Research Article

A Multimode Dynamic Short-Term Traffic Flow Grey Prediction Model of High-Dimension Tensors

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Received 18 February 2019; Revised 21 April 2019; Accepted 5 May 2019; Published 3 June 2019

Academic Editor: Eric Campos-Canton

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Short-term traffic flow prediction is an important theoretical basis for intelligent transportation systems, and traffic flow data contain abundant multimode features and exhibit characteristic spatiotemporal correlations and dynamics. To predict the traffic flow state, it is necessary to design a model that can adapt to changing traffic flow characteristics. Thus, a dynamic tensor rolling nonhomogeneous discrete grey model (DTRNDGM) is proposed. This model achieves rolling prediction by introducing a cycle truncation accumulated generating operation; furthermore, the proposed model is unbiased, and it can perfectly fit nonhomogeneous exponential sequences. In addition, based on the multimode characteristics of traffic flow data tensors and the relationship between the cycle truncation accumulated generating operation and matrix perturbation to determine the cycle of dynamic prediction, the proposed model compensates for the periodic verification of the RSDGM and SGM grey prediction models. Finally, traffic flow data from the main route of Shaoshan Road, Changsha, Hunan, China, are used as an example. The experimental results show that the simulation and prediction results of DTRNDGM are good.

1. Introduction

Intelligent transportation systems (ITSs), which are traffic control management information systems, have been developed over the past few years. Traffic control systems can provide accurate real-time traffic information for traffic management and traffic guidance systems and consequently represent important components of intelligent traffic control networks. Short-term traffic flow predictions based on real-time information constitute preconditions for implementing real-time traffic control and management, and thus, they form an important theoretical basis for ITSs. Accordingly, traffic data provide an important foundation for the research and development of ITSs, and such data contain rich multimode features. However, determining how to fully use the multimodal features of traffic data under a unified framework remains a challenge.

Tensor models are generalizations of both vector models and matrix models as multidimensional models. In recent years, the use of tensors to represent multidimensional data with multimode features [1] has been shown to overcome the deficiencies of vector and matrix data forms with which it is difficult to characterize multidimensional features [2]. However, the multimode information of traffic flow can be analyzed in a tensor framework. The multiple modes can reflect the characteristics of the traffic system in many respects, although traditional time series and matrix modeling analysis methods are limited when analyzing the features of traffic data in multiple modes (e.g., at various temporal and spatial scales) in a unified framework. By constructing a “week-day-time” model of traffic data, better results can be obtained than those yielded by vector and matrix data formats [3–7]. This approach is conducive to maintaining the structure and characteristics of traffic data, which can be observed at different spatial and temporal scales and exhibit remarkable interconnected patterns. Accordingly, researchers have found that traffic data can exhibit multiple strong correlations among these patterns [8–11]. Additionally, the inherent correlations in traffic data can be analyzed according to the multimode tensor characteristics of traffic,
and the potential trends of traffic data can be determined to provide a basis upon which traffic systems process traffic data.

To date, many forecasting methods have been used to predict resource problems. These methods are divided into three categories: short-term traffic flow prediction techniques based on vector data flow (such as the wavelet analysis [12], support vector machine [13], the chaotic prediction model [14, 15], and neural network [16, 17]), short-term traffic flow prediction techniques based on matrix data flow [18–21] (such as the multivariate time series prediction model [18] and the Kalman filtering method [20]), and short-term traffic flow prediction techniques based on tensor data flow (such as the seasonal autoregressive integrated moving average + generalized autoregressive conditional heteroscedasticity (SARIMA + GARCH) model [22] and seasonal self-vector regression (Seasonal-SVR) prediction model [23]). The abovementioned prediction models are usually based on a large sample size and thus cannot be used to solve small-scale problems. However, short-term traffic flow prediction, which is employed to forecast the traffic flow state in a future period, is based on the dynamic data of road traffic flow states. The time interval and prediction period of these data are generally within 15 minutes. Consequently, the traffic flow data in the preceding time period will have the greatest impact on the subsequent period. The sampling rate also has a considerable impact on the prediction; for example, if the interval is one hour and the sampling interval is 5 minutes, only 12 groups of data will be collected in one hour, which constitutes a small sample of data. In contrast, the grey model (GM) involves a small sample size and very poor information, while the prediction model is adaptable and capable of more effectively handling changes in parameters.

Since grey theory was initially proposed in 1982 [24], the grey prediction model has formed the core component of grey system theory, and grey theory has been widely investigated. Accordingly, the development of GM (1,1), which constitutes the core grey prediction model, has been constantly improved and optimized [25–30]. This model has been widely applied in various fields [31–38], among which short-term traffic flow prediction is particularly important. For example, Hsu et al. [39] proposed an adaptive GM (1,1) model for traffic prediction in non detector intersections. In addition, Wen et al. [40] used the GM (1,1) model to predict air traffic flow and showed that the prediction capability of the GM (1,1) model is better than those of the ARIMA and multiple regression models through an experimental comparison. Guo et al. [41] established a grey nonlinear delay GM (1,1) model to predict short-term traffic flow on urban roads, and Lu et al. [42] used the nonlinear grey Bernoulli equation to obtain a grey prediction model for traffic flow prediction and achieved good results. Furthermore, Mao et al. [43] constructed a grey triangular GM (1,1) model to predict traffic flow fluctuations, while Xiao et al. [44] proposed a seasonal grey GM (1,1) rolling prediction model based on the cycle truncation accumulated generating operation (CTAGO). Yang et al. [45] established a coupling model with a close grey relational degree for weight determination based on the ARIMA model and the seasonal grey DGM (1,1) rolling model that obtained time sequence data and cross-section data at the intersection point. Bezuglov and Comert [46] used the Fu Liye error to improve the coupled prediction using the GM (1,1) and grey Verhulst models and applied the combined model to traffic flow prediction. Additionally, Ren et al. [47] established an improved grey GM (1,4) model for road traffic safety predictions in Germany.

The abovementioned grey models, which are based on either vector data flow (i.e., single-variable prediction models) or matrix data flow (i.e., multivariable prediction models), are used to predict short-term traffic flow. Unfortunately, it is often difficult to characterize traffic data in a unified framework with these linear or planar representations due to the limitations of data dimensionality (e.g., space, week, day, and time). Moreover, the prediction of future short-term traffic flow needs to take advantage of data collected in real time, and it requires traffic data that are acquired continuously (i.e., rolled over from preceding prediction periods) to predict traffic flow during future periods. In addition, data such as social network data streams and video and network data streams are often dynamic and have multiple modes. Consequently, researchers often construct these data sets as dynamic tensors. Dynamic tensors are used to characterize dynamic data with multiple patterns; furthermore, they can retain the original multimode characteristics of the data set and reflect the dynamic characteristics of the data. Dynamic tensors have achieved good results in practical applications.

Therefore, to better extract the multimode characteristics (e.g., the week, day, and time) of traffic data, this paper presents a rolling approximate nonhomogeneous discrete grey model (DTRNDGM model) based on the characteristics of dynamic tensor data and the characteristics of traffic flow data. The purpose of this paper is to utilize the dynamic tensor. Under the proposed framework, we can fully exploit the multimodal and dynamic characteristics of traffic data and improve the accuracy of short-term traffic flow forecasting. This model can fully employ the characteristics of real data; accordingly, affine transformation is utilized to study the unbiased prediction of an approximate nonhomogeneous exponential sequence by using the proposed model and to verify that the proposed model is completely fit for nonhomogeneous exponential sequences. The information priority principle and the optimal rolling cycle according to a correlation analysis of the "week-day-time" model are used in dynamic tensor model, thereby promoting the accuracy of the rolling cycle of the model composing the rolling grey SGM model [44] and the RSDGM [45] model cycles through data testing and promoting a model with a higher accuracy through calculations and predictions. Therefore, the research results in this paper are of great significance for the data structure of traditional grey prediction models and for improving their performance.

In the following chapters, we use abbreviations for different grey prediction models; these abbreviations and their meanings are listed in Table 1.

The remainder of this paper is arranged as follows. In the third chapter, the rolling nonhomogeneous discrete grey model (RNDGM) is established. In the third chapter, the multimode DTRNDGM is established. In the fourth chapter, the dynamic cycle of the new model is studied with the
### Table 1: Abbreviations and Corresponding Definitions for Grey Prediction Models

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Dynamic characteristics of the tensor model and traffic flow data using matrix perturbation analysis. Finally, the fifth chapter provides the conclusions.

## 2. Rolling Nonhomogeneous Discrete Grey Model (RNDGM)

In this section, a rolling NDGM is constructed based on the periodic truncated accumulating sequences of the original sequence, and the properties of the model are studied. Because the NDGM model has a good prediction effect on the approximate nonhomogeneous exponential series, an affine transformation is performed to investigate the parametric properties of the RNDGM, which can fully fit the sequences of nonhomogeneous exponential growth. The rolling NDGM model has the same effect on the approximate nonhomogeneous exponential series.

### 2.1. NDGM Model

Let $X^{(0)}(k)$ be the original data sequence:

$$X^{(0)}(k) = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)).$$  \(1\)

Meanwhile, $X^{(1)}$ generates the 1-AGO sequence for a single accumulation:

$$X^{(1)}(k) = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)),$$  \(2\)

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n$.

**Definition 1.** Let the sequences $X^{(0)}(k), X^{(1)}(k)$ follow (1) and (2), respectively; then

$$x^{(1)}(k + 1) = \beta_1 x^{(1)}(k) + \beta_2 k + \beta_3$$  \(3\)

is a grey system predictive model composed of first-order equations with one variable, abbreviated as the NDGM (1,1) model \([18]\), and the recursive function is

$$\tilde{x}^{(1)}(k + 1) = \beta_1 \tilde{x}^{(1)}(1) + \beta_2 \sum_{j=1}^{k} j \rho_1^{k-j} + \frac{1 - \beta_1}{1 - \beta_1} \beta_3,$$  \(4\)

$$k = 1, 2, \ldots, n - 1.$$

The restored NDGM (1,1) value is as follows:

$$\tilde{x}^{(0)}(k + 1) = \tilde{x}^{(1)}(k + 1) - x^{(1)}(k)$$  \(5\)

### 2.2. Rolling Forecast NDGM Model

**Definition 2** (see \([44]\)). Let $x^{(0)}(k) = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ be the original sequence, and

$$y^{(0)}(k) = \text{CTAGO}(x^{(0)}(k)) = \sum_{j=1}^{q} x^{(0)}(k + j - 1)$$  \(6\)

∀$k = 1, 2, \ldots, n - q + 1,$

where CTAGO has been defined above, and $q$ is the number of elements contained in each cycle. If $r = n - q + 1$, then $y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(r))$ is a CTAGO sequence.

**Definition 3** (see \([44]\)). Let $y^{(1)}(k) = (y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(r))$, where

$$y^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i), \quad k = 1, 2, \ldots, r,$$  \(7\)

where $y^{(1)}(k)$ is called the 1-AGO sequence of CTAGO sequence $y^{(0)}(k)$.

The following lemmas describe the properties of CTAGO sequence $y^{(0)}(k)$ and 1-AGO sequence $y^{(1)}(k)$.

**Lemma 4.** Let $x^{(0)}(k) = ac^k + b$ have nonhomogeneous exponential data features; then, the CTAGO transform sequence $y^{(0)}(k)$, which is $y^{(0)}(k) = ac^k + d$, has the same data characteristics.

**Proof.** Bring $x^{(0)}(k) = ac^k + b$ into $y^{(0)}(k)$:

$$y^{(0)}(k) = \sum_{j=1}^{q} x^{(0)}(k + j - 1) = \sum_{j=1}^{q} (ac^{k+j-1} + b)$$

$$= \sum_{j=1}^{q} ac^{k+j-1} + qb = ac^k + \left( \sum_{j=2}^{q} ac^{k+j-1} + qb \right)$$  \(8\)

$$= ac^k + d$$

The following properties can be obtained from \([44]\).

**Lemma 5** (see \([44]\)). The primary accumulated sequence of CTAGO transform sequence $y^{(0)}(k)$ of the original sequence
\( x^{(0)}(k) \) is associated with a grey index trend and has a positive grey index value of 1.

**Definition 6.** Let the sequences \( y^{(0)}(k), y^{(1)}(k) \), follow Definitions 2 and 3; then

\[
y^{(1)}(k + 1) = \beta_1 y^{(1)}(k) + \beta_2 k + \beta_3
\]

where

\[
\begin{pmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{pmatrix} = (B^T B)^{-1} B^T Y,
\]

\[
B = \begin{pmatrix}
y^{(1)}(1) & 1 & 1 \\
y^{(1)}(2) & 2 & 1 \\
\vdots & \vdots & \vdots \\
y^{(1)}(r - 1) & r - 1 & 1
\end{pmatrix}_{(r-1) \times 3}
\]

\[
Y = \begin{pmatrix}
y^{(1)}(2) \\
y^{(1)}(3) \\
\vdots \\
y^{(1)}(r)
\end{pmatrix}_{r-1}
\]

This rolling approximation nonhomogeneous discrete grey model based on the CTAGO transformation sequence is referred to as RNDGM. Let

\[
A = \begin{pmatrix}
1 & 0 & 0 & \cdots & 0 \\
1 & 1 & 0 & \cdots & 0 \\
1 & 1 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \cdots & 1
\end{pmatrix}_{r}
\]

\[
G = \begin{pmatrix}
1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & \cdots & 1 & \cdots & 1
\end{pmatrix}_{r \times r}
\]

According to Definitions 2, 3, and 6, the following properties can be defined.

**Property 7.** The RNDGM matrices B and Y can be written as follows:

\[
B = \begin{pmatrix}
y^{(1)}(1) & 1 & 1 \\
y^{(1)}(2) & 2 & 1 \\
\vdots & \vdots & \vdots \\
y^{(1)}(r - 1) & r - 1 & 1
\end{pmatrix}_{(r-1) \times 3}
\]

\[
= \begin{pmatrix}
1 & 0 & \cdots & 0 \\
1 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 1 & \cdots & 1
\end{pmatrix}_{r-1}
\]

\[
Y = \begin{pmatrix}
\frac{y^{(0)}(1)}{q} & \frac{1}{q} \\
\frac{y^{(0)}(2)}{q} & 0 \\
\vdots & \vdots \\
\frac{y^{(0)}(n)}{q} & 0
\end{pmatrix}_{r-1}
\]

\[
= A_1 G \begin{pmatrix}
\frac{x^{(0)}(1)}{q} & \frac{1}{q} \\
\frac{x^{(0)}(2)}{q} & 0 \\
\vdots & \vdots \\
\frac{x^{(0)}(n)}{q} & 0
\end{pmatrix}_{r-1}
\]
\[
\begin{pmatrix}
1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 1 & 1 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & \cdots & 1
\end{pmatrix}_{m \times n}
\]
\[
\begin{pmatrix}
x^{(0)} (1) \\
x^{(0)} (2) \\
\vdots \\
x^{(0)} (n)
\end{pmatrix}
= A_2 G_2
\begin{pmatrix}
x^{(0)} (1) \\
x^{(0)} (2) \\
\vdots \\
x^{(0)} (n)
\end{pmatrix}
\]

(12)

**Theorem 8.** (1) The time response of RNDGM is
\[
y^{(1)} (k + 1) = \beta_1 y^{(1)} (1) + \beta_2 \sum_{j=1}^{k} j \beta_1^{k-j} \frac{1 - \beta_1^k}{1 - \beta_1},
\]
where \( y^{(1)} (1) = y^{(1)} (1) \).

(2) The time response of the CTAGO sequence \( y^{(0)} (k) \) is
\[
y^{(0)} (k + 1) = y^{(1)} (k + 1) - y^{(1)} (k),
\]
where \( k = 1, 2, \ldots, n - 1 \).

(3) The time response of \( x^{(0)} (k) \) is
\[
x^{(0)} (k + 1) = y^{(0)} (k - q + 2) - y^{(0)} (k - q + 1) + x^{(0)} (k - q + 1),
\]
where \( k = q, q + 1, \ldots, n \).

**Proof.** (1) is obviously available from (4).

(2) is known from the definition of the sequence.

(3) \( y^{(0)} (k + 1) - y^{(0)} (k) = \sum_{j=1}^{q} x^{(0)} (k + j) - \sum_{j=1}^{q} x^{(0)} (k + j - 1) = x^{(0)} (k + 1) + \cdots + x^{(0)} (k + q - 1) + x^{(0)} (k + q) - \left[ x^{(0)} (k + 1) + \cdots + x^{(0)} (k + q - 1) \right] = x^{(0)} (k + q) - x^{(0)}(k) \) [9].

Thus
\[
x^{(0)} (k + 1) = y^{(0)} (k - q + 2) - y^{(0)} (k - q + 1) + x^{(0)} (k - q + 1),
\]
where \( k = q, q + 1, \ldots, n \).

The least squares estimation from Definition 6 can directly get Property 9.

**Property 9.** The parameter package of RNDGM is as follows:
\[
P_I = (\beta_1, \beta_2, \beta_3)^T = \frac{1}{M_4} \left[ M_1, M_2, M_3 \right]^T
\]
\[
P_{II} = \left[ M_4, M_1, M_2, M_3 \right]
\]

where
\[
C = \sum_{k=1}^{n-1} y^{(1)} (k)^2,
\]
\[
D = \sum_{k=1}^{n-1} y^{(1)} (k),
\]
\[
E = \sum_{k=1}^{n-1} y^{(1)} (k),
\]
\[
F = \sum_{k=1}^{n-1} k,
\]
\[
G = \sum_{k=1}^{n-1},
\]
\[
H = \sum_{k=1}^{n-1} y^{(1)} (k) y^{(1)} (k + 1),
\]
\[
I = \sum_{k=1}^{n-1} k y^{(1)} (k + 1),
\]
\[
M = \sum_{k=1}^{n-1} y^{(1)} (k + 1),
\]
\[
N = n - 1.
\]

\[
M_4 = CFN + 2DGE - CG^2 - ND^2 - FE^2.
\]
\[
M_1 = \left( FN - G^2 \right) H + \left( EG - DN \right) I + \left( DG - EF \right) M,
\]
\[
M_2 = \left( EG - DN \right) H + \left( CN - E^2 \right) I + \left( DE - CG \right) M,
\]
\[
M_3 = \left( DG - EF \right) H + \left( DE - CG \right) I + \left( CF - D^2 \right) M.
\]

In this section, the parameter characteristics of RNDGM starting from the affine transformation are investigated. Then, according to the intrinsic links among the model parameters and the sequence forms, the simulation prediction accuracy of the model is determined. The specific properties of this method are given by the following theorem.

**Theorem 10.** For \( x^{(0)}(k) = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), an affine transformation is performed to obtain the sequence \( z^{(0)}(k) = (\rho x^{(0)}(1) + \omega, \rho x^{(0)}(2) + \omega, \ldots, \rho x^{(0)}(n) + \omega) \), and the parameters before and after the change are set to \( (\beta_1, \beta_2, \beta_3)^T, (\beta_1', \beta_2', \beta_3')^T \) where \( y^{(0)}(k) \) is a CTAGO sequence of \( x^{(0)}(k) \), \( y_1^{(1)}(k) \) is the 1-AGO sequence of \( y^{(0)}(k) \), \( y_1^{(0)}(k) \) is
a CTAGO sequence of $z^{(0)}(k)$, $y^{(1)}(k)$ is called the 1-AGO sequence of $y^{(0)}(k)$, and

$$
\begin{pmatrix}
\beta'_1 \\
\beta'_2 \\
\beta'_3
\end{pmatrix} = 
\begin{pmatrix}
\beta_1 \\
\beta_2 + (1 - \beta_1)q\omega \\
\rho\beta_3 + q\omega
\end{pmatrix}.
\quad (21)
$$

Proof. The affine sequence can yield the transformed parameter sequence as follows:

$$
C' = \sum_{k=1}^{n-1} y^{(1)}(k)^2 
= \sum_{k=1}^{n-1} \left[ \sum_{i=1}^{q} (\rho \sum_{j=1}^{q} \beta^{(0)}(i+j-1) + \omega) \right]^2 
= \sum_{k=1}^{n-1} \left[ \rho y^{(1)}(k) + k\omega \right]^2 
= \rho^2 C + 2\rho q\omega D + q^2 \omega^2 F.
\quad (22)
$$

Similarly, we can see that

$$
\beta'_1 = \frac{M'_1}{M'_4} = \frac{\rho^2 M_1}{\rho^2 M_4} = \frac{M_1}{M_4} = \beta_1,
\beta'_2 = \frac{M'_2}{M'_4} = \frac{\rho^3 M_2 + \rho^2 q\omega (M_4 - M_1)}{\rho^2 M_4} = \rho \beta_2 + q\omega (1 - \beta_1),
\beta'_3 = \frac{M'_3}{M'_4} = \frac{\rho^3 M_3 + \rho^2 q\omega M_4}{\rho^2 M_4} = \rho \frac{M_3}{M_4} + q\omega
= \rho \beta_3 + q\omega.
\quad (26)
$$

This proves that the conclusion is true.

\[ \square \]

**Theorem 11.** Let $\tilde{x}^{(0)}(k)$ and $\tilde{z}^{(0)}(k)$ be the RNDGM fitting values of the original sequence $x^{(0)}(k)$ and the affine transformation sequence $z^{(0)}(k)$, respectively. Thus,

$$
\tilde{z}^{(0)}(k) = \rho \tilde{x}^{(0)}(k) + \omega.
\quad (27)
$$

Proof. Based on Theorem 8(2), (3), $y^{(0)}(k)$ is a CTAGO sequence of $x^{(0)}(k)$ and $y^{(1)}(k)$ is called the 1-AGO sequence of $y^{(0)}(k)$. We have

$$
\tilde{x}^{(0)}(k+1) = \tilde{y}^{(0)}(k - q + 2) - y^{(0)}(k - q + 1) + x^{(0)}(k - q + 1) + y^{(0)}(k - q + 1) + x^{(0)}(k - q + 1)
= \beta_3^{k-q+1} y^{(1)}(1) + \beta_2 \sum_{j=1}^{k-q+1} \beta_1^{k-q+1-j} + \frac{1 - \beta_1^{k-q+1}}{1 - \beta_1}.
\quad (24)
$$

Similarly, we can see that

$$
M'_1 = \rho^2 M_1,
M'_2 = \rho^3 M_2 + \rho^2 q\omega (M_4 - M_1),
M'_3 = \rho^3 M_3 + \rho^2 q\omega M_4
\quad (25)
$$

Therefore,

$$
\begin{align*}
\beta'_1 &= \frac{M'_1}{M'_4} = \frac{\rho^2 M_1}{\rho^2 M_4} = \frac{M_1}{M_4} = \beta_1, \\
\beta'_2 &= \frac{M'_2}{M'_4} = \frac{\rho^3 M_2 + \rho^2 q\omega (M_4 - M_1)}{\rho^2 M_4} = \rho \beta_2 + q\omega (1 - \beta_1), \\
\beta'_3 &= \frac{M'_3}{M'_4} = \frac{\rho^3 M_3 + \rho^2 q\omega M_4}{\rho^2 M_4} = \rho \frac{M_3}{M_4} + q\omega
= \rho \beta_3 + q\omega.
\end{align*}
\quad (26)
$$

This proves that the conclusion is true. \[ \square \]
\[
\beta_3 = \left( \beta_1^{k-q-1} \cdot y_1^{(1)}(1) + \beta_2 \sum_{j=1}^{k-1} j \beta_1^{k-q-1-j} \right)
\]
\[
+ \frac{1 - \beta_1^{k-q-1}}{1 - \beta_1} \cdot \beta_3 - y^{(0)}(k - q + 1) + x^{(0)}(k - q + 1)
\]
\[
+ 1) = \left[ \left( 1 - \frac{1}{\beta_1} \right) \bar{y}_1^{(1)}(1) + \left( \frac{\beta_3}{\beta_1} - \frac{\beta_2}{1 - \beta_1} \right) \right]
\]
\[
\beta_1^{k-q+1} + \frac{\beta_2}{1 - \beta_1} - y^{(0)}(k - q + 1) + x^{(0)}(k - q + 1)
\]
\[
+ 1).
\]
\[
(28)
\]

According to Definition 3 and Theorem 8 (1), after the affine transformation, we can obtain the following equation from formula (13):
\[
\bar{y}_1^{(1)}(1) = y_1^{(1)}(1) = z^{(0)}(1) = \sum_{j=1}^{q} (\rho x^{(0)}(j) + \omega)
\]
\[
(29)
\]

Based on Theorem 8(2), (3), \( y_1^{(0)}(k) \) is a CTAGO sequence of \( z^{(0)}(k) \) and \( y_1^{(1)}(k) \) is called the 1-AGO sequence of \( y_1^{(0)}(k) \). We have
\[
\bar{z}^{(0)}(k + 1) = \bar{y}_1^{(0)}(k - q + 2) - y_1^{(0)}(k - q + 1)
\]
\[
+ z^{(0)}(k - q + 1) = \bar{y}_1^{(1)}(k - q + 2) - \bar{y}_1^{(1)}(k - q)
\]
\[
+ 1) - y_1^{(0)}(k - q + 1) + z^{(0)}(k - q + 1)
\]
\[
= \beta_1^{k-q+1} \cdot \bar{y}_1^{(1)}(1) + \beta_2 \sum_{j=1}^{k-q+1} j \beta_1^{k-q-1-j}
\]
\[
+ \frac{1 - \beta_1^{k-q+1}}{1 - \beta_1} \cdot \beta_3 - \left( \beta_1^{k-q-1} \cdot \bar{y}_1^{(1)}(1) \right)
\]
\[
+ \frac{\beta_2}{1 - \beta_1} \sum_{j=1}^{k-q-1-j} j \beta_1^{k-q-1-j} + \frac{1 - \beta_1^{k-q-1}}{1 - \beta_1} \cdot \beta_3 - y_1^{(0)}(k - q)
\]
\[
+ 1) + x^{(0)}(k - q + 1) = \left[ \left( 1 - \frac{1}{\beta_1} \right) \bar{y}_1^{(1)}(1) \right]
\]
\[
\bar{z}^{(0)}(k + 1) = \left( \left( 1 - \frac{1}{\beta_1} \right) \bar{y}_1^{(1)}(1) + \frac{\beta_3}{\beta_1} - \frac{\beta_2}{1 - \beta_1} \right)
\]
\[
\beta_1^{k-q+1} + \frac{\beta_2}{1 - \beta_1} - y_1^{(0)}(k - q + 1)
\]
\[
+ x^{(0)}(k - q + 1) + \omega = \rho \bar{x}^{(0)}(k + 1) + \omega.
\]
\[
(30)
\]

Theorems 10 and 11 show that the predicted values obtained before and after the affine transformation of RNDGM exhibit an affine transformation relationship, indicating that RNDGM shows no bias in the prediction of nonhomogeneous index sequences. The following inferences can be derived from this finding.

Inference 12. For a nonhomogeneous index sequence \( x^{(0)}(k) \), the restored value of RNDGM can be determined as follows:
\[
\hat{x}^{(0)}(k) = \lambda a^k + \delta, \quad k = 1, 2, \ldots, n.
\]
\[
\lambda = \left( 1 - \frac{1}{\beta_1} \right) \bar{y}_1^{(1)}(1) + \left( \beta_3 - \frac{\beta_2}{1 - \beta_1} \right),
\]
\[
(32)
\]
\[
\delta = \frac{\beta_2}{1 - \beta_1} - y^{(0)}(k - q + 1) + x^{(0)}(k - q + 1).
\]

Lemmas 4 and 5 reflect the data characteristics of the original sequence \( x^{(0)}(k) \) and the CTAGO transformed sequence \( \bar{y}^{(0)}(k) \) in RNDGM. Theorem 8 and the related inferences reflect the parameter characteristics of RNDGM. According
to the direct intrinsic relationships among the model parameters and the sequence index forms in a nonhomogeneous exponential sequence, RNDGM can fully fit a sequence of nonhomogeneous exponential growth.

3. Dynamic Tensor Rolling Nonhomogeneous Discrete Grey Model (DTRNDGM)

In practical applications, multimode correlations exist among traffic data at various scales. Thus, multimode correlations of traffic data can be used to construct different traffic data tensor models, which should fully consider the multimode information associated with traffic data.

3.1. Establishing a Traffic Data Tensor Model. Because traffic data exhibit strong spatiotemporal correlations, multiple traffic flow data tensor models can be constructed in a combined model. For example, the traffic volume of a certain road segment can be expressed as tensor data in a “collector-week-day-time” model as follows:

\[ \chi \in R^{L \times W \times D \times T}, \quad L = 11, \ W = 11, \ D = 7, \ T = 288 \]  \hspace{1cm} (33)

where \( L \) represents a loop/collector (i.e., the model has 11 collectors), \( W \) represents the number of weeks of data, \( D \) represents the number of days with data per week, and \( T \) represents the time. In this case, 288 traffic flows per day are analyzed.

If the above model is composed of traffic data collected by a single collector, it can be expressed in a “week-day-time” format as a third-order tensor: \( \chi \in R^{W \times D \times T} \). If the tensor \( T = 288 = 24 \times 12 \) described above includes data collected every five minutes over the course of a day (i.e., 12 times an hour), then a fifth-order tensor \( \chi \in R^{11 \times 1 \times 7 \times 24 \times 12} \) model can be constructed. For a time series, different tensor models can be constructed based on different acquisition times. For example, for data collected once every 10 minutes over the course of a day (i.e., \( 24 \times 12 \) minutes), a fifth-order tensor \( \chi \in R^{11 \times 1 \times 7 \times 24 \times 12} \) model can be built. The time series can also be expanded to two or three weeks. To intuitively describe the traffic flow tensor model, a typical third-order tensor can be constructed based on a combination of transverse time series, longitudinal time series, and spatial time series. A schematic diagram for a third-order tensor is shown in Figure 1. Thus, different tensor models can be constructed according to different requirements.

3.2. Establishing DTRNDGM. As discussed above, traffic data exhibit multimodal correlations. The correlation of the weekly model is the highest in the traffic tensor model. However, the data for the RSDGM require that the length of the data sequence be kept constant (i.e., when new data are introduced, old data must be eliminated). Replacing old information with new information allows a one-step dynamic rolling prediction with the grey model. Furthermore, the length of the model is closely related to the prediction accuracy, and the correlation of the cycle model in the traffic tensor model is the highest. Accordingly, a higher precision is associated with a higher correlation when the cycle length is

\( q = 7 \). During the rolling prediction of traffic flow, when the cycle length is \( q = 7 \), which is the same as the length of the data test in [44, 45], the simulation precision is the highest. According to the cycle mode of the traffic flow tensor model and a forecast of the traffic flow data with a cycle length of \( q = 7 \), the dynamic tensor model is defined as follows.

Definition 13. Consider the traffic flow during the \( [t, t + \alpha] \) time period, in which \( [D_{2q+1} - \alpha, D_{2q+1} - 2\alpha, \ldots, D_{2q+1} - ma,] \) are the traffic flow data observed in real time, where \( m \) is the length of the interval; here, \( m = 2q = 14 \). These traffic data can be used to predict the traffic flow of the matrix \( [D_{2q+1}, D_{2q+1} + \alpha, \ldots, D_{2q+1} + h\alpha,] \), where \( D_{2q+1} \) indicates the starting point of the prediction, \( \beta_i \) indicates the interval of time, and \( h \) indicates the range of the prediction. If the matrix \( [D_{2q+1} - \alpha, D_{2q+1} - 2\alpha, \ldots, D_{2q+1} + h\alpha,] \) for every day during the same interval \( [t, t + \alpha] \) is regarded as a dynamic matrix, then it is called a dynamic tensor model.

Definition 14. Establish a dynamic tensor rolling nonhomogeneous discrete grey model (DTRNDGM) by combining the dynamic tensor model of Definition 13 and RNDGM of Definition 6.

During the prediction and simulation processes with DTRNDGM, real-time observations of traffic flow data over an interval of \( m = 2q \) affect the simulation and prediction accuracy of the model. According to the strongest correlation of the tensor data, we can select a cycle length of \( q = 7 \). If parameter \( q \) is small, then the resulting lack of information will distort the prediction. In contrast, if parameter \( q \) is large, then it may cause data redundancy, following which an optimal prediction cannot be obtained. From the characteristics of the model itself, we will illustrate the prediction effect of parameter \( q \) and introduce new information priority and modeling concepts derived mainly from the perturbation [44, 45] of the matrix. First, we introduce several related lemmas [48].

Lemma 15 (see [48]). Let \( A \in C^{m \times n}, b \in C^m \), \( A^t \) be the generalized inverse of the matrix \( A \), \( B_0 = A + E \), and \( c = b + k \in C^m \). Additionally, the linear least square problems are
\[ \|B_1x-c\|_2 = \min \|Ax-b\|_2 = \min, \text{ and the corresponding solutions to these problems are } x + \Delta x \text{ and } x. \text{ If } \text{rank}(A) = \text{rank}(B_1) = n, \text{ and } \|A^T\|_2\|E\|_2 < 1, \text{ then} \]
\[ \|h\| \leq \frac{\kappa_+}{\gamma_1} \left( \frac{\|E\|_2}{\|A\|} \|x\| + \frac{\|k\|}{\|A\|} \frac{\|r_x\|}{\|A\|} \right), \tag{34} \]

where \( \kappa_+ = \|A^T\|_2\|A\| \), \( \gamma_1 = 1 - \|A^T\|_2\|E\|_2 \), and \( r_x = b - Ax \).

**Lemma 16 (see [44]).** The length of the original data sequence \( x^{(0)}(k) \) is \( n \), and the period is \( q \). Therefore, the length of the periodic truncated accumulated sequence \( y^{(0)}(k) \) is \( r = n-q+1 \). To satisfy the new information priority principle of the grey model, \( n = 2q-1 \).

The following subsection discusses whether the corresponding weights of the original data in DTRNDGM satisfy the weight priority principle of the data corresponding to the previous period. The following theorems are presented.

**Theorem 17.** If the length of \( x^{(0)}(k) \) is \( n = 2q-1 \), then the length of the periodic truncated accumulated sequence \( y^{(0)}(k) \) is \( r = q \). According to the least squares method, if only disturbances \( \hat{x}^{(0)}(t) = x^{(0)}(t) + \epsilon(t=1,2,\ldots,n-1,n) \) occur, then \( B \) is the same as in Property 7, and \( \|B^T\|_2\|\Delta B\|_2 < 1 \). Additionally, the perturbation bound of the RNDGM parameter estimates is \( L(x^{(0)}(t)) \); therefore,

\[ L\left(x^{(0)}(1)\right) < L\left(x^{(0)}(2)\right) < \cdots < L\left(x^{(0)}(q)\right), \]
\[ L\left(x^{(0)}(q)\right) > L\left(x^{(0)}(q+1)\right) > \cdots > L\left(x^{(0)}(2q-1)\right). \tag{35} \]

In other words,

\[ L\left(x^{(0)}(q)\right) = \max \{L\left(x^{(0)}(1)\right), L\left(x^{(0)}(2)\right), \ldots, L\left(x^{(0)}(2q-2)\right), L\left(x^{(0)}(2q-1)\right)\}. \tag{36} \]

**Proof.** If only disturbances \( \hat{x}^{(0)}(1) = x^{(0)}(1) + \epsilon \) occur, then the following is true according to Lemma 15:

\[ Y + \Delta Y_1 = A_1G_1 \begin{pmatrix} x^{(0)}(1) + \epsilon \\ x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{pmatrix} = Y + A_1G_1 \begin{pmatrix} \epsilon \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \]

\[ B + \Delta B_1 = A_1G_1 \begin{pmatrix} 1 \& 1 \\ 1 \& 0 \\ \vdots \\ 1 \& 0 \end{pmatrix} = B + A_1G_1 \begin{pmatrix} \epsilon \& 0 \\ 0 \& 0 \\ \vdots \end{pmatrix}, \]

\[ \Delta Y_1 = A_1G_1 \begin{pmatrix} \epsilon \\ \vdots \\ \epsilon \end{pmatrix}, \]

\[ \Delta B_1 = A_1G_1 \begin{pmatrix} \epsilon \\ \vdots \\ \epsilon \end{pmatrix}, \]

\[ \|\Delta Y_1\|_2 = \sqrt{r-1} |\epsilon|, \]

\[ \|\Delta B_1\|_2 = \sqrt{\lambda_{\max}(\Delta B_1^T\Delta B_1)} = \sqrt{(r-1)\epsilon^2} = \sqrt{r-1} |\epsilon|. \tag{37} \]

Letting \( r = q \),

\[ \|\Delta B_1\|_2 = \sqrt{q-1} |\epsilon|, \]

\[ \|\Delta Y_1\|_2 = \sqrt{q-1} |\epsilon|. \tag{38} \]
Therefore,

\[
L(x^{(0)}(1)) = \frac{k_t}{\gamma_t} \left( \frac{\|\Delta B\|}{\|B\|} \|x\| + \frac{\|\Delta Y\|}{\|B\|} \right) + \frac{k_t}{\gamma_t} \|r_x\| \|B\| \quad (39)
\]

\[
= \sqrt{q-1} |\epsilon| k \quad (40)
\]

Therefore,

\[
L(x^{(0)}(1)) = \frac{k_t}{\gamma_t} \left( \frac{\|\Delta B\|}{\|B\|} \|x\| + \frac{\|\Delta Y\|}{\|B\|} \right) + \frac{k_t}{\gamma_t} \|r_x\| \|B\| \quad (39)
\]

\[
= \sqrt{q-1} |\epsilon| k \quad (40)
\]

If only disturbances \(\tilde{x}^{(0)}(t) = x^{(0)}(t) + \epsilon, 2 \leq t \leq q, t \in \mathbb{Z}\) occur, then \(\Delta Y\) and \(\Delta B\) change accordingly.

\[
B + \Delta B = A_1 G_1 \quad (39)
\]

\[
\Delta B_1 = A_1 G_1 \quad (39)
\]

\[
\Delta Y_1^T \Delta Y_1 = 4\epsilon^2 + t\epsilon^2 + \cdots + t^2 \epsilon^2
\]

\[
= \left[ (4 + 9 + \cdots + t) \right] \epsilon^2 = M \epsilon^2, \quad (40)
\]

\[
\Delta B_1^T \Delta B_1 = (\epsilon^2 + 4\epsilon^2 + \cdots + t^2 \epsilon^2 + \cdots + t^2 \epsilon^2) \quad (40)
\]

\[
= \epsilon^2 + M \epsilon^2 - t^2 \epsilon^2 \quad (40)
\]

Therefore,

\[
\|\Delta Y_1\|_2 = \sqrt{M} |\epsilon|, \quad 2 \leq t \leq q.
\]

\[
\|\Delta B_1\|_2 = \sqrt{1 + M - t^2} |\epsilon|, \quad 2 \leq t \leq q. \quad (41)
\]

Obviously, \(\sqrt{M}\) is included in this equation, and \(\sqrt{1 + M - t^2}\) increases with \(t\). Thus,

\[
L(x^{(0)}(1)) < L(x^{(0)}(2)) < \cdots < L(x^{(0)}(q)). \quad (42)
\]

Similarly, we can see that, when disturbances occur,

\[
\tilde{x}^{(0)}(t) = x^{(0)}(t) + \epsilon, \quad t \in \mathbb{Z}, \quad q + 1 \leq t \leq 2q - 2. \quad (43)
\]
Next, let

\[
N = \sum_{j=1}^{n-t+1} j^2,
\]

\[
\|\Delta B\|_2 = \sqrt{N + (n-t)^2} |\epsilon|,
\]

\[
\|\Delta Y\|_2 = \sqrt{N} |\epsilon|,
\]

and let \(\|\Delta B_{2q-1}\|_2 = 0\). Thus,

\[
L(x^{(0)}(t)) = \frac{\kappa_1}{\gamma_1} |\epsilon| \left( \frac{\sqrt{N - (n-t)^2}}{|B|} \|x\| + \frac{\sqrt{N}}{|B|} \right),
\]

\[
L(x^{(0)}(2q-1)) = \frac{\kappa_1}{\gamma_1} |\epsilon| \left( \frac{\sqrt{N - (n-t)^2}}{|r_0|} \|x\| + \frac{\sqrt{N}}{|r_0|} \right),
\]

From the above formula, we can obtain

\[
L(x^{(0)}(q)) > L(x^{(0)}(q+1)) > \cdots > L(x^{(0)}(2q-1)).
\]  \(\Box\)

Because \(\tilde{x}^{(0)}(2q) = j^{(0)}(q+1) - x^{(0)}(q) + x^{(0)}(q)\), Theorem 17 shows that, in the case of the same disturbance, the parameter perturbation bounds are the largest when \(\tilde{x}^{(0)}(2q)\), indicating that the influence of the parameter estimation in \(x^{(0)}(q)\) greater and can be understood as the maximum weight of the data. In brief, DTRNDGM considers the new information priority principle of the grey model and the periodic correlation of traffic flow data. Therefore, the number of original data points in the sequence is kept unchanged at \(m = 2q = 14\), and new data are introduced continuously while old data are excluded. The DTRNDGM process can be described as follows.

Step 1. Select the tensor model based on the raw data.

Step 2. Extract data according to traffic tensor “week-day-time” model

\[
x_1^{(0)} = x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(q), x^{(0)}(q+1), \ldots,
\]

\[
x^{(0)}(2q).
\]  \(47\)

Step 3. Process CTAGO sequences, \(y^{(0)}(q)\), and accumulated CTAGO sequences \(y^{(1)}(q)\).


Step 5. For the first time, model and calculate the above sequence, and then use DTRNDGM to predict \(\tilde{x}^{(0)}(2q+1)\).

Step 6. Update the information in real time, remove the old information in \(x^{(0)}(1)\), add new observations in \(x^{(0)}(2q+1)\), and then obtain data in the sequence

\[
x_2^{(0)} = x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(q), x^{(0)}(q+1), \ldots,
\]

\[
x^{(0)}(2q),
\]  \(48\)

using DTRNDGM to predict \(\tilde{x}^{(0)}(2q+2)\).

Step 7. Repeat Step 3 for all data points that need to be predicted.

Step 8. Calculate the error of the simulated predictions. Substitute the values in Steps 2–5 to calculate the simulated and predicted values. The sequences \(\tilde{x}^{(0)}(k)\) and \(x^{(0)}(k)\) constitute the raw data of Step 1.

The above rolling prediction process is shown schematically in Figure 2.


4.1. Data Analysis Using the Traffic Tensor Model. The data used in this study were obtained from the Institute of Urban Transport, School of Transportation Engineering, Central South University [49]. The data were collected from the four straight lanes (from south to north) of Shaoshan Road in Changsha, China, by collectors every five minutes. The traffic flow data were acquired from 8:00 to 9:00 for 22 days from October 14, 2013, to November 4, 2013. The details are presented in Table 2.

The data in Table 2 are only the data with a five-minute sampling interval from 8:00–9:00 in the morning. Over the course of one day, data are collected 288 times. Thus, the third-order tensor form of the traffic data “week-day-time” model can be obtained as \(x \in R^{3\times7\times24\times88}\). Alternatively, the fourth-order tensors \(x \in R^{3\times7\times24\times12}\), \(x \in R^{3\times7\times24\times12}\), \(x \in R^{3\times7\times24\times3}\) can be obtained. Among them, the number 3 represents 3 weeks of data, 7 represents 7 days a week, 24 represents 24 hours a day, 12 represents a 5-minute collection, 4 represents a 15-minute collection, 3 represents a 20-minute collection, and 1 represents an hourly collection. The four tensor models described above are shown in Figures 3(a)–3(d).

4.2. Analysis of the DTRNDGM Rolling Prediction Method Based on Tensor Data. Figure 3 shows four typical tensor models for 22 days of data from October 14, 2013, to November 4, 2013. The data were collected from Shaoshan Road, Changsha, China. The first dynamic tensor model that corresponds to Figure 3(a) is \(x \in R^{3\times7\times24\times1}\), and the period \([t_j, t_j + \alpha]\) represents the morning peak period from 8:00 to 9:00. The element \(D_{2q+1} - \alpha_\epsilon, D_{2q+1} - 2\alpha_\epsilon, \ldots, D_{2q+1} + h_\alpha\) reflects the traffic flow in the same period on October 14, 2013, and \(D_{2q+1} + h_\alpha\) represents the traffic flow in the same period on the 22nd
day (November 4). The duration of the real-time data set is 14 days. After the prediction, the oldest data are promptly removed, and new data are reentered. The computational complexity of this algorithm is minimal. Therefore, compared with other grey models, rolling prediction does not increase the complexity of the calculations. This approach uses the latest information and prioritizes new information. According to the modeling process shown in Figure 2, the dynamic tensor model in Figure 3(a) is simulated and predictions are made.
Table 2: Traffic flow data from 8:00 to 9:00 at a sampling interval of 5 minutes.

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<th>8:05-8:10</th>
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<td>92</td>
<td>63</td>
<td>90</td>
<td>103</td>
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<tr>
<td>1034</td>
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<td>152</td>
<td>166</td>
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<td>157</td>
<td>117</td>
<td>79</td>
<td>119</td>
<td>129</td>
</tr>
</tbody>
</table>

(I) Data Processing. Obtain the original sequence $X^{(0)}(k)$ based on the characteristics of the model data:

$$X_1^{(0)} = (1371, 1435, 1442, 1407, 1548, 1147, 983, 1338, 1378, 1380, 1396, 1246, 1093)$$

The CTAGO sequence $Y_1^{(0)}(k)$ is as follows:

$$Y_1^{(0)} = (9333, 9300, 9243, 9181, 9170, 9011, 9110, 9220)$$

Similarly, the 1-AGO sequence of CTAGO sequence $Y_1^{(1)}(k)$ can be written as follows:

$$Y_1^{(1)} = (9333, 18633, 27876, 37057, 46227, 55238, 64348, 73568)$$

(II) Parameter Estimation. The parameters obtained by a least squares method are as follows:

$$\beta_1 = 0.39968,$$
$$\beta_2 = 5477.44472,$$
$$\beta_3 = 9453.60908$$

(III) Constructing the RNDGM Model. The parameter value obtained in step (II) is substituted into Theorem 8 to obtain the 8th observation value $\hat{x}^{(0)}(8) = 1337.50$.

(IV) Simulating and Forecasting other Data for the Next Two Weeks. The information in $x^{(0)}(1)$ is removed, and new information from the data in $x^{(0)}(15) = 1444$ is added to constitute the sequence

The above modeling process is repeated to forecast the sequences $\hat{x}^{(0)}(8), \hat{x}^{(0)}(9), \ldots, \hat{x}^{(0)}(21)$.

(V) Computing and Comparing the Forecasted Values and Errors. The mean relative simulation percentage error
14 Complexity

24 hours

7 Days

3 weeks

1 time/hour

(a) \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 1} \)

(b) \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 12} \)

(c) \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 4} \)

(d) \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 3} \)

Figure 3: Four tensor models with different combinations of traffic data.

(MRSPE), also known as the mean relative prediction percentage error (MRPPE), is calculated as follows:

\[
\text{MRSPE (MRPPE)} = \frac{1}{N} \sum_{k=2}^{N} \frac{|\hat{x}(k) - x(k)|}{x(k)} \times 100\%
\]

(54)

where \( x(k) \) is the value of traffic flow, \( \hat{x}(k) \) is the predicted value of traffic flow, and \( N \) is the number of data points.

The predictive values and errors for traffic flow based on DTRNDGM, RSDGM, SGM, and NDGM are given in Table 3.

For the third week of the data shown in Table 3, with regard to the predictions during this period, the accuracy of DTRNDGM is much better than those of RSDGM [45], SGM [44], and classic NDGM. Moreover, the rolling predictions of RSDGM and SGM yield similar simulation results; the simulation results of RSDGM and SGM are consistent, and these models perform best in simulations of exponential sequenced data. According to the prediction for November 4, DTRNDGM performs much better than the other three models. Based on Table 1, the absolute MRSPE and MRPPE values for the above four models of traffic flow and the real data are illustrated in Figures 4(a)–4(d).

Figure 4 reveals that the simulation and prediction performances of DTRNDGM, RSGM, SGM, and NDGM are the best among the three models, which demonstrates that the optimization and reform of DTRNDGM, RDGM, SGM, and NDGM are effective.

To determine the prediction capabilities of DTRNDGM with the three tensor data models (Figures 3(b), 3(c) and 3(d)), predictions on two consecutive days (October 31 and November 1) are analyzed. Tensor model \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 12} \) (Figure 3(b)) uses the 5-minute prediction period from 8:00–8:05, whereas tensor model \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 4} \) (Figure 3(c)) uses the 15-minute prediction period from 8:00–8:15, and tensor model \( \mathbf{x} \in \mathbb{R}^{3 \times 7 \times 24 \times 3} \) (Figure 3(d)) uses the 20-minute prediction period from 8:00–8:20 to determine the associated effects. The data used range from October 17 to October 31, and both modeling and rolling forecasting are performed. The specific analysis results are shown in Tables 4–6.

As presented in Tables 4–6, the DTRNDGM results are the best among the three tensor models for two-day rolling prediction. Additionally, the prediction results of RSDGM, SGM, and NDGM are extremely similar, thereby illustrating the consistency of the predictions of these models and the factors that influence each model. DTRNDGM is therefore better for simulating nonhomogeneous sequences.

5. Conclusions

The multimode traffic flow data prediction technique proposed in this paper is characterized by a high-dimensional tensor. According to the temporal correlations of the traffic flow data and the nonhomogeneous exponential properties and dynamics of the data, a dynamic tensor rolling nonhomogeneous discrete grey forecasting model (DTRNDGM) suitable for multimode state traffic flow data is proposed. Accordingly, the properties of the model are studied, and a comparative experiment is performed. The following points summarize the findings of this research:

(1) According to the approximately nonhomogeneous exponential properties and dynamics of traffic flow data, a new dynamic and approximate nonhomogeneous grey prediction model is proposed. According to an affine transformation, the model is completely fitted to the nonhomogeneous exponential sequence, and the cycle truncation accumulated generating operation is introduced to achieve the prediction or simulation of dynamic traffic flow data.

(2) The multimode characteristics of traffic data are expressed by a high-dimensional tensor, and a grey
Table 3: Simulated or forecasted values and errors of the traffic flow in the $\chi \in R^{37 \times 7 \times 24 \times 1}$ model.

<table>
<thead>
<tr>
<th>Time</th>
<th>Real data</th>
<th>Simulated data</th>
<th>DTRNDGM</th>
<th>RSDGM</th>
<th>SGM</th>
<th>NDGM</th>
<th>Simulated data</th>
<th>MRSPE</th>
<th>Forecasted data</th>
<th>Forecasted data</th>
<th>Forecasted data</th>
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<tr>
<td>1028</td>
<td>1444</td>
<td>1337.50</td>
<td>1197.45</td>
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<td>1429</td>
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<td>1270.64</td>
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<td>1030</td>
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<td>1506.46</td>
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<tr>
<td>1103</td>
<td>962</td>
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<td>1241.69</td>
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<tr>
<td>MRSPE</td>
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<td>0.1260</td>
<td>0.2255</td>
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</tbody>
</table>

(a) Simulated and forecasted effects of DTRNDGM

(b) Simulated and forecasted effects of RSDGM

(c) Simulated and forecasted effects of SGM

(d) Simulated and forecasted effects of NDGM

Figure 4: Simulated and forecasted effects of the four models.
model (i.e., the DTRNDGM) is proposed based on multimode short-term traffic data forecasting with high-dimension tensor models. According to a temporal correlation analysis of the “week-day-time” model in the dynamic tensor models and the theory of matrix perturbation analysis, the optimal rolling cycle of the new model is studied.

(3) Finally, traffic flow data from 8:00–9:00 for the main route of Shaoshan Road in Changsha, Hunan, China, are collected, and an example analysis is performed. Four typical “week-day-time” tensor models are constructed, and experiments under these four dynamic tensor models are conducted to perform a contrast analysis. The experimental results show that DTRNDGM predicts traffic flow much better than RSDGM, SGM, and NDGM.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

This work is supported by the National Natural Science Foundation of China (71871174); Project of Humanities and Social Sciences Planning Fund of Ministry of Education of China (18YJA630022); the Science and Technology Research Project of Chongqing Municipal Education Commission (KJQN201800624).

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