Hindawi
Complexity
Volume 2019, Article ID 8523748, 16 pages
https://doi.org/10.1155/2019/8523748

Research Article

Looking for Accurate Forecasting of Copper TC/RC Benchmark Levels

Francisco J. Díaz-Borrego, María del Mar Miras-Rodríguez, and Bernabé Escobar-Pérez

Universidad de Sevilla, Seville, Spain

Correspondence should be addressed to María del Mar Miras-Rodríguez; mmiras@us.es

Received 22 November 2018; Revised 24 February 2019; Accepted 5 March 2019; Published 27 March 2019

Guest Editor: Marisol B. Correia

Forecasting copper prices has been the objective of numerous investigations. However, there is a lack of research about the price at which mines sell copper concentrate to smelters. The market reality is more complex since smelters obtain the copper that they sell from the concentrate that mines produce by processing the ore which they have extracted. It therefore becomes necessary to thoroughly analyse the price at which smelters buy the concentrates from the mines, besides the price at which they sell the copper. In practice, this cost is set by applying discounts to the price of cathodic copper, the most relevant being those corresponding to the smelters' benefit margin (*Treatment Charges-TC* and *Refining Charges-RC*). These discounts are agreed upon annually in the markets and their correct forecasting will enable making more adequate models to estimate the price of copper concentrates, which would help smelters to duly forecast their benefit margin. Hence, the aim of this paper is to provide an effective tool to forecast copper TC/RC annual benchmark levels. With the annual benchmark data from 2004 to 2017 agreed upon during the LME Copper Week, a three-model comparison is made by contrasting different measures of error. The results obtained indicate that the LES (*Linear Exponential Smoothing*) model is the one that has the best predictive capacity to explain the evolution of TC/RC in both the long and the short term. This suggests a certain dependency on the previous levels of TC/RC, as well as the potential existence of cyclical patterns in them. This model thus allows us to make a more precise estimation of copper TC/RC levels, which makes it useful for smelters and mining companies.

1. Introduction

1.1. Background. The relevance of copper trading is undeniable. In 2016 exports of copper ores, concentrates, copper matte, and cement copper increased by 1.5%, reaching 47.3 b $USD, while imports attained 43.9 b $USD [1]. In addition, the global mining capacity is expected to rise by 10% from the 23.5 million tonnes recorded in 2016 to 25.9 million tonnes in 2020, with smelter production having reached the record figure of 19.0 million tonnes in 2016 [2].

The world's copper production is essentially achieved through alternative processes which depend on the chemical and physical characteristics of the copper ores extracted. According to the USGS' 2017 Mineral Commodity Summary on Copper [3], global identified copper resources contained 2.1 billion tonnes of copper as of 2014, of which about 80% are mainly copper sulphides, whose copper content has to be extracted through pyrometallurgical processes [4]. In 2010, the average grades of ores being mined ranged from 0.5% to 2% Cu, which makes direct smelting unfeasible for economic and technical reasons. So, copper sulphide ores undergo a process known as froth-flotation to obtain concentrates containing ≈ 30% Cu, which makes concentrates the main products offered by copper mines [2, 5]. Concentrates are later smelted and, in most cases, electrochemically refined to produce high-purity copper cathodes (Figure 1). Copper cathodes are generally regarded as pure copper, with a minimum copper content of 99.9935% Cu [6]. Cathodes are normally produced by integrated smelters that purchase concentrates at a discounted price of the copper market price and sell cathodic copper at the market price, adding a premium when possible.

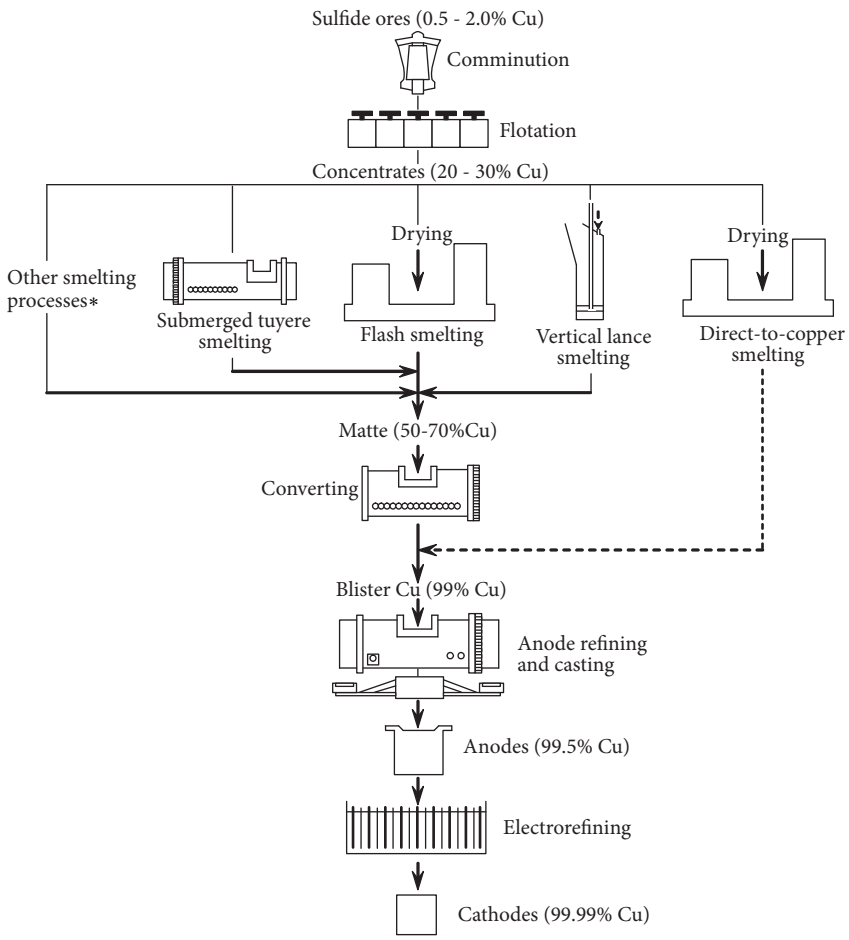

Figure 1: Industrial processing of copper sulphides ore to obtain copper cathodes. (Source: Extractive Metallurgic of Copper, pp.2 [4]).

1.2. TC/RC and Their Role in Copper Concentrates Trade. The valuation of these copper concentrates is a recurrent task undertaken by miners or traders following processes in which market prices for copper and other valuable metals such as gold and silver are involved, as well as relevant discounts or coefficients that usually represent a major part of the revenue obtained for concentrate trading, smelting, or refining. The main deductions are applied to the market value of the metal contained by concentrates such as *Copper Treatment Charges (TC)*, *Copper Refining Charges (RC)*, *the Price Participation Clause (PP)*, and *Penalties for Punishable Elements* [7]. These are fixed by the different parties involved in a copper concentrates long-term or spot supply contract, where TC/RC are fixed when the concentrates are sold to a copper smelter/refinery. The sum of TC/RC is often viewed as the main source of revenue for copper smelters along with copper premiums linked to the selling of copper cathodes. Furthermore, TC/RC deductions pose a concern for copper mines as well as a potential arbitrage opportunity for traders, whose strong financial capacity and indepth knowledge of the market make them a major player [8].

Due to their nature, TC/RC are discounts normally agreed upon taking a reference which is traditionally set on an annual basis at the negotiations conducted by the major market participants during *LME Week* every October and, more recently, during the *Asia Copper Week* each November, an event that is focused more on Chinese smelters. The TC/RC levels set at these events are usually taken as benchmarks for the negotiations of copper concentrate supply contracts throughout the following year. Thus, as the year goes on, TC/RC average levels move up and down depending on supply and demand, as well as on concentrate availability and smelters' available capacity. Consultants, such as *Platts, Wood Mackenzie*, and *Metal Bulletin,* regularly carry out their own market surveys to estimate the current TC/RC levels. Furthermore, *Metal Bulletin* has created the first TC/RC index for copper concentrates [9].

1.3. The Need for Accurate Predictions of TC/RC. The current information available for market participants may be regarded as sufficient to draw an accurate assumption of market sentiment about current TC/RC levels, but not enough to foresee potential market trends regarding these crucial discounts, far less as a reliable tool which may be ultimately applicable by market participants to their decision-making framework or their risk-management strategies. Hence, from an organisational standpoint, providing accurate forecasts of copper TC/RC benchmark levels, as well as an accurate

mathematical model to render these forecasts, is a research topic that is yet to be explored in depth. This is a question with undeniable economic implications for traders, miners, and smelters alike, due to the role of TC/RC in the copper trading revenue stream.

Our study seeks to determine an appropriate forecasting technique for TC/RC benchmark levels for copper concentrates that meets the need of reliability and accuracy. To perform this research, three different and frequently-applied techniques have been preselected from among the options available in the literature. Then, their forecasting accuracy at different time horizons will be tested and compared. These techniques (Geometric Brownian Motion -GBM-; the Mean Reversion -MR-; Linear Exponential Smoothing -LES-), have been chosen primarily because they are common in modelling commodities prices and their future expected behaviour, as well as in stock indices' predictive works, among other practical applications [10–13]. The selection of these models is further justified by the similarities shared by TC/RC with indices, interest rates, or some economic variables that these models have already been applied to. Also in our view, the predictive ability of these models in commodity prices such as copper is a major asset to take them into consideration. The models have been simulated using historical data of TC/RC annual benchmark levels from 2004 to 2017 agreed upon during the LME Copper Week. The dataset employed has been split into two parts, with two-thirds as the in-sample dataset and one-third as the out-of-sample one.

The main contribution of this paper is to provide a useful and applicable tool to all parties involved in the copper trading business to forecast potential levels of critical discounts to be applied to the copper concentrates valuation process. To carry out our research, we have based ourselves on the following premises: (1) GBM would deliver good forecasts if copper TC/RC benchmark levels vary randomly over the years, (2) a mean-reverting model, such as the OUP, would deliver the best forecasts if TC/RC levels were affected by market factors and consequently they move around a long-term trend, and (3) a moving average model would give a better forecast than the other two models if there were a predominant factor related to precedent values affecting the futures ones of benchmark TC/RC. In addition, we have also studied the possibility that a combination of the models could deliver the most accurate forecast as the time horizon considered is increased, since there might thus be a limited effect of past values, or of sudden shocks, on future levels of benchmark TC/RC. So, after some time, TC/RC levels could be "normalized" towards a long-term average.

The remainder of this article is structured as follows: Section 2 revises the related work on commodity discounts forecasting and commodity prices forecasting techniques, as well as different forecasting methods; Section 3 presents the reasoning behind the choice of each of the models employed, as well as the historic datasets used to conduct the study and the methodology followed; Section 4 shows the results of simulations of different methods; Section 5 indicates error comparisons to evaluate the best forecasting alternative for TC/RC among all those presented; Section 6 contains the study's conclusions and proposes further lines of research.

2. Related Work

The absence of any specific method in the specialised literature in relation to copper TC/RC leads us to revisit previous literature in order to determine the most appropriate model to employ for our purpose, considering those that have already been used in commodity price forecasting as the logical starting point due to the application similarities that they share with ours.

Commodity prices and their forecasting have been a topic intensively analysed in much research. Hence, there are multiple examples in literature with an application to one or several commodities, such as Xiong et al. [14], where the accuracy of different models was tested to forecast the interval of agricultural commodity future prices; Shao and Dai [15], whose work employs the Autoregressive Integrated Moving Average (ARIMA) methodology to forecast food crop prices such as wheat, rice, and corn; and Heaney [16], who tests the capacity of commodities future prices to forecast their cash price were the cost of carry to be included in considerations, using the LME Lead contract as an example study. As was done in other research [17–19], Table 1 summarizes similar research in the field of commodity price behaviours.

From the summary shown in Table 1, it is seen that moving average methods have become increasingly popular in commodity price forecasting. The most broadly implemented moving average techniques are ARIMA and Exponential Smoothing. They consider the entire time series data and do not assign the same weight to past values than those closer to the present as they are seen as affecting greater to future values. The Exponential Smoothing models' predictive accuracy has been tested [20, 21], concluding that there are small differences between them (Exponential Smoothing) and ARIMA models.

Also, GBM and MR models have been intensively applied to commodity price forecasting. Nonetheless, MR models present a significant advantage over GBM models which allows them to consider the underlying price trend. This advantage is of particular interest for commodities that, according to Dixit and Pindyck [22]—pp. 74, regarding the price of oil "*in the short run, it might fluctuate randomly up and down (in responses to wars or revolutions in oil producing countries, or in response to the strengthening or weakening of the OPEC cartel), in the longer run, it ought to be drawn back towards the marginal cost of producing oil. Thus, one might argue that the price of oil should be modelled as a mean-reverting process.*"

3. Methodology

This section presents both the justification of the models chosen in this research and the core reference dataset used as inputs for each model compared in the methodology, as well as the steps followed during the latter to carry out an effective comparison in terms of the TC/RC forecasting ability of each of these models.

3.1. Models in Methodology. GBM (see *Appendix A* for models definition.) has been used in much earlier research as a

Table 1: Summary of previous literature research.

AUTHOR	RESEARCH
Shafiee & Topal [17]	Validates a modified version of the long-term trend reverting jump and dip diffusion model for forecasting commodity prices and estimates the gold price for the next 10 years using historical monthly data.
Li *et al.* [18]	Proposes an ARIMA-Markov Chain method to accurately forecast mineral commodity prices, testing the method using mineral molybdenum prices.
Issler *et al.* [19]	Investigates several commodities' co-movements, such as Aluminium, Copper, Lead, Nickel, Tin, and Zinc, at different time frequencies, and uses a bias-corrected average forecast method proposed by Issler and Lima [43] to give combined forecasts of these metal commodities employing RMSE as a measure of forecasting accuracy.
Hamid & Shabri [44]	Models palm oil prices using the Autoregressive Distributed Lag (ARDL) model and compares its forecasting accuracy with the benchmark model ARIMA. It uses an ARDL bound-testing approach to co-integration in order to analyse the relationship between the price of palm oil and its determinant factors.
Duan *et al.* [45]	Predicts China's crude oil consumption for 2015-2020 using the fractional-order FSIGM model.
Brennan & Schwartz [46]	Employs the Geometric Brownian Motion (GBM) to analyse a mining project's expected returns assuming it produces a single commodity.
McDonald & Siegel [47]	Uses GBM to model the random evolution of the present value of an undefined asset in an investment decision model.
Zhang *et al.* [48]	Models gold prices using the Ornstein-Uhlenbeck Process (OUP) to account for a potentially existent long-term trend in a Real Option Valuation of a mining project.
Sharma [49]	Forecasts gold prices in India with the Box Jenkins ARIMA method.

way of modelling prices that are believed not to follow any specific rule or pattern and hence seen as *random*. Black and Scholes [23] first used GBM to model stock prices and since then others have used it to model asset prices as well as commodities, these being perhaps the most common of all, in which prices are expected to increase over time, as does their variance [11]. Hence, following our first premise, concerning whether TC/RC might vary randomly, there should not exist a main driving factor that would determine TC/RC future benchmark levels and therefore GBM could to a certain extent be a feasible model for them.

However, although GBM or "random walk" may be well suited to modelling immediate or short-term price paths for commodities, or for TC/RC in our case, it lacks the ability to include the underlying long-term price trend should we assume that there is one. Thus, in accordance with our second premise on benchmark TC/RC behaviour, levels would move in line with copper concentrate supply and demand as well as the smelters' and refineries' available capacity to transform concentrates into metal copper. Hence, a relationship between TC/RC levels and copper supply and demand is known to exist and, therefore, is linked to its market price, so to some extent they move together coherently. Therefore, in that case, as related works on commodity price behaviour such as Foo, Bloch, and Salim [24] do, we have opted for the MR model, particularly the OUP model.

Both GBM and MR are Markov processes which means that future values depend exclusively on the current value, while the remaining previous time series data are not considered. On the other hand, moving average methods employ the average of a pre-established number of past values in different ways, evolving over time, so future values do not rely exclusively on the present, hence behaving as though they had only a limited memory of the past. This trait of the moving average model is particularly interesting when past prices are believed to have a certain, though limited, effect on present values, which is another of the premises for this research. Existing alternatives of moving average techniques pursue considering this "memory" with different approaches. As explained by Kalekar [25], Exponential Smoothing is suitable only for the behaviours of a specific time series; thus Single Exponential Smoothing (SES) is reasonable for short-term forecasting with no specific trend in the observed data, whereas Double Exponential Smoothing or Linear Exponential Smoothing (LES) is appropriate when data shows a cyclical pattern or a trend. In addition, seasonality in observed data can be computed and forecasted through the usage of Exponential Smoothing by the Holt-Winters method, which adds an extra parameter to the model to handle this characteristic.

3.2. TC/RC Benchmark Levels and Sample Dataset. TC/RC levels for copper concentrates continuously vary throughout the year, relying on private and individual agreements between miners, traders, and smelters worldwide. Nonetheless, the TC/RC benchmark fixed during the *LME week* each October is used by market participants as the main reference to set actual TC/RC levels for each supply agreed upon for the following year. Hence, the year's benchmark TC/RC is taken here as a good indicator of a year's TC/RC average levels. Analysed time series of benchmark TC/RC span from 2004 through to 2017, as shown in Table 2, as well as the source each value was obtained from. We have not intended to reflect the continuous variation of TC/RC for the course of any given year, though we have however considered benchmark prices alone as we intuitively assume that annual variations of actual TC/RC in contracts will eventually be reflected in the benchmark level that is set at the end of the year for the year to come.

3.3. TC/RC Relation. TC and RC normally maintain a 10:1 relation with different units, with TC being expressed in US

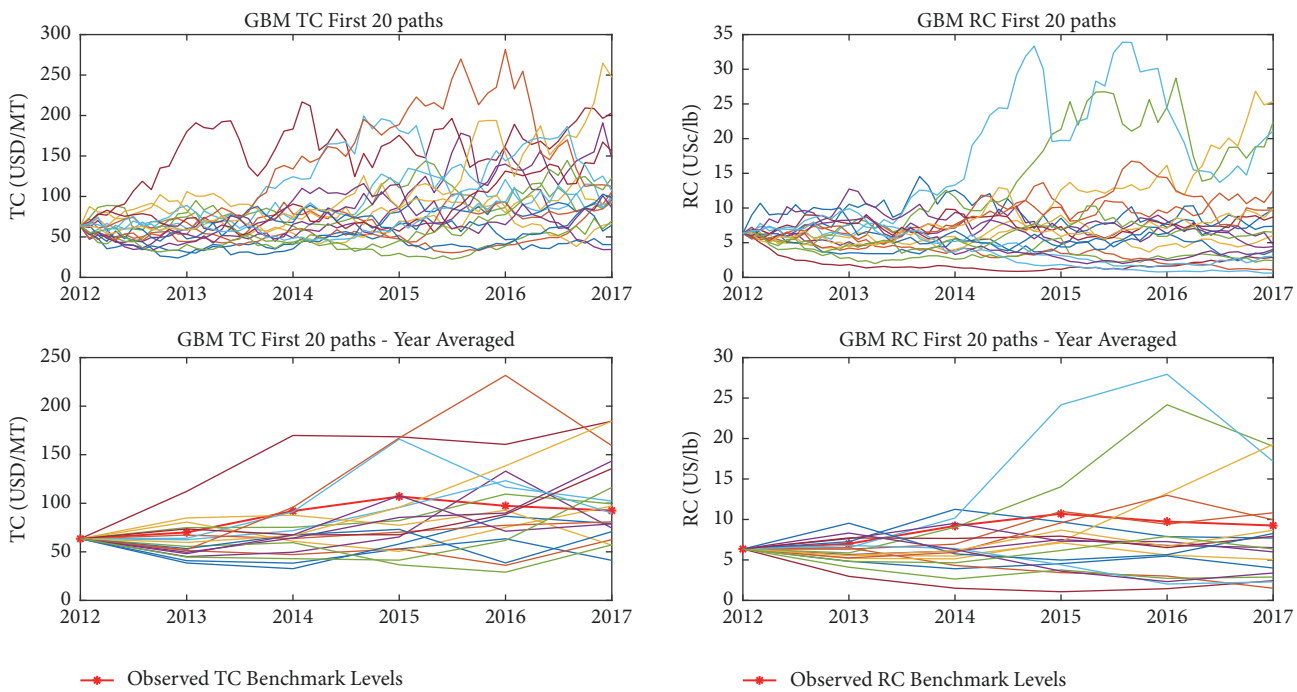

Figure 2: The upper illustrations show the first 20 Monte Carlo paths for either TC or RC levels using monthly steps. The illustrations below show the annual averages of monthly step forecasts for TC and RC levels.

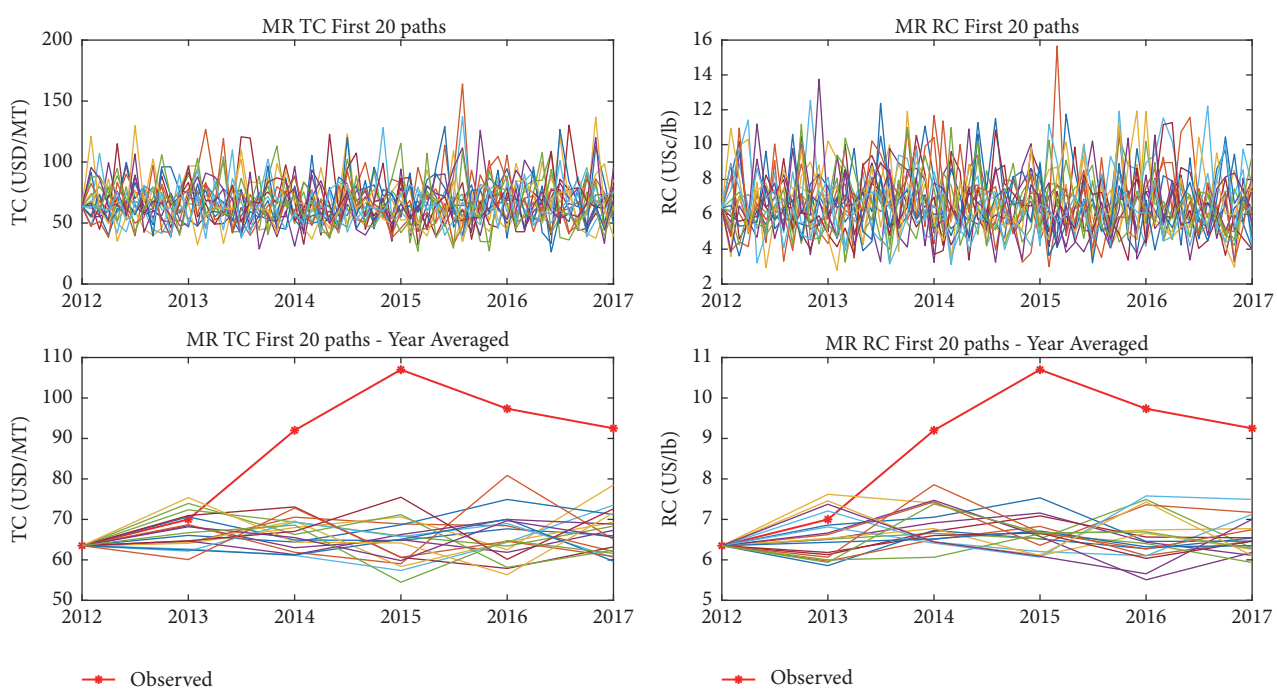

Figure 3: The first 20 Monte Carlo paths following the OUP model using monthly steps are shown on the upper illustrations for either TC or RC. The annual averaged simulated paths every 12 values are shown below.

dollars per metric tonne and RC in US cents per pound of payable copper content in concentrates. In fact, benchmark data show that, of the last 14 years, it was only in 2010 that values of TC and RC did not conform to this relation, though it did remain close to it (46.5/4.7). Nonetheless, this relation has not been observed in the methodology herein, thus treating TC and RC independently, developing separate unrelated forecasts for TC and RC to understand whether

Table 2: TC/RC year benchmark levels (Data available free of charge. Source: Johansson [50], Svante [51], Teck [52], Shaw [53], Willbrandt and Faust [54, 55], Aurubis AG [56], Drouven and Faust [57], Aurubis AG [58], EY [59], Schachler [60], Nakazato [61].).

YEAR	TC (USD/MT)	RC (USc/lb)
2004	45	4.5
2005	85	8.5
2006	95	9.5
2007	60	6.0
2008	45	4.5
2009	75	7.5
2010	46.5	4.7
2011	56	5.6
2012	63.5	6.35
2013	70	7.0
2014	92	9.2
2015	107	10.7
2016	97.35	9.735
2017	92.5	9.25

these individual forecasts for TC and RC would render better results than a single joint forecast for both TC/RC levels that would take into account the 10:1 relation existent in the data.

3.4. Models Comparison Method. The works referred to in the literature usually resort to different measures of errors to conduct the testing of a model's forecasting capacity regardless of the specific nature of the forecasted value, be it cotton prices or macroeconomic parameters. Thus, the most widely errors used include *Mean Squared Error (MSE), Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE)*, and *Root Mean Square Error (RMSE)* [14, 16, 17, 21, 25–29]. In this work Geometric Brownian Motion (GBM), Ornstein-Uhlenbeck Process (OUP) and Linear Exponential Smoothing (LES) models have been used to forecast annual TC/RC benchmark levels, using all the four main measures of error mentioned above to test the predictive accuracy of these three models. The GBM and OUP models have been simulated and tested with different step sizes, while the LES model has been analysed solely using annual time steps.

The GBM and OUP models were treated separately to the LES model; thus GBM and OUP were simulated using Monte Carlo (MC) simulations, whereas LES forecasts were simple calculations. In a preliminary stage, the models were first calibrated to forecast values from 2013 to 2017 using available data from 2004 to 2012 for TC and RC separately, hence the disregarding of the well-known inherent 10:1 relation (*see Appendix B for Models Calibration*). The GBM and OUP models were calibrated for two different step sizes, monthly steps and annual steps, in order to compare forecasting accuracy with each step size in each model. Following the calibration of the models, Monte Carlo simulations (MC) were carried out using Matlab software to render the pursued forecasts of GBM and OUP models, obtaining 1000 simulated paths for each step size. MC simulations using monthly steps for the 2013–2017 timespan were averaged every 12 steps to deliver year forecasts of TC/RC benchmark levels. On the other hand, forecasts obtained by MC simulations taking annual steps for the same period were considered as year forecasts for TC/RC annual benchmark levels without the need for extra transformation. Besides, LES model forecasts were calculated at different time horizons to be able to compare the short-term and long-term predictive accuracy. LES forecasts were obtained for one-year-ahead, hence using known values of TC/RC from 2004 to 2016; for two years ahead, stretching the calibrating dataset from 2004 to 2015; for five-year-ahead, thus using the same input dataset as for the GBM and OUP models, from 2004 to 2012.

Finally, for every forecast path obtained, we have calculated the average of the squares of the errors with respect to the observed values, *MSE*, the average distance of a forecast to the observed mean, *MAD*, the average deviation of a forecast from observed values, *MAPE*, and the square root of MSE, *RMSE* (see *Appendix C* for error calculations.). The results of *MSE, MAD, MAPE*, and *RMSE* calculated for each forecast path were averaged by the total number of simulations carried out for each case. Averaged values of error measures of all simulated paths, $\overline{\text{MSE}}$, $\overline{\text{MAD}}$, $\overline{\text{MAPE}}$, and $\overline{\text{RMSE}}$, for both annual-step forecasts and monthly step forecasts have been used for cross-comparison between models to test predictive ability at every step size possible.

Also, to test each model's short-term forecasting capacity against its long-term forecasting capacity, one-year-ahead forecast errors of the LES model were compared with the errors from the last year of the GBM and OUP simulated paths. Two-year-ahead forecast errors of LES models were compared with the average errors of the last two years of GBM and OUP, and five-year-ahead forecast errors of LES models were compared with the overall average of errors of the GBM and OUP forecasts.

4. Analysis of Results

Monte Carlo simulations of GBM and OUP models render 1000 different possible paths for TC and RC, respectively, at each time step size considered. Accuracy errors for both annual time steps and averaged monthly time steps, for both GBM and OUP forecasts, were first compared to determine the most accurate time step size for each model. In addition, the LES model outcome for both TC and RC at different timeframes was also calculated and measures of errors for all the three alternative models proposed at an optimum time step were finally compared.

4.1. GBM Forecast. The first 20 of 1000 Monte Carlo paths for the GBM model with a monthly step size using Matlab software may be seen in Figure 2 for both the TC and the RC levels compared to their averaged paths for every 12 values obtained. The tendency for GBM forecasts to steeply increase over time is easily observable in the nonaveraged monthly step paths shown.

The average values of error of all 1000 MC paths obtained through simulation for averaged monthly step and annual-step forecasts are shown in Table 3 for both TC and RC

Table 3: Average of main measures of error for GBM after 1000 MC simulations from 2013 to 2017.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC Averaged Monthly Steps	$5.60x10^3$	46.98	0.50	56.38
TC Annual Steps	$5.20x10^3$	49.02	0.52	58.57
RC Averaged Monthly Steps	58.74	4.55	0.48	5.46
RC Annual Steps	50.85	4.91	0.52	5.84

Table 4: Number of paths for which measures of error are higher for monthly steps than for annual steps in GBM.

	MSE	*MAD*	*MAPE*	*RMSE*
TC	457/1000	455/1000	453/1000	453/1000
RC	439/1000	430/1000	429/1000	429/1000

discounts over the period from 2013 to 2017, which may lead to preliminary conclusions in terms of an accuracy comparison between averaged monthly steps and annual steps.

However, a more exhaustive analysis is shown in Table 4, where the number of times the values of each error measure are higher for monthly steps is expressed over the total number of MC simulations carried out. The results indicate that better values of error are reached the majority of times for averaged monthly step simulations rather than for straight annual ones.

In contrast, short-term accuracy was also evaluated by analysing the error measures of one-year-ahead forecasts (2013) in Table 5 and of two-year-ahead forecasts (2013–2014) in Table 6. The results indicate, as one may expect, that accuracy decreases as the forecasted horizon is widened, with the accuracy of averaged monthly step forecasts remaining higher than annual ones as found above for the five-year forecasting horizon.

4.2. OUP Forecast. Long-term levels for TC and RC, μ, the speed of reversion, λ, and the volatility of the process, σ, are the parameters determined at the model calibration stage (see *Appendix B* for the model calibration explanation.), which define the behaviour of the OUP, shown in Table 7. The calibration was done prior to the Monte Carlo simulation for both TC and RC, with each step size using available historical data from 2004 to 2012. The OUP model was fitted with the corresponding parameters for each case upon MC simulation.

Figure 3 shows the Mean Reversion MC estimations of the TC/RC benchmark values from 2013 to 2017 using monthly steps. The monthly forecasts were rendered from January 2012 through to December 2016 and averaged every twelve values to deliver a benchmark forecast for each year. The averaged results can be comparable to actual data as well as to the annual Monte Carlo simulations following Mean Reversion. The lower-side figures show these yearly averaged monthly step simulation outcomes which clearly move around a dash-dotted red line, indicating the long-term run levels for TC/RC to which they tend to revert.

The accuracy of monthly steps against annual steps for the TC/RC benchmark levels forecast was also tested by determining the number of simulations for which average error measures became higher. Table 8 shows the number of times monthly simulations have been less accurate than annual simulations for five-year-ahead OUP forecasting by comparing the four measures of errors proposed. The results indicate that only 25-32% of the simulations drew a higher average error, which clearly results in a better predictive accuracy for monthly step forecasting of TC/RC annual benchmark levels.

The averaged measures of errors obtained after the MC simulations of the OUP model for both averaged monthly steps and annual steps giving TC/RC benchmark forecasts from 2013 to 2017 are shown in Table 9.

The error levels of the MC simulations shown in Table 9 point towards a higher prediction accuracy of averaged monthly step forecasts of the OUP Model, yielding an averaged MAPE value that is 12.9% lower for TC and RC 5-step-ahead forecasts. In regard to MAPE values, for monthly steps, only 26.6% of the simulations rise above annual MC simulations for TC and 25% for RC 5-step-ahead forecasts, which further underpins the greater accuracy of this OUP set-up for TC/RC level forecasts. A significant lower probability of higher error levels for TC/RC forecasts with monthly MC OUP simulations is reached for the other measures provided. In addition, short-term and long-term prediction accuracy were tested by comparing errors of forecasts for five-year-ahead error measures in Table 10, one-year-ahead in Table 11, and two-year-ahead in Table 12.

With a closer forecasting horizon error, measures show an improvement of forecasting accuracy when average monthly steps are used rather than annual ones. For instance, the MAPE values for 2013 forecast for TC are 68% lower for averaged monthly steps than for annual steps, and also MAPE for 2013–2014 were 30% lower for both TC and RC forecasts. Similarly, better values of error are achieved for the other measures for averaged monthly short-term forecasts than in other scenarios. In addition, as expected, accuracy is increased for closer forecasting horizons where the level of errors shown above becomes lower as the deviation of forecasts is trimmed with short-term predictions.

4.3. LES Forecast. In contrast to GBM and OUP, the LES model lacks any stochastic component, so nondeterministic

Table 5: Average for main measures of error for GBM after 1000 MC simulations for 2013.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC Averaged Monthly Steps	336.58	14.55	0.21	14.55
TC Annual Steps	693.81	20.81	0.30	20.81
RC Averaged Monthly Steps	2.97	1.38	0.20	1.38
RC Annual Steps	6.57	2.05	0.29	2.05

Table 6: Average for main measures of error for GBM after 1000 MC simulations for 2013-2014.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC Averaged Monthly Steps	994.92	21.87	0.31	24.35
TC Annual Steps	$1.33\text{x}10^3$	28.33	0.40	31.04
RC Averaged Monthly Steps	10.35	2.40	0.34	2.71
RC Annual Steps	13.13	2.84	0.34	3.11

Table 7: OUP parameters obtained in the calibration process.

	μ	λ	σ
TC Monthly	63.45	$4.792\text{x}10^5$	2.534
TC Annual	63.45	2.974	1.308
RC Monthly	6.35	$4.763\text{x}10^5$	2.519
RC Annual	6.35	2.972	1.305

methods such as the Monte Carlo are not required to obtain a forecast. Nonetheless, the LES model relies on two *smoothing constants* which must be properly set in order to deliver accurate predictions; hence the values of the smoothing constants were first optimised (see *Appendix B* for LES Model fitting and smoothing constants optimisation.). The optimisation was carried out for one-year-ahead forecasts, two-year-ahead forecasts, and five-year-ahead forecasts by minimising the values of MSE for both TC and RC. The different values used for smoothing constants, as well as the initial values for level and trend obtained by the linear regression of the available dataset from 2004 through to 2012, are shown in Table 12.

Compared one-year-ahead, two-year-ahead, and five-year-ahead LES forecasts for TC and RC are shown in Figure 4, clearly indicating a stronger accuracy for shorter-term forecasts as the *observed* and *forecasted* plotted lines overlap.

Minimum values for error measures achieved through the LES model parameter optimisation are shown in Table 13. The values obtained confirm the strong accuracy for shorter-term forecasts of TC/RC seen in Figure 4.

5. Discussion and Model Comparison

The forecasted values of TC/RC benchmark levels could eventually be applied to broader valuation models for copper concentrates and their trading activities, as well as to the copper smelters' revenue stream, thus the importance of delivering as accurate a prediction as possible in relation to these discounts to make any future application possible. Each of the models presented may be a feasible method on its own with eventual later adaptations to forecasting future values of benchmark TC/RC. Nonetheless, the accuracy of these models as they have been used in this work requires, firstly, a comparison to determine whether any of them could be a good standalone technique and, secondly, to test whether a combination of two or more of them would deliver more precise results.

When comparing the different error measures obtained for all the three models, it is clearly established that results for a randomly chosen simulation of GBM or OUP would be more likely to be more precise had a monthly step been used to deliver annual forecasts instead of an annual-step size. In contrast, average error measures for the entire population of simulations with each step size employed showing that monthly step simulations for GBM and OUP models are always more accurate than straight annual-step forecasts when a shorter time horizon, one or two-year-ahead, is taken into consideration. However, GBM presents a higher level of forecasting accuracy when the average for error measures of all simulations is analysed, employing annual steps for long-term horizons, whereas OUP averaged monthly step forecasts remain more accurate when predicting long-term horizons. Table 14 shows the error improvement for the averaged monthly step forecasts of each model. Negative values indicate that better levels of error averages have been found in straight annual forecasts than for monthly step simulations.

Considering the best results for each model and comparing their corresponding error measures, we can opt for the best technique to employ among the three proposed in this paper. Hence, GBM delivers the most accurate one-year forecast when averaging the next twelve-month predictions for TC/RC values, as does the MR-OUP model. Table 15 shows best error measures for one-year-ahead forecasts for GBM, OUP, and LES models.

Unarguably, the LES model generates minimal error measures for one-year-ahead forecasts, significantly less than the other models employed. A similar situation is found for two-year-ahead forecasts where minimum error measures are also delivered by the LES model, shown in Table 16.

Finally, accuracy measures for five-year-ahead forecasts of the GBM model might result in somewhat contradictory

Table 8: Number of paths for which measures of error are higher for monthly steps than for annual steps in MR-OUP 5-Steps MC simulation.

	MSE	*MAD*	*MAPE*	*RMSE*
TC	311/1000	283/1000	266/1000	266/1000
RC	316/1000	281/1000	250/1000	250/1000

Table 9: Average for main measures of error for MR-OUP after 1000 MC simulations 2013-2017.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC Averaged Monthly Steps	842.54	26.14	0.27	28.95
TC Annual Steps	$1.14\text{x}10^{3}$	29.31	0.31	33.25
RC Averaged Monthly Steps	8.40	2.61	0.27	2.89
RC Annual Steps	11.48	2.94	0.31	3.33

Table 10: Average for main measures of error for MR-OUP after 1000 MC simulations 2013.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC Averaged Monthly Steps	39.14	5.22	0.07	5.22
TC Annual Steps	366.43	15.55	0.22	15.55
RC Averaged Monthly Steps	0.39	0.51	0.07	0.51
RC Annual Steps	3.63	1.54	0.22	1.54

Table 11: Average for main measures of error for MR-OUP after 1000 MC simulations 2013-2014.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC Averaged Monthly Steps	371.65	15.67	0.18	19.00
TC Annual Steps	682.44	21.85	0.26	23.24
RC Averaged Monthly Steps	3.76	1.57	0.18	1.91
RC Annual Steps	6.89	2.19	0.26	2.43

Table 12: LES Model Optimised Parameters.

	L_0	T_0	α	β
TC 1 year	71.3611	-1.5833	-0.2372	0.1598
RC 1 year	7.1333	-0.1567	-0.2368	0.1591
TC 2 years	71.3611	-1.5833	$-1.477\text{x}10^{-4}$	777.4226
RC 2 years	7.1333	-0.1567	$-1.448\text{x}10^{-4}$	789.8336
TC 5 years	71.3611	-1.5833	-0.2813	0.1880
RC 5 years	7.1333	-0.1567	-0.2808	0.1880

Table 13: LES error measures for different steps-ahead forecasts.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
TC 1 year	$1.0746\text{x}10^{-5}$	0.0033	$4.6831\text{x}10^{-5}$	0.0033
RC 1 year	$5.1462\text{x}10^{-8}$	$2.2685\text{x}10^{-4}$	$3.2408\text{x}10^{-5}$	$2.2685\text{x}10^{-4}$
TC 2 years	189.247	9.7275	0.1057	13.7567
RC 2 years	1.8977	0.9741	0.1059	1.3776
TC 5 years	177.5531	7.8881	0.0951	10.8422
RC 5 years	1.1759	0.7889	0.0952	1.0844

terms for MSE reaching better values for annual steps than for averaged monthly steps, while the other figures do better on averaged monthly steps. Addressing the definition of MSE, this includes the variance of the estimator as well as its bias, being equal to its variance in the case of unbiased estimators. Therefore, MSE measures the quality of the estimator but also magnifies estimator deviations from actual values since both positive and negative values are squared and averaged. In contrast, RMSE is calculated as the square root of MSE and, following the previous analogy, stands for the standard

Table 14: Error Average Improvement for averaged monthly steps before annual steps.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
GBM TC/RC 1 Year	53.26%/56.53%	32.72%/35.87%	33.33%/35.48%	32.72%/35.87%
GBM TC/RC 2 Years	26.95%/26.35%	25.75%/15.97%	26.19%/2.86%	24.34%/13.69%
GBM TC/RC 5 Years	-11.73%/-11.04%	8.73%/7.09%	9.26%/7.69%	7.47%/5.65%
OUP TC/RC 1 Year	88.08%/87.08%	63.71%/62.50%	62.91%/63.19%	63.68%/62.24%
OUP TC/RC 2 Years	46.28%/44.21%	27.71%/26.17%	29.99%/30.75%	22.16%/20.75%
OUP TC/RC 5 Years	26.02%/25.53%	10.93%/9.97%	13.12%/12.18%	13.06%/12.69%

Table 15: TC/RC best error measures for 2013 forecasts after 1000 MC simulations.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
GBM TC/RC 1 Year∗	331.11/3.36	14.25/1.43	0.20/0.20	14.25/1.43
OUP TC/RC 1 Year∗	41.05/0.42	5.36/0.54	0.0766/0.0777	5.36/0.54
LES TC/RC 1 Year	$1.07\text{x}10^{-5}/5.14\text{x}10^{-8}$	$0.003/2.26\text{x}10^{-4}$	$4.68\text{x}10^{-5}/3.24\text{x}10^{-5}$	$0.003/2.26\text{x}10^{-4}$

∗Averaged monthly steps.

Table 16: TC/RC Best error measures for 2013–2014 forecasts after 1000 MC simulations.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
GBM TC/RC 2 Steps∗	$1.03\text{x}10^{3}/10.06$	22.00/2.42	0.31/0.34	24.47/2.71
OUP TC/RC 2 Steps∗	373.31/3.76	15.73/1.58	0.1802/0.1813	19.00/1.91
LES TC/RC 2 Steps	189.24/1.89	9.72/0.97	0.1057/0.1059	13.75/1.37

∗Averaged monthly steps.

deviation of the estimator if MSE were considered to be the variance. Though RMSE overreacts when high values of MSE are reached, it is less prone to this than MSE since it is calculated as its squared root, thus not accounting for large errors as disproportionately as MSE does. Furthermore, as we have compared an average of 1000 measures of errors corresponding to each MC simulation performed, the values obtained for average RMSE stay below the square root of average MSE, which indicates that some of these disproportionate error measures are, to some extent, distorting the latter. Hence, RMSE average values point towards a higher accuracy for GBM five-year forecasts with averaged monthly steps, which is further endorsed by the average values of MAD and MAPE, thus being the one used for comparison with the other two models as shown in Table 17.

The final comparison clearly shows how the LES model outperforms the other two at all average measures provided, followed by the OUP model in accuracy, although the latter more than doubles the average MAPE value for LES.

The results of simulations indicate that measures of errors tend to either differ slightly or not at all for either forecasts of any timeframe. A coherent value with the 10:1 relation can then be given with close to the same level of accuracy by multiplying RC forecasts or dividing TC ones by 10.

6. Conclusions

Copper TC/RC are a keystone for pricing copper concentrates which are the actual feedstock for copper smelters. The potential evolution of TC/RC is a question of both economic and technical significance for miners, as their value decreases the potential final selling price of concentrates. Additionally, copper miners' revenues are more narrowly related to the market price of copper, as well as to other technical factors such as ore dilution or the grade of the concentrates produced. Smelters, on the contrary, are hugely affected by the discount which they succeed in getting when purchasing the concentrates, since that makes up the largest part of their gross revenue, besides other secondary sources. In addition, eventual differences between TC/RC may give commodity traders ludicrous arbitrage opportunities. Also, differences between short- and long-term TC/RC agreements offer arbitrage opportunities for traders, hence comprising a part of their revenue in the copper concentrate trading business, as well copper price fluctuations and the capacity to make economically optimum copper blends.

As far as we are aware, no rigorous research has been carried out on the behaviour of these discounts. Based on historical data on TC/RC agreed upon in the LME Copper Week from 2004 to 2017, three potentially suitable forecasting models for TC/RC annual benchmark values have been compared through four measures of forecasting accuracy at different horizons. These models were chosen, firstly, due to their broad implementation and proven capacity in commodity prices forecasting that they all share and, secondly, because of the core differences in terms of price behaviour with each other.

Focusing on the MAPE values achieved, those obtained by the LES model when TC and RC are treated independently have been significantly lower than for the rest of the models. Indeed, one-year-ahead MAPE measures for TC values for the GBM model (20%) almost triple those of

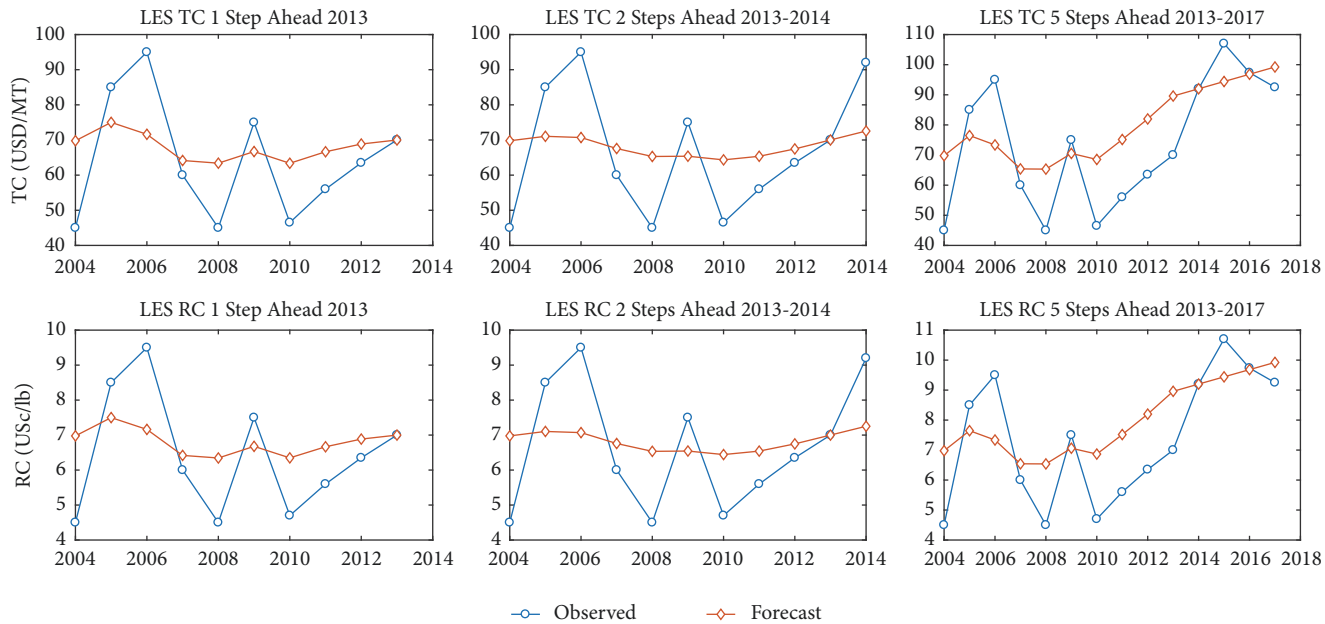

Figure 4: One-step-ahead forecasts (2013), two-step-ahead forecasts (2013-2014) and five-step-ahead forecasts (2013-2017) for TC and RC using the Linear Exponential Smoothing model (LES).

Table 17: TC/RC Best error measures for 2013–2017 forecasts after 1000 MC simulations.

	$\overline{MSE}$	$\overline{MAD}$	$\overline{MAPE}$	$\overline{RMSE}$
GBM TC/RC 5 Steps∗	6.00x10^3/61.65	46.51/4.59	0.49/0.48	55.71/5.51
OUP TC/RC 5 Steps∗	843.34/8.43	26.16/2.62	0.2715/0.2719	28.96/2.89
LES TC/RC 5 Steps	177.55/1.17	7.88/0.78	0.0951/0.0952	10.84/1.08

∗Averaged monthly steps.

the OUP model (7.66%), in contrast with the significantly lower values from the LES model (0.0046%). This gap tends to be narrowed when TC/RC values are forecasted at longer horizons, when most measures of error become more even. The GBM and OUP models have proven to deliver better accuracy performance when the TC/RC values are projected monthly and then averaged to obtain annual benchmark forecasts. Even so, the LES model remains the most accurate of all with MAPE values of 10% at two-year-ahead forecasts, with 18% and 31% for TC for OUP and GBM, respectively.

Finally, despite TC and RC being two independent discounts applied to copper concentrates, they are both set jointly with an often 10:1 relation as our data reveals. This relation also transcends to simulation results and error measures, hence showing a negligible discrepancy between the independent forecasting of TC and RC, or the joint forecasting of both values, keeping the 10:1 relation. This is, for instance, the case of the five-year-ahead OUP MAPE values (0.2715/0.2719) which were obtained without observing the 10:1 relation in the data. A similar level of discrepancy was obtained at any horizon with any model, which indicates that both values could be forecasted with the same accuracy using the selected model with any of them and then applying the 10:1 relation.

Our findings thus suggest that both at short and at long-term horizons TC/RC annual benchmark levels tend to exhibit a pattern which is best fit by an LES model. This indicates that these critical discounts for the copper trading business do maintain a certain dependency on past values. This would also suggest the existence of cyclical patterns in copper TC/RC, possibly driven by many of the same market factors that move the price of copper.

This work contributes by delivering a formal tool for smelters or miners to make accurate forecasts of TC/RC benchmark levels. The level of errors attained indicates the LES model may be a valid model to forecast these crucial discounts for the copper market. In addition, our work further contributes to helping market participants to project the price of concentrates with an acceptable degree of uncertainty, as now they may include a fundamental element for their estimation. This would enable them to optimise the way they produce or process these copper concentrates. Also, the precise knowledge of these discounts' expected behaviour contributes to letting miners, traders, and smelters alike take the maximum advantage from the copper concentrate trading agreements that they are part of. As an additional contribution, this work may well be applied to gold or silver RC, which are relevant deduction concentrates when these have a significant amount of gold or silver.

Once TC/RC annual benchmark levels are able to be forecasted with a certain level of accuracy, future research should go further into this research through the exploration of the potential impact that other market factors may have on these discounts.

As a limitation of this research, we should point out the timespan of the data considered, compared to those of other forecasting works, on commodity prices for example, which use broader timespans. For our case, we have considered the maximum available sufficiently reliable data on TC/RC benchmark levels, starting back in 2004, as there is no reliable data beyond this year. Also, as another limitation, we have used four measures of error which are among the most frequently used to compare the accuracy of different models. However, other measures could have been used at an individual level to test each model's accuracy.

Appendix

A. Models

A.1. Geometric Brownian Motion (GBM). GBM can be written as a generalisation of a Wiener (continuous time-stochastic Markov process, with independent increments and whose changes over any infinite interval of time are normally distributed, with a variance that increases linearly with the time interval [22].) process:

$$dx = \alpha x dt + \sigma x dz \tag{A.1}$$

where according to Marathe and Ryan [30] the first term is known as the Expected Value, whereas the second is the Stochastic component, with α being the drift parameter and σ the volatility of the process. Also, dz is the Wiener process which induces the abovementioned stochastic behaviour in the model:

$$dz = \epsilon_t \sqrt{dt} \longrightarrow \epsilon_t \sim N(0,1) \tag{A.2}$$

The GBM model can be expressed in discrete terms according to

$$\Delta x = x_t - x_{t-1} = \alpha x_{t-1} \Delta t + \sigma x_{t-1} \epsilon \sqrt{\Delta t} \tag{A.3}$$

In GBM percentage changes in x, $\Delta x/x$ are normally distributed; thus absolute changes in x, Δx are *lognormally* distributed. Also, the expected value and variance for $x(t)$ are

$$\mathbb{E}[x(t)] = x_0 e^{\alpha t} \tag{A.4}$$

$$\operatorname{var}[x(t)] = x_0^2 e^{2\alpha t}\left(e^{\sigma^2 t} - 1\right) \tag{A.5}$$

A.2. Orstein-Uhlenbeck Process (OUP). The OUP process was first defined by Uhlenbeck and Orstein [31] as an alternative to the regular Brownian Motion to model the velocity of the diffusion movement of a particle that accounts for its losses due to friction with other particles. The OUP process can be regarded as a modification of Brownian Motion in continuous time where its properties have been changed (Stationary, Gaussian, Markov, and stochastic process.) [32]. These modifications cause the process to move towards a central position, with a stronger attraction the further it is from this position. As mentioned above, the OUP is usually employed to model commodity prices and is the simplest version of a *mean-reverting process* [22]:

$$dS = \lambda(\mu - S)\, dt + \sigma dW_t \tag{A.6}$$

where S is the level of prices, μ the long-term average to which prices tend to revert, and λ the speed of reversion. Additionally, in a similar fashion to that of the GBM, σ is the volatility of the process and dW_t is a Wiener process with an identical definition. However, in contrast to GBM, time intervals in OUP are not independent since differences between current levels of prices, S, and long-term average prices, μ, make the expected change in prices, dS, more likely either positive or negative.

The discrete version of the model can be expressed as follows:

$$S_t = \mu\left(1 - e^{-\lambda \Delta t}\right) + e^{-\lambda \Delta t} S_{t-1} + \sigma \sqrt{\frac{1 - e^{-2\lambda \Delta t}}{2\lambda}}\, dW_t \tag{A.7}$$

where the expected value for $x(t)$ and the variance for $(x(t) - \mu)$ are

$$\mathbb{E}[x(t)] = \mu + (x_0 - \mu)\, e^{-\lambda t} \tag{A.8}$$

$$\operatorname{var}[x(t) - \mu] = \frac{\sigma^2}{2\lambda}\left(1 - e^{-2\lambda t}\right) \tag{A.9}$$

It can be derived from previous equations that as time increases prices will tend to long-term average levels, μ. In addition, with large time spans, if the speed of reversion, λ, becomes high, variance tends to 0. On the other hand, if the speed of reversion is 0 then $\operatorname{var}[x(t)] \longrightarrow \sigma^2 t$, making the process a simple Brownian Motion.

A.3. Holt's Linear Exponential Smoothing (LES). Linear Exponential Smoothing models are capable of considering both levels and trends at every instant, assigning higher weights in the overall calculation to values closer to the present than to older ones. LES models carry that out by constantly updating local estimations of levels and trends with the intervention of one or two smoothing constants which enable the models to dampen older value effects. Although it is possible to employ a single smoothing constant for both the level and the trend, known as Brown's LES, to use two, one for each, known as Holt's LES, is usually preferred since Brown's LES tends to render estimations of the trend "unstable" as suggested by authors such as Nau [33]. Holt's LES model comes defined by the level, trend, and forecast updating equations, each of these expressed as follows, respectively:

$$L_t = \alpha Y_t + (1 - \alpha)\left(L_{t-1} + T_{t-1}\right) \tag{A.10}$$

$$T_t = \beta\left(L_t - L_{t-1}\right) + (1 - \beta)\, T_{t-1} \tag{A.11}$$

$$\widehat{Y}_{t+k} = L_t + kT_t \tag{A.12}$$

With α being the first smoothing constant for the levels and β the second smoothing constant for the trend. Higher values for the smoothing constants imply that either levels or trends are changing rapidly over time, whereas lower values imply the contrary. Hence, the higher the constant, the more uncertain the future is believed to be.

B. Model Calibration

Each model has been calibrated individually using two sets of data containing TC and RC levels, respectively, from 2004 to 2012. The available dataset is comprised of data from 2004 to 2017, which make the calibration data approximately 2/3 of the total.

B.1. GBM. Increments in the logarithm of variable x are distributed as follows:

$$\Delta(\ln x) \sim N\left(\left(\alpha - \frac{\sigma^2}{2}\right)t, \sigma t\right) \tag{B.1}$$

Hence, if m is defined as the sample mean of the difference of the natural logarithm of the time series for TC/RC levels considered for the calibration and n as the number of increments of the series considered, with n=9,

$$m = \frac{1}{n}\sum_{t=1}^{n}(\ln x_t - \ln x_{t-1}) \tag{B.2}$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\ln x_t - \ln x_{t-1} - m)^2} \tag{B.3}$$

$$m = \alpha - \frac{\sigma^2}{2} \tag{B.4}$$

$$s = \sigma \tag{B.5}$$

B.2. OUP. The OUP process is an AR1 process [22] whose resolution is well presented by Woolridge [34] using *OLS* techniques, fitting the following:

$$y_t = a + by_{t-1} + \varepsilon_t \tag{B.6}$$

Hence, the estimators for the parameters of the OUP model are obtained by OLS for both TC and RC levels independently:

$$\widehat{\lambda} = -\frac{\ln b}{\Delta t} \tag{B.7}$$

$$\widehat{\mu} = \frac{a}{1-b} \tag{B.8}$$

$$\widehat{\sigma} = \varepsilon_t\sqrt{\frac{2\ln(1+b)}{(1+b)^2 - 1}} \tag{B.9}$$

B.3. LES. A linear regression is conducted on the input dataset available to find the starting parameters for the LES model, the initial Level, L_0, and the initial value of the trend, T_0, irrespective of TC values and RC values. Here, as recommended by Gardner [35], the use of OLS is highly advisable due to the erratic behaviour shown by trends in the historic data, so the obtaining of negative values of S_0 is prevented. Linear regression fulfils the following:

$$Y_t = at + b \tag{B.10}$$

$$L_0 = b \tag{B.11}$$

$$T_0 = a \tag{B.12}$$

By fixing the two smoothing constants, the values for the forecasts, $\widehat{Y}_{t+k}$, can be calculated at each step using the model equations. There are multiple references in the literature on what the optimum range for each smoothing constant is; Gardner [35] speaks of setting moderate values for both parameter less than 0.3 to obtain the best results. Examples pointing out the same may be found in Brown [36], Coutie [37], Harrison [38], and Montgomery and Johnson [39]. Also, for many applications, Makridakis and Hibon [20] and Chatfield [40] found that parameter values should fall within the range of 0.3-1. On the other hand, McClain and Thomas [41] provided a condition of stability for the nonseasonal *LES* model given by

$$\begin{aligned} &0 < \alpha < 2 \\ &0 < \beta < \frac{4 - 2\alpha}{\alpha} \end{aligned} \tag{B.13}$$

Also, the largest possible value of α that allows the avoidance of areas of oscillation is proposed by McClain and Thomas [41] and McClain [42]:

$$\alpha < \frac{4\beta}{(1+\beta)^2} \tag{B.14}$$

However, according to Gardner, there is no tangible proof that this value improves accuracy in any form. Nonetheless, we have opted to follow what Nau [33] refers to as "*the usual way*", namely, minimising the *Mean Squared Error (MSE)* of the one-step-ahead forecast of TC/RC for each input data series previously mentioned. To do so, Matlab's *fminsearch* function has been used with function and variable tolerance levels of $1\text{x}10^{-4}$ as well as a set maximum number of function iterations and function evaluations of $1\text{x}10^{6}$ to limit computing resources. In Table 18 the actual number of necessary iterations to obtain optimum values for smoothing constants is shown. As can be seen, the criteria are well beyond the final results, which ensured that an optimum solution was reached with assumable computing usage (the simulation required less than one minute) and with a high degree of certainty.

Table 18: Iterations on LES smoothing constants optimisation.

	Iterations	Function Evaluations
TC 1-Step	36	40
TC 2-Steps	249945	454211
TC 5-Steps	40	77
RC 1-Step	32	62
RC 2-Steps	254388	462226
RC 5-Steps	34	66

C. Error Calculations

C.1. Mean Square Error (MSE)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\widehat{Y}_i - Y_i\right)^2 \tag{C.1}$$

where $\widehat{Y}_i$ are the forecasted values and Y_i those observed.

C.2. Mean Absolute Deviation (MAD)

$$MAD = \frac{1}{n}\sum_{i=1}^{n}\left|\widehat{Y}_i - \overline{Y}\right| \tag{C.2}$$

where $\widehat{Y}_i$ are the forecasted values and $\overline{Y}$ the average value of all the observations.

C.3. Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|\widehat{Y}_i - Y_i\right|}{Y_i} \tag{C.3}$$

The above formula is expressed in parts-per-one and is the one used in the calculations conducted here. Hence, multiplying the result by 100 would deliver percentage outcomes.

C.4. Root Mean Square Error (RMSE)

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\widehat{Y}_i - Y_i\right)^2} \tag{C.4}$$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Supplementary Materials

The Matlab codes developed to carry out the simulations for each of the models proposed, as well as the datasets used and the Matlab workspaces with the simulations outcomes as they have been reflected in this paper, are provided in separate folders for each model. GBM folder contains the code to make the simulations for the Geometric Brownian Motion model, "GBM.m", as well as a separate script which allows determining the total number of monthly step simulations for which errors levels have been lower than annual-step simulations, "ErrCouter.m". The datasets used are included in two excel files: "TC.xlsx" and "RC.xlsx". "GBM_OK_MonthlySteps.mat" is the Matlab workspace containing the outcome of monthly step simulations, whereas "GBM_OK_AnnualSteps.mat" contains the outcome for annual-step simulations. Also, "GBM_OK.mat" is the workspace to be loaded prior to performing GBM simulations. MR folder contains the code file to carry out the simulations for the Orstein-Uhlenbeck Process, "MR.m", as well as the separate script to compare errors of monthly step simulations with annual-step simulations, "ErrCounter.m". "TC.xlsx" and "RC.xlsx" contain the TC/RC benchmark levels from 2004 to 2017. Finally, monthly step simulations' outcome has been saved in the workspace "MR_OK_MonthlySteps.mat", while annual-step simulations' outcome has been saved in the Matlab workspace "MR_OK_AnnualSteps.mat". Also, "MR_OK.mat" is the workspace to be loaded prior to performing MR simulations. LES folder contains the code file to carry out the calculations necessary to obtain the LES model forecasts, "LES.m". Three separate Matlab functions are included: "mapeLES.m", "mapeLES1.m", and "mapeLES2.m", which define the calculation of the MAPE for the LES' five-year-ahead, one-year-ahead, and two-year-ahead forecasts. Also, "LES_OK.mat" is the workspace to be loaded prior to performing LES simulations. Finally, graphs of each model have been included in separate files as well, in a subfolder named "Graphs" within each of the models' folder. Figures are included in Matlab's format (.fig) and TIFF format. TIFF figures have been generated in both colours and black and white for easier visualization. *(Supplementary Materials)*

References

[1] United Nations, *International Trade Statistics Yearbook*, vol. 2, 2016.

[2] International Copper Study Group, *The World Copper Factbook*, 2017.

[3] K. M. Johnson, J. M. Hammarstrom, M. L. Zientek, and C. L. Dicken, "Estimate of undiscovered copper resources of the world, 2013," U.S. Geological Survey, 2017.

[4] M. E. Schlesinger and W. G. Davenport, *Extractive Metallurgy of Copper*, Elsevier, 5th edition, 2011.

[5] S. Glöser, M. Soulier, and L. A. T. Espinoza, "Dynamic analysis of global copper flows. Global stocks, postconsumer material flows, recycling indicators, and uncertainty evaluation," *Environmental Science & Technology*, vol. 47, no. 12, pp. 6564–6572, 2013.

[6] The London Metal Exchange, "Special contract rules for copper grade A," https://www.lme.com/en-GB/Trading/Brands/Composition.

[7] U. Soderstrom, "Copper smelter revenue stream," 2008.

[8] F. K. Crundwell, *Finance for Engineers (Evaluation and Funding of Capital Projects)*, 2008.

[9] Metal Bulletin TC/RC Index, "Meteoritical Bulletin," https://www.metalbulletin.com/Article/3319574/Copper/PRICING-NOTICE-Specification-of-Metal-Bulletin-copper-concentrates-TCRC-index.html.

[10] J. Savolainen, "Real options in metal mining project valuation: review of literature," *Resources Policy*, vol. 50, pp. 49–65, 2016.

[11] W. J. Hahn, J. A. DiLellio, and J. S. Dyer, "Risk premia in commodity price forecasts and their impact on valuation," *Energy Econ*, vol. 7, p. 28, 2018.

[12] C. Alberto, C. Pinheiro, and V. Senna, "Price forecasting through multivariate spectral analysis: evidence for commodities of BM&Fbovespa," *Brazilian Business Review*, vol. 13, no. 5, pp. 129–157, 2016.

[13] Z. Hloušková, P. Ženíšková, and M. Prášilová, "Comparison of agricultural costs prediction approaches," *AGRIS on-line Papers in Economics and Informatics*, vol. 10, no. 1, pp. 3–13, 2018.

[14] T. Xiong, C. Li, Y. Bao, Z. Hu, and L. Zhang, "A combination method for interval forecasting of agricultural commodity futures prices," *Knowledge-Based Systems*, vol. 77, pp. 92–102, 2015.

[15] Y. E. Shao and J.-T. Dai, "Integrated feature selection of ARIMA with computational intelligence approaches for food crop price prediction," *Complexity*, vol. 2018, Article ID 1910520, 17 pages, 2018.

[16] R. Heaney, "Does knowledge of the cost of carry model improve commodity futures price forecasting ability? A case study using the london metal exchange lead contract," *International Journal of Forecasting*, vol. 18, no. 1, pp. 45–65, 2002.

[17] S. Shafiee and E. Topal, "An overview of global gold market and gold price forecasting," *Resources Policy*, vol. 35, no. 3, pp. 178–189, 2010.

[18] Y. Li, N. Hu, G. Li, and X. Yao, "Forecasting mineral commodity prices with ARIMA-Markov chain," in *Proceedings of the 2012 4th International Conference on Intelligent Human-Machine Systems and Cybernetics (IHMSC)*, pp. 49–52, Nanchang, China, August 2012.

[19] J. V. Issler, C. Rodrigues, and R. Burjack, "Using common features to understand the behavior of metal-commodity prices and forecast them at different horizons," *Journal of International Money and Finance*, vol. 42, pp. 310–335, 2014.

[20] S. Makridakis, M. Hibon, and C. Moser, "Accuracy of forecasting: an empirical investigation," *Journal of the Royal Statistical Society. Series A (General)*, vol. 142, no. 2, p. 97, 1979.

[21] S. Makridakis, A. Andersen, R. Carbone et al., "The accuracy of extrapolation (time series) methods: Results of a forecasting competition," *Journal of Forecasting*, vol. 1, no. 2, pp. 111–153, 1982.

[22] A. K. Dixit and R. S. Pindyck, *Investment under uncertainty*, vol. 256, 1994.

[23] F. Black and M. Scholes, "The pricing of options corporate liabilities," *Journal of Political Economy*, vol. 81, pp. 637–659, 1973.

[24] N. Foo, H. Bloch, and R. Salim, "The optimisation rule for investment in mining projects," *Resources Policy*, vol. 55, pp. 123–132, 2018.

[25] P. Kalekar, "Time series forecasting using Holt-Winters exponential smoothing," *Kanwal Rekhi School of Information Technology*, Article ID 04329008, pp. 1–13, 2004, http://www.it.iitb.ac.in/~praj/acads/seminar/04329008_Exponential-Smoothing.pdf.

[26] L. Gómez-Valle and J. Martínez-Rodríguez, "Advances in pricing commodity futures: multifactor models," *Mathematical and Computer Modelling*, vol. 57, no. 7-8, pp. 1722–1731, 2013.

[27] M. Khashei and M. Bijari, "A novel hybridization of artificial neural networks and ARIMA models for time series forecasting," *Applied Soft Computing*, vol. 11, no. 2, pp. 2664–2675, 2011.

[28] M. Khashei and M. Bijari, "An artificial neural network (p, d, q) model for timeseries forecasting," *Expert Systems with Applications*, vol. 37, no. 1, pp. 479–489, 2010.

[29] A. Choudhury and J. Jones, "Crop yield prediction using time series models," *The Journal of Economic Education*, vol. 15, pp. 53–68, 2014.

[30] R. R. Marathe and S. M. Ryan, "On the validity of the geometric brownian motion assumption," *The Engineering Economist*, vol. 50, no. 2, pp. 159–192, 2005.

[31] G. E. Uhlenbeck and L. S. Ornstein, "On the theory of the Brownian motion," *Physical Review A: Atomic, Molecular and Optical Physics*, vol. 36, no. 5, pp. 823–841, 1930.

[32] Á. Tresierra Tanaka and C. M. Carrasco Montero, "Valorización de opciones reales: modelo ornstein-uhlenbeck," *Journal of Economics, Finance and Administrative Science*, vol. 21, no. 41, pp. 56–62, 2016.

[33] R. Nau, *Forecasting with Moving Averages*, Duke's Fuqua School of Business, Duke University, Durham, NC, USA, 2014, http://people.duke.edu/128.~rnau/Notes_on_forecasting_with_moving_averages–Robert_Nau.pdf.

[34] J. M. Woolridge, *Introductory Econometrics*, 2011.

[35] E. S. Gardner Jr., "Exponential smoothing: the state of the art-part II," *International Journal of Forecasting*, vol. 22, no. 4, pp. 637–666, 2006.

[36] R. G. Brown, *Smoothing, Forecasting and Prediction of Discrete Time Series*, Dover Phoenix Editions, Dover Publications, 2nd edition, 1963.

[37] G. Coutie et al., *Short-Term Forecasting, ICT Monogr. No. 2*, Oliver Boyd, Edinburgh, Scotland, 1964.

[38] P. J. Harrison, "Exponential smoothing and short-term sales forecasting," *Management Science*, vol. 13, no. 11, pp. 821–842, 1967.

[39] D. C. Montgomery and L. A. Johnson, *Forecasting and Time Series Analysis*, New York, NY, USA, 1976.

[40] C. Chatfield, "The holt-winters forecasting procedure," *Journal of Applied Statistics*, vol. 27, no. 3, p. 264, 1978.

[41] J. O. McClain and L. J. Thomas, "Response-variance tradeoffs in adaptive forecasting," *Operations Research*, vol. 21, pp. 554–568, 1973.

[42] J. O. McClain, "Dynamics of exponential smoothing with trend and seasonal terms," *Management Science*, vol. 20, no. 9, pp. 1300–1304, 1974.

[43] J. V. Issler and L. R. Lima, "A panel data approach to economic forecasting: the bias-corrected average forecast," *Journal of Econometrics*, vol. 152, no. 2, pp. 153–164, 2009.

[44] M. F. Hamid and A. Shabri, "Palm oil price forecasting model: an autoregressive distributed lag (ARDL) approach," in *Proceedings of the 3rd ISM International Statistical Conference 2016 (ISM-III): Bringing Professionalism and Prestige in Statistics*, Kuala Lumpur, Malaysia, 2017.

[45] H. Duan, G. R. Lei, and K. Shao, "Forecasting crude oil consumption in China using a grey prediction model with an optimal fractional-order accumulating operator," *Complexity*, vol. 2018, 12 pages, 2018.

[46] M. J. Brennan and E. S. Schwartz, "Evaluating natural resource investments," *The Journal of Business*, vol. 58, no. 2, pp. 135–157, 1985.

[47] R. McDonald and D. Siegel, "The value of waiting to invest," *The Quarterly Journal of Economics*, vol. 101, no. 4, pp. 707–727, 1986.

[48] K. Zhang, A. Nieto, and A. N. Kleit, "The real option value of mining operations using mean-reverting commodity prices," *Mineral Economics*, vol. 28, no. 1-2, pp. 11–22, 2015.

[49] R. K. Sharma, "Forecasting gold price with box jenkins autoregressive integrated moving average method," *Journal of International Economics*, vol. 7, pp. 32–60, 2016.

[50] J. Johansson, "Boliden capital markets day 2007," 2007.

[51] N. Svante, "Boliden capital markets day September 2011," 2011.

[52] Teck, "Teck modelling workshop 2012," 2012.

[53] A. Shaw, "AngloAmerican copper market outlook," in *Proceedings of the Copper Conference Brisbane*, vol. 24, Nanning, China, 2015, http://www.minexconsulting.com/publications/CRU.

[54] P. Willbrandt and E. Faust, "Aurubis DVFA analyst conference report 2013," 2013.

[55] P. Willbrandt and E. Faust, "Aurubis DVFA analyst conference report 2014," 2014.

[56] A. G. Aurubis, "Aurubis copper mail December 2015," 2015.

[57] D. B. Drouven and E. Faust, "Aurubis DVFA analyst conference report 2015," 2015.

[58] A. G. Aurubis, "Aurubis copper mail February 2016," 2016.

[59] EY, "EY mining and metals commodity briefcase," 2017.

[60] J. Schachler, *Aurubis AG interim report 2017*, Frankfurt, Germany, 2017.

[61] Y. Nakazato, "Aurubis AG interim report," 2017.