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Research Article

Robust Control of Disturbed Fractional-Order Economical Chaotic Systems with Uncertain Parameters

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This paper focuses on the robust control of fractional-order economical chaotic system (FOECS) with parametric uncertainties and external disturbances. The dynamical behavior of FOECS is studied by numerical simulation, and circuit implementations of FOECS are also given. Based on fractional-order Lyapunov stability theorems, a robust adaptive controller, which can guarantee that all signals remain bounded and the tracking error tends to a small region, is designed. The proposed method can be used to control a large range of fractional-order systems with system uncertainties. Fractional-order adaptation laws are constructed to update the estimation of adaptive parameters. Finally, the robustness and effectiveness of our control method are indicated by simulation results.

1. Introduction

It has been shown that fractional-order nonlinear systems (FONSs) have been investigated by a lot of engineers and physicists because FONSs have wide potential applications in many domains [1–3]. In fact, the fractional calculus brings some advantages in modeling nonlinear systems. The fractional calculus can model real-world models in the whole-time domain, and it has memory. It should be mentioned that the integer-order one does not have these abilities. Thus, the fractional calculus will play a great role in modeling many actual systems, for example, stochastic diffusion, molecular spectroscopy, control theory, viscoelastic dynamics, quantum mechanics, and many research results can be seen in [4–8] and the references therein. On the contrary, it is well known that chaotic system is a supremely intricate nonlinear system that has been widely investigated due to its successful applications in signal processing, combinatorial optimization, secure communication, and many others. Especially, a chaotic system has the property that it is sensitive to the changing of initial conditions and the variations of the system parameters. Consequently, a large number of meritorious results on control and synchronization of fractional-order chaotic dynamics of nonlinear systems have become a hot research topic and a lot of interesting results have been reported, for example, in [9–13].

In the last two decades, the study of economical system has become more and more popular [14–24]. A lot of works have been done to describe properties of economical date and the dynamic behavior of economical systems. Recently, many researchers have made a lot of efforts to investigate main features of economic theory, e.g., overlapping waves of structural changes or commercial demand and irregular and erratic economic fluctuations. In fact, economists usually consider a model that has a simple behavior and composed of only endogenous variables. Thus, they can consider exogenous shock variables based on weather variables, political events, and other human factors. To describe the complicated economical behavior, some mathematical models were also introduced, for example, the van der Pol model [25] and

the IS-LM model [26]. Actually, there are many kinds of nonlinear systems that show chaotic behavior [27]. Thus, if economical systems show chaotic phenomenon, it is hard for people to provide feasible economic decision making. That is to say, it is advisable to study and control economical chaotic systems.

The remainder of our work is organized as follows: Section 2 presents the development of research in this field, the existing research gaps, and the contributions of this study. Section 3 gives some preliminaries about the fractional calculus and the description and dynamical behavior of fractional-order economical chaotic systems (FOECSs). Section 4 presents the controller design procedure and the stability analysis. Simulation results are shown in Section 5. Finally, Section 6 gives a brief conclusion of this paper.

2. Literature Review

The chaotic dynamics in economical systems were first founded in 1985, and after that, many control and synchronization methods for economical chaotic systems have been reported [16, 28–30]. In [16], a robust adaptive controller was given to control chaos in FOECSs, where the matched system uncertainties were considered, whereas in [16], the sign(·) function was used in the controller design which will lead to chattering phenomenon. In order to control a representative chaotic fractional finance system, an adaptive fuzzy control approach was given in [31] where fuzzy systems were used to approximate nonlinear functions. A new aspect of robust synchronization of a FOECS has been addressed in [32]. The fixed points and chaotic and periodic motions are given in [33], and dynamical behavior of a FOECS with time delay was studied in [34]. Dadras and Momeni [28] provided an adaptive control method to study the synchronization problem of FOECSs based on a sliding surface. The system studied in [28] was known, but it is impossible to accurately model an actual system in real life. And in the controller design, in order to make the sliding mode exist at every point of the sliding mode surface, the control law was constructed by using the sign function so that the chattering was unavoidable. In the final stability analysis, the integer-order Lyapunov stability theory was applied. Therefore, compared with [16], Dadras and Momeni [28] did not completely study the fractional-order economic chaotic system with the fractional-order stability theory. It should be mentioned that in the above literature considering the control or synchronization of FOECSs, the system model should be known in advance. However, it is well known that most systems suffer from system uncertainties and disturbances in nature. In actual life, we know that economical systems may suffer from weather changes, the limited size of transport, political influence, monetary policy, and many other human factors. Consequently, we should take system uncertainties and external disturbances into consideration when we investigate the control of FOECSs.

Due to limitations of available theoretical tools for analyzing the stability of nonlinear fractional systems, the number of research studies in this field is still low in comparison to that of integer-order systems. Based on the above discussion, in this paper, we investigate the control of FOECSs with unmatched system uncertainties and external disturbances. The fractional Lyapunov stability method is utilized to design the robust controller and analyze the system's stability. Compared with some related works, the main contribution of our work can be concluded as follows. (1) A robust adaptive controller is designed for FOECSs with unmatched system uncertainties. The system uncertainties model we considered is representative, and many models used in the literature, for example, in [16], can be seen as a special case of our model. (2) The stability analysis is proven strictly. The stability analysis method we used is very similar to that of the integer-order systems. It should be pointed out that our main result (see, Theorem 1) of our method provides a framework which can be easily referenced to analyze stability of fractional-order systems.

3. Preliminaries

3.1. Fractional Calculus. The α-th fractional-order integral is

$$\mathbb{D}_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau, \tag{1}$$

where $\Gamma(\cdot)$ represents Euler's function.

Caputo's α-th derivative is given as

$$\mathbb{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau, \tag{2}$$

where $n-1 \le \alpha < n$.

For fractional calculus, we give the following results to facilitate the controller design as well as the stability analysis.

Definition 1 (see [1]). The Mittag-Leffler function is given as

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \tag{3}$$

with $\alpha, \beta > 0$ and $z \in \mathbb{C}$.

The Laplace transform of (3) can be given as

$$\mathscr{L}\left\{t^{\beta-1} E_{\alpha,\beta}\left(-at^{\alpha}\right)\right\} = \frac{s^{\alpha-\beta}}{s^{\alpha}+a}, \tag{4}$$

where $a \in \mathbb{R}^+$ is a constant.

Lemma 1 (see [1]). *Let $\beta \in \mathbb{C}$, $0 < \alpha < 2, \mu \in \mathbb{R}$, and*

$$\frac{\pi\alpha}{2} < \mu < \min\{\pi, \pi\alpha\}, \tag{5}$$

and then it holds

$$E_{\alpha,\beta}(z) = -\sum_{j=1}^{n} \frac{1}{\Gamma(\beta-\alpha j)z^j} + o\left(\frac{1}{|z|^{n+1}}\right), \quad |z| \longrightarrow \infty, \tag{6}$$

where $n \in \mathbb{N}$ *and* $\mu \le |\arg(z)| \le \pi$.

Lemma 2 (see [1]). *Suppose that* $0 < \alpha < 2$ *and* $\beta \in \mathbb{R}$. *For* $\mu > 0$ *and* $(\pi\alpha/2) < \mu \le min\{\pi, \pi\alpha\}$, *it holds*

$$\left|E_{\alpha,\beta}(z)\right| \le \frac{a}{1+|z|}, \tag{7}$$

with $a > 0$ *and* $\mu \le |arg(z)| \le \pi$ *and* $|z| \ge 0$.

Lemma 3 (see [35]). *Let* $x(t)$ *be a smooth function. Suppose that* $x(t) = 0$ *is an equilibrium of*

$$\mathbb{D}_t^{\alpha} x(t) = f(x), \tag{8}$$

where $f(x)$ *is a continuous nonlinear function. If*

$$\begin{aligned} g_1(\|x\|) \le V(t, x(t)) \le g_2(\|x\|), \\ \mathbb{D}_t^{\beta} V(t, x(t)) \le -g_3(\|x\|), \end{aligned} \tag{9}$$

with $0 < \beta < 1$ *and* $g_1(\cdot), g_2(\cdot)$, *and* $g_3(\cdot)$ *being class-k functions, then system (8) is asymptotically stable.*

Lemma 4 (see [7, 36]). *Suppose that* $x(t) \in \mathbb{R}^n$ *is a smooth function. For* $0 < \alpha < 1$, *it holds*

$$\frac{1}{2}\mathbb{D}_t^{\alpha} x^T(t) x(t) \le x^T(t) \mathbb{D}_t^{\alpha} x(t). \tag{10}$$

Lemma 5 (see [1]). *Let* $x(t) \in \mathbb{C}^1[0, T]$ *with* $T > 0$ *and* $0 < \alpha \le 1$, *then it holds*

$$\begin{aligned} \mathbb{D}_t^{-\alpha} \mathbb{D}_t^{\alpha} x(t) = x(t) - x(0), \\ \mathbb{D}_t^{\alpha} \mathbb{D}_t^{-\alpha} x(t) = x(t). \end{aligned} \tag{11}$$

In the following parts, we will use an algorithm to solve fractional-order differential equations. A brief explanation of this algorithm is given below.

Consider

$$\begin{cases} \mathbb{D}_t^{\alpha} y(t) = f(t, y(t)), \\ y(0) = y_0. \end{cases} \tag{12}$$

Based on Lemma 5, (12) can be rewritten as

$$y(t) = y_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau, y(\tau)) d\tau. \tag{13}$$

Define $h = (T/N)$, $N \in Z$, $t_n = nh$, $n = 0, 1, \ldots, N$. Thus, (13) is estimated as [1]

$$\begin{aligned} y_h(t_{n+1}) = y_0 + \frac{h^{\alpha}}{\Gamma(\alpha+2)} f\left(t_n + 1, y_h^p(t_{n+1})\right) \\ + \frac{h^{\alpha}}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, y_h(t_j)\right), \end{aligned} \tag{14}$$

with $a_{j,n+1} = n^{\alpha+1} - (n-\alpha)(n+1)^{\alpha}$ for $j = 0$ and $a_{j,n+1} = (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}$ for $1 \le j \le n$, $y_h^p(t_{n+1}) = y_0 + (1/\Gamma(\alpha)) \sum_{j=0}^{n} b_{j,n+1} f(t_j, y_h(t_j))$, $p = \alpha + 1$, and $b_{j,n+1} = (h^{\alpha}/\alpha)((n+1-j)^{\alpha} - (n-j)^{\alpha})$.

The approximation error can be obtained as $\max|y(t_j) - y_h(t_j)| = o(h^p)$ [1].

3.2. The FOECS. The FOECS can be described by

$$\begin{cases} \mathbb{D}_t^{\alpha} \zeta_1(t) = \zeta_3(t) + (\zeta_2(t) - a)\zeta_1(t) + a_1 \zeta_4(t), \\ \mathbb{D}_t^{\alpha} \zeta_2(t) = 1 - b\zeta_2(t) - \zeta_1^2(t) + a_2 \zeta_4(t), \\ \mathbb{D}_t^{\alpha} \zeta_3(t) = -\zeta_1(t) - c\zeta_3(t) + a_3 \zeta_4(t), \\ \mathbb{D}_t^{\alpha} \zeta_4(t) = -\zeta_1(t)\zeta_2(t)\zeta_3(t), \end{cases} \tag{15}$$

where $0 < \alpha \le 1$ is the fractional order, $\zeta_1(t)$ represents the interest rate of the market, $\zeta_2(t)$ is the investment demand, $\zeta_3(t)$ corresponds to the price index, $\zeta_4(t)$ is the confidence of the market, a stands for the saving amount, b is the cost in each investment, c represents the demand elasticity, and a_1, a_2, and a_3 are impact parameters. Let $\boldsymbol{\zeta}(t) = [\zeta_1(t), \zeta_2(t), \zeta_3(t), \zeta_4(t)]^T$ be the state vector.

Let $a = 2.10$, $b = 0.01$, $c = 2.61$, $a_1 = 8.41$, $a_2 = 6.40$, $a_3 = 2.21$, and the initial condition be $\zeta(0) = [1.0, 5.0, 4.1, 3.0]^T$.

It has been shown in [37] that, under above parameters, when $\alpha > 0.88$, the system (15) exhibits chaotic behavior. Figure 1 shows the chaotic behavior of system (15) for $\alpha = 0.90$.

Now, let us consider the circuit implementation of the FOECS. Just as the results in [38], the approximation of $(1/s^{0.9})$ can be given by

$$\frac{1}{s^{0.9}} \approx \frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+2.154)(s+359.4)}. \tag{16}$$

Thus, an unit circuit is designed to implement the function $(1/s^{0.9})$ which is shown in Figure 2. Here, the chain fractance consists of three resistors R_a, R_b, and R_d and three capacitors C_a, C_b, and C_d. The transfer function $\mathrm{FC}(s)$ of this chain can be given by

$$FC(s) = \frac{(C_0/C_a)}{s + (1/R_a C_a)} + \frac{(C_0/C_b)}{s + (1/R_b C_b)} + \frac{(C_0/C_d)}{s + (1/R_d C_d)}. \tag{17}$$

Let $C_0 = 1\mu\mathrm{F}$; from (16) and (17), we can gain $R_a = 62.84\,M\Omega$, $R_b = 250\,k\Omega$, $R_d = 205\,k\Omega$, $C_a = 1.23\,\mu\mathrm{F}$, $C_b = 1.835\,\mu\mathrm{F}$, $C_d = 1.1\,\mu\mathrm{F}$, and $\mathrm{FC}(t) \approx (1/t^{0.9})$.

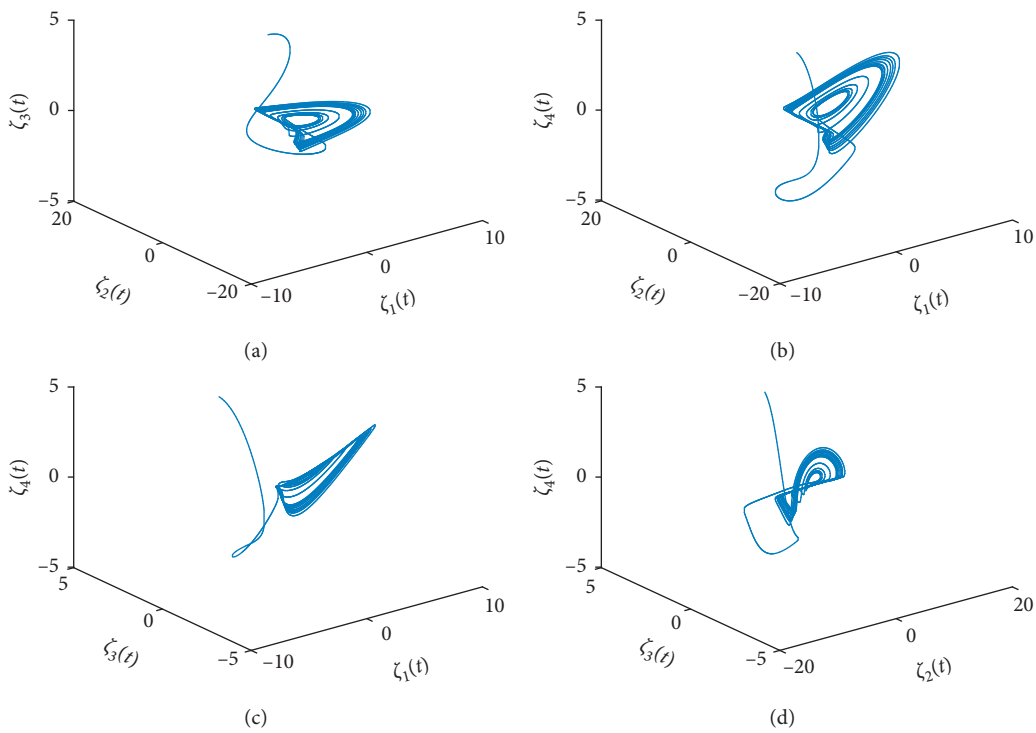

Figure 1: Chaotic phenomenon of (15) with $\alpha = 0.92$.

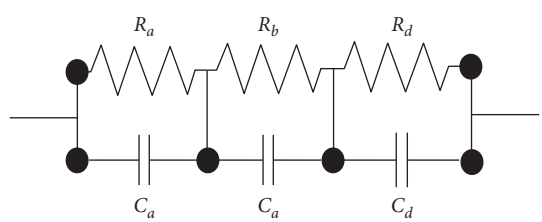

Figure 2: Unit circuit for realizing $(1/s^{0.9})$.

To implement the FOECS, an electronic circuit is constructed by using R-C components, analog multipliers, and operational amplifiers. The circuit diagram is depicted in Figure 3, whose mathematical equations are given as follows:

$$\begin{cases} \mathbb{D}_t^{\alpha}\zeta_1(t) = \dfrac{1}{R_6C_1}\zeta_3(t) + \dfrac{1}{R_1R_2C_1}\zeta_1(t)\zeta_2(t) + \dfrac{1}{R_5R_{12}C_1}\zeta_4(t) - \dfrac{1}{R_3R_4C_1}\zeta_1(t), \\ \mathbb{D}_t^{\alpha}\zeta_2(t) = 1 - \dfrac{1}{R_7C_2}\zeta_2(t) - \dfrac{1}{R_8C_2}\zeta_1^2(t) + \dfrac{1}{R_9R_{13}C_2}\zeta_4(t), \\ \mathbb{D}_t^{\alpha}\zeta_3(t) = -\dfrac{1}{R_{10}C_3}\zeta_1(t) - \dfrac{1}{R_{11}R_{14}C_3}\zeta_3(t) + \dfrac{1}{R_{15}R_{16}C_3}\zeta_4(t), \\ \mathbb{D}_t^{\alpha}\zeta_4(t) = -\dfrac{1}{R_{17}C_4}\zeta_1(t)\zeta_2(t)\zeta_3(t), \end{cases} \tag{18}$$

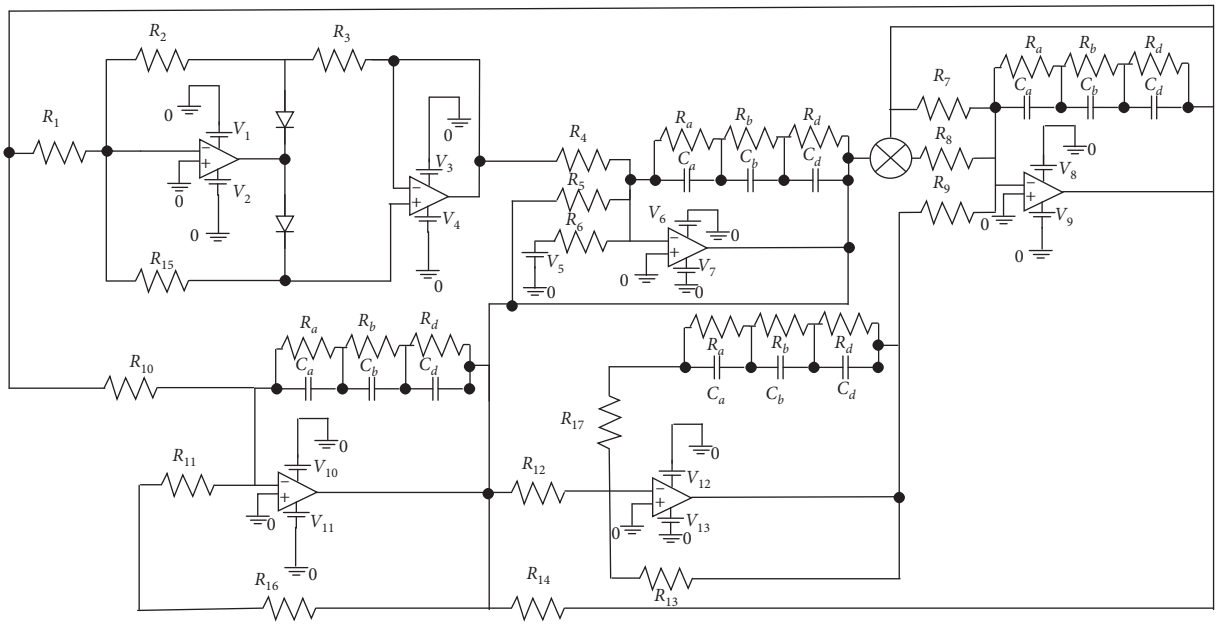

Figure 3: Simulation circuit of the FOECS.

where C_1, C_2, and C_3 are three fractional capacitors, and they are implemented by R_a, R_b, R_d, C_a, C_b, and C_d. In order to satisfy the FOECS, we choose the values of resistors and capacitors as $R_1 = R_2 = 1\,k\Omega$, $R_3 = R_5 = 4\,k\Omega$, $R_4 = 120\,\Omega$, $R_6 = R_8 = R_{10} = R_{17} = 1\,M\Omega$, $R_7 = 100\,M\Omega$, $R_{16} = 90\,\Omega$, $R_9 = R_{11} = R_{12} = 30\,\Omega$, $R_{13} = R_{15} = 5\,k\Omega$, $R_{14} = 12\,k\Omega$, and $C_1 = C_2 = C_3 = C_4 = 1\mu F$.

Remark 1. In the circuit implementation of the FOECS, the order of the system is set as $\alpha = 0.9$. Noting that the system parameters in (15) are very hard to be approximated by the circuit, the parameters in (18) are different with those in (15) (in fact, we used the approximate quantity of the parameters in (15)).

Remark 2. Based on [18], we know that the financial variables of financial systems could have long memory, such as stock market prices, gross domestic product, interest rates, and foreign exchange rates. Compared with the integer-order calculus, the fractional-order calculus introduced into the system (15) can more accurately describe the long-term memory effect in the financial system.

4. Controller Design and Stability Analysis

Taking uncertain parameters and disturbances into consideration, the controlled FOECS can be described as

$$\begin{cases} \mathbb{D}_t^\alpha \zeta_1(t) = \zeta_3(t) + (\zeta_2(t) - a)\zeta_1(t) + a_1\zeta_4(t) + u_1(t) + d_1(t) + \varphi_1^T(\zeta(t))\theta_1, \\ \mathbb{D}_t^\alpha \zeta_2(t) = 1 - b\zeta_2(t) - \zeta_1^2(t) + a_2\zeta_4(t) + u_2(t) + d_2(t) + \varphi_2^T(\zeta(t))\theta_2, \\ \mathbb{D}_t^\alpha \zeta_3(t) = -\zeta_1(t) - c\zeta_3(t) + a_3\zeta_4(t) + u_3(t) + d_3(t) + \varphi_3^T(\zeta(t))\theta_3 \\ \mathbb{D}_t^\alpha \zeta_4(t) = -\zeta_1(t)\zeta_2(t)\zeta_3(t) + u_4(t) + d_4(t) + \varphi_4^T(\zeta(t))\theta_4, \end{cases} \tag{19}$$

where $u_i(t) \in \mathbb{R}$ represents the controller, $d_i(t) \in \mathbb{R}$ denotes the external disturbance, $\varphi_i(\zeta(t))$: $\mathbb{R}^4 \mapsto \mathbb{R}^{m_i}$ is a known bounded nonlinear function, and $\theta_i \in \mathbb{R}^{m_i}$ is an unknown constant vector with $i = 1, 2, 3, 4$ and $m_i \in \mathbb{N}$.

Remark 3. In fact, the economical systems usually suffer from system uncertainties and unknown disturbances, for example, unknown price and cost fluctuation, market's human intervention, and system model uncertainties. Note that we assume that $\varphi_1(\cdot), \varphi_2(\cdot), \varphi_3(\cdot)$, and $\varphi_4(\cdot)$ can have different dimensions, and it is easy to know that the uncertain terms considered in system (19) stand for a large range of system uncertainties. Thus, the economical model considered in some recent literature, for example, [10, 16, 18, 37–39], is a special case of our model (19).

Our objective here is to design a proper controller $u_i(t)$ such that the state vector $\zeta(t)$ could track the desired signal $\zeta_d(t) = [\zeta_{d1}(t), \zeta_{d2}(t), \zeta_{d3}(t), \zeta_{d4}(t)]^T \in \mathbb{R}^4$. The tracking error is defined as $e(t) = [e_1(t), e_2(t), e_3(t), e_4(t)]^T = \zeta(t) - \zeta_d(t)$. To proceed, we need the following assumption.

Assumption 1. The nonlinear function $d_i(t)$ is bounded, i.e., there exists a constant $d_i^* > 0$ such that

$$\left|d_i(t)\right| \le d_i^*. \tag{20}$$

Assumption 2. The referenced signal $\zeta_d(t)$ is a known continuous bounded function and has bounded known continuous first-order derivative.

Lemma 6 (see [40]). *If a is a positive constant, it holds $|x| - x\tanh(x/a) \le 0.2785a = a'$.*

According to the definition of $e(t)$, it follows from (15) that

$$\begin{cases} \mathbb{D}_t^\alpha e_1(t) = \zeta_3(t) + (\zeta_2(t) - a)\zeta_1(t) + a_1\zeta_4(t) + u_1(t) + \varphi_1^T(\zeta(t))\theta_1 - D_t^\alpha \zeta_{d1}(t) + d_1(t), \\ \mathbb{D}_t^\alpha e_2(t) = 1 - b\zeta_2(t) - \zeta_1^2(t) + a_2\zeta_4(t) + u_2(t) + d_2(t) - D_t^\alpha \zeta_{d2}(t) + \varphi_2^T(\zeta(t))\theta_2, \\ \mathbb{D}_t^\alpha e_3(t) = -\zeta_1(t) - c\zeta_3(t) + a_3\zeta_4(t) + u_3(t) + d_3(t) + \varphi_3^T(\zeta(t))\theta_3 - D_t^\alpha \zeta_{d3}(t), \\ \mathbb{D}_t^\alpha e_4(t) = -\zeta_1(t)\zeta_2(t)\zeta_3(t) + u_4(t) + d_4(t) + \varphi_4^T(\zeta(t))\theta_4 - D_t^\alpha \zeta_{d4}(t). \end{cases} \tag{21}$$

Thus, the controller can be designed as

$$\begin{cases} u_1(t) = -k_1e_1(t) - \zeta_3(t) - (\zeta_2(t) - a)\zeta_1(t) - a_1\zeta_4(t) - \varphi_1^T(\zeta(t))\widehat{\theta}_1(t) + D_t^\alpha \zeta_{d1}(t) - \tanh\left(\dfrac{e_1(t)}{a_1}\right), \\ u_2(t) = -k_2e_2(t) + \zeta_1^2(t) - a_2\zeta_4(t) - \tanh\left(\dfrac{e_2(t)}{a_2}\right) - \varphi_2^T(\zeta(t))\widehat{\theta}_2(t) + D_t^\alpha \zeta_{d2}(t) - 1 + b\zeta_2(t), \\ u_3(t) = -k_3e_3(t) + \zeta_1(t) - \tanh\left(\dfrac{e_3(t)}{a_3}\right) - \varphi_3^T(\zeta(t))\widehat{\theta}_3(t) + D_t^\alpha \zeta_{d3}(t) + c\zeta_3(t) - a_3\zeta_4(t), \\ u_4(t) = -k_4e_4(t) + \zeta_1(t)\zeta_2(t)\zeta_3(t) - \tanh\left(\dfrac{e_4(t)}{a_4}\right) - \varphi_4^T(\zeta(t))\widehat{\theta}_4(t) + D_t^\alpha \zeta_{d4}(t), \end{cases} \tag{22}$$

where $k_i > 0$ is a design parameter, $a_i > 0$ is a constant, and $\widehat{d}_i^*(t)$ and $\widehat{\theta}_i(t)$ are the estimation of d_*^i and θ_i, respectively.

Remark 4. In this work, an adaptive controller (22) which seems complicated is designed. In this controller, all inputs, i.e., $u_i(t), i = 1, 2, 3, 4$, have the same structure. Take $u_1(t)$ as an example; it contains four parts, i.e., a nonlinear function $-\zeta_3(t) - (\zeta_2(t) - a)\zeta_1(t) - a_1\zeta_4(t)$ consisting of system variables, a feedback term of the tracking error $-k_1e_1(t) - \tanh(e_1(t)/a_1)$, an adaptive term $-\varphi_1^T(\zeta(t))\widehat{\theta}_1(t)$, and a smooth function of the referenced signal $D_t^\alpha \zeta_{d1}(t)$. It can be observed that all inputs are easy to be implemented.

Substituting (22) into (21) gives

$$\mathbb{D}_t^\alpha e_i(t) = -k_ie_i(t) + d_i(t) - \tanh\left(\frac{e_i(t)}{a_i}\right) - \phi_i^T(\zeta(t))\widetilde{\theta}_i(t), \tag{23}$$

with $i = 1, 2, 3, 4$, and

$$\widetilde{\theta}_i(t) = \widehat{\theta}_i(t) - \theta_i, \tag{24}$$

is the estimation error of the unknown constant vector θ_i.

Multiplying $e_i(t)$ to both sides of (23), and according to Assumption 1 and Lemma 6, we have

$$\begin{aligned} e_i(t)\mathbb{D}_t^\alpha e_i(t) &= -k_ie_i^2(t) + e_i(t)d_i(t) - e_i(t)\tanh\left(\frac{e_i(t)}{a_i}\right) \\ &\quad - e_i(t)\varphi_i^T(\zeta(t))\widetilde{\theta}_i(t) \\ &\le -k_ie_i^2(t) + \left|e_i(t)\right|d_i^* - e_i(t)\tanh\left(\frac{e_i(t)}{a_i}\right) \\ &\quad - e_i(t)\varphi_i^T(\zeta(t))\widetilde{\theta}_i(t) \\ &\le -k_ie_i^2(t) + \left|e_i(t)\right| - e_i(t)\tanh\left(\frac{e_i(t)}{a_i}\right) \\ &\quad - e_i(t)\varphi_i^T(\zeta(t))\widetilde{\theta}_i(t) \\ &= -k_ie_i^2(t) - a_i' - e_i(t)\varphi_i^T(\zeta(t))\widetilde{\theta}_i(t), \end{aligned} \tag{25}$$

where $a_i' > 0$.

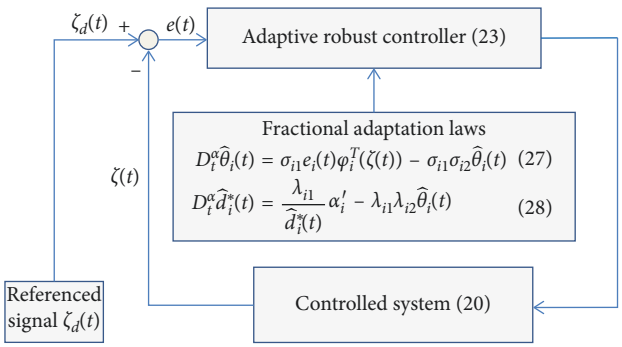

Figure 4: The structure of the proposed method.

Let the fractional-order adaptation laws be

$$\mathbb{D}_t^\alpha \widehat{\theta}_i(t) = \sigma_{i1} e_i(t) \varphi_i^T(\zeta(t)) - \sigma_{i1}\sigma_{i2}\widehat{\theta}_i, \tag{26}$$

$$\mathbb{D}_t^\alpha \widehat{d}_i^*(t) = \frac{\lambda_{i1}}{\widehat{d}_i^*(t)} a_i' - \lambda_{i1}\lambda_{i2}\widehat{d}_i^*(t), \tag{27}$$

with $i = 1, 2, 3, 4, \lambda_{i1}, \lambda_{i2}, \sigma_{i1}, \sigma_{i2} > 0$, and $\widetilde{d}_i^*(t) = \widehat{d}_i^*(t) - d_i^*$ is the estimation error of the unknown constant d_i^*. The structure of the proposed method is shown in Figure 4.

Based on above discussion, we can give the following theorem.

Theorem 1. *Based on Assumptions 1 and 2, the controller (22) of the system (19) together with the fractional-order adaptation laws (26) and (27) can guarantee that all variables remain bounded and the tracking error $e(t)$ converges to a small neighborhood of the origin.*

Proof. Let the Lyapunov function be

$$V(t) = \frac{1}{2}\sum_{i=1}^{4} e_i^2(t) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\sigma_{i1}}\widetilde{\theta}_i^T(t)\widetilde{\theta}_i(t) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\lambda_{i1}}\widetilde{d}_i^{*2}(t). \tag{28}$$

From (25)–(27) and Lemma 4, one has

$$\begin{aligned}
\mathbb{D}_t^\alpha V(t) &= \frac{1}{2}\sum_{i=1}^{4}\mathbb{D}_t^\alpha e_i^2(t) + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\sigma_{i1}}\mathbb{D}_t^\alpha \widetilde{\theta}_i^T(t)\widetilde{\theta}_i(t) \\
&\quad + \frac{1}{2}\sum_{i=1}^{4}\frac{1}{\lambda_{i1}}\mathbb{D}_t^\alpha \widetilde{d}_i^{*2}(t) \\
&\le \sum_{i=1}^{4} e_i(t)\mathbb{D}_t^\alpha e_i(t) + \sum_{i=1}^{4}\frac{1}{\sigma_{i1}}\widetilde{\theta}_i^T(t)\mathbb{D}_t^\alpha\widetilde{\theta}_i(t) \\
&\quad + \sum_{i=1}^{4}\frac{1}{\lambda_{i1}}\widetilde{d}_i^*(t)\mathbb{D}_t^\alpha\widetilde{d}_i^*(t) \\
&= -\sum_{i=1}^{4} k_i e_i^2(t) - \sum_{i=1}^{4} e_i(t)\varphi_i^T(\zeta(t))\widetilde{\theta}_i(t) \\
&\quad + \sum_{i=1}^{4}\frac{1}{\sigma_{i1}}\widetilde{\theta}_i^T(t)\mathbb{D}_t^\alpha\widetilde{\theta}_i(t) + \sum_{i=1}^{4}\frac{1}{\lambda_{i1}}\widetilde{d}_i^*(t)\mathbb{D}_t^\alpha\widetilde{d}_i^*(t) - \sum_{i=1}^{4} a_i'.
\end{aligned} \tag{29}$$

It follows from (29) that

$$\begin{aligned}
\mathbb{D}_t^\alpha V(t) &\le -\sum_{i=1}^{4} a_i' - \sum_{i=1}^{4} e_i(t)\phi_i^T(\zeta(t))\widetilde{\theta}_i(t) + \sum_{i=1}^{4}\widetilde{\theta}_i^T(t) \\
&\quad \cdot \left[e_i(t)\phi_i^T(\zeta(t)) - \sigma_{i2}\widehat{\theta}_i(t)\right] - \sum_{i=1}^{4} k_i e_i^2(t) \\
&\quad + \sum_{i=1}^{4}\widetilde{d}_i^*(t)\left[\frac{a_i'}{\widetilde{d}_i^*(t)} - \lambda_{i2}\widehat{d}_i^*(t)\right] \\
&= -\sum_{i=1}^{4} k_i e_i^2(t) - \sum_{i=1}^{4}\lambda_{i2}\widetilde{d}_i^*(t)\widehat{d}_i^*(t) - \sum_{i=1}^{4}\sigma_{i2}\widetilde{\theta}_i^T(t)\widehat{\theta}_i(t) \\
&= -\sum_{i=1}^{4} k_i e_i^2(t) - \sum_{i=1}^{4}\lambda_{i2}\widetilde{d}_i^*(t)\left(\widetilde{d}_i^*(t) + d_i^*\right) \\
&\quad - \sum_{i=1}^{4}\sigma_{i2}\widetilde{\theta}_i^T(t)\left(\widetilde{\theta}_i(t) + \theta_i\right) \\
&= -\sum_{i=1}^{4} k_i e_i^2(t) - \sum_{i=1}^{4}\lambda_{i2}\widetilde{d}_i^{*2}(t) - \sum_{i=1}^{4}\lambda_{i2}\widetilde{d}_i^*(t)d_i^* \\
&\quad - \sum_{i=1}^{4}\sigma_{i2}\widetilde{\theta}_i^T(t)\theta_i - \sum_{i=1}^{4}\sigma_{i2}\widetilde{\theta}_i^T(t)\widetilde{\theta}_i(t) \\
&\le -\sum_{i=1}^{4} k_i e_i^2(t) - \sum_{i=1}^{4}\frac{\lambda_{i2}}{2}\widetilde{d}_i^{*2}(t) + \sum_{i=1}^{4}\frac{\lambda_{i2}}{2}d_i^{*2} \\
&\quad + \sum_{i=1}^{4}\frac{\sigma_{i2}}{2}\theta_i^T\theta_i - \sum_{i=1}^{4}\frac{\sigma_{i2}}{2}\widetilde{\theta}_i^T(t)\widetilde{\theta}_i(t) \\
&\le -\underline{k}\sum_{i=1}^{4} e_i^2(t) - \frac{\underline{\lambda}_2}{2} i = \sum_{1}^{4}\widetilde{d}_i^{*2}(t) \\
&\quad - \frac{\underline{\sigma}_2}{2}\sum_{i=1}^{4}\widetilde{\theta}_i^T(t)\widetilde{\theta}_i(t) + A \\
&\le -BV(t) + A,
\end{aligned} \tag{30}$$

where $\underline{k} = \min_{1\le i\le 4}\{k_i\}$, $\underline{\sigma}_2 = \min_{1\le i\le 4}\{\lambda_{i2}\}$, $\underline{\sigma}_2 = \min_{1\le i\le 4}\{\sigma_{i2}\}$, $B = \min\{2\underline{k}, \underline{\lambda}_2\overline{\lambda}_1, \underline{\sigma}_2\overline{\sigma}_1\}$, $\overline{\lambda}_1 = \max_{1\le i\le 4}\{\lambda_{i1}\}$, $\overline{\sigma}_1 = \max_{1\le i\le 4}\{\sigma_{i1}\}$, and $A = \sum_{i=1}^{4}(\lambda_{i2}/2)d_i^{*2} + \sum_{i=1}^{4}(\sigma_{i2}/2)\theta_i^T\theta_i$.

According to (30), we have

$$\mathbb{D}_t^\alpha V(t) + y(t) = -BV(t) + A, \tag{31}$$

where $y(t) > 0$.

The Laplace transform of (31) is given as

$$V(s) = \frac{s^{\alpha-1}}{s^{\alpha}+B}V(0) + \frac{A}{s(s^{\alpha}+B)} - \frac{Y(s)}{s^{\alpha}+B} = \frac{s^{\alpha-1}}{s^{\alpha}+B}V(0) + \frac{s^{\alpha-(1+\alpha)}A}{(s^{\alpha}+B)} - \frac{Y(s)}{s^{\alpha}+B}. \tag{32}$$

According to (4), we can solve (32) as

$$V(t) = V(0)E_{\alpha,1}(-Bt^{\alpha}) + At^{\alpha}E_{\alpha,1+\alpha}(-Bt^{\alpha}) - y(t) * t^{-1}E_{\alpha,0}(-Bt^{\alpha}), \tag{33}$$

where $*$ represents the convolution operator. Noting $t^{-1}E_{\alpha,0}(-Bt^{\alpha}) \geq 0$ and $y(t) \geq 0$, we have $y(t) * t^{-1}E_{\alpha,0}(-Bt^{\alpha}) \geq 0$.

Then, we obtain

$$V(t) \leq V(0)E_{\alpha,1}(-Bt^{\alpha}) + At^{\alpha}E_{\alpha,1+\alpha}(-Bt^{\alpha}). \tag{34}$$

Because $\arg(-Bt^{\alpha}) = -\pi$, $|-Bt^{\alpha}| \geq 0$, and $0 < \alpha \leq 1$, based on Lemma 2, it holds

$$\left|E_{\alpha,1}(-Bt^{\alpha})\right| \leq \frac{C}{1+Bt^{\alpha}}, \tag{35}$$

where $C > 0$. Then, we have

$$\lim_{t\longrightarrow\infty} V(0)E_{\alpha,1}(-Bt^{\alpha}) = 0. \tag{36}$$

That is to say, with respect to any $\mu > 0$, one can find $t_1 > 0$ so that

$$V(0)E_{\alpha,1}(-Bt^{\alpha}) < \mu. \tag{37}$$

It follows from Lemma 1 that

$$t^{\alpha}E_{\alpha,\alpha+1}(-Bt^{\alpha}) \leq \frac{A}{B} + \mu. \tag{38}$$

Noting that we can find proper design parameters such that $(A/B) \leq \mu$, according to (34), (37), and (38), we know

$$V(t) < 3\mu. \tag{39}$$

Based on the definition of $V(t)$, it is easy to know that the boundedness of all signals can be guaranteed. On the contrary, the tracking error can be arbitrarily small. □

Remark 5. In this paper, we design adaptation laws (26) and (27) to update $\widehat{\theta}_i(t)$ and $\widehat{d}_i^*(t)$, respectively. These laws update the parameters by using fractional differential equations. In fact, this kind of laws can also be seen in some literature, e.g., in [41–44]. However, in the abovementioned literature, the fractional adaptation laws only have one term, i.e., the positive term. Noting that the fractional derivatives of the updated parameters are greater than zero, and thus, according to the properties of the fractional derivative, the updated parameters are monotonically increasing. That is to say, the boundedness of the updated parameters cannot be guaranteed. However, in this work, by introducing the second term $-\sigma_{i1}\sigma_{i2}\widehat{\theta}_i$ in (26), we can guarantee the boundedness of the updated parameter.

Remark 7. In the proof of Theorem 1, we can see that $\|e(t)\| \leq \sqrt{6\mu}$. To drive the tracking error $e(t)$ as small as possible, we can set (A/B) as small as possible. To obtain this objective, according to the form of A and B, we can choose large k_i, λ_{i1}, and σ_{i1} and small σ_{i2} and λ_{i2}.

Remark 8. To analyze the fractional-order system's stability, a commonly used Lyapunov function is $V(t) = e^T(t)e(t)$. It should be emphasized that its fractional derivative is [45]

$$\mathbb{D}_t^{\alpha}V(t) = \sum_{i=1}^{\infty}\frac{\Gamma(1+\alpha)}{\Gamma(1+i)\Gamma(1-i+\alpha)}\mathbb{D}_t^{i}e(t)\mathbb{D}_t^{\alpha-i}e(t) + 2e^T(t)\mathbb{D}_t^{\alpha}e(t). \tag{40}$$

In fact, this complicated form is very hard to be used. However, in this paper, by using our method, it is not necessary to use (40) in the stability analysis.

5. Simulation Studies

In the simulation, the parameters are $a = 2.10$, $b = 0.01$, $c = 2.61$, $a_1 = 8.41$, $a_2 = 6.40$, and $a_3 = 2.21$ and the disturbances are chosen as $d_1(t) = \cos(\zeta_1(t))$, $d_2(t) = \cos(\zeta_2(t))$, $d_3(t) = \cos(\zeta_3(t))$, and $d_4(t) = \cos(\zeta_4(t))$ which are activated when $t = 5$. Let the initial condition be $\zeta(0) = [1.5, -2, 3, -3]^T$. The fractional order is $\alpha = 0.95$.

The parametric uncertain vectors are chosen as $\theta_1(t) = [1.1, -0.5, 2, 1.5]^T$, $\theta_3(t) = [2, 1, -0.5]^T$, and $\theta_2(t) \equiv \theta_4(t) \equiv 0$, and the basis functions are chosen as $\varphi_1(\zeta(t)) = [\zeta_1(t), \zeta_2(t), \zeta_3(t), \zeta_4(t)]^T$ and $\varphi_3(\zeta(t)) = [\zeta_1(t)\sin\zeta_2(t), \cos\zeta_3(t), \sin\zeta_4(t)]^T$.

The controller design parameters are given as $k_i = 0.2$, $\lambda_{i1} = \sigma_{i1} = 5$, and $\lambda_{i2} = \sigma_{i2} = 0.005$ with $i = 1, 2, 3, 4$.

First, let the desired signal be $\zeta_d(t) \equiv [0, 0, 0, 0]^T$. The simulation results are given in Figures 5–9. Figure 5 shows the time response of the state variables $\zeta_1(t), \zeta_2(t)$, $\zeta_3(t)$, and $\zeta_4(t)$. We can see these variables tend to zero in about 2 s and stay in a very small neighborhood of zero thereafter. The control inputs are depicted in Figure 6. The estimations for $\theta_1(t)$ as well $\theta_3(t)$ are given in Figures 8 and 9, respectively. According to Figures 7–9, we can see that the boundedness of the updated parameters can be guaranteed, just like the results in Theorem 1 and the statements in Remark 5. These simulation results indicate that our control method has satisfactory control performance and good robustness.

Let the desired signal be $\zeta_d(t) = [\cos t, \sin t, \cos t, \sin t]^T$. The other parameters and values are chosen as the same as above. The simulation results can be seen in Figures 10–12.

Remark 9. The simulation results of [16] are that the designed controllers were activated at $t = 30$ s, whereas in our paper, the proposed controllers are activated at about $t = 2$ s. Compared with the simulation results in [16], it is not difficult to see that the controller we designed does not use the symbolic function cited in [16]. Therefore, there is no

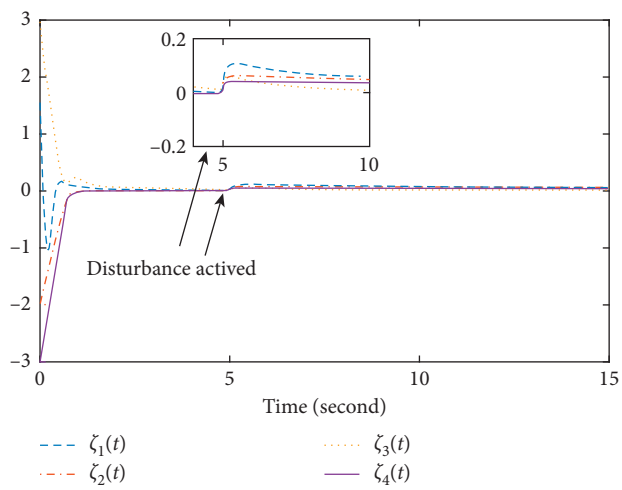

Figure 5: Time response of state variables.

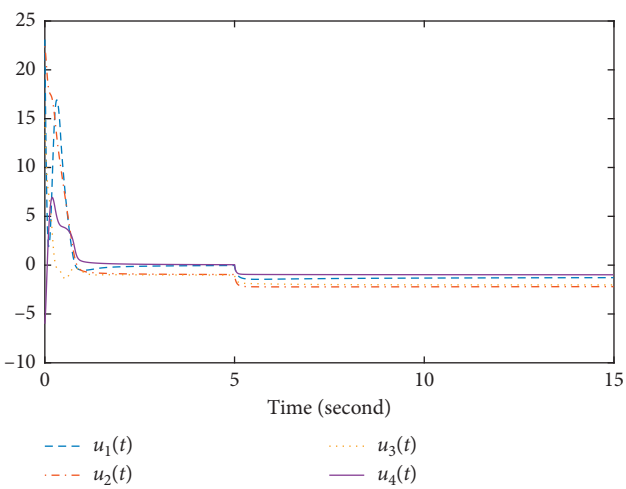

Figure 6: Time response of control inputs.

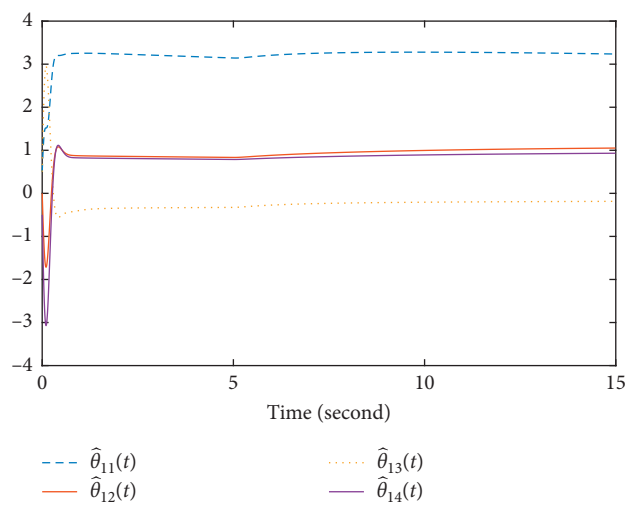

Figure 8: The estimation of $\theta_1(t)$.

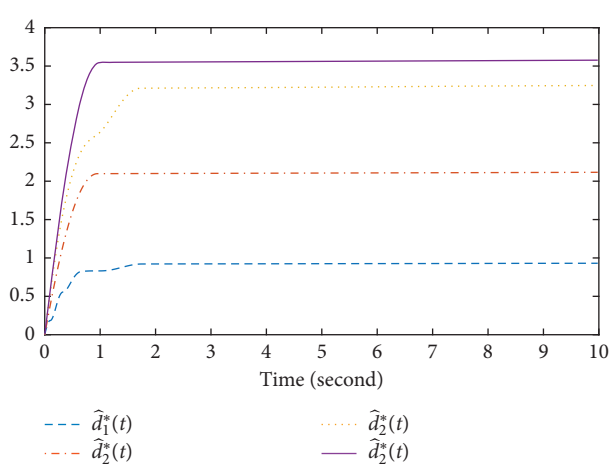

Figure 7: The estimations of d_1^*, d_2^*, d_3^*, and d_4^*.

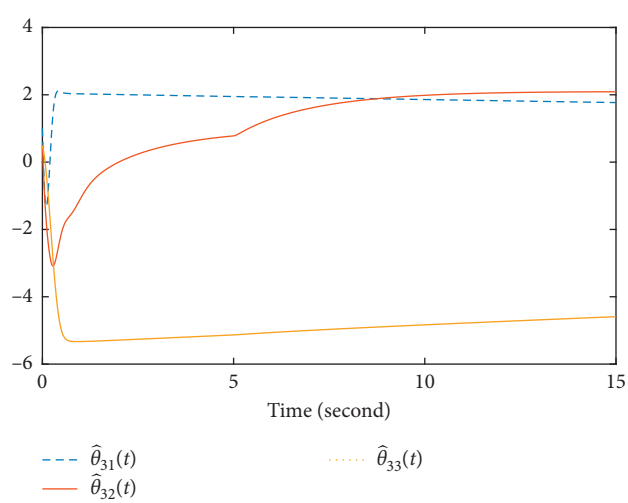

Figure 9: The estimation of $\theta_3(t)$.

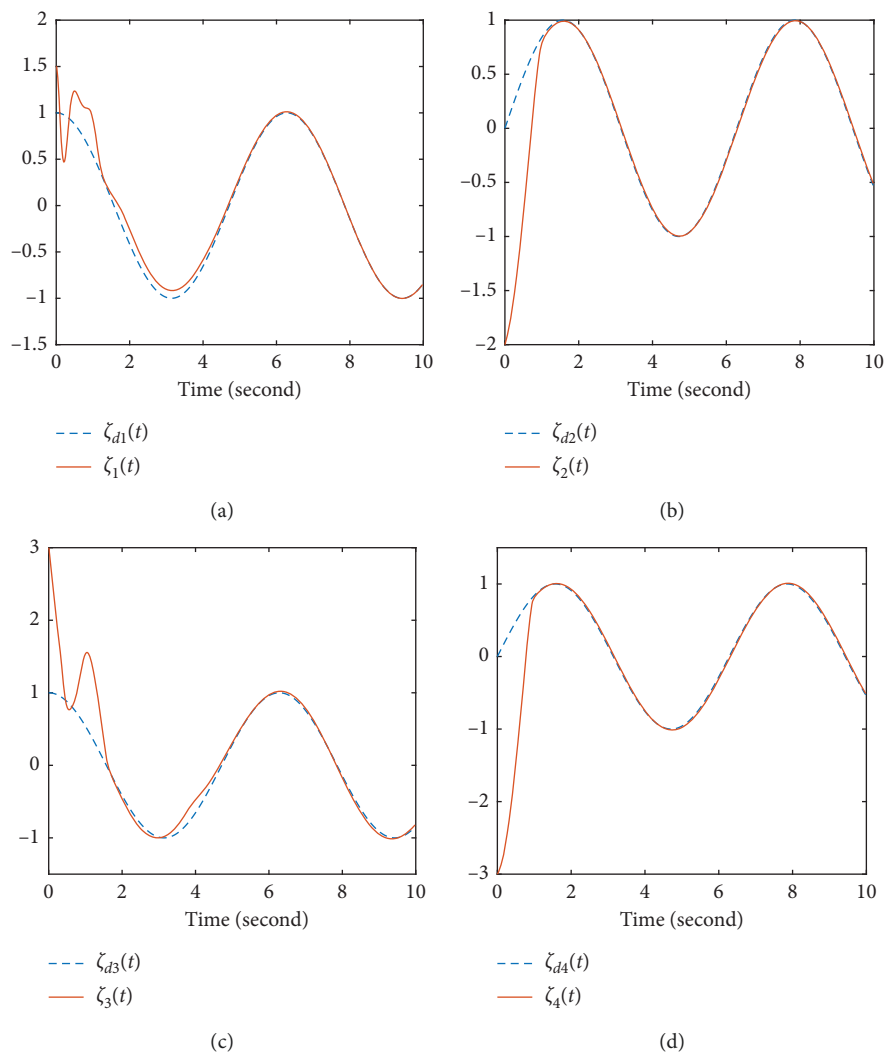

Figure 10: Tracking performance of state variables.

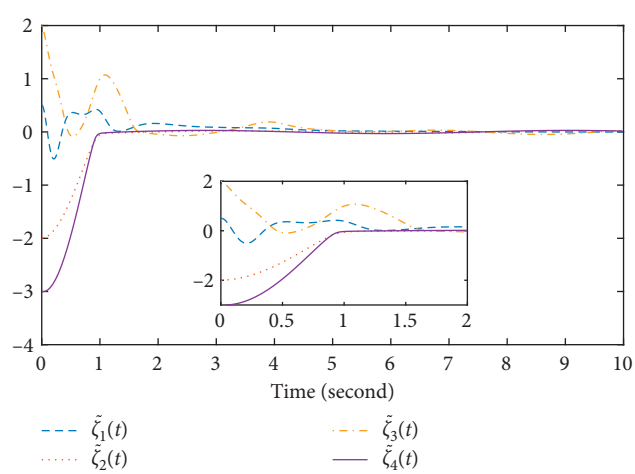

Figure 11: Tracking errors.

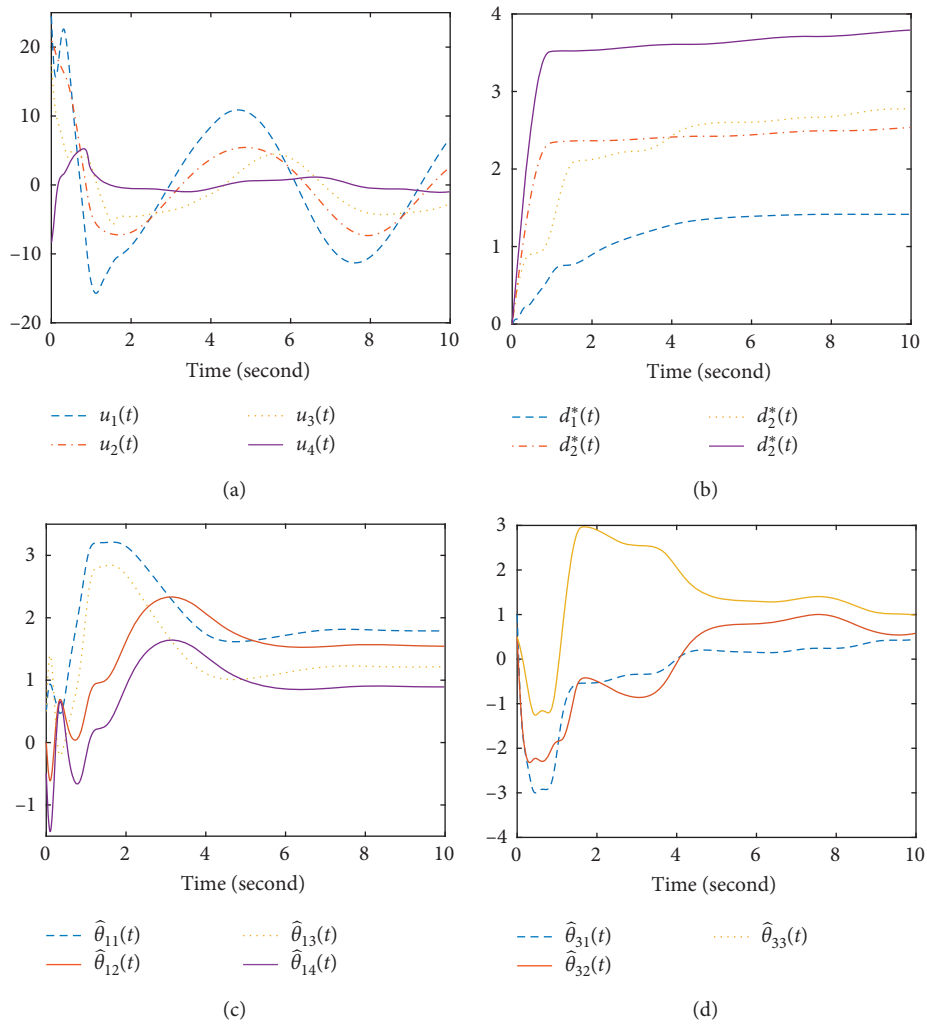

Figure 12: Control inputs and the updated parameters.

chattering phenomenon in our simulation results. Although arctan (10·) was used to approximate sign(·) in the simulation of [16], the controller designed by us can work more effectively in terms of comparing the convergence rate of simulation results. Since we directly use the proposed controller to obtain simulation results, our simulation results can more effectively prove the theory of this paper.

6. Conclusions

In this paper, an adaptive robust control method of FOECSs subject to unmatched system uncertainties as well as external disturbances is investigated. It has been shown by our work that (1) the quadratic Lyapunov function can be used in the stability analysis of FOECSs based on the fractional stability criterion and (2) the parameters can be updated by fractional-order differential equations. The proposed method gives an easy way to analyze the stability of fractional-order systems. Simulation results have verified the feasibility of our control method. On the contrary, it is assumed that the unknown parameter is a constant vector, i.e., when the uncertain term is time-varying, the method presented in this paper is powerless. In addition, four control inputs are used, which reduce the real-world applications of our approach. How to solve aforementioned problems is one of our future research directions.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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