$, $\theta_e^{(j)}$ is the j-th vertex in $\aleph_e$.

(3) For any $\theta \in \aleph$, $PA(\theta) - QC$ is a Metzler matrix, then systems (23) and (24) are the interval observer of system (13) with the gain of $L = P^{-1}Q$.

Proof. In (18) and (19), θ_e^+ stands for the maximum operator of the optimal decision variable. In (23) and (24), the state equation observer x^+ can be expressed as

$$\dot{x}^+ = \left[\overline{A}\left(\theta_e^+\right) - LC\right]x^+ + \overline{B}\left(\theta_e^+\right)u + Ly \quad (33)$$

According to Lemma 4, θ_e^+ is located at the vertex h^n of multicellular body $\aleph_e$. This means the state equation observer x^+ in (30) is a transformed LPV system. In the theorem, (1) and (2) ensure that there exists a quadratic Lyapunov function in the observer error state matrix $\overline{A}(\theta_e^+) - LC, \forall\theta_e^{(j)} \in \aleph_e$. Since u, y and state vector x^+ are all bounded, the state function of observer error $e = x^+ - x$ can be expressed as

$$\begin{aligned}\dot{x}^+ - \dot{x} = {} & [A(\theta) - LC]\left(x^+ - x\right) + w^+\left(x^+, \theta_e^+, u\right) \\ & - \left[A(\theta) - A(\theta_0)\right]x^+ - \left[B(\theta) - B(\theta_0)\right]u\end{aligned} \quad (34)$$

The observer error in (34) $e = x^+ - x$ is a time-varying linear system with positive bounded inputs and bounded outputs, due to the following reasons:

(a) It can be inferred from Lemma 4 that $A(\theta) - LC$ is a Hurwitz matrix.

(b) According to Lemma 5 , $PA(\theta) - QC$ is a Metzler matrix for any $\theta \in \aleph$.

(c) In (18) and (19), it is defined that

$$\begin{aligned}& w^+\left(x^+, \theta_e^+, u\right) - \left[A(\theta) - A(\theta_0)\right]x^+ \\ & \quad - \left[B(\theta) - B(\theta_0)\right]u\end{aligned} \quad (35)$$

is a positive bounded input and that x^+ is bounded.

Thus, it is proved that x^+ is the finite upper bound of the true state x. Similarly, we can prove the positive definiteness of $x - x^-$: if the initial state satisfies $x^-(0) \le x(0) \le x^+(0)$, then $x^+(t)$ and $x^-(t)$ constitute the interval estimate of $x(t)$.

$$x^-(t) \le x(t) \le x^+(t) \quad (36)$$

Thus, (23) and (24) are the interval observer of system (13). □

4. Simulation

In system (13), $x(t) \in R^2$ stands for system state and $u(t)$ stands for the known input of the system. The following definitions are given for the simulation:

$$\begin{aligned}u(t) &= 1, \\ B(\theta) &= B_0 = [0.1; 0.1] \in R^{2\times 1}, \\ C &= [0, 1] \in R^{1\times 2} \\ A(\theta) &= \begin{bmatrix} -0.632 - 0.16\sin(t) & 0.1\cos(3t) \\ -0.14\cos(2t) & 0.06\sin(t)\end{bmatrix} \\ &= A_0 + \sum_{i=1}^{3} A_i\theta_i\end{aligned} \quad (37)$$

where

$$\begin{aligned}\theta &= \left[\theta_1; \theta_2; \theta_3\right] = [\sin(t); \cos(3t); \cos(2t)] \\ A_0 &= [-0.632, 0; 0, 0], \\ A_1 &= [-0.16, 0; 0, 0.06] \\ A_2 &= [0, 0.1; 0, 0], \\ A_3 &= [0, 0; -0.14, 0]\end{aligned} \quad (38)$$

where A_0 is not a Hurwitz matrix. Using the linear matrix inequality toolbox of Matlab, we have the observer gain:

$$L = [0; 4.368] \quad (39)$$

Figure 1 shows the interval state estimation of the system. It can be seen that $x_i^-(t) \le x_i(t) \le x_i^+(t)$, $i = 1, 2, 3$, is valid at any time. Thus, it is proved that the proposed interval observers (23) and (24) can realize the interval estimation of the system state.

The simulation results show that the state of the linear multicellular system (13) always fell between the results of the interval observers (23) and (24), achieving the expected estimation effect. Thus, the proposed design algorithm is proved valid.

5. Conclusions

This chapter explores the interval state estimation problem of linear multicellular systems and develops an interval observer capable of estimating the real-time upper and lower bounds of the state at any time. The proposed design method can

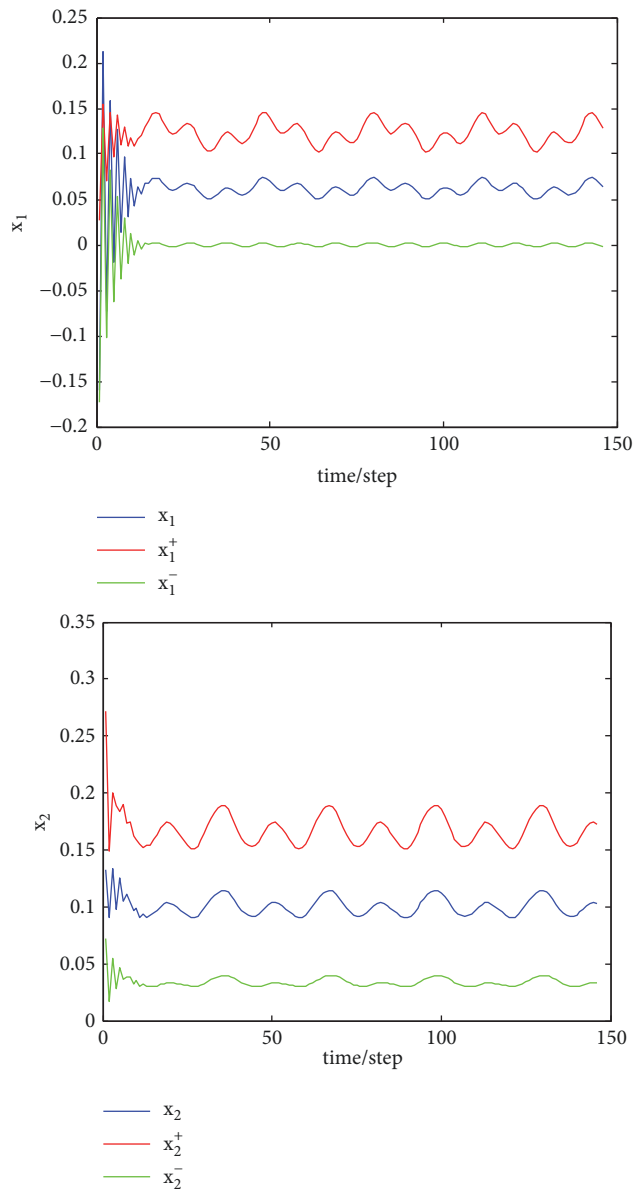

FIGURE 1: Interval state estimation of linear multicelluar systems.

solve many practical control problems. Specifically, the linear multicellular system was converted into an LPV system through convex analysis; then, the author proposed a design method for interval state observer of linear multicellular systems according to the positive system theory. Through strict reasoning, the proposed design method was proved correct and effective.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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