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Research Article

Formation Control of Multi-Agent Systems with Region Constraint

Zhengquan Yang,[1,2] **Qing Zhang**,[1] **and Zengqiang Chen**[3]

[1]*College of Science, Civil Aviation University of China, Tianjin 300300, China*
[2]*Air Traffic Management Research Base, Civil Aviation University of China, Tianjin 300300, China*
[3]*College of Computer and Control Engineering, Nankai University, Tianjin 300350, China*

Correspondence should be addressed to Zhengquan Yang; zquanyang@163.com

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In this paper, the formation problem for multi-agent systems with region constraint is studied while few researchers consider this problem. The goal is to control all multi-agents to enter the constraint area while reaching formation. Each agent is constrained by a common convex set. A formation control law is presented based on local information of the neighborhood. It is proved that the positions of all the agents would converge to the set constraint while reaching formation. Finally, two numerical examples are presented to illustrate the validity of the theoretical results.

1. Introduction

In recent years, more and more researchers have discussed the formation control of multi-agent systems (MASs). The formation of MASs is widely used in load transportation, satellite formation flying, and unmanned aerial vehicle formation [1]. The purpose of multi-agent formation control is to control all agents to realize and maintain a prespecified geometric shape.

Many control strategies have been used for formation control, such as distance-based, displacement-based, and position-based strategies [2, 3]. The consensus theory has also been widely used to solve the formation control [4–7]. Lin et al. [8] studied the time-varying formation having shape constraints, which lie in the null space of a complex Laplacian. The necessary conditions for the formation of a multi-agent under fixed and undirected graphs are given in [9]. The leader-following formation control problem of second-order MASs with time-varying delay and nonlinear dynamics was investigated [10]. Xiao et al. [11] developed a new control framework to solve the finite-time formation of MASs. Sun et al. [12] gave a modified gradient control algorithm which can achieve finite time formation stabilization of first-order MASs. Lee and Ahn [13] proposed a formation control method which includes a combination of global orientation estimation and formation control law. In Ref. [14], adaptive formation control of MASs is proposed, which has an unknown leader. The time-varying formation problem was researched in previous studies [15–17]. Xia et al. [18] proposed an optimal formation control strategy using the estimated position information. In Ref. [19], a combination of attractive-repulsive artificial potential field and a control term related to the angular information between robots was used to reduce undesired local minima. In Ref. [20], an adaptive control approach was developed to solve such a problem by using the volume condition constraints. Inspired by distance rigidity theory and bearing rigidity theory, Jing [21] developed an angle rigidity theory to study whether the shape of a planar graph can be uniquely determined by angles only.

The above research into formation control relies on formation maintenance or stabilization. Recently, formation control with constraints has been studied by some researchers. Ref. [22] designed a space constrained controller based on neural network, which provides a tool for tracking control or path planning in nonomniscient constrained space. Hernandez-Martinez et al. [23] considered the desired formation specification with signed area constraints, and designed a tracking control strategy using distance and area

terms. A leader-follower formation control method for under-actuated autonomous underwater vehicles with line-of-sight and angle constraints is proposed in Ref. [24]. Liu [25] proposed a formation control algorithm which uses distance and signed area information to ensure convergence to the required formation shape.

To the best of our knowledge, there has not been a formal article to systematically address the formation protocols in the presence of region constraints. But in real control systems, we need to control agents to the desired area. That is, to consider situations where the only concern of the final state is a constraint rather than some specific geometry. We focus on the formation control problem of second-order MASs with region constraints. We know that the leader-following formation can solve this problem by imposing that one robot (leader) converges to the goal position in the plane (within a desired area). However, the main advantages of this scheme are that: the controller does not depend on the leader, and has good robustness; the agent can converge to the constraint region quickly; all agents can enter the constrained area accurately. Compared with previous results, the features of this paper are as follows: First, we propose a new control law with region constraints, which has the sum of a formation part and a projection part. Second, an asymptotic convergence is proved via a common Lyapunov function method.

The remainder of this paper is organized as follows: Section 2 introduces the basic principles and problem statement of graph theory. In Section 3, a formation protocol with region constraint is presented. In Section 4, numerical examples are given. Section 5 contains the conclusion.

2. Preliminaries

Notations. $\|s\|$ denotes the Euclidean norm of the vector s. The 1-norm of the vector s is denoted by $\|s\|_1$. $P_\Omega(s)$ denotes the projection of the vector s onto the closed convex set Ω.

In this paper, an undirected graph $\mathscr{G}$ is considered with node set $\mathscr{V} = \{1, 2, \ldots, N\}$ and edges set $\mathscr{E}$. j is called a neighbor of node i if $(i, j) \in \mathscr{E}$. The neighbors of vertex i are given by $N_i = \{j :\in V : (i, j) \in \mathscr{E}\}$. The adjacency matrix of the graph $\mathscr{G}$ is $\mathscr{A} = \left[a_{ij}\right] \in R^{N\times N}$ which is denoted as $a_{ii} = 0, a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathscr{E}$, otherwise $a_{ij} = 0$.

In this paper, a system that is made up of N agents (indexed by $1, 2, \ldots, N$) is considered. The state of agent i is denoted by $x_i \in R^2$ $(i = 1, \ldots, N)$. The dynamics of each agent is described by the integrators

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= u_i(t), \quad i \in \mathscr{N}, \end{aligned} \tag{1}$$

where $v_i \in R^2$ is the velocity vector, and $u_i \in R^2$ is the control input acting on agent i.

A framework is defined as a pair $(\mathscr{G}, X)$ where $X = \left[x_1^T, x_2^T, \ldots x_N^T\right]^T \in R^{2N}$. Ordering edges of $\mathscr{E}$ in some way, the rigidity function $r_G(X) : R^{2N} \to R^{|\mathscr{E}|}$ associated with the framework $(\mathscr{G}, X)$ is defined as:

$$r_G(X) = \frac{1}{2}\left[\ldots, \left\|x_i - x_j\right\|^2, \ldots\right]^T, \quad (i, j) \in \mathscr{E}. \tag{2}$$

Definition 2.1 (see [26]). A framework $(\mathscr{G}, X)$ is rigid if there exists a neighborhood $\mathscr{U}$ of R^{2N} such that $r_G^{-1}(r_G(x)) \cap \mathscr{U} = r_{\mathscr{K}}^{-1}(r_{\mathscr{K}}(x)) \cap \mathscr{U}$, where $\mathscr{K}$ is the complete graph on N-vertices.

In this paper, we consider the region constraint on the formation control of multi-agents. That is, all agents are given a region constraint $\Omega \in R^2$, which limits all multi-agents to enter the constraint area while reaching formation. To do that, give a point $X^* = \left[x_1^{*T}, x_2^{*T}, \ldots, x_N^{*T}\right]^T, x_i^* \in R^2$ such that $(\mathscr{G}, X)$ is infinitesimally rigid, the target formation is defined as

$$E_X := \left\{X : x_i - x_j = x_i^* - x_j^*, j \in N_i, i \in \mathscr{K}\right\}. \tag{3}$$

That is, E_X is the set of all formations congruent with X^*.

Lemma 1 (see [27]). *Given a closed convex set $\Omega \subset R^2, \forall y \in R^2, \forall x \in \Omega$, it has:*

$$\left(P_\Omega(y) - x\right)^T\left(y - P_\Omega(y)\right) \geq 0. \tag{4}$$

Problem 2.1. For the system (1), design the control u_i for each agent i in terms of $x_i - x_j$, $j \in N_i$ such that $x_i - x_j$ converges to $x_i^* - x_j^*$, and $x_i \in \Omega$ as $t \to \infty$.

3. Main Results

In this section, we propose a new controller to make multi-agent realize asymptotic formation. The controller has a projection part and the sum of the formation part. The new distributed formation controller is as follows:

$$\begin{aligned} u_i(t) = &-\alpha v_i(t) - \sum_{j\in N_i} a_{ij}\left(x_i(t) - x_j(t) - \gamma_{ij}^*\right) \\ &- \beta\left(x_i(t) - P_\Omega(x_i(t))\right), \end{aligned} \tag{5}$$

where $\alpha > 0$ and $\beta > 0$ are two constants, $\gamma_{ij}^* = x_i^* - x_j^*$.

We now propose the important result of this paper.

Theorem 1. *For all $\alpha > 0$, and $\beta > 0$, for the system (1) with the control algorithm (5), all agents achieve the asymptotical convergence of the formation shape, and $x_i \in \Omega$ as $t \to \infty$.*

Proof. Consider the Lyapunov function of system (1) as

$$\begin{aligned} V(t) = &\frac{1}{2}\sum_{i=1}^N v_i^T(t)v_i(t) + \frac{1}{4}\sum_{i=1}^N\sum_{j=1}^N a_{ij}\left\|x_i(t) - x_j(t) - \gamma_{ij}^*\right\|^2 \\ &+ \frac{\beta}{2}\sum_{i=1}^N \left\|x_i(t) - P_\Omega(x_i(t))\right\|^2. \end{aligned} \tag{6}$$

Its derivative along the system (1) is

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{N} v_i^T(t)\Big[-\alpha v_i(t) - \sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad - \beta\big(x_i(t)-P_\Omega(x_i(t))\big)\Big] + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big)^T \\ &\quad \cdot\left(v_i(t)-v_j(t)+\beta\sum_{i=1}^{N}(x_i)(t)-P_\Omega(x_i(t))\right)^T v_i(t) \\ &= -\alpha\sum_{i=1}^{N} v_i^T(t)v_i(t) - \sum_{i=1}^{N} v_i^T(t)\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big)^T\big(v_i(t)-v_j(t)\big).\end{aligned} \tag{7}$$

Because $a_{ij} = a_{ji} = 1$, we have

$$\begin{aligned}&\sum_{i=1}^{N} v_i^T(t)\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\Big[v_i^T(t)\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + v_j^T(t)\big(x_j(t)-x_i(t)-\gamma_{ji}^*\big)\Big] \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\Big[v_i^T(t)\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad - v_j^T(t)\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big)\Big] \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\big(v_i(t)-v_j(t)\big)^T\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big).\end{aligned} \tag{8}$$

Equation (7) can be rewritten as

$$\dot{V}(t) = -\alpha\sum_{i=1}^{N} v_i^T(t)v_i(t) < 0. \tag{9}$$

Therefore, based on Lasalle's invariance principle, we have, $t \to \infty$,

$$v_i(t) \equiv 0 \Rightarrow \dot{v}_i(t) \equiv 0, \quad \forall i = 1, 2, \ldots, N. \tag{10}$$

It then follows from (1) and (5) that for all $i \in \mathscr{V}$,

$$\begin{aligned}u_i(t) \equiv 0 \Rightarrow &\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &+ \beta\big(x_i(t)-P_\Omega(x_i(t))\big) \equiv 0.\end{aligned} \tag{11}$$

Then, we get

$$\begin{aligned}&\big(x_i(t)-x_i^*\big)^T\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + \beta\big(x_i(t)-x_i^*\big)^T\big(x_i(t)-P_\Omega(x_i(t))\big) \equiv 0.\end{aligned} \tag{12}$$

Thus, we have

$$\begin{aligned}&\sum_{i=1}^{N}\big(x_i(t)-x_i^*\big)^T\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + \beta\sum_{i=1}^{N}\big(x_i(t)-x_i^*\big)^T\big(x_i(t)-P_\Omega(x_i(t))\big) \equiv 0.\end{aligned} \tag{13}$$

Moreover, it follows that

$$\begin{aligned}&\sum_{i=1}^{N}\big(x_i(t)-x_i^*\big)^T\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\Big[\big(x_i(t)-x_i^*\big)^T\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + \big(x_j(t)-x_j^*\big)^T\big(x_j(t)-x_i(t)-\gamma_{ij}^*\big)\Big] \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\Big[\big(x_i(t)-x_i^*\big)^T\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + \big(x_j(t)-x_j^*\big)^T\big(x_j(t)-x_i(t)+\gamma_{ij}^*\big)\Big] \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big)^T\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big).\end{aligned} \tag{14}$$

This implies that Equation (7) can be written as follows:

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big)^T\big(x_i(t)-x_j(t)-\gamma_{ij}^*\big) \\ &\quad + \beta\sum_{i=1}^{N}\big(x_i(t)-x_i^*\big)^T\big(x_i(t)-P_\Omega(x_i(t))\big) \equiv 0.\end{aligned} \tag{15}$$

By using the inequality $(P_\Omega(x_i(t)) - x_i^*)^T(x_i(t) - P_\Omega(x_i(t))) \geq 0$, we get

$$\begin{aligned}&\big(x_i(t)-x_i^*\big)^T\big(x_i(t)-P_\Omega(x_i(t))\big) \\ &= \big(x_i(t)-P_\Omega(x_i(t))\big)^T\big(x_i(t)-P_\Omega(x_i(t))\big) \\ &\quad + \big(P_\Omega(x_i(t))-x_i^*\big)^T\big(x_i(t)-P_\Omega(x_i(t))\big) \\ &\geq \|x_i(t)-P_\Omega(x_i(t))\|^2 \\ &\geq 0.\end{aligned} \tag{16}$$

This shows

$$\sum_{i=1}^{N}\sum_{j\in N_i} a_{ij}\left\|x_i(t)-x_j(t)-\gamma_{ij}^*\right\|^2 \equiv 0. \tag{17}$$

$$\left\|x_i(t)-P_\Omega(x_i(t))\right\| \equiv 0. \tag{18}$$

Therefore, we get $\lim_{t\to\infty}\left[x_i(t)-x_j(t)\right] = \gamma_{ij}^*$, $\lim_{t\to\infty}\left[x_i(t)-P_\Omega(x_i(t))\right] = 0$, that is, all agents achieve the asymptotical convergence of the formation shape, and $x_i \in \Omega$ as $t \to \infty$. □

4. Simulation Results

In this section, we present two instances to illustrate the effectiveness of the method presented in this paper.

Example 1. Simulation is performed with six agents moving in a 2D plane. In simulation, the parameters in algorithm (4) are $\alpha = 1, \beta = 1$. We assume the adjacency matrix $A = \left[a_{ij}\right]$ as follows:

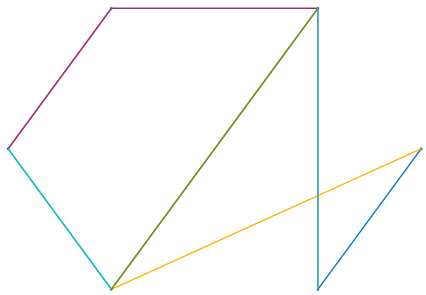

Figure 1: The target formation shape.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}. \tag{19}$$

The constraint area is a circle, which is $\Omega = \{(x, y) : (x-5)^2 + (y-5)^2 \le 4\}$. The target formation shape is a regular hexagon which is shown in Figure 1. In the simulation, the projection $P_\Omega(x_i(t))$ is implemented as follows: Find the shortest distance $\min_{y\in\Omega}\|x_i(t) - y\|$ from $x_i(t)$ to the constraint set Ω, and the point $P_\Omega(x_i(t))$ satisfying $\|x_i(t) - P_\Omega(x_i(t))\| = \min_{y\in\Omega}\|x_i(t) - y\|$ is the projection onto Ω.

The initial positions and velocities of all multi-agents are shown in Figure 2. The initial positions are set randomly, and the initial velocities are chosen randomly in the unit square. Figure 3 shows the motion trajectory of the multi-agent. Figure 4 depicts the final formation shape of the multi-agent. The convergence of the errors of the position and the region constraint are plotted in Figure 5. So we can see that the six-agent group moves into the constraint set and then achieves the desired formation shape. The errors converge to zero, but they jump to other values in a discontinuous manner. So, the convergence of the errors is asymptotic. The control inputs of the simulation are plotted in Figure 6. We can see that the control input converges to zero asymptotically.

Example 2. Simulation is performed with 155 agents moving in a 2D plane. In simulation, the parameters in algorithm (4) are also $\alpha = 1, \beta = 1$. The initial positions are set randomly and the initial velocities are chosen randomly in the unit square. We take the random network [28] as the initial configuration of the MAS. The construction method of the random network is as follows: starting with a set of N isolated vertices and adding successive edges between them according to probability p. We take the adjacency matrix of the random network as the matrix $A = [a_{ij}]$. Here, we choose $N = 155, p = 0.05$.

We choose the Olympic rings as the target configurations of the group. The constraint area is a rectangle, which is $\Omega = \{(x, y) : 3 \le x \le 15, 2 \le y \le 7\}$. Figure 7 depicts the motion trajectory of the multi-agent. Figure 8 depicts the final formation shape of the multi-agent. So we can see that the

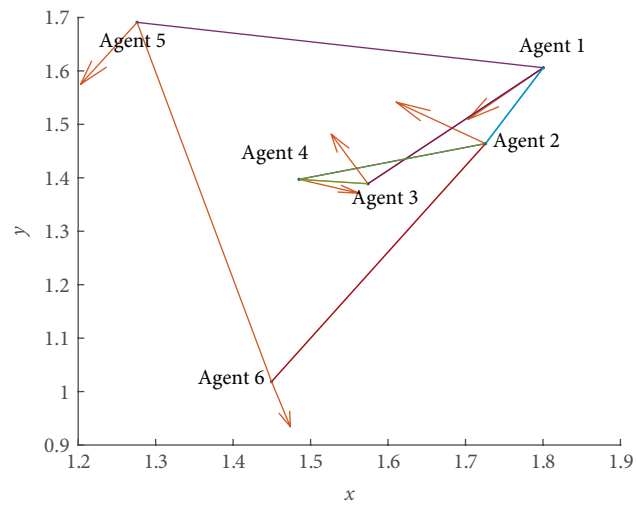

Figure 2: The initial positions and velocities of a six-agent.

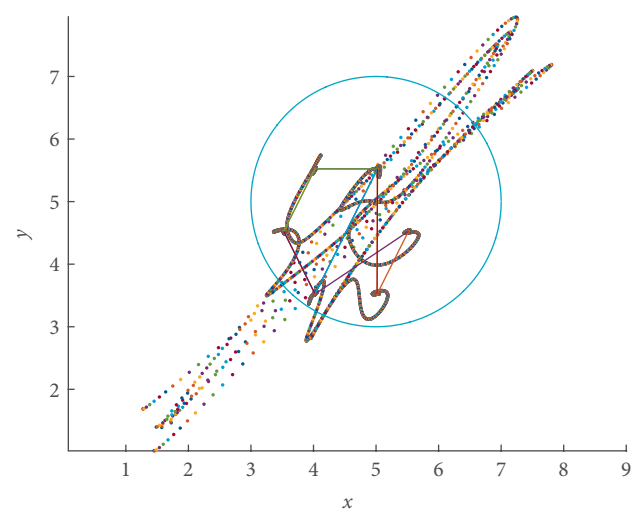

Figure 3: The motion trajectory of a six-agent.

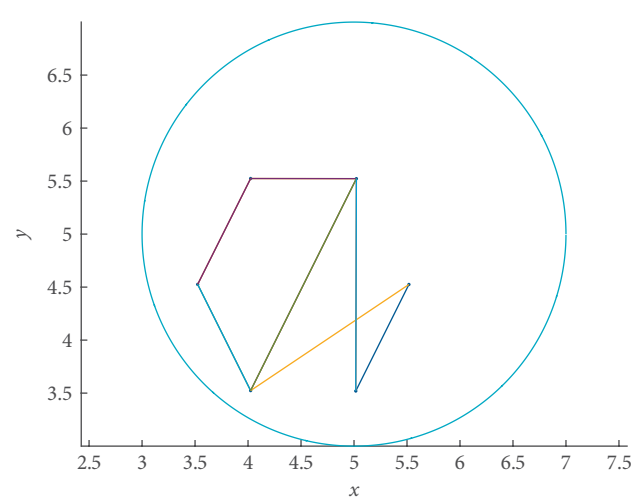

Figure 4: The final formation shape of a six-agent.

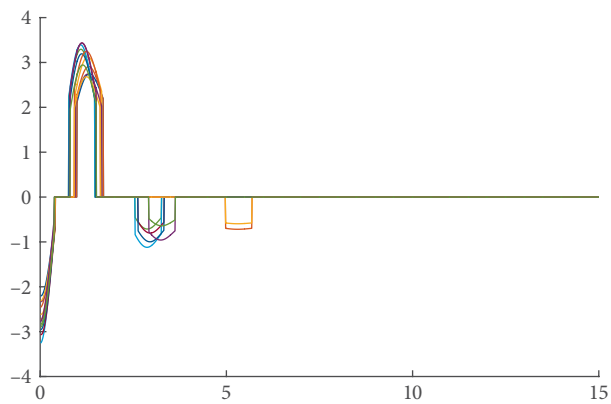

Figure 5: The error convergence of the position and the area constraint of a six-agent.

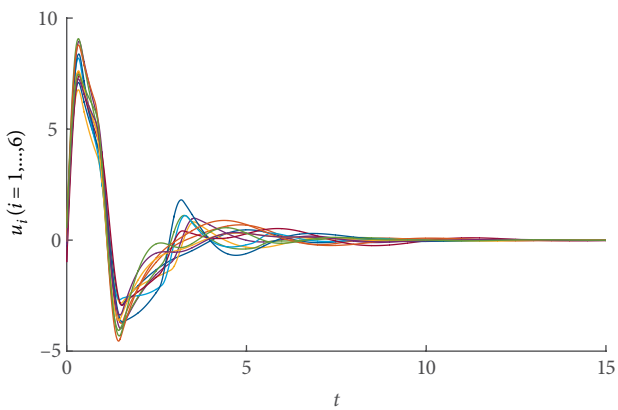

Figure 6: The control inputs u_i of a six-agent.

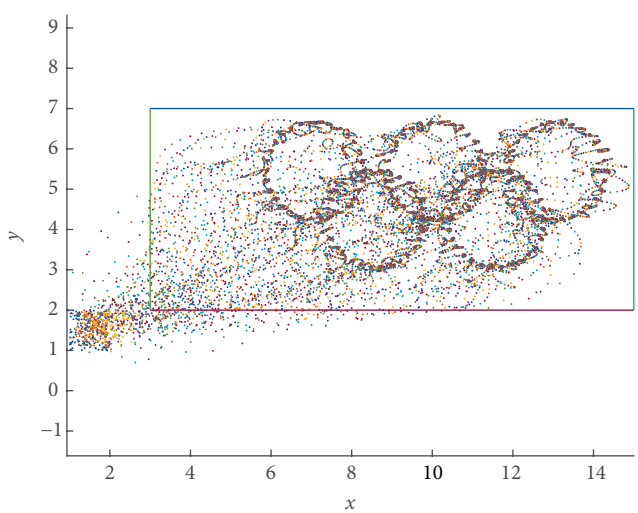

Figure 7: The motion trajectory of a 155-agent.

155-agent group moves into the constraint set and then achieves the desired formation shape. The control inputs of the simulation are plotted in Figure 9. We get the control input converges to zero.

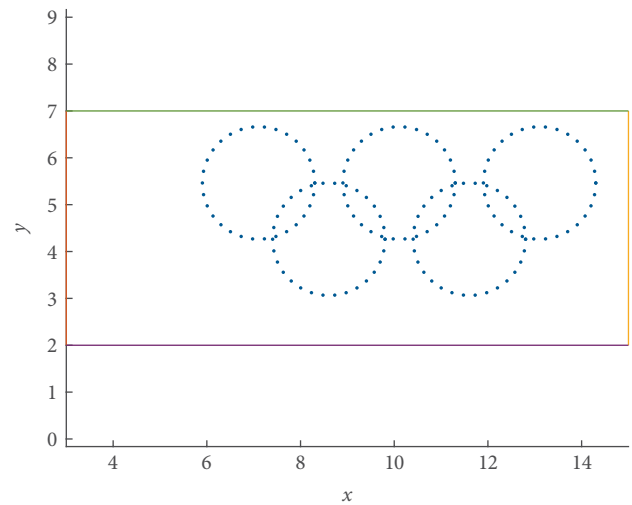

Figure 8: The final formation shape of a 155-agent.

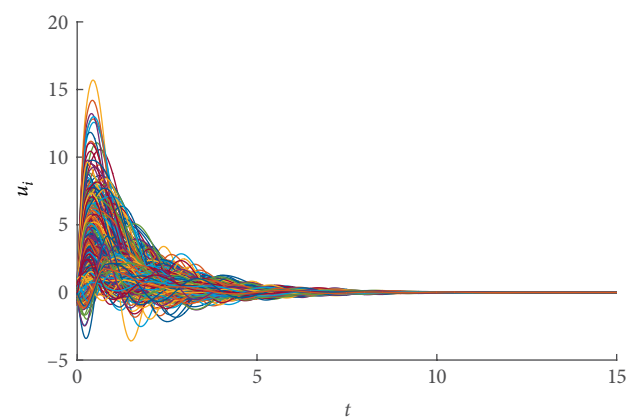

Figure 9: The control inputs u_i of a 155-agent.

5. Conclusion

This paper studied the formation problem for MASs with region constraint. A formation controller law was proposed to enable all multi-agents to enter the constraint area while reaching formation. Each agent was constrained by a common convex set. We proved that the positions of all the agents would converge to the set constraint while reaching formation. Finally, two numerical simulations were carried out to illustrate the validity of the theoretical results.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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