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Research Article

The Exponential Stabilization of a Class of n-D Chaotic Systems via the Exact Solution Method

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This paper treats the exponential stabilization of a class of n-D chaotic systems. A new control approach which is called the exact solution method is presented. The most important feature of this method is that the solution of the system under consideration can be carefully designed to converge exponentially to the origin. Based on this method, the exponential stabilization of a class of n-D chaotic systems and its application in controlling chaotic system with unknown parameter are presented. The Genesio-Tesi system is taken to give the numerical simulation which is completely consistent with the theoretical analysis presented in this paper.

1. Introduction

The chaotic system is a special class of nonlinear system whose dynamic behavior is extremely dependent on initial conditions. Owing to the result of high sensitivity to initial conditions, the behavior of the chaotic system appears to be unpredictable and stochastic, even if the model of the considered chaotic system is deterministic. Since small differences of initial input may yield dramatically different results, the accurate prediction of long-term development of chaotic system is impossible. Now, it is generally agreed that this complex and irregular phenomenon is useful because it has many applications in some areas such as secure communications and information sciences [1]. For the purpose of utilizing chaotic signals, the chaos control and chaos synchronization of dynamical systems have attracted a wide range of research activities for over two decades [2–8]. A wide variety of approaches have been proposed for achieving chaos control and synchronization which include adaptive control method [9], sliding mode control [10, 11], predictive control method [12], and backstepping method [13].

Most of the approaches for dealing with the chaos control and synchronization are based on the Lyapunov method. The Lyapunov method consists of two steps. First, design a proper Lyapunov function. Second, calculate its derivative along the trajectories of chaotic system. If the derivative of the Lyapunov candidate is negative, then the equilibrium of the considered system is globally asymptotically stable. Because it determines the stability of the system based on an energy function (called as the Lyapunov function) rather than solving the differential equations, Lyapunov method is an important method to analyze the stability of the system. The downside of this method is that it does not tell us whether the system is globally exponentially stable at the origin and what is the rate of the convergence.

In this paper, a new control approach which is called the exact solution method is presented. Contrary to the Lyapunov method, the exact solution method needs to construct an exact solution of the considered system. Based on this method, the exponential stabilization of a class of n-D chaotic systems is considered. Some novel controllers are presented to make the controlled system be exponentially stabilized. A numerical example is given to show the validity and feasibility of the proposed controller.

The main contents of this paper are described as follows. The system description is introduced in Section 2. By using the exact solution method, the stabilization of a class of n-D chaotic systems and its application in controlling chaotic system with unknown parameter are presented in Section 3. To show the effectiveness of the proposed approach, simulation results are given in Section 4. Finally, some concluding remarks are summarized in Section 5.

2. System Description

In this paper, the control of a class of n–D chaotic systems which can be described as system (1) is considered.

The considered system is given as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ &\vdots \\ \dot{x}_{n-1} &= x_n, \\ \dot{x}_n &= f(x), \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in R^{n\times 1}$ is the state vector of system (1) and $f(x)$ is a continuous function of x.

3. The Main Results

In this section, we consider the exponential stabilization of system (1) at the origin by using the exact solution method. In order to control chaotic behaviors in system (1), the control input u is added in the last state equation. Then, the controlled system is rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ &\vdots \\ \dot{x}_{n-1} &= x_n, \\ \dot{x}_n &= f(x) + u, \end{aligned} \tag{2}$$

where u is a controller to be designed later.

Before giving the main results, we introduce the following essential definition.

Definition 1. The controlled system (2) is said to be globally exponentially stable at the origin if there exist constants $\rho(>0)$ and $\lambda(<0)$ such that $|x_i| \le \rho e^{\lambda t}$, $t \ge 0$, $i = 1, 2, \ldots, n$, hold for any initial values.

Now, we construct an exact solution of system (2) and propose the following Theorem 2.

Theorem 2. *Suppose that the controller u is chosen as*

$$u = \lambda_1^n c_1^* + \lambda_2^n c_2^* + \cdots + \lambda_n^n c_n^* - f(x), \tag{3}$$

and then system (2) is globally exponentially stable at the origin, where $\lambda_i < 0$ $(i = 1, 2, \ldots, n)$, $\lambda_i \ne \lambda_j$ for $i \ne j$, and

$$c_1^* = \omega \begin{vmatrix} x_1 & 1 & 1 & \cdots & 1 \\ x_2 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & \lambda_2^{n-1} & \lambda_3^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix},$$

$$c_2^* = \omega \begin{vmatrix} 1 & x_1 & 1 & \cdots & 1 \\ \lambda_1 & x_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & x_n & \lambda_1^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix},$$

$$\vdots$$

$$c_n^* = \omega \begin{vmatrix} 1 & 1 & \cdots & 1 & x_1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{n-1} & x_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_{n-1}^{n-1} & x_n \end{vmatrix},$$

$$\begin{aligned} \omega &= \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}^{-1} \\ &= \frac{1}{\prod_{1\le i<j\le n}(\lambda_j - \lambda_i)}. \end{aligned} \tag{4}$$

Proof. For the sake of making the origin of system (2) globally exponentially stable, we suppose that

$$x_1 = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \cdots + c_n e^{\lambda_n t}, \tag{5}$$

where $c_1, c_2, \cdots, c_n$ are constants that are related to initial conditions. Since $\lambda_i < 0$, it is obvious that $\lim_{t\to\infty} x_1 = 0$. In view of $\dot{x}_1 = x_2$, we have

$$x_2 = c_1\lambda_1 e^{\lambda_1 t} + c_2\lambda_2 e^{\lambda_2 t} + \cdots + c_n\lambda_n e^{\lambda_n t}. \tag{6}$$

Obviously, we obtain $\lim_{t\to\infty} x_2 = 0$.

Repeating this process for $n-2$ times yields

$$x_n = \dot{x}_{n-1} = c_1\lambda_1^{n-1} e^{\lambda_1 t} + c_2\lambda_2^{n-1} e^{\lambda_2 t} + \cdots + c_n\lambda_n^{n-1} e^{\lambda_n t}. \tag{7}$$

Similarly, we get $\lim_{t\to\infty} x_n = 0$. According to the last equation of (2), we derive

$$\begin{aligned} u &= \dot{x}_n - f(x) \\ &= c_1\lambda_1^n e^{\lambda_1 t} + c_2\lambda_2^n e^{\lambda_2 t} + \cdots + c_n\lambda_n^n e^{\lambda_n t} - f(x). \end{aligned} \tag{8}$$

Based on the above deduction process, we come to the conclusion that if u is chosen as (8), then system (2) has a solution:

$$\begin{aligned} x_1 &= c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \cdots + c_n e^{\lambda_n t}, \\ x_2 &= c_1\lambda_1 e^{\lambda_1 t} + c_2\lambda_2 e^{\lambda_2 t} + \cdots + c_n\lambda_n e^{\lambda_n t}, \\ &\vdots \\ x_n &= c_1\lambda_1^{n-1} e^{\lambda_1 t} + c_2\lambda_2^{n-1} e^{\lambda_2 t} + \cdots + c_n\lambda_n^{n-1} e^{\lambda_n t}. \end{aligned} \tag{9}$$

Because the solution of system (2) is uniqueness, we know that (9) are the unique solution of system (2) and satisfy $\lim_{t\longrightarrow\infty} x_1 = \lim_{t\longrightarrow\infty} x_2 = \cdots = \lim_{t\longrightarrow\infty} x_n = 0$.

In order to use x_i to represent u, by using Cramer's Rule, the term $c_i e^{\lambda_i t}$ in (9) can be obtained as follows:

$$c_1 e^{\lambda_1 t} = \omega \begin{vmatrix} x_1 & 1 & 1 & \cdots & 1 \\ x_2 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & \lambda_2^{n-1} & \lambda_3^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}, \tag{10}$$

$$c_2 e^{\lambda_2 t} = \omega \begin{vmatrix} 1 & x_1 & 1 & \cdots & 1 \\ \lambda_1 & x_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & x_n & \lambda_1^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}, \tag{11}$$

$$\vdots$$

$$c_n e^{\lambda_n t} = \omega \begin{vmatrix} 1 & 1 & \cdots & 1 & x_1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{n-1} & x_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_{n-1}^{n-1} & x_n \end{vmatrix}, \tag{12}$$

$$\begin{aligned} \omega &= \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \lambda_3^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}^{-1} \\ &= \frac{1}{\prod_{1\le i<j\le n}\left(\lambda_j - \lambda_i\right)}. \end{aligned} \tag{13}$$

Substituting $c_i e^{\lambda_i t}$ into (8), we get

$$u = \lambda_1^n c_1^* + \lambda_2^n c_2^* + \cdots + \lambda_n^n c_n^* - f(x). \tag{14}$$

Let $\lambda = \max\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. By (9), one can easily derive that there exists a constant $\rho > 0$ such that

$$\left|x_i\right| \le \rho e^{\lambda t}, \quad i = 1, 2, \ldots, n. \tag{15}$$

According to Definition 1, we know that system (2) is globally exponentially stable at the origin and the speed of convergence is relevant to λ, and the larger the number λ, the faster the rate of the convergence. This ends the proof of Theorem 2. □

Remark 3. The controller in (8) is strongly dependent on x_1, and different x_1 will lead to different controller u.

Remark 4. Let

$$\begin{aligned} V = &\begin{vmatrix} x_1 & 1 & 1 & \cdots & 1 \\ x_2 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & \lambda_2^{n-1} & \lambda_3^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}^2 \\ &+ \begin{vmatrix} 1 & x_1 & 1 & \cdots & 1 \\ \lambda_1 & x_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & x_n & \lambda_1^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}^2 + \cdots \\ &+ \begin{vmatrix} 1 & 1 & \cdots & 1 & x_1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{n-1} & x_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_{n-1}^{n-1} & x_n \end{vmatrix}^2. \end{aligned} \tag{16}$$

By using (10)-(12), it is easy to prove that V is a proper Lyapunov function. In fact, (16) is equivalent to

$$V = \left(\frac{c_1}{\omega} e^{\lambda_1 t}\right)^2 + \left(\frac{c_2}{\omega} e^{\lambda_2 t}\right)^2 + \cdots + \left(\frac{c_n}{\omega} e^{\lambda_n t}\right)^2. \tag{17}$$

The derivative of V is

$$\begin{aligned} V &= 2\lambda_1 \left(\frac{c_1}{\omega} e^{\lambda_1 t}\right)^2 + 2\lambda_2 \left(\frac{c_2}{\omega} e^{\lambda_2 t}\right)^2 + \cdots \\ &\quad + 2\lambda_n \left(\frac{c_n}{\omega} e^{\lambda_n t}\right)^2 \\ &= 2\lambda_1 \begin{vmatrix} x_1 & 1 & 1 & \cdots & 1 \\ x_2 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & \lambda_2^{n-1} & \lambda_3^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}^2 \\ &\quad + 2\lambda_2 \begin{vmatrix} 1 & x_1 & 1 & \cdots & 1 \\ \lambda_1 & x_2 & \lambda_3 & \cdots & \lambda_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & x_n & \lambda_1^{n-1} & \cdots & \lambda_n^{n-1} \end{vmatrix}^2 + \cdots \\ &\quad + 2\lambda_n \begin{vmatrix} 1 & 1 & \cdots & 1 & x_1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{n-1} & x_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_{n-1}^{n-1} & x_n \end{vmatrix}^2 \le 0. \end{aligned} \tag{18}$$

By (18), one can see that system (2) is globally stable at the origin, but in this case we do not know whether the state of the system converges exponentially to the origin and what is the convergence rate.

Remark 5. Many papers have investigated the control or synchronization problem of system (1). For example, papers [14, 15] considered the synchronization of system (1) by using the derivative control and backstepping method, respectively. Similarly, the authors did not tell us whether the system is globally exponentially stable at the origin and what is the rate of the convergence.

In the following, we discuss the application of Theorem 2. Consider the (n+1)-D chaotic system which is given as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ &\vdots \\ \dot{x}_n &= x_{n+1}, \\ \dot{x}_{n+1} &= f(x) + \theta g(x) + u, \end{aligned} \tag{19}$$

where $x = (x_1, x_2, \ldots, x_n, x_{n+1})^T \in R^{(n+1)\times 1}$ is the state vector of system (19), $f(x)$, $g(x)$ are two continuous functions of x, $\theta \in R$ is the unknown parameter, and u is the controller.

Now, with the help of Theorem 2, we can consider the stabilization of system (19) and derive the following Theorem 6.

Theorem 6. *Suppose that the controller u in system (19) is chosen as*

$$u = -f(x) + \dot{u}_0 - (x_{n+1} - u_0) - \hat{\theta} g(x), \tag{20}$$

and the updated law of $\hat{\theta}$ is

$$\dot{\hat{\theta}} = (x_{n+1} - u_0) g(x), \tag{21}$$

and then system (19) is globally stable at the origin; that is, $\lim_{t\to\infty} x_1 = \lim_{t\to\infty} x_2 = \cdots = \lim_{t\to\infty} x_{n+1} = 0$, where $\hat{\theta}$ is the estimated value of θ and $u_0 = \lambda_1^n c_1^* + \lambda_2^n c_2^* + \cdots + \lambda_n^n c_n^*$.

Proof. By Theorem 2, we know that if $x_{n+1} = u_0$, then we get $\lim_{t\to\infty} x_1 = \lim_{t\to\infty} x_2 = \cdots = \lim_{t\to\infty} x_n = 0$. Furthermore, we can see that $\lim_{t\to\infty} u_0 = 0$. Since we assume that $x_{n+1} = u_0$, we have $\lim_{t\to\infty} x_{n+1} = 0$. Therefore, in the following, we only need to prove that $\lim_{t\to\infty} x_{n+1} = u_0$. From the last equation of system (19), we have

$$\dot{x}_{n+1} - \dot{u}_0 = f(x) + \theta g(x) + u - \dot{u}_0. \tag{22}$$

Substituting u defined in (20) into (22), we obtain

$$\frac{d(x_{n+1} - u_0)}{dt} = -(x_{n+1} - u_0) + (\theta - \hat{\theta}) g(x). \tag{23}$$

Take

$$V = \frac{(x_{n+1} - u_0)^2}{2} + \frac{(\theta - \hat{\theta})^2}{2}. \tag{24}$$

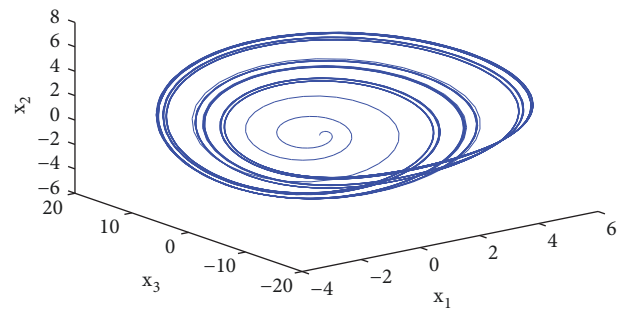

Figure 1: The chaotic attractor of system (27) with $x_1(0) = 1$, $x_2(0) = -1$, $x_3(0) = 3$.

The derivative of V along system (23) is

$$\begin{aligned} \dot{V} &= (x_{n+1} - u_0)\left(-(x_{n+1} - u_0) + (\theta - \hat{\theta}) g(x)\right) \\ &\quad - (\theta - \hat{\theta})\dot{\hat{\theta}} = -(x_{n+1} - u_0)^2 \le 0. \end{aligned} \tag{25}$$

Obviously, we have $\lim_{t\to\infty} x_{n+1} = u_0$. This ends the proof of Theorem 6. □

In what follows, we consider the special case where θ is a known parameter in advance. In this case, we have the following Corollary whose proof is omitted.

Corollary 7. *If θ is a known parameter and we suppose the controller u in system (19) is chosen as*

$$u = -f(x) + \dot{u}_0 - (x_{n+1} - u) - \theta g(x), \tag{26}$$

then system (19) is globally stable at the origin; that is, $\lim_{t\to\infty} x_1 = \lim_{t\to\infty} x_2 = \cdots = \lim_{t\to\infty} x_{n+1} = 0$.

4. Simulation Results

Note that most of the chaotic attractors are 3-D chaotic systems, so in this section we take a 3-D system, that is, the Genesio-Tesi system, as an example to verify the effectiveness of the proposed scheme.

The Genesio-Tesi system [16] is one of the famous chaotic systems, which is given as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -c_1 x_1 - b_1 x_2 - a_1 x_3 + x_1^2 + u, \end{aligned} \tag{27}$$

where u is the controller, $a_1 > 0, b_1 > 0$, and $c_1 > 0$ are constants satisfying $a_1 b_1 < c_1$. System (27) is chaotic for $c_1 = 6$, $b_1 = 2.92$, $a_1 = 1.2$, and $u = 0$. The chaotic attractor of system (27) is shown in Figure 1. In the following, we suppose that $c_1 = 6$, $b_1 = 2.92$, and $a_1 = 1.2$ so that system (27) has a chaotic attractor.

Case 1. a_1, b_1, and c_1 are known parameters in advance.

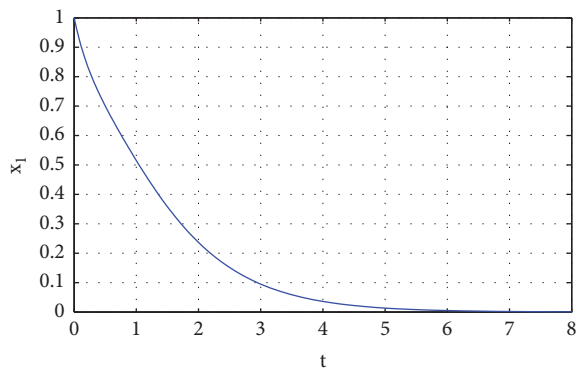

FIGURE 2: The time response of states x_1 of system (27) with controller (31).

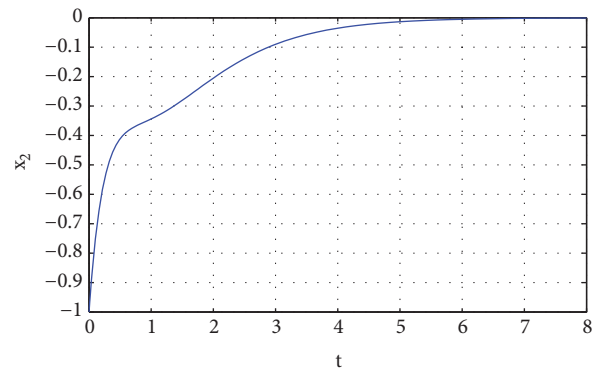

FIGURE 3: The time response of system (27) with controller (31).

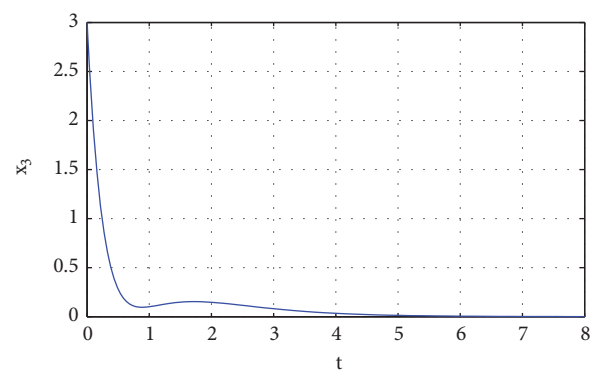

FIGURE 4: The time response of states x_3 of system (27) with controller (31).

For simplicity's sake, we assume that $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_2 = -3$; by Theorem 2, we have

$$\begin{aligned} c_1 e^{-t} + c_2 e^{-2t} + c_3 e^{-3t} &= x_1, \\ -c_1 e^{-t} - 2c_2 e^{-2t} - 3c_3 e^{-3t} &= x_2, \\ c_1 e^{-t} + 4c_2 e^{-2t} + 9c_3 e^{-3t} &= x_3, \end{aligned} \tag{28}$$

and

$$u = \dot{x}_3 - f(x) = -c_1 e^{-t} - 8c_2 e^{-2t} - 27c_3 e^{-3t} - f(x), \tag{29}$$

where $f(x) = -c_1 x_1 - b_1 x_2 - a_1 x_3 + x_1^2$.

After simple calculation, we get

$$\begin{aligned} c_1 e^{-t} &= \frac{-6x_1 - 5x_2 - x_3}{-2}, \\ c_2 e^{-2t} &= \frac{6x_1 + 8x_2 + 2x_3}{-2}, \\ c_3 e^{-3t} &= \frac{-2x_1 - 3x_2 - x_3}{-2}. \end{aligned} \tag{30}$$

Putting them into (29) yields

$$\begin{aligned} u = \frac{1}{2}&\left(-6x_1 - 5x_2 - x_3 + 8\left(6x_1 + 8x_2 + 2x_3\right)\right. \\ &\left. + 27\left(-2x_1 - 3x_2 - x_3\right)\right) - f(x) = -6x_1 - 11x_2 \\ &- 6x_3 - \left(-c_1 x_1 - b_1 x_2 - a_1 x_3 + x_1^2\right). \end{aligned} \tag{31}$$

The chaotic trajectories of system (27) with controller (31) and $x_1(0) = 1$, $x_2(0) = -1$, and $x_3(0) = 3$ are shown in Figures 2–4.

Case 2. a_1, b_1, and c_1 are unknown parameters in advance.

For simplicity's sake, we suppose that $\lambda_1 = -1$ and $\lambda_2 = -2$. By Theorem 2, we have

$$\begin{aligned} c_1 e^{-t} + c_2 e^{-2t} &= x_1, \\ -c_1 e^{-t} - 2c_2 e^{-2t} &= x_2, \end{aligned} \tag{32}$$

and according to Cramer's Rule, we obtain

$$\begin{aligned} c_1 e^{-t} &= 2x_1 + x_2, \\ c_2 e^{-2t} &= -\left(x_1 + x_2\right), \end{aligned} \tag{33}$$

and then we have

$$u_0 = \dot{x}_2 = c_1 e^{-t} + 4c_2 e^{-2t}. \tag{34}$$

Thus, we have

$$u_0 = 2x_1 + x_2 - 4\left(x_1 + x_2\right) = -2x_1 - 3x_2. \tag{35}$$

Now, we suppose that a_1, b_1, and c_1 are three unknown parameters. Obviously, $f(x) = x_1^2$. Based on Theorem 6, the controller u can be chosen as

$$\begin{aligned} u = &-x_1^2 + \left(-2x_2 - 3x_3\right) - \left(x_3 - \left(-2x_1 - 3x_2\right)\right) \\ &+ \hat{c}_1 x_1 + \hat{b}_1 x_2 + \hat{a}_1 x_3. \end{aligned} \tag{36}$$

The updated laws are given as

$$\dot{\hat{c}}_1 = \left(x_3 - \left(-2x_1 - 3x_2\right)\right) x_1, \tag{37}$$

$$\dot{\hat{b}}_1 = \left(x_3 - \left(-2x_1 - 3x_2\right)\right) x_2, \tag{38}$$

$$\dot{\hat{a}}_1 = \left(x_3 - \left(-2x_1 - 3x_2\right)\right) x_3. \tag{39}$$

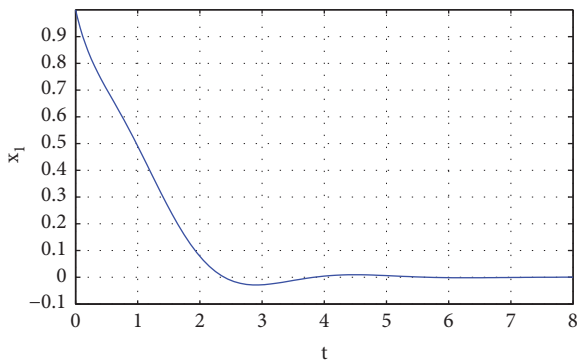

FIGURE 5: The time response of state x_1 of system (27) with controller (36).

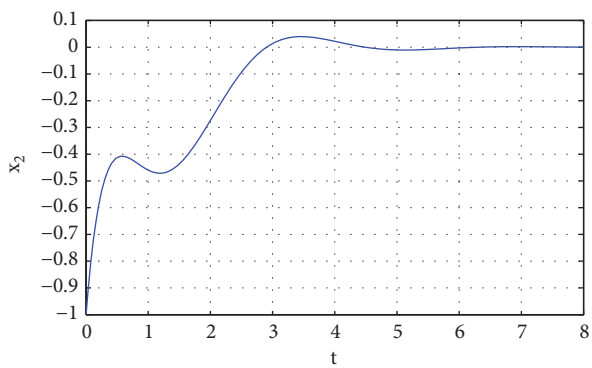

FIGURE 6: The time response of state x_2 of system (27) with controller (36).

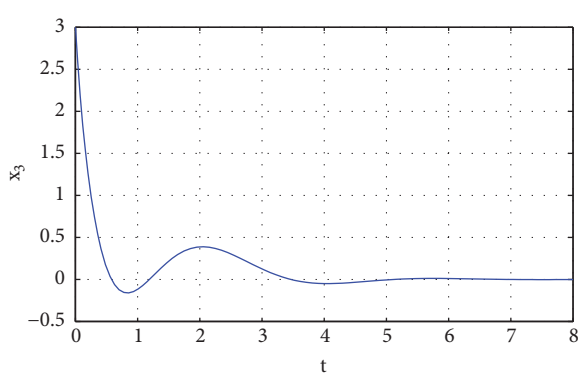

FIGURE 7: The time response of state x_3 of system (27) with controller (36).

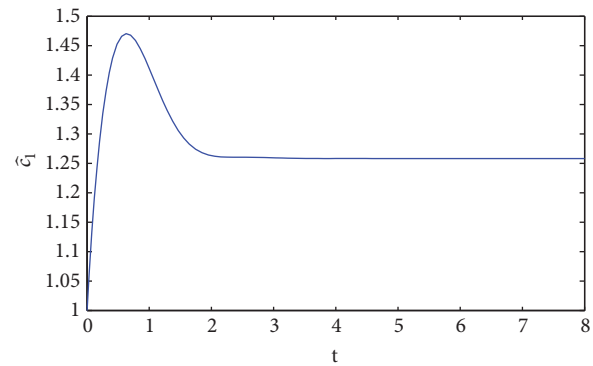

FIGURE 8: The time response of states $\hat{c}_1$ of system (27) with controller (36).

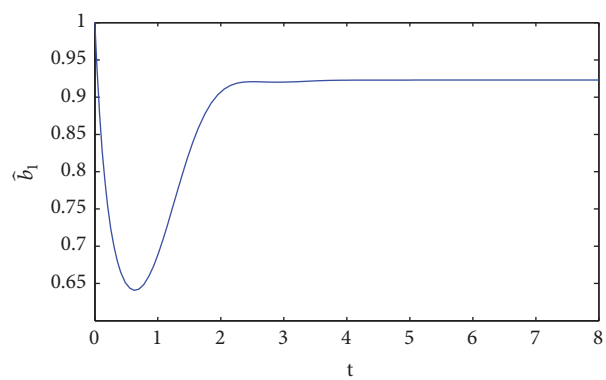

FIGURE 9: The time response of states $\hat{b}_1$ of system (27) with controller (36).

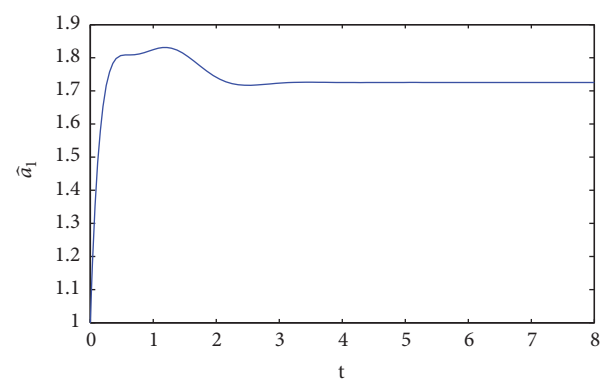

FIGURE 10: The time response of states $\hat{a}_1$ of system (27) with controller (36).

The simulation results with $x_1(0) = 1, x_2(0) = -1, x_3(0) = 3$, and $\dot{\hat{c}}_1(0) = \dot{\hat{b}}_1(0) = \dot{\hat{a}}_1(0) = 1$ are shown in Figures 5–10. Figures 5–7 show the time response of states x_1, x_2, and x_3 of system (27) with controller (36). Figures 8–10 display the time response of states $\hat{c}_1$, $\hat{b}_1$, and $\hat{a}_1$ of system (27) with controller (36).

From Figures 2–7, one can observe that the trajectories of system (27) under controllers (31) and (36) converge to the origin rapidly which is completely consistent with the theoretical analysis presented in this paper.

5. Conclusions

In this paper, we use the exact solution method to investigate the exponential stabilization of a class of n-D chaotic systems. This method possesses two advantages: firstly, the solution can be designed to converge exponentially to the origin; secondly, the speed of convergence is known, which is determined by $\lambda = \max\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. Therefore, by taking proper value of λ, the rapid convergence can be obtained. Based on this method, the exponential stabilization of a class

of n-D chaotic systems and its application in controlling chaotic system with unknown parameter are presented. The simulation results reveal that the proposed novel control strategy is effective.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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