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Research Article

Stability of the Evolutionary Game System and Control Strategies of Behavior Instability in Coal Mine Safety Management

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In this paper, we try to find the right control method for the game behavior instability in coal mine safety management. Through the analysis and comparison of the system stability with inflexible and flexible costs and penalties, it can be concluded that the dynamical game system with flexible costs (incentive rewards) and flexible penalty mechanism can significantly reduce the dynamics of unsafe behaviors in coal mine safety supervision. A combined mechanism of incentive rewards and flexible penalty is put forward to improve the stability of the dynamical system and control the instability of behaviors effectively. The results of model simulation show that the combined mechanism has very good property and can optimize and control the instability of behaviors and strategies of the interested parties. Based on the theoretical conclusions, some control strategies and policy advice are proposed for the improvement of the system and measures of safety management for government departments.

1. Introduction

The frequent occurrence of coal mine safety accidents always makes the safety production situation in China become the focus of attention at home and abroad. Management chaos, illegal operations, poor policy implementation, and regulatory dislocation have long become the prominent factors affecting the safety of coal mine production. The traditional safety management is mainly based on experience-oriented administrative regulations and legislations. However, due to the differences of region, staff and technical level, and so on, coal mine safety management has great complexity. The effectiveness of some rules and regulations is temporary and limited in the implementation process.

In order to ensure that the safety legislation is closely integrated with management practice, many scholars have deeply analyzed the influence factors of coal mine safety supervision and studied the relationship between the occurrences of unsafe behaviors and the interest distribution of coal mine safety [1–6].

Through the previous studies on the game of safety supervision, it is found that the dynamical system of safety management game model hardly has the stability and self-control generally, and the game behaviors of the players always have repeated volatility [7, 8]. The volatility of the behaviors may sometimes provide wrong information and cause unrealistic or even error related decisions and measures of coal mine safety management, which will seriously affect the efficiency of safety management.

In the previous studies of evolutionary game model in coal mine safety management, we have obtained the following replicator dynamics [9, 10]:

$$\begin{aligned} \dot{x_1} &= x_1(1-x_1)\left[(x_2+(1-x_2)x_3)f_1-c_1\right]; \\ \dot{x_2} &= x_2(1-x_2)\left[(1-x_1)(f_1+x_3f_2)-c_2\right]; \\ \dot{x_3} &= x_3(1-x_3)\left[(1-x_1)(1-x_2)(f_1+f_2)-c_3\right] \end{aligned} \tag{1}$$

where $c_i (i = 1, 2, 3)$ is the safety cost (input) of coal miners (denoted as player I), safety inspection groups inside the coal

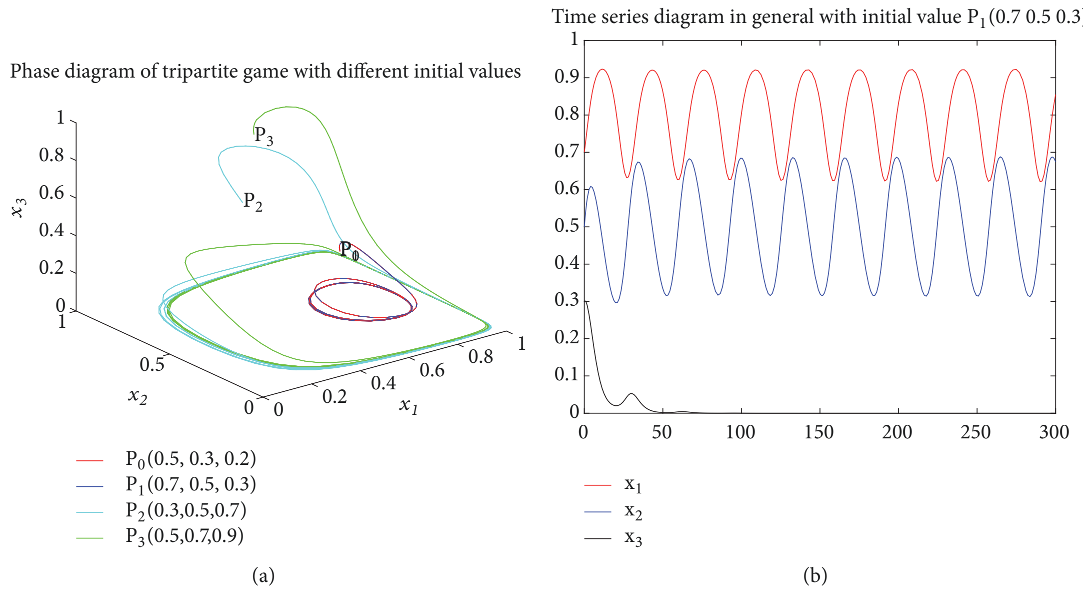


FIGURE 1: Phase graph of dynamical system (1) and time series diagram of players' strategy selection probabilities. Values of model parameters: $(c_1, c_2, c_3, f_1, f_2) = (0.5, 0.2, 0.3, 1, 1.2)$.

mine (player II), and safety regulatory departments of local government (player III). $f_i (i = 1, 2)$ is the penalty index (fine) when player I or player II has made mistakes (unsafe behaviors) and is checked out, respectively.

According to the actual situation of China's coal mine production and coal mine safety regulation, we assume that the parameters described above satisfy the following conditions:

(i) $c_2 \leq c_3 \leq c_1 \leq 1$;

(ii) $f_i \geq 1, i = 1, 2$.

Phase diagram analysis and time series diagram are commonly used methods to study the stability of nonlinear dynamical systems. Phase diagram analysis method has the characteristic of vivid intuition, which represents all possible states of the dynamic system. Phase diagram analysis greatly promotes the wide application of differential dynamical system in many disciplines [11–15]. Time series graph depicts the process of function changing with time variable t, which can simply and clearly describe the evolution process, chaos, and synchronization of differential dynamical systems [16–19]. In addition to the eigenvalue method, a large part of model analysis in this study uses these two methods.

The game model contains too many variables and parameters, it is difficult to obtain the model's analytical solution [10]. In this paper, we mainly use numerical simulation method to analyze the evolutionary process. So, it is very important to properly set the initial values of parameters and variables. Coal mine safety management involves various types of coal production enterprises with different technological development levels in different regions, and the coal production process and safety management status of these enterprises themselves have great differences. The corresponding parameters and initial values of variables will not be the same and changeless. Supposing that the relevant method is applied in a specific coal production enterprise or a specific mining area, the model parameters and initial values of variables should be deduced and estimated according to the actual situation of coal production. In this way, the model analysis will be more pertinent and the corresponding conclusions will be more accurate. In this paper, we merely set the values of model parameters and the initial value of the variables theoretically reasonably but do not make detailed and specific statistical reasoning.

In order to reflect the dependence of the stability of dynamic system on the initial value, we choose several different initial values of variables (each player's strategy selection probabilities of x_1, x_2, and x_3) in the system. For example, we set the value of $(x_1(0), x_2(0), x_3(0))$ to $P_0(0.5, 0.3, 0.2)$, $P_1(0.7, 0.5, 0.3)$, $P_2(0.3, 0.5, 0.7)$, $P_3(0.5, 0.7)$, and $P_3(0.5, 0.7, 0.9)$, respectively, in Figure 1(a).

Through the stability analysis of the dynamical system (1) (see Figure 1, where Figure 1(a) is phase graph of dynamical system and Figure 1(b) demonstrates the time series of players' strategy selection probabilities), we can find that there are repeated fluctuations in the game process of multiparty safety management. It agrees with the actual situation of safety management to a certain extent. If the coal miners do not adopt a safety production strategy (or the proportion of unsafe strategies in the production worker group is larger) at the beginning, the regulators will increase the level of safety supervision. With the increase of supervision intensity, coal miners will take safe production behaviors (or increase the proportion of safety behavior choice) or even completely do not choose unsafe behavior. Then coal production safety conditions will improve, and then regulators will reduce supervision, and then the unsafe behaviors will be more likely to happen. Over time, coal mine safety will always be in the process of cyclical fluctuations. This phenomenon appears not only in coal mine safety management but also in other processes of production safety management [20–23].

The game model of safety regulation does not have global stability under general conditions [10]. Most of the existing studies proposed the system and measures for safety management from the points of view of the policy makers, according to the specific situation (such as types of accidents or illegal behaviors) [24–28]. In this way, even if the relevant policies and measures have played a certain role in a short period, in a short period, the long-term effectiveness can not be guaranteed because of the lack of control (inhibition or avoidance) of behavior volatility of the interested groups. The repeated fluctuations and volatility (instability) often provide the decision-makers with wrong information, which will lead to unrealistic or even error decisions and measures of coal mine safety management, and seriously affect the efficiency of coal mine safety management.

In order to definitely improve the efficiency of safety management, some control methods are needed for restraining the instability of players's dynamical behavior and strategies in the process of coal mine safety supervision. So, in this paper, we proceed from the relationship between the model parameters (or variables) and explore the functionary mechanism of those model parameters (or variables) on the stability of game behaviors. As the design mechanism of model parameters and variables (i.e., the formulation of incentive and penalty measures in the process of safety supervision) is found out, then we could put forward the control methods and control strategies for the behavior instability. First, the stability of the dynamical system of the game model is studied in the cases of the generally inflexible cost subsidies and penalties index. Second, the stability of the dynamical system is considered under the flexible cost functions and flexible penalty functions. We believe that the relevant dynamic control methods and strategies will provide a theoretical basis for the regulation and control of coal mine safety management system and measures.

The rest of this paper is organized as follows. In Section 2, we analyze and compare the system stability of evolutionary game models in four cases: inflexible cost subsidies, flexible costs (incentive rewards), inflexible penalties, and flexible penalties. In Section 3, we carry out further theoretical analysis under the combined mechanism of incentive rewards and flexible penalties. Then, in Section 4, we put forward some control strategies to suppress the behavior instability in safety management system. Our paper is finally concluded in Section 5.

2. Analysis on Stability of the Dynamical Game System

2.1. Stability of the Dynamical Game System with Inflexible Cost Subsidies. The inflexible cost subsidy mentioned in this article refers to those policy oriented fixed subsidies for the workers which are undifferentiated and averagely shared, such as the underground allowances and the post allowances, etc. The increase of inflexible subsidies is equivalent to when the reduction of the safety production costs of coal miners, or the safety inspection cost of safety inspection team, or the safety supervision cost of the government department ($c_i, i = 1, 2, 3$) becomes smaller. Therefore, based on the general situation as shown in Figure 1, we decrease the values of c_1, c_2, and c_3, respectively, or synchronously to comparatively analyze the stability of dynamical system of the game model.

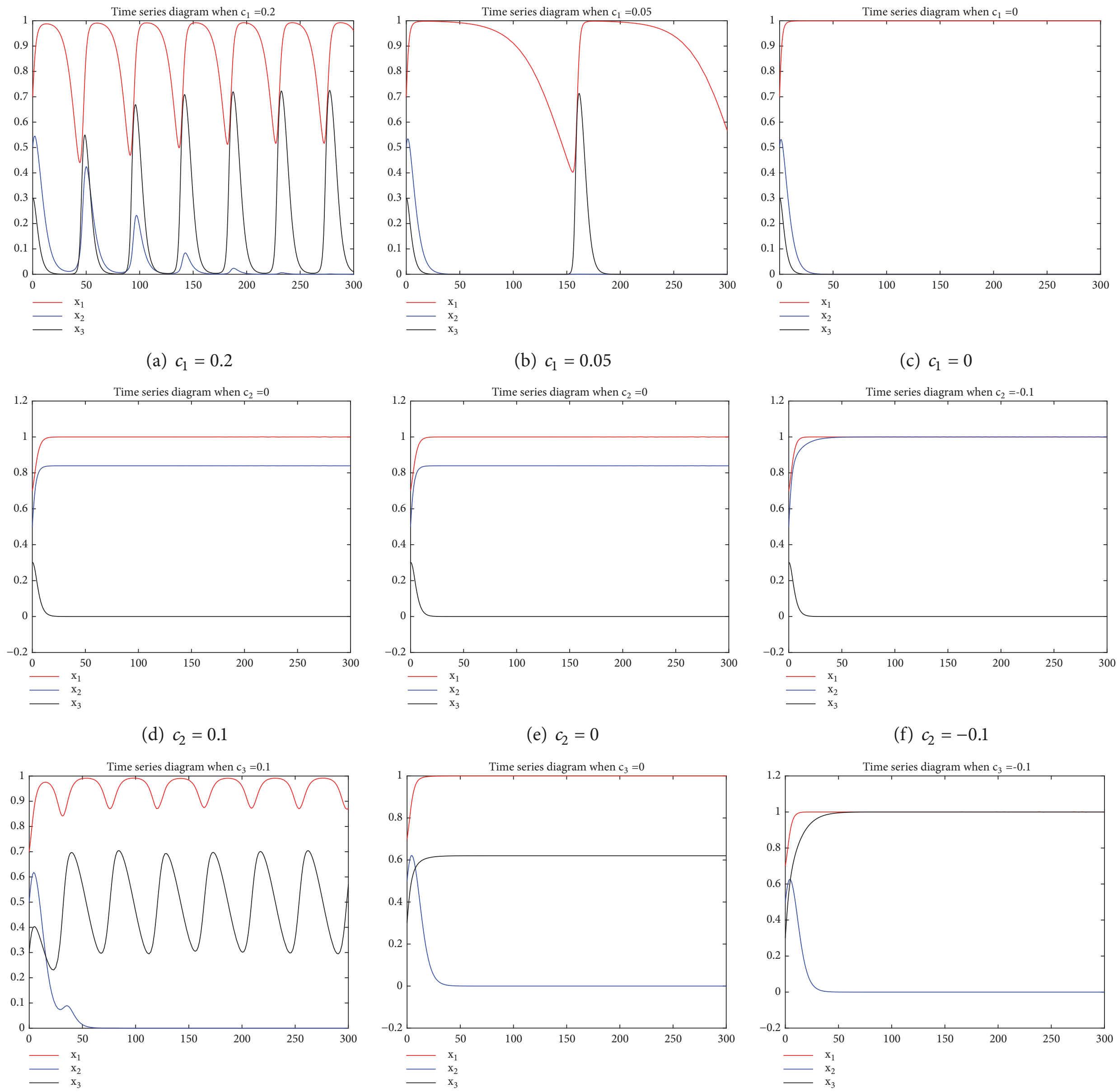
Here, Figures 2(a), 2(b), and 2(c) are time series graphs of strategy selection probabilities (x_1, x_2, x_3) when c_1 decreases gradually. Similarly, we get Figures 2(d), 2(e), and 2(f) when c_2 decreases gradually and Figures 2(g), 2(g), and 2(i) when c_3 decreases gradually. Figures 3(a), 3(b), and 3(c) show the time series of strategy selection probabilities when c_1, c_2, and c_3 decreases synchronously.

As can be seen from Figures 2 and 3, the volatility of the game system has been reduced when the value of c_1, c_2, or c_3 becomes smaller. However, the stability of the game system has not changed substantially. Under normal circumstances (see Figures 2(a), 2(d), and 2(g)), x_1 still has no asymptotic stability (controllability). Only under some extreme conditions such as $c_1 = 0, c_2 < 0$, or $c_3 < 0$ which are difficult to reach in practice, will x_1 (probability of coal miners selecting safe production behaviors) quickly and steadily approach 1 (see Figures 2(c), 2(f), 2(i), and 3(c)).

So, on the premise of guaranteeing the economic benefits of coal enterprises, it is necessary to suitably raise the policy subsidies and the welfare level of the employees, which will relevantly reduce the safety input cost of the interested groups and promote them to increase investment in production. Then the production safety of coal enterprises and safety management work can be run at a higher level.

2.2. Stability of the Dynamical Game System with Flexible Cost (Incentive Rewards). In Section 2.1, we found that general cost reductions do not completely bring down the volatility of the dynamical system of game models. Salary is the sum of various forms of remuneration or return obtained by the employees of the enterprise for providing labor or labor to their work units, including job salary, performance salary, bonus (month award, quarterly award, annual award, etc.), allowance, labor bonus, welfare, and so on. Salary management is the key to reflect the vital interests and social values of employees, and also it is the internal power of enterprise operation [29, 30], so that the incentive salary system must adhere to the principle of pay according to performance. The implementation of performance compensation should be based on a scientific performance evaluation system. Otherwise, the fairness of performance compensation will not reach the purpose of motivating employees. In recent years, various enterprises (including various types of enterprises in the coal industry) pay more and more the wages of employees, quarterly awards, year-end awards, and other variable income to the total income of workers. The discretionary variable part (performance salary, bonus, etc.) plays a very important role in stimulating the working enthusiasm in coal enterprises and improving safety efficiency.

Therefore, performance salary and bonus and some other incentive rewards should be taken into account of the players' costs of safety production. Those who perform better will get more rewards, and their safety cost inputs will be reduced



(a) $c_1 = 0.2$ (b) $c_1 = 0.05$ (c) $c_1 = 0$

(d) $c_2 = 0.1$ (e) $c_2 = 0$ (f) $c_2 = -0.1$

(g) $c_3 = 0.1$ (h) $c_3 = 0$ (i) $c_3 = -0.1$

Figure 2: Time series graph of players' strategy selection probabilities when c_1, c_2, c_3 have smaller values separately. Initial values of variables: $(x_1, x_2, x_3)|_{t=0} = P_1(0.7, 0.5, 0.3)$.

accordingly. So we construct some flexible cost functions, such as $c_i(x_i) = c_i(1-x_i), c_i(x_i) = c_i(1-x_i^2), c_i(x_i) = c_i(1-\sqrt{x_i})$, to analyze the stability of dynamical system with incentive rewards.

If $c_i(x_i) = c_i(1-x_i), i = 1, 2, 3$, are taken, income functions of the three players in the game model are

$$
\begin{aligned}
\pi_1 &= -x_1 (1 - x_1) c_1 - (1 - x_1) (x_2 + x_3 - x_2 x_3) f_1; \\
\pi_2 &= -x_2 (1 - x_2) c_2 \\
&\quad + (1 - x_1) [x_2 f_1 - (1 - x_2) x_3 f_2]; \\
\pi_3 &= -x_3 (1 - x_3) c_3 + (1 - x_1) (1 - x_2) x_3 (f_1 + f_2).
\end{aligned} \tag{2}
$$

The replicator dynamics corresponding to the game model are as follows:

$$
\begin{aligned}
\dot{x_1} &= x_1 (1 - x_1) [x_1 c_1 + (x_2 + (1 - x_2) x_3) f_1]; \\
\dot{x_2} &= x_2 (1 - x_2) [x_2 c_2 + (1 - x_1) (f_1 + x_3 f_2)]; \\
\dot{x_3} &= x_3 (1 - x_3) [x_3 c_3 + (1 - x_1) (1 - x_2) (f_1 + f_2)].
\end{aligned} \tag{3}
$$

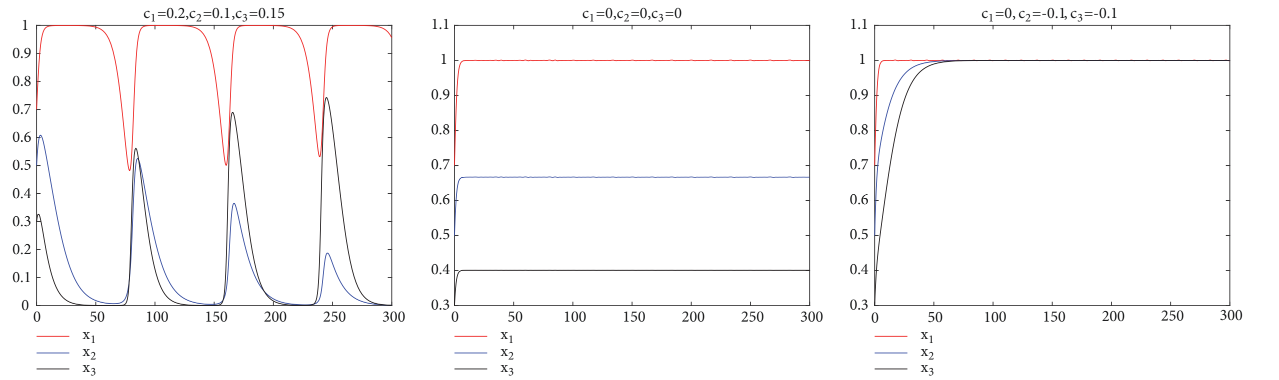


FIGURE 3: Time series graph of players' strategy selection probability when c_1, c_2, c_3 become smaller simultaneously. $(x_1, x_2, x_3)|_{t=0} = P_1(0.7, 0.5, 0.3)$.

The Jacobian matrix of the dynamical system (3) is

$$J = \frac{\partial(\dot{x_1}, \dot{x_2}, \dot{x_3})}{\partial(x_1, x_2, x_3)} = \begin{pmatrix} l_1 & x_1(1-x_1)(1-x_3)f_1 & x_1(1-x_1)(1-x_2)f_1 \\ x_2(1-x_2)(f_1+f_2x_3) & l_2 & x_2(1-x_1)(1-x_2)f_2 \\ -x_3(1-x_3)(1-x_2)(f_1+f_2) & -x_3(1-x_3)(1-x_1)(f_1+f_2) & l_3 \end{pmatrix} \quad (4)$$

where

$$\begin{aligned} l_1 &= (1-2x_1)(x_2+x_3-x_2x_3)f_1 + (2x_1-3x_1^2)c_1; \\ l_2 &= (1-2x_2)(1-x_1)(f_1+x_3f_2) + (2x_2-3x_2^2)c_2; \\ l_3 &= (1-2x_3)(1-x_1)(1-x_2)(f_1+f_2) \\ &\quad + (2x_3-3x_3^2)c_3. \end{aligned} \quad (5)$$

Let $\dot{x_1} = \dot{x_2} = \dot{x_3} = 0$, and we get eight pure strategy equilibrium points of system (3), $X_i (i = 0, 1, \ldots, 8)$:

$$\begin{aligned} &(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), \\ &(0,1,1), (1,1,1). \end{aligned} \quad (6)$$

According to Formula (4), all of the Jacobian matrices at the equilibrium points are diagonal matrices (see Table 1). And all of the eigenvalues are not larger than zero at the four points with $x_1 = 1$ (i.e., coal miners will choose safety production behaviors), and the first eigenvalues of them are negative.

According to the eigenvalue judgment method of evolutionary game strategy (ESS) [31, 32], the ESS of coal miners is $x_1 = 1$, which means they will surely choose the safety production strategy in the case of this function form of flexible costs.

As can be shown in Figures 4 and 5, dynamical system (3) goes into evolutionary stable state despite different initial values. Especially for some reasonable initial state values, the evolutionary process of the game system is very anticipated and preferable (see Figures 4(a) and 4(b)).

Therefore, if the incentive salary system is established in coal enterprise, the proportion of unsafe behaviors will greatly be reduced and coal mine safety will be maintained in a stable and controllable state. An incentive salary system means that the salary should be linked to the performance of coal workers with the efficiency of safety production and the frequency of unsafe behavior, and the variable part of coal enterprise workers' performance wages, bonuses, welfare, and so on should be increased in their personal incomes.

2.3. Stability of the Dynamical Game System with Inflexible Penalties. As can be seen in Figure 6, the volatility of the dynamical system would be reduced when value of f_1 or f_2 increases. In contrast to Figures 6(a), 6(b), and 6(c), we find that x_1 approaches 1 and x_2 increases with smaller amplitudes when f_1 becomes larger. That is, the increase in the intensity of player I's penalty can make coal workers increase the willingness to choose the behaviors of safe production, and the enterprise safety inspection department is also willing to increase the intensity of safety inspection.

By contrasting Figure 6(d) with Figure 6(e), we also find that x_1 approaches 1 with smaller amplitudes when f_2 becomes larger. However, the volatilities of x_2 and x_3 have no significant reduction when f_1 or f_2 becomes larger (even when f_1 and f_2 become large synchronously; see Figure 6(f)), while the values of x_2 and x_3 have the trends of increasing. In addition, the increase in the intensity of player II's penalty may increase the workload and tension in safety regulation.

So the conclusion is that although the behavior instability will decrease with the increase of the values of f_1 and

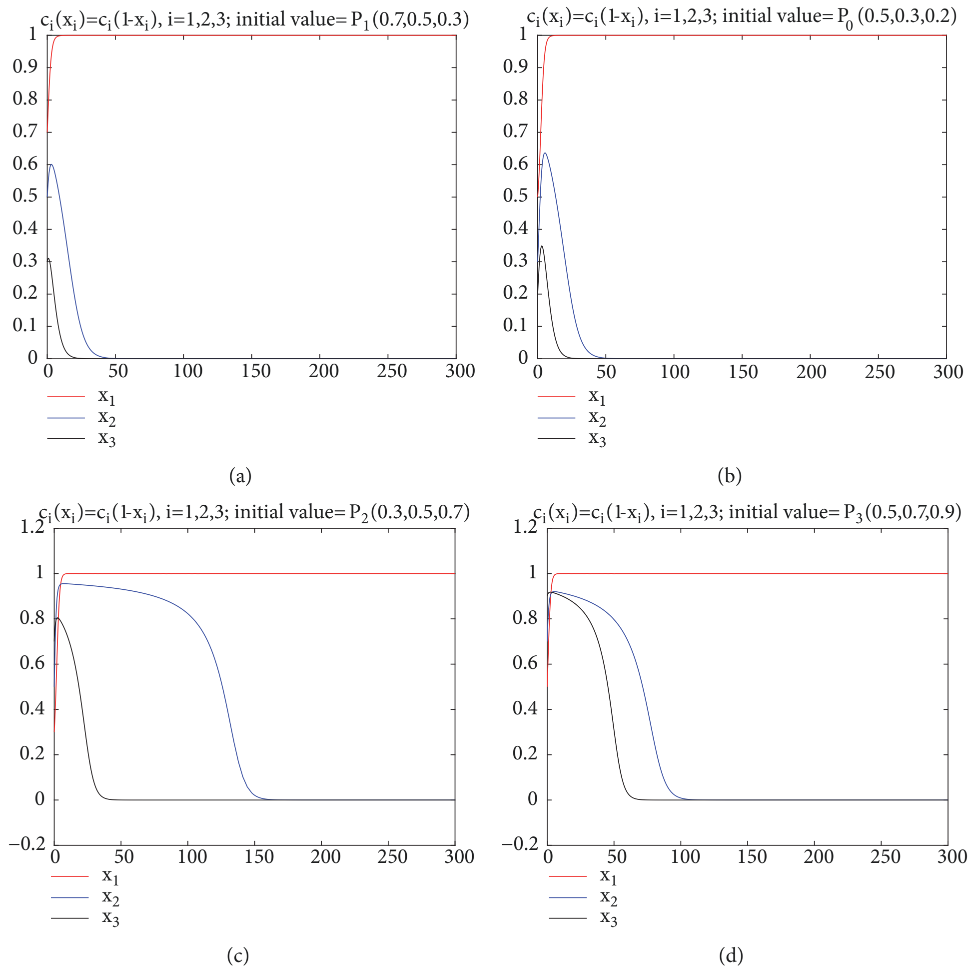


Figure 4: Time series graph of players' strategy selection probability when costs become flexible. $c_i(x_i) = c_i(1 - x_i), i = 1, 2, 3$.

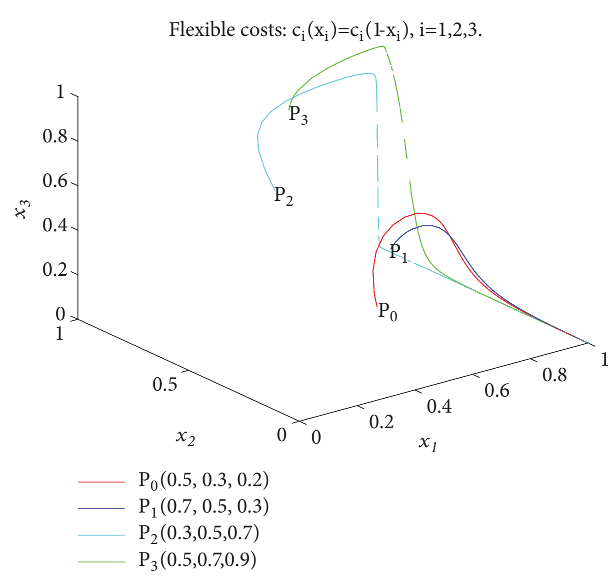
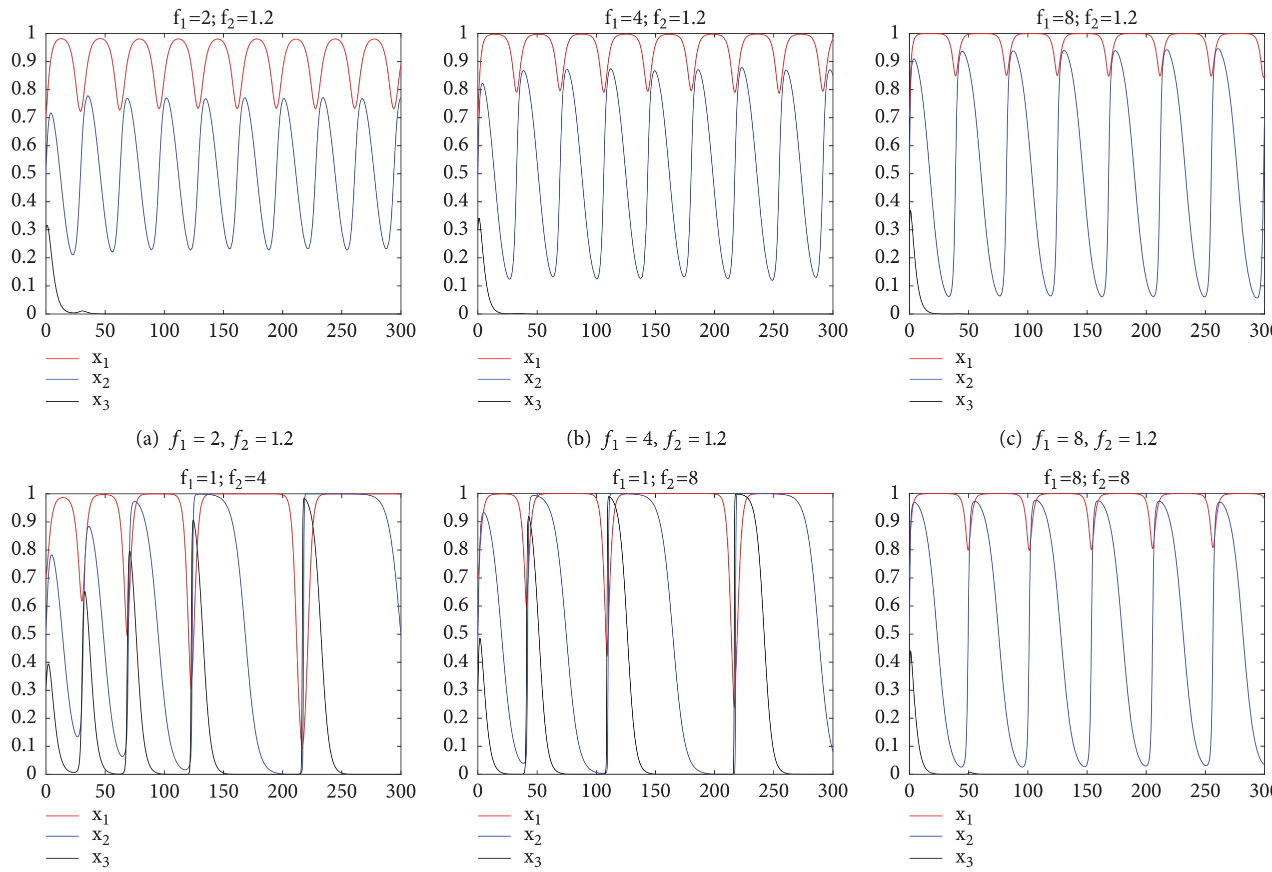


Figure 5: Phase diagram of the tripartite game system with incentive rewards.

Table 1: Jacobian Matrix of Equilibrium Points with Dynamical costs.

Equilibrium points	(0,0,0)	(0,1,0)	(0,0,1)	(0,1,1)
J	$\mathrm{diag}(0, f_1, f_1 + f_2)$	$\mathrm{diag}(f_1, -c_2 - f_1, 0)$	$\mathrm{diag}(f_1, f_1 + f_2, -c_3 - f_1 - f_2)$	$\mathrm{diag}(f_1, -c_2 - f_1 - f_2, -c_3)$
Equilibrium points	(1,0,0)	(1,1,0)	(1,0,1)	(1,1,1)
J	$\mathrm{diag}(-c_1, 0, 0)$	$\mathrm{diag}(-c_1, -c_2, 0)$	$\mathrm{diag}(-c_1, 0, -c_3)$	$\mathrm{diag}(-c_1 - f_1, -c_2, -c_3)$



(a) $f_1 = 2$, $f_2 = 1.2$ (b) $f_1 = 4$, $f_2 = 1.2$ (c) $f_1 = 8$, $f_2 = 1.2$

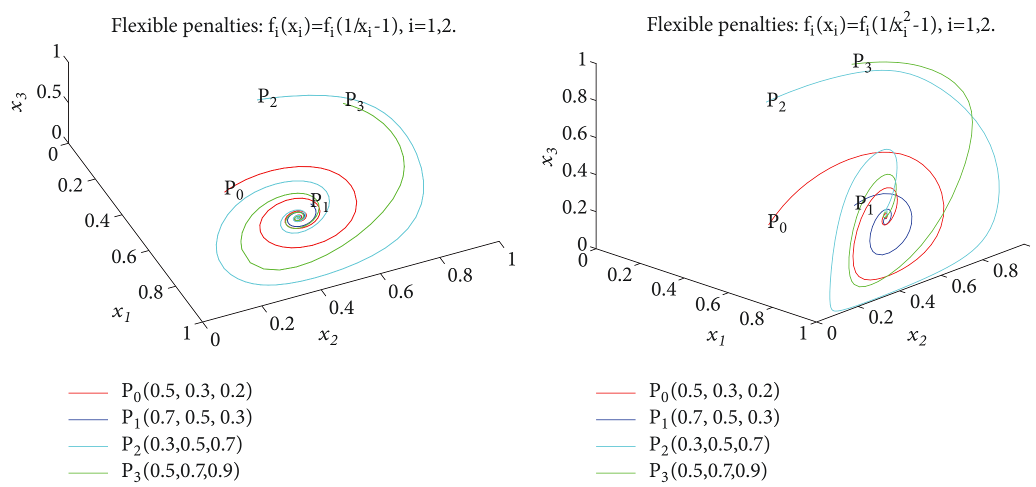
(d) $f_1 = 1$, $f_2 = 4$ (e) $f_1 = 1$, $f_2 = 8$ (f) Both f_1 and f_2 are large

Figure 6: Time series graph of players' strategy selection probability when f_1 or f_2 has larger values. Initial value of variables $(x_1, x_2, x_3)|_{t=0} = P_1(0.7, 0.5, 0.3)$.

f_2, there is no essential change of the volatility of the dynamical system. That is, the ordinary increase of penalty index can not completely eliminate the behavior instability of game models. So, on the one hand, it is necessary to make proper punishment for unsafe behaviors strictly in the coal mine safety management. On the other hand, too, excessively severe penalty measures may lead to the instability of the working state of the supervised objects, which may bring negative effects on the efficiency of coal mine safety supervision.

2.4. Stability of the Dynamical Game System with Flexible Penalties. According to the conclusion of Section 2.3, the stability of players' selections has not been completely controlled as the values of f_1, f_2 become larger simply. Therefore, we consider some flexible penalty functions $f_i(x_i)$ to substitute for general penalty index f_i and analyze the influence of flexible penalty mechanism on system stability.

The design of the flexible penalty function in this part is based on the following considerations: for the players who do not often make mistakes, the penalty for accidental mistakes is relatively light; for the players who often make mistakes (repeatedly taught not to change), the more frequent the mistakes are, the larger the corresponding penalty will be. Therefore, the flexible penalty functions $f_i(x_i)$ mainly discuss three functional forms to analyze and be compared with each other: $M_i(1 - x_i), M_i(x_i^{-1} - 1), M_i(x_i^{-2} - 1), i = 1, 2$. Set the values of function parameters $M_1 = 1$ and $M_2 = 1.2$ (separately equivalent to the preceding parameter values of f_1 and f_2), and numerical simulation results indicate that flexible penalty mechanism can significantly improve the stability of dynamical systems of evolutionary game models



(a) $f_i(x_i) = f_i(x_i^{-1} - 1)$, $i = 1, 2$ (b) $f_i(x_i) = f_i(x_i^{-2} - 1)$, $i = 1, 2$

Figure 7: Phase diagram of dynamical system of tripartite game with flexible penalty strategy.

(see Figures 7, 8, and 9). In particular the latter two types of flexible penalty function can effectively control the evolutionary stability of game model, which can make the probability of players' strategy selection x_1, x_2, and x_3 stabilize quickly. In addition, the latter two types can theoretically avoid the unwanted situations like any equilibrium point x_i equals zero (where x_i appears on the denominator).

The comparison between Figures 7(a) and 7(b) demonstrates that the last function form $M_i(x_i^{-2} - 1)$ is even better one because x_1 can quickly and steadily tends to 1. Suppose that $f_i(x_i) = f_i(x_i^{-2} - 1), i = 1, 2$, and then we get the income function of the three players in the game model as follows:

$$
\begin{aligned}
\pi_1 &= -x_1 c_1 - (1 - x_1)(x_2 + x_3 - x_2 x_3) f_1(x_1^{-2} - 1);\\
\pi_2 &= -x_2 c_2 + (1 - x_1)\\
&\quad \cdot \left[x_2 f_1(x_1^{-2} - 1) - (1 - x_2) x_3 f_2(x_2^{-2} - 1)\right];\\
\pi_3 &= -x_3 c_3 + (1 - x_1)(1 - x_2)\\
&\quad \cdot x_3 \left(f_1(x_1^{-2} - 1) + f_2(x_2^{-2} - 1)\right).
\end{aligned}
\tag{7}
$$

Replicator dynamic equations corresponding to game models are

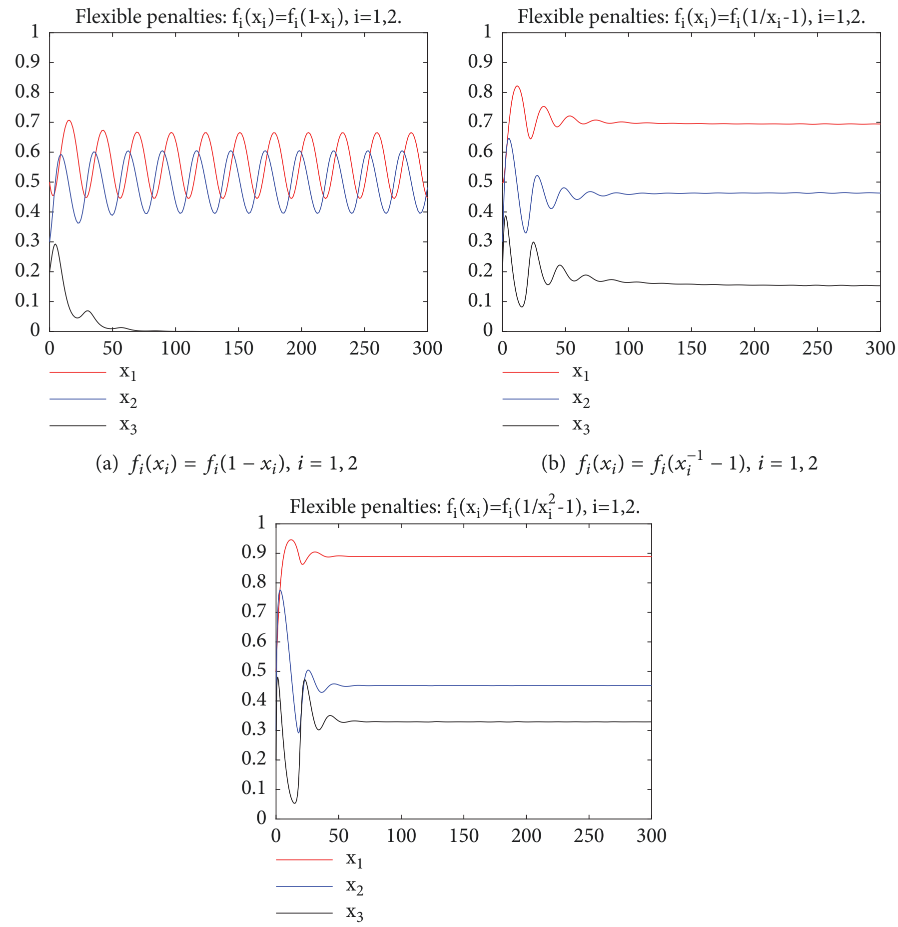
$$
\begin{aligned}
\dot{x_1} &= x_1(1 - x_1)\left[-c_1 + (x_2 + x_3 - x_2 x_3) f_1(x_1^{-2} - 1)\right];\\
\dot{x_2} &= x_2(1 - x_2)\left[-c_2\right.\\
&\quad \left.+ (1 - x_1)\left(f_1(x_1^{-2} - 1) + x_3 f_2(x_2^{-2} - 1)\right)\right];\\
\dot{x_3} &= x_3(1 - x_3)\left[-c_3\right.\\
&\quad \left.+ (1 - x_1)(1 - x_2)\left(f_1(x_1^{-2} - 1) + f_2(x_2^{-2} - 1)\right)\right].
\end{aligned}
\tag{8}
$$

The Jacobian matrix of the dynamical system of the game model is

$$
J = \frac{\partial(\dot{x_1}, \dot{x_2}, \dot{x_3})}{\partial(x_1, x_2, x_3)} = (j_{ij})_{3\times 3} \tag{9}
$$

where

$$
\begin{aligned}
j_{11} &= (1 - 2x_1)\left[(x_2 + x_3 - x_2 x_3) f_1(x_1^{-2} - 1) - c_1\right]\\
&\quad - \frac{2(1 - x_1)(x_2 + x_3 - x_2 x_3) f_1}{x_1^2};\\
j_{12} &= x_1(1 - x_1)(1 - x_3) f_1(x_1^{-2} - 1);\\
j_{13} &= x_1(1 - x_1)(1 - x_2) f_1(x_1^{-2} - 1);\\
j_{21} &= x_2(1 - x_2)\left(-f_1(x_1^{-2} - 1) - \frac{2(1 - x_1) f_1}{x_1^3}\right.\\
&\quad \left.+ f_2(x_2^{-2} - 1) x_3\right);\\
j_{22} &= (1 - 2x_2)\\
&\quad \cdot \left[(1 - x_1)\left(f_1(x_1^{-2} - 1) + x_3 f_2(x_2^{-2} - 1)\right) - c_2\right]\\
&\quad - \frac{2(1 - x_1)(1 - x_2) x_2 x_3 f_2}{x_2^3};\\
j_{23} &= x_2(1 - x_1)(1 - x_2) f_2(x_2^{-2} - 1);\\
j_{31} &= -x_3(1 - x_3)(1 - x_2)\left[-f_1(x_1^{-2} - 1)\right.\\
&\quad \left.- \frac{2(1 - x_1) f_1}{x_1^3} + f_2(x_2^{-2} - 1)\right];\\
j_{32} &= -x_3(1 - x_3)(1 - x_1)\left[-f_2(x_2^{-2} - 1)\right.\\
&\quad \left.- \frac{2(1 - x_2) f_2}{x_2^3} + f_1(x_1^{-2} - 1)\right];\\
j_{33} &= (1 - 2x_3)\\
&\quad \cdot \left[(1 - x_1)(1 - x_2)\left(f_1(x_1^{-2} - 1) + f_2(x_2^{-2} - 1)\right)\right.\\
&\quad \left.- c_3\right].
\end{aligned}
\tag{10}
$$



(a) $f_i(x_i) = f_i(1 - x_i)$, $i = 1, 2$

(b) $f_i(x_i) = f_i(x_i^{-1} - 1)$, $i = 1, 2$

(c) $f_i(x_i) = f_i(x_i^{-2} - 1)$, $i = 1, 2$

Figure 8: Time series graph of dynamical system under flexible penalty mechanism. Initial value is $P_0(0.5, 0.3, 0.2)$.

Let $\dot{x}_1 = \dot{x}_2 = \dot{x}_3 = 0$, it is found that there are only two pure policy equilibrium points in the system, $X_0 = (1, 1, 0)$; $X_1 = (1, 1, 1)$, and there may be a mixed policy equilibrium point X_2 satisfying the following conditions:

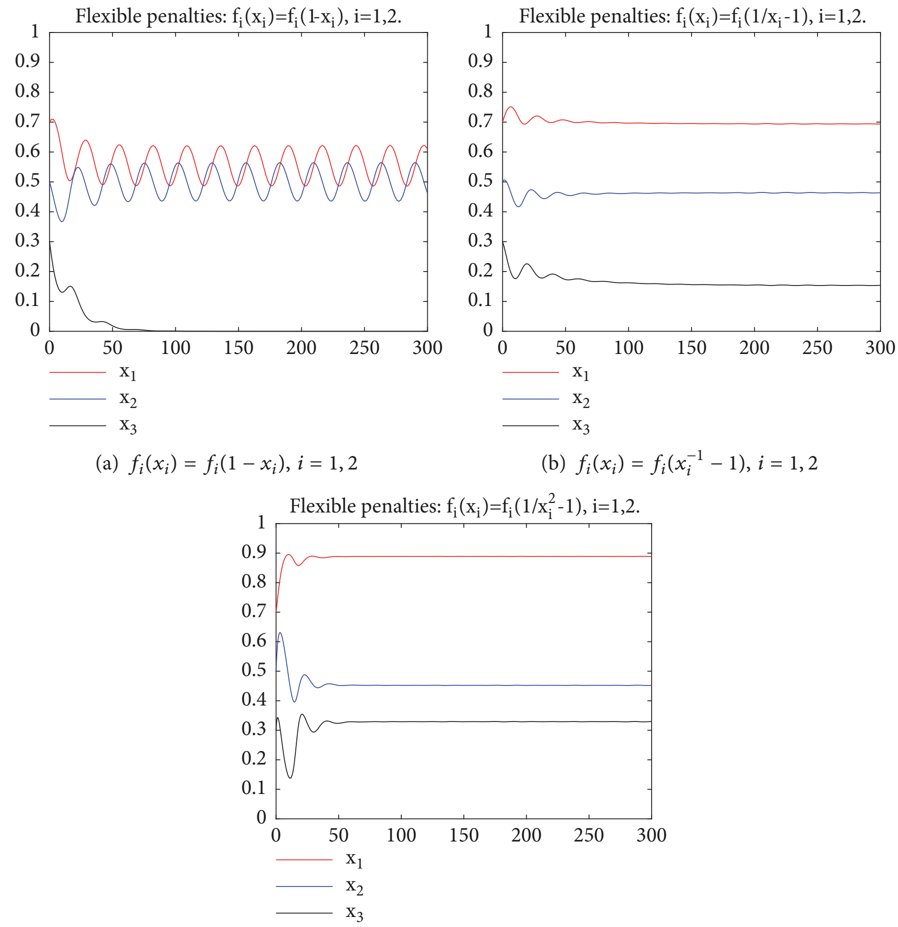
$$\begin{aligned}
(x_2 + x_3 - x_2 x_3) f_1 \left(x_1^{-2} - 1\right) &= c_1;\\
(1 - x_1)\left[f_1\left(x_1^{-2} - 1\right) + x_3 f_2\left(x_2^{-2} - 1\right)\right] &= c_2; \qquad (11)\\
(1 - x_1)(1 - x_2)\left[f_1\left(x_1^{-2} - 1\right) + f_2\left(x_2^{-2} - 1\right)\right] &= c_3.
\end{aligned}$$

Through calculating, we get $J(X_0) = \mathrm{diag}(c_1, c_2, -c_3)$; $J(X_1) = \mathrm{diag}(c_1, c_2, c_3)$. The dynamical systems do not have evolutionary stability in the two equilibrium fields because both Jacobian matrices at the two pure strategy equilibrium points have positive eigenvalues. In addition, Formula (11) shows that the solution of x_1 must be less than 1 even if the equilibrium point of mixed strategy X_2 exists.

So it can be concluded from the numerical simulations that flexible penalty mechanism can significantly reduce the dynamics of unsafe behaviors in coal mine safety supervision. Once the penalties can be closely correlated with the frequency of their mistakes, the players would like to consistently choose the safe behaviors (or improve the efficiency of safety supervision) with great probability. Then the controllability of safety supervision system and the performance efficiency of safety management measures can be guaranteed. However, a shortcoming of flexible penalties is that although x_1 is an ESS, it does not regularly converge to 1 (see Figure 9(c)).

3. Stability of the Dynamical Game System under Combined Mechanism of Incentive Rewards and Flexible Penalties

Through the previous analysis in Sections 2.2 and 2.4, we find that incentive rewards (flexible costs) can raise players' selection probabilities of safe behaviors and flexible penalties can bring system stability. Considering the actual situation of coal production, we certainly prefer to have an ESS of $x_1 = 1$ in the dynamical safety management system. So we propose a combined mechanism of incentive rewards and flexible penalties and then analyze the stability of the evolutionary dynamical system on this basis.



(a) $f_i(x_i) = f_i(1 - x_i)$, $i = 1, 2$

(b) $f_i(x_i) = f_i(x_i^{-1} - 1)$, $i = 1, 2$

(c) $f_i(x_i) = f_i(x_i^{-2} - 1)$, $i = 1, 2$

FIGURE 9: Time series graph of dynamical system under flexible penalty mechanism. Initial value is $P_1(0.7, 0.5, 0.3)$.

Two cases are chosen as an example to be analyzed and compared in this part. One case is all players unified incentive rewards and flexible penalties; i.e., $c_i(x_i) = c_i(1 - x_i), i = 1, 2$, and $f_i(x_i) = f_i(x_i^{-2} - 1), i = 1, 2, 3$. The other case is that player I (coal miners) is subjected to incentive reward mechanism, and player II is subjected to flexible penalty mechanism, and player III is not given any incentive reward and flexible penalty (i.e., $c_1(x_1) = c_1(1 - x_1)$, $f_2(x_2) = f_2(x_2^{-2} - 1)$, and the rest of the parameters remain unchanged). As shown in the phase diagrams (see Figure 10), both results of the two cases of combined mechanisms are very satisfying ($x_1 = 1$ is ESS).

The evolutionary processes of more cases of combined mechanisms are demonstrated by the time series graphs (see Figure 11). Through mutual comparison among them, we can find that some of them bring more satisfying results (see Figures 11(a) and 11(b)). Although starting from different initial values, the dynamical system always tends to be ideally stable in very short time (player I chooses safe production strategy with $x_1 = 1$ and the values of x_2 and x_3 are also stable in some reasonable values). Therefore, we can conclude that this combined mechanism of incentive reward and flexible penalties can optimize and control the instability of the behavior strategy selection in the safety supervision model. This conclusion is undoubtedly a highlight of this study. These combined mechanisms may be the key measure to solve the behavior fluctuations in the safety management system.

4. Control Strategies for the Improvement of the System and Measures of Safety Management

At present, the formulation of safety management policies and measures is mostly based on the temporary measures taken according to the occurrence of accidents [1, 33]. The policies are adjusted with the occurrence of accidents. The relevant supervision measures are often characterized by accident types and lack of comprehensive analysis of safety system, which is one of the important reasons for frequent accidents. There are many factors that affect the game behavior and game strategy of each interested group. The study of the regulatory mechanism of influencing factors is an important guarantee for the rationality of the formulation of management rules. Only when the management rules can regulate the bad game behavior of the players and the game rules can the management rules be effective.

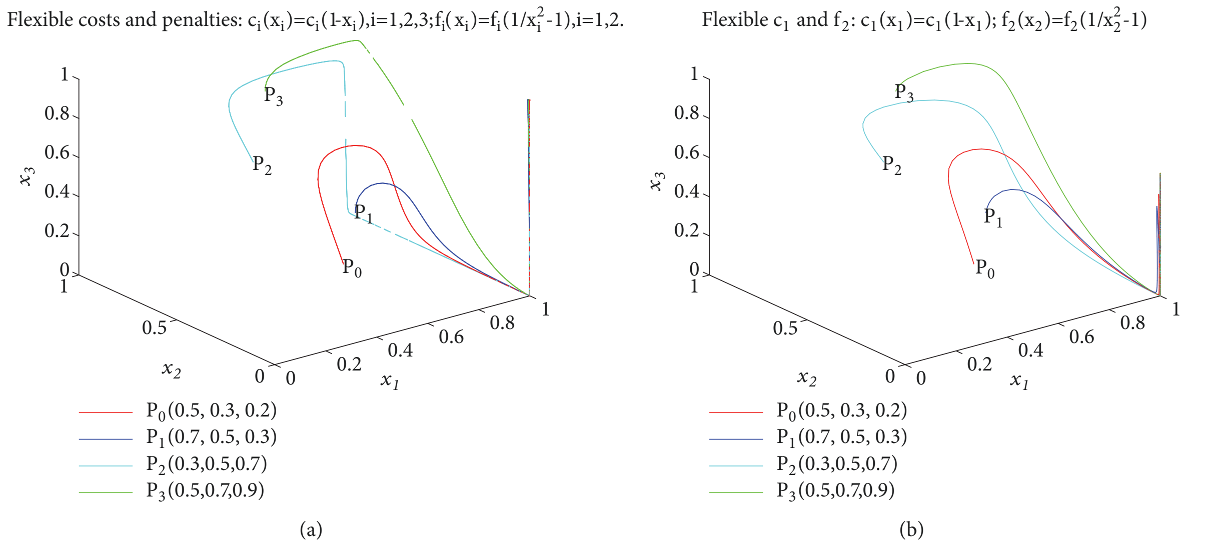


Figure 10: Phase diagrams of dynamical system of tripartite game under combined mechanisms of incentive reward and flexible penalty.

According to the researches of the coal mine safety supervision model in Sections 2 and 3, we finally put forward the following regulation and control strategies of coal mine safety management system and supervision measures.

(1) Restraint Measures in Safety Supervision System of Coal Mine Safety Management. In Section 2.1, it is concluded that raising policy subsidies and the welfare level of the employees would relevantly reduce the safety input cost and make the production safety of coal enterprises and safety management work run at a higher level. So the governmental supervision department should formulate some policy-based or industry-specific constraints for safety supervision behaviors, including mandatory provisions on the supervision strategies of internal and external safety supervision and inspection departments. In order to better improve the overall production safety level and safety management level, the majority of front-line production workers in coal enterprises can be called upon to exercise self-restraint on their own safety production activities spontaneously (strictly disciplined and supervised by each other). It is more important especially in large and medium-sized state-owned coal enterprises, where the quality of coal mine workers is very good and the technology of safety production is more advanced.

It should be noted that the so-called subsidies and employee benefits are not necessarily limited to the economic level. For ordinary employees, especially the coal miners working in the production line, enterprises should increase investment in regular safety knowledge training and safety skills drills to improve the safety quality and safety awareness of employees and inhibit the occurrence of unsafe behavior. For enterprise safety managers and government personnel in charge of safety supervision departments, it is necessary to carry out regular or irregular study of safety skills and safety management knowledge in coal mine safety, so as to improve the safety knowledge and safety management level of safety management personnel and ensure the implementation of safety supervision policies and measures.

(2) Energetically Promoting the Incentive Salary System in the Coal Industry. In Section 2.2, we found that incentive salary system can greatly reduce the proportion of unsafe behaviors and impels coal mine safety to be stable and controllable. Incentive reward system will reform the traditional unreasonable wage and salary distribution in coal enterprises and significantly increase wage distribution ratio of safety-related posts and production links, such as high technical requirements, high production safety intensity, and heavy management responsibility. Then the staff and workers of coal enterprises can be guided to spontaneously strengthen the improvement of safety production technology and the exertion of safety production capacity.

There are still some backward phenomena in the salary distribution system of nowadays' coal enterprises in China, such as the serious tendency of internal income distribution equalitarianism and unreasonable internal income distribution relationship, which seriously affect the efficiency of safety supervision in coal enterprises and the level of safety management [34]. Safety production is the most important task in the coal enterprises. So we should vigorously promote the incentive salary system in the coal industry, especially in large and medium-sized state-owned coal enterprises. Post-performance pay system, year-end accident-free incentives, and other ways should be adopted properly to improve the correlation between the income of workers and their safety production efficiency and safety behavior.

(3) Establishing the Flexible Penalty Mechanism of Unsafe Behavior in Coal Mine Safety System. It can be seen from the research results in Sections 2.4 and 3 that the flexible penalty mechanism based on the frequency of unsafe behavior can better reflect the management mode of tight integration in safety management. More closely related to the occurrence of coal mine safety behaviors, the flexible penalty mechanism can better guarantee the stability of coal mine safety management.

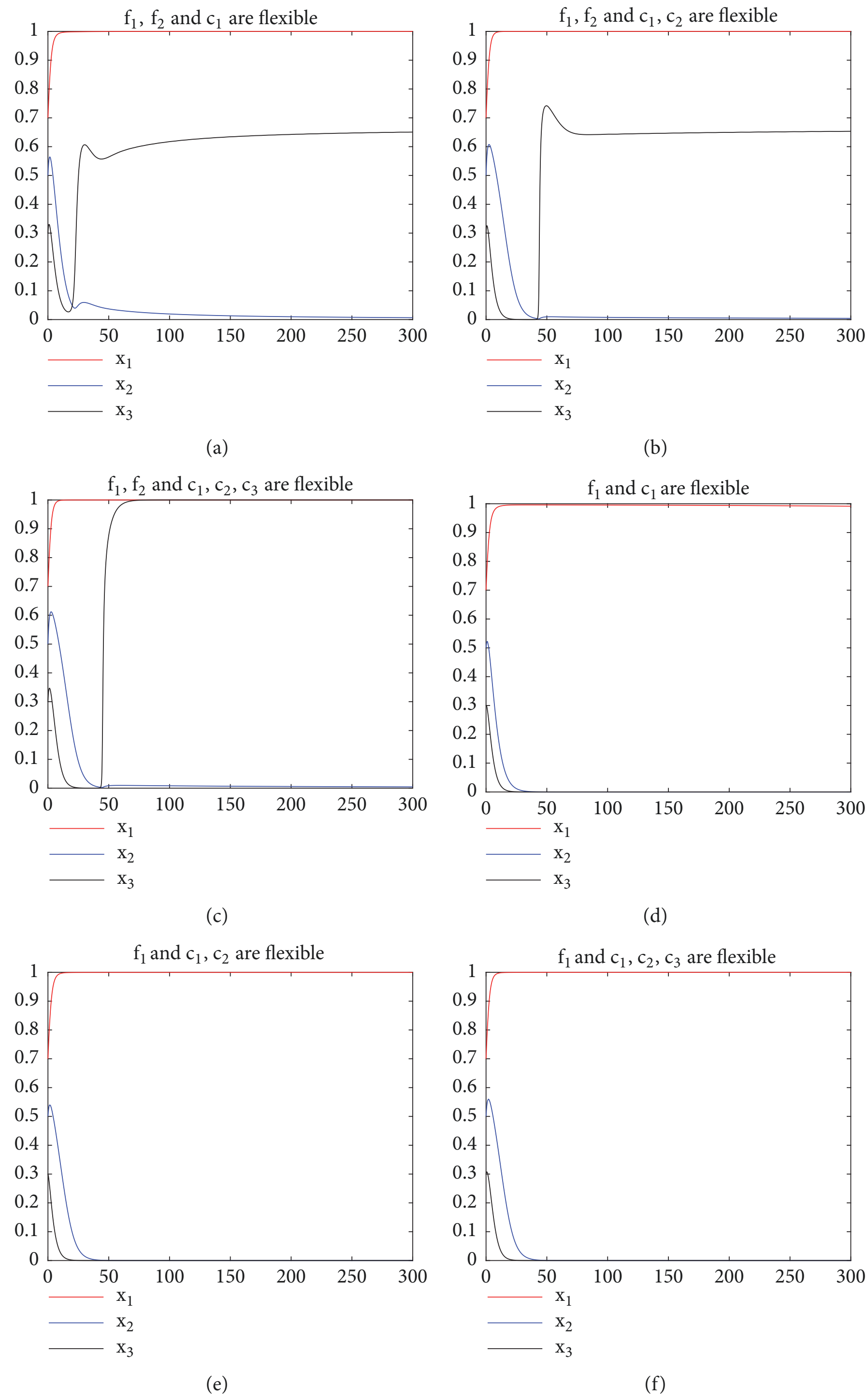


FIGURE 11: Time series graphs of dynamical system under combined mechanisms of incentive reward and flexible penalty. Initial value is $P_1(0.7, 0.5, 0.3)$.

In coal mine safety management, the penalty measures against unsafe behaviors of coal miners are very important. The difficulty and inadequacy of flexible cost mechanism are to have certain requirements for the operation profitability of coal mine enterprises. Actually, for many slow-growing small and medium-sized coal mine enterprises, they may prefer to motivate employees through punitive measures.

5. Conclusions

To control players' behavior instability in coal mine safety management, this study starts from the stability analysis of the dynamical game system under the four cases of inflexible costs (static subsidies), flexible costs (incentive rewards), inflexible penalties, and flexible penalties. Then, we discuss the stability of the dynamical system under combined mechanism of cost and penalty and find out which method is more effective to control the instability of dynamical system. Through the comparative analysis of inflexible and flexible rewards and penalties mechanism, it can be found that if the salary and penalty mechanism of coal enterprises can be closely related to their safety production behaviors (or safety supervision behaviors), the stability of the game dynamical system of coal enterprises' safety supervision can be enhanced. We propose a combined mechanism of incentive reward and flexible penalties for the improvement of the safety management system to restrain the emergence and spread of unsafe behaviors in the coal mine safety system. Some results of this research provide a theoretical basis for more reasonable and more effective policies of coal mine safety management.

It is worth noting that it is necessary to carry out the modern financial management of coal enterprises. The coal industry should vigorously improve the performance wages, quarterly awards, year-end bonuses, welfare, and other variable income in the proportion of employees'salaries and strive to build and improve the incentive salary system for coal enterprise employees. And, more importantly, more close connection should be established between the system of staff safety performance evaluation and the system of rewards and punishment.

In the future, the corresponding tuning mechanism for the parameters will be investigated in different safety management systems. In order to improve the accuracy of model parameters, we should investigate and analyze the actual situation of coal mine safety management by means of questionnaire survey and field visit, so as to improve the practicability of the model method in this paper. Besides, some optimal techniques should also be considered for improving control performance of the dynamical system, such as extremum seeking control (ESC) method [35, 36]. We will continue our efforts to study the optimization of the dynamical system stability.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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