Research Article

The Evolution of Collective Strategies in SMEs’ Innovation: A Tripartite Game Analysis and Application

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Received 27 June 2019; Revised 12 September 2019; Accepted 26 September 2019; Published 30 October 2019

1. Introduction

Technology innovation is widely believed to be the main driving force for economic growth [1]. The enterprises may require a substantial capital investment, since technology innovation is a long-term and sustainable process. However, due to the high risk and information asymmetry, external investors are reluctant to invest in technology innovation [2, 3]. Therefore, insufficient funds have been bothering the innovation enterprises and hindering their ability to innovate [4].

Small- and medium-sized enterprises (SMEs), the largest innovation group in China, play an irreplaceable role in promoting economic growth, enabling innovation, increasing taxes, creating employment, and improving people’s livelihood. According to China’s Ministry of Industry and Information Technology (MIIT), SMEs increase more than 50% of tax revenue, create more than 60% of GDP, complete more than 70% of invention patents, and provide more than 80% of urban jobs, accounting for more than 99% of the total number of enterprises. The financing channels for SMEs compared to large enterprises are relatively narrow, and SMEs have more serious difficulty in raising money [5–7]. The research on SMEs’ innovation financing is of great significance both in theory and in practice.

Government subsidies have become the common measures adopted by many countries to support enterprises’ innovation. The widely accepted reason for the government to subsidize R&D activities is the existence of market failures, which create a gap between private benefits and social benefits derived from R&D activities [8]. The positive spillover effect of R&D activities will lead to lower R&D
investment of enterprises than the optimal level. Although a large number of studies have confirmed the value of government subsidies, to our knowledge, few scholars pay attention to the impact of government subsidies on the SMEs’ innovation [9]. No one tries to examine the interrelationship between the government, SMEs, and external investors in innovation activities.

In this paper, we seek to contribute to the literature on SMEs’ innovation in four ways. First, we focus purely on SMEs and hope this research can help to understand how public instruments affect these enterprises. We also consider the effects of government subsidies on the investment decisions of external investors to SMEs’ innovation. So far, there is little empirical evidence on the effectiveness of public tools on SMEs’ innovation [10–14]. In our paper, we examine the effects of government subsidies on the strategy choices of SMEs and external investors.

Second, this paper mainly focuses on the cooperation mechanism among the government, SMEs, and external investors in the decision-making process of SMEs’ innovation. Prior studies only consider the relationship between the government and subsidized enterprises, or between the government and external investors. Moreover, the academics have made considerable effort to understand and evaluate the effect of government subsidies on enterprises’ R&D investment but paid less attention to the effect of government subsidies on enterprises’ innovation willingness and the use of R&D investment. And there is no consistent conclusion on whether government subsidies can actually alleviate the financing constraints of enterprises and bring significant increase in innovation output [4, 15]. To solve this problem, it is necessary to sort out the decision-making behavior, disclose the conflicts of interest among groups of stakeholders, and find a practical solution. The cooperation strategy among the government, external investors, and SMEs has attracted virtually no scrutiny. Our paper seeks to fill this gap by examining the interaction mechanism of the three parties.

The third contribution of this paper is to try to construct a tripartite evolutionary game model of the relationship among the government, SMEs, and external investors. We regard the government, SMEs, and external investors as the players with bounded rationality, who will constantly adjust their strategies in SMEs’ innovation process. In recent years, the application of game theory in the field of enterprise innovation has been increasing, but the application of evolutionary game theory is rare [16–18]. This study can provide new ideas for the scholars in this field.

Lastly, we attempt to explore the interaction mechanism and the antecedents influencing the strategy selections. Using Matlab software, the effects of government subsidies, external investors’ appraisal costs, external investors’ return rate, and external investors’ investment amount on the players are simulated. This paper will enable us to determine the conditions and ranges under which the relevant factors work. The results of this study can provide a theoretical guidance for solving the financing problem of SMEs’ innovation.

The remainder of this paper is organized as follows. Section 2 provides a brief view of the extant literature. Section 3 establishes a trilateral evolutionary game theory model of the interaction among the government, investors, and SMEs. Section 4 discusses the replicator dynamic equation and the equilibrium points. Section 5 describes the results of evolutionary game simulation. Section 5 presents the conclusion and some policy suggestions.

2. Literature Review

2.1. Effects of Government Subsidies on Enterprises’ Innovation. Research on the effect of government subsidies on enterprises innovation has been a hot issue in the academic community. There is a great deal of research on the relationship of government subsidies and enterprises’ innovation investment. Theoretically, government subsidies on the one hand can directly increase enterprises’ R&D funds and on the other hand can enhance the confidence of the enterprises’ innovation and promote the increase of corporate R&D investment [19–21]. Busom [22], Lee and Hwang [23], Hud and Hussinger [24], Radas et al. [9], and Hottenrott and Lopes-Bento [11] considered that government subsidies are positively correlated with enterprises’ R&D investment. However, Wallsten [14] and Marino et al. [25] believed that government subsidies had a substitution effect on enterprise R&D investment, i. e., government subsidies will reduce enterprise R&D investment. Marino et al. empirically showed that there was a significant substitution between private and public funds to R&D, especially for medium-high levels of government subsidies [25].

In the past few years, the effect of government subsidies on the external investors funding the enterprises’ innovation has attracted significant scholarly attention. Asymmetric information may be the reason for the external investors not willing to fund R&D due to its inherent risk, even if the innovation enterprises promised high expected returns [2]. The government agency identifies and evaluates the R&D projects and then decides whether to subsidize innovation enterprises. Enterprises funded by the government are often those with relatively high innovation ability and relatively low risk. Therefore, government subsidies could provide a certification effect about enterprises quality and improve enterprises’ access to external finance [13, 26–28].

In recent years, some scholars investigated the effect of government subsidies on enterprises’ innovation output, and most of them believed that government subsidies had a significant role in promoting innovation output [19, 29, 30]. Bérubé and Pierre found that the R&D subsidy program implemented in northern Italy had a significant impact on the amount of patents, but the increment was markedly greater in the case of smaller firms [29]. Some scholars also paid attention to the effect of government R&D subsidies on enterprises’ innovation efficiency [31, 32]. Jin et al. found that government subsidies had a negative influence on innovation efficiency of China’s high-tech industries [32].

Game theory has proven to be an effective method for analyzing the firms’ innovation strategies [33–39]. Existing studies usually assume that all participants are rational and static, which is inconsistent with the facts. Enterprises’ innovation is a dynamic process. The participants in enterprises’ innovation, such as enterprises, governments, scientific research institutions, and banks, cannot be completely rational. It is often unreasonable to apply general game theory to analyze innovation decision-making problems.

Since 1980s, evolutionary game has quickly grown into an active area of research in social economy, which is based on the theories of biological evolutionism, nonlinear dynamics, and game theory. Evolutionary game is an increasingly popular approach among the research of enterprises’ innovation. Ying et al. proposed an evolutionary game model and analyzed the effects of cluster informal contracts on innovation cooperation among cluster enterprises [40]. Yang et al. established an evolutionary game model among the government, enterprises, universities, and research institutes and explored the mechanism of intellectual property cooperation [41]. Shen used evolutionary game to examine the enterprise decision-making behavior in the process of open innovation from the perspective of endogenous knowledge spillovers [42]. Lin et al. set up an evolutionary game model of human innovation behaviors and discussed the impact of the heterogeneous structure on the evolution of innovation behaviors based on the scale-free network [43–48].

Today, the financing mechanisms to support SMEs’ innovation have been a subject of great interest and a major challenge to policy makers as SMEs are considered as the key element to promote economic growth and stability. Following the above research trends, we specifically study the investment decision-making behavior of the government, external investors, and SMEs in the process of SMEs’ innovation and discuss how to maximize the benefits of the government, SMEs, and external investors. To the best of our knowledge, this problem is not solved in the existing literature. Different from previous research, we construct a trilateral evolutionary game model among the government, SMEs, and external investors and formulate the replicator dynamic equations to analyze the evolutionarily stable strategies (ESSs) of multiple stakeholders. Finally, the theoretical results are verified by simulation and experiments. Our paper not only provides a different perspective for the existing literature to analyze the effects of public subsidies but also provides ideas for how to optimize public subsidies.

3. Tripartite Evolutionary Game Model in SMEs’ Innovation

The government, external investors, and SMEs are three critical stakeholders in the process of SMEs’ innovation investment. The government represents the public interest and mainly guides SMEs to carry out innovation activities through innovation policies. It acts as the promoter and direct beneficiary of SMEs’ innovation. External investors who pursue profit maximization mainly refer to financial institutions and often decide whether to invest in innovation projects after rigorous evaluations. SMEs are regarded as the main driving forces for innovation. Accounting irregularity, lacking collateral for banks, and information asymmetry are common issues inherent to SMEs. SMEs have to choose between adopting an innovation and maintaining the status quo when making business decisions. Although innovation is an important way for SMEs’ growth, all have increased the demand for talent, increased financial burdens and costs, and created uncertainties that could only make SMEs more reluctant to innovate. It can be seen that in the process of SMEs’ innovation, the government, external investors, and SMEs will make behavioral choices in their own interests. The government has the responsibility for supporting access to finance for SMEs’ innovation. Only by finding an equilibrium mechanism under incomplete information can the three parties form a virtuous circle system in innovation game and fundamentally solve the financing problem of SMEs’ innovation. Therefore, this paper makes the following assumptions:

(1) The government, external investors, and SMEs are the main stakeholders in the process of SMEs’ innovation investment. The three parties interact with each other in the decision-making process and finally reach the evolutionary equilibrium.

(2) The government, external investors, and SMEs all have bounded rationality and incomplete information.

(3) Due to information asymmetry, the government is the dominant force in the decision-making process, and the external investors are its subordinates. Under the conditions of the government subsidizing SMEs’ innovation, external investors can invest directly in SMEs without spending costs.

We assume that the strategies of the government are {Subsidize, Not subsidize}. The optional strategies of external investors are {Invest, Not invest}, while the strategies of SMEs are {Innovate, Not innovate}. Suppose the probability that the government chooses the “Subsidize” strategy is \( x (x \in [0,1]) \) and the probability that the government chooses the “Not subsidize” strategy is \( 1 - x \). Suppose the probability that the external investors adopt the “Invest” strategy is \( y (y \in [0,1]) \) and the probability that the external investors adopt the “Not invest” strategy is \( 1 - y \). The probability of selecting the “Innovate” strategy for SMEs is assumed to be \( z (z \in [0,1]) \), and the probability of selecting the “Not innovate” strategy is \( 1 - z \).

\( S \) represents the amount of government subsidies for SMEs’ innovation. \( V_1 \) and \( V_2 \), respectively, represent the social benefit obtained by the government, when SMEs adopt “Innovate” and “Not innovate” strategies. Under the condition of selecting “Subsidize” strategy for the government, when the external investors adopt the “Invest” strategy, there will be some social benefit increase for the government, compared to the case where the external investors adopt the “Not invest” strategy. This benefit increase is denoted as \( \Delta V_{11} \). Similarly, under the condition of selecting the “Not subsidize” strategy for the government, \( \Delta V_{12} \) represents the social benefit increase that
the government can obtain, when the external investors adopt the “Invest” strategy instead of the “Not invest” strategy.

\( P \) represents the investment amount of choosing “Invest” strategy for external investors. If the average annual return rate obtained by external investors through innovation investment is \( i \), their annual return is \( P \times i \). It is assumed that due to the existence of the certification effect, when the government adopts the “Subsidize” strategy, the external investors can choose to invest directly without spending money to identify the same enterprise. When the government adopts the “Not subsidize” strategy, the appraisal costs of the external investors for SMEs are \( C_1 \).

\( Q_1 \) and \( Q_2 \) represent the benefits obtained by the SMEs, when they adopt “Innovate” and “Not innovate” strategies. \( \Delta Q \) represents the benefit increase that the SMEs can obtain, when the external investors adopt the “Innovate” strategy instead of the “Not invest” strategy. According to the actual situation in emerging economies, we suppose that \( V_1 \geq V_2 \) and \( Q_1 \geq Q_2 \). The corresponding parameters are described in Table 1.

Based on the above analysis, we can establish the payoff matrix among the government, investors, and SMEs, as shown in Table 2.

### 4. Game Model Solution and Analysis

#### 4.1. Replicator Dynamic Equation

Let \( U_x \) and \( U_{1-x} \) represent, respectively, the expected earnings of “Subsidize” and “Not subsidize” for the government. According to the payoff matrix, the fitness of the government with two different strategies can be calculated as follows:

\[
U_x = V_2 - S + yz\Delta V_{11} + z(V_1 - V_2), \\
U_{1-x} = V_2 + yz\Delta V_{12} + z(V_1 - V_2). 
\]  
(1)

The average expected earnings of the government can be calculated as

\[
\bar{U}_A = xU_x + (1-x)U_{1-x}. 
\]
(2)

The replicator dynamic equation for the government can be achieved as follows:

\[
A(x) = \frac{dx}{dt} = x(U_x - \bar{U}_A) = x(1-x)[-S + yz(\Delta V_{11} - \Delta V_{12})].
\]
(3)

Let \( U_y \) and \( U_{1-y} \) represent, respectively, the expected earnings of “Invest” and “Not invest” for external investors. According to the payoff matrix, the fitness of the external investors with two different strategies can be calculated as follows:

\[
U_y = -P - C_1 + xC_1 + zP(i + 1), \\
U_{1-y} = 0. 
\]
(4)

The average expected earnings of external investors can be calculated as

\[
\bar{U}_B = yU_y + (1-y)U_{1-y}. 
\]
(5)

#### Table 1: Parameter description.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>The probability that the government subsidizes SMEs’ innovation</td>
</tr>
<tr>
<td>( y )</td>
<td>The probability that the external investors invest SMEs’ innovation</td>
</tr>
<tr>
<td>( z )</td>
<td>The probability that SMEs implement technological innovation</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>The social benefit of SMEs with general production management</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>The social benefit of SMEs with technological innovation</td>
</tr>
<tr>
<td>( \Delta V_{11} )</td>
<td>The social benefit increase of SMEs’ innovation when the government adopts the “Subsidize” strategy and external investors adopt the “Invest” strategy</td>
</tr>
<tr>
<td>( \Delta V_{12} )</td>
<td>The social benefit increase of SMEs’ innovation when the government adopts the “Not subsidize” strategy and the external investors adopt the “Invest” strategy</td>
</tr>
<tr>
<td>( S )</td>
<td>The amount of government subsidies for SMEs’ innovation</td>
</tr>
<tr>
<td>( P )</td>
<td>The investment amount of the external investors in SMEs’ innovation</td>
</tr>
<tr>
<td>( i )</td>
<td>The investment return rate of the external investors in SMEs’ innovation</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>Investors’ appraisal costs for SMEs</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>The benefits of SMEs when they adopt the “Innovate” strategy</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>The benefits of SMEs when they adopt the “Not innovate” strategy</td>
</tr>
<tr>
<td>( \Delta Q )</td>
<td>The increase in benefits of SMEs’ innovation when the external investors adopt the “Invest” strategy</td>
</tr>
</tbody>
</table>

#### Table 2: The payoff matrix among the government, investors, and SMEs.

<table>
<thead>
<tr>
<th>External investors</th>
<th>External investors</th>
<th>Smes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Government</strong></td>
<td><strong>Invest (( z ))</strong></td>
<td><strong>Not innovate (1-( z ))</strong></td>
</tr>
<tr>
<td><strong>Subsidize</strong> (( x ))</td>
<td>( V_1 - S + \Delta V_{11} )</td>
<td>( V_1 - S )</td>
</tr>
<tr>
<td><strong>Invest</strong> (( y ))</td>
<td>( P \times i )</td>
<td>( Q_1 + \Delta Q + S )</td>
</tr>
<tr>
<td><strong>Not invest</strong> (1-( y ))</td>
<td>( Q_1 + S )</td>
<td>( Q_1 )</td>
</tr>
<tr>
<td><strong>Not subsidize</strong> (1-x)</td>
<td>( V_1 + \Delta V_{12} )</td>
<td>( V_2 )</td>
</tr>
<tr>
<td><strong>Invest</strong> (( y ))</td>
<td>( P \times i - C_1 )</td>
<td>( Q_1 + \Delta Q )</td>
</tr>
<tr>
<td><strong>Not invest</strong> (1-( y ))</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Not invest</strong> (1-x)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The replicator dynamic equation for external investors can be achieved as follows:

\[
B(y) = \frac{dy}{dt} = y(U_y - \bar{U}_B) = y(1-y)[-P - C_1 + xC_1 + zP(i + 1)]. 
\]
(6)
Let \( U_z \) and \( U_{1-z} \) represent, respectively, the expected earnings of “Innovate” and “Not innovate” for SMEs. According to the payoff matrix, the fitness of SMEs with two different strategies can be calculated as follows:

\[
U_z = Q_1 + xS + yΔQ_1, \\
U_{1-z} = Q_2 + xS + yP.
\]

The average expected earnings of SMEs can be calculated as

\[
\mathbb{U}_C = zU_z + (1-z) U_{1-z}.
\]

The replicator dynamic equation for SMEs can be achieved as follows:

\[
C(z) = \frac{dz}{dt} = z(U_z - \mathbb{U}_C) = z(1-z)[(Q_1 - Q_2) + y(ΔQ - P)].
\]

Therefore, the replicator dynamic equation of the SMEs’ innovation system can be obtained from combining equations (3), (6), and (9):

\[
\begin{align*}
A(x) &= x(1-x)[-S + yz(ΔV_{11} - ΔV_{12})], \\
B(y) &= y(1-y)[-P - C_1 + xC_1 + zP(i+1)], \\
C(z) &= z(1-z)[(Q_1 - Q_2) + y(ΔQ - P)].
\end{align*}
\]

4.2. Replicator Dynamic Analysis of Each Stakeholder. As for the government, it can be inferred from equation (3) that

1. When \( y = y^* = S/z(ΔV_{11} - ΔV_{12}) \), \( A(x) = 0 \), all game strategies are at a stable state (see Figure 1(a)).
2. When \( y \neq S/z(ΔV_{11} - ΔV_{12}) \), \( x = 0 \) and \( x = 1 \) are two stable points of \( A(x) = 0 \). The stability strategies of the government need to be further analyzed. The derivative of equation (3) can be calculated as

\[
A'(x) = \frac{∂A(x)}{∂x} = (1 - 2x)[-S + yz(ΔV_{11} - ΔV_{12})].
\]

Then, we discuss two circumstances according to equations (3) and (11):

1. When \( S > (ΔV_{11} - ΔV_{12}) \), under the constraints of \( 0 < x < 1 \), \( 0 < y < 1 \), and \( 0 < z < 1 \), we can prove \(-S + yz(ΔV_{11} - ΔV_{12}) < 0\). Therefore, \( A(x)/\partial x (x = 0) < 0 \) and \( A(x)/\partial x (x = 1) > 0 \). Thus, \( x = 0 \) is the ESS, as shown in Figure 1(b).
2. If \( S < (ΔV_{11} - ΔV_{12}) \), then
   (i) When \( y > y^* \), \( A(x)/\partial x (x = 0) > 0 \) and \( A(x)/\partial x (x = 1) < 0 \). Therefore, \( x = 1 \) is the ESS, as shown in Figure 1(b).
   (ii) When \( y < y^* \), \( A(x)/\partial x (x = 0) < 0 \) and \( A(x)/\partial x (x = 1) > 0 \). Therefore, \( x = 0 \) is the ESS, as shown in Figure 1(b).

As for external investors, it can be inferred from equation (6) that

1. When \( x = x^* = P + C_1 - zP(i+1)/C_1 \), \( B(y) = 0 \), all game strategies are at a stable state (see Figure 2(a)).
2. When \( x \neq P + C_1 - zP(i+1)/C_1 \), \( y = 0 \) and \( y = 1 \) are two stable points of \( B(y) = 0 \). The stability strategies of the government need to be further analyzed. The derivative of equation (6) can be calculated as

\[
B'(y) = \frac{∂A(y)}{∂y} = (1 - 2y)[-P - C_1 + xC_1 + zP(i+1)].
\]

Then, we discuss two circumstances according to equations (6) and (12):

1. When \( P + C_1 > xC_1 + zP(i+1) \), under the constraints of \( 0 < x < 1 \), \( 0 < y < 1 \), and \( 0 < z < 1 \), we can prove \(-P - C_1 + xC_1 + zP(i+1) < 0\). Therefore, \( B(y)/\partial y (y = 0) < 0 \) and \( B(y)/\partial y (y = 1) > 0 \). Thus, \( y = 0 \) is the ESS, as shown in Figure 2(b).
2. If \( P + C_1 < xC_1 + zP(i+1) \), then
   (i) When \( x > x^* \), \( B(y)/\partial y (y = 0) > 0 \) and \( B(y)/\partial y (y = 1) < 0 \). Therefore, \( y = 1 \) is the ESS, as shown in Figure 2(b).
   (ii) When \( x < x^* \), \( B(y)/\partial y (y = 0) < 0 \) and \( B(y)/\partial y (y = 1) > 0 \). Therefore, \( y = 0 \) is the ESS, as shown in Figure 2(b).

As for SMEs, it can be inferred from equation (9) that

1. When \( y = y^* = (Q_1 - Q_2)/(P - ΔQ) \), \( C(z) = 0 \), all game strategies are at a stable state (see Figure 3(a)).
2. When \( y \neq (Q_1 - Q_2)/(P - ΔQ) \), \( z = 0 \) and \( z = 1 \) are two stable points of \( C(z) = 0 \). The stability strategies of the government need to be further analyzed. The derivative of equation (9) can be calculated as

\[
C'(z) = \frac{∂C(z)}{∂z} = (1 - 2z)[(Q_1 - Q_2) + y(ΔQ - P)].
\]

Then, we discuss two circumstances according to equations (9) and (13):

1. When \( Q_2 - Q_1 > y(ΔQ - P) \), under the constraints of \( 0 < x < 1 \), \( 0 < y < 1 \), and \( 0 < z < 1 \), we can prove \((Q_2 - Q_1) + y(ΔQ - P) < 0\). Therefore, \( C(z)/\partial z (z = 0) < 0 \) and \( C(z)/\partial z (z = 1) > 0 \). Thus, \( y = 0 \) is the ESS, as shown in Figure 3(b).
2. If \( Q_2 - Q_1 < y(ΔQ - P) \), then
   (i) When \( y > y^* \), \( C(z)/\partial z (z = 0) > 0 \) and \( C(z)/\partial z (z = 1) < 0 \). Therefore, \( z = 1 \) is the ESS, as shown in Figure 3(b).
Figure 1: Replicator dynamic phase diagram of the government. (a) \( y = y^* \). (b) \( y > y^* \), \( x \rightarrow 1 \); \( y < y^* \), \( x \rightarrow 0 \).

Figure 2: Replicator dynamic phase diagram of external investors. (a) \( z = z^* \). (b) \( z > z^* \), \( y \rightarrow 1 \); \( z < z^* \), \( y \rightarrow 0 \).

Figure 3: Replicator dynamic phase diagram of SMEs. (a) \( y = y^* \). (b) \( y > y^* \), \( z \rightarrow 1 \); \( y < y^* \), \( z \rightarrow 0 \).
Complexity

(ii) When \( y < y^* \), \( \partial C(z)/\partial z (z = 0) < 0 \) and \( \partial C(z)/\partial z (z = 1) > 0 \). Therefore, \( z = 0 \) is the ESS, as shown in Figure 3(b).

4.3. Stability Analysis of the Dynamic Systems. According to the replicator dynamic analysis of three stakeholders mentioned above, the strategy choice of SMESs is only related to external investors, but the strategy choice of the government and external investors depends on the decision-making behavior of the other two groups. Therefore, the evolutionary stability strategy can be analyzed by stepwise analysis. First, taking \( x \) as a constant, we analyze the evolution of external investors and the enterprise. Then, taking \( z \) as a constant, we discuss the evolution strategy of the government and investors.

Under the condition that \( x \) is regarded as constant, let the replicator dynamic equation be \( B(\gamma) = C(z) = 0 \), we can get equilibrium points \( (0, 0), (0, 1), (1, 0) \), \( (1, 1) \). When \( y^* = (Q_1 - Q_2)/(P - \Delta Q) \), \( z^* = (P + C_1 - xC_1)/(P(i + 1)) \), and \( y^* \in [0, 1], z^* \in [0, 1] \), we can get equilibrium points \( (y^*, z^*) \). According to the research of Friedman, the stability of the equilibrium points can be judged from the Jacobian matrix [44]. The Jacobian matrix of replicator dynamic equation is as follows:

\[
J_1 = \begin{bmatrix}
\frac{\partial F_X}{\partial y} & \frac{\partial F_X}{\partial z} \\
\frac{\partial F_Y}{\partial y} & \frac{\partial F_Y}{\partial z}
\end{bmatrix} = \begin{bmatrix}
(1 - 2y)[xC_1 + zP(i + 1) - P - C_1]
\end{bmatrix}
\].

The determinants \( \det(J_1) \) and trace \( \text{tr}(J_1) \) of the matrix \( J_1 \) are as follows:

\[
\det(J_1) = (1 - 2y)[xC_1 + zP(i + 1) - P - C_1]
\].

\[
\text{tr}(J_1) = (1 - 2y)[xC_1 + zP(i + 1) - P - C_1]
\].

If the equilibrium point satisfies the conditions of \( \det(J_1) > 0 \) and \( \text{tr}(J_1) < 0 \), it is an ESS. According to the stability analysis method, we analyze the local stability of five equilibrium points. To facilitate the observation of the calculation results, we set \( v_1 = xC_1 - P - C_1 \), \( v_2 = xC_1 + P + C_1, v_3 = Q_1 - Q_2 \), and \( v_4 = \Delta Q - P + Q_1 - Q_2 \). Under the constraint of \( 0 < v_1 < 1 \), we can prove \( v_1 < 0 \). Based on the hypothesis of \( Q_1 > Q_2 \), we can deduce \( v_3 > 0 \). The results of stability analysis between external investors and SMEs are shown in Table 3 and Figure 4. According to the criteria, the equilibrium points \( (0, 0), (y^*, z^*) \) are saddle points. When \( v_4 < 0 \), the equilibrium point \( (1, 0) \) is a saddle point; otherwise, when \( v_4 > 0 \), the equilibrium point \( (1, 0) \) is an unstable point, and the equilibrium points \( (0, 1), (1, 1) \) are an ESS. The results indicate that in the case of external investors participating in SMEs’ innovation, when the earnings obtained by SMEs adopting the “Innovate” strategy are greater than the earnings obtained by SMEs adopting the “Not innovate” strategy, i.e., \( Q_1 + \Delta Q > Q_2 + P \), the equilibrium point \( (1, 1) \) is an ESS. The decisions of external investors will be greatly influenced by the probability of government subsidies. When the probability of government subsidies is high, external investors tend to choose the “Invest” strategy.

Under the condition that \( z \) is regarded as constant, let the replicator dynamic equation be \( A(x) = B(\gamma) = 0 \), and we can get equilibrium points \( (0, 0), (0, 1), (1, 0) \), \( (1, 1) \). When \( x^* = P + C_1 - z(P + C_1) \), \( y^* = S/z(\Delta V_{11} - \Delta V_{12}) \), and \( x^* \in [0, 1], y^* \in [0, 1] \), we can get equilibrium points \( (x^*, y^*) \). The Jacobian matrix of replicator dynamic equation is as follows:

\[
J_2 = \begin{bmatrix}
\frac{\partial F_X}{\partial x} & \frac{\partial F_X}{\partial y} \\
\frac{\partial F_Y}{\partial x} & \frac{\partial F_Y}{\partial y}
\end{bmatrix} = \begin{bmatrix}
(1 - 2x)[yS(\Delta V_{11} - \Delta V_{12}) - S] \\
C_1 y(1 - y)
\end{bmatrix}
\].

\[
\text{tr}(J_2) = (1 - 2x)[yS(\Delta V_{11} - \Delta V_{12}) - S] + C_1 y(1 - y)
\].

\[
\det(J_2) = (1 - 2x)[yS(\Delta V_{11} - \Delta V_{12}) - S]C_1 y(1 - y)
\].
can deduce that the decisions of the government and external investors are shown in Table 4. When the probability of enterprise innovation is low, the government tends to choose the “Invest” strategy. Otherwise, when the probability of enterprise innovation is high, the government tends to choose the “Subsidize” strategy and the external investors tend to choose the “Innovate” strategy. We analyze the local stability of five equilibrium points. To facilitate the observation of the calculation results, we set $v_5 = -S_1$, $v_6 = z(\Delta V_{11} - \Delta V_{12}) - S$, $v_2 = zP(i + 1) - P - C_1$, and $v_8 = zP(i + 1) - P$. Based on the hypothesis of $S > 0$, we can deduce $v_5 < 0$. The results of stability analysis between the government and external investors are shown in Table 4 and Figure 5. According to the criteria, when $v_5 < 0$, the equilibrium point (0, 0) is an ESS. When $v_6 < 0$ and $v_7 > 0$, (0, 1) is an ESS. When $v_8 > 0$, the equilibrium point (1, 0) is an unstable point, and when $v_8 < 0$, the equilibrium point (1, 0) is a saddle point. When $v_6 > 0$ and $v_9 > 0$, (1, 1) is an ESS. The equilibrium points ($x^*, y^*$) are saddle points. The results indicate that the decisions of the government and external investors will be greatly influenced by the probability of SMEs’ innovation. When the probability of enterprise innovation is high, the government tends to choose the “Subsidize” strategy and the external investors tend to choose the “Innovate” strategy. Otherwise, when the probability of enterprise innovation is low, the government tends to choose the “Not subsidize” strategy and the investors tend to choose the “Not invest” strategy.

As discussed in Tables 3 and 4, the evolutionary stable strategy in trilateral evolutionary game needs to be subjected to the local stable conditions in both stages. Therefore, we can obtain 3 stable strategy combinations (0, 0, 1), (0, 1, 1), and (1, 1, 1). When $P * i < C_1$, (0, 0, 1) is an ESS. When $Q_1 + \Delta Q > Q_2 + P$, $(\Delta V_{11} - \Delta V_{12}) < S$, and $P * i > C_1$, (0, 1, 1) is an ESS. When $Q_1 + \Delta Q > Q_2 + P$, $(\Delta V_{11} - \Delta V_{12}) > S$, and $i > 0$, (1, 1, 1) is an ESS. According to the analysis results, when the return on investment of external investors is greater than 0 ($i > 0$), external investors will invest in enterprises’ innovation, regardless of whether the government supports it. In fact, due to the high risks of SMEs’ innovation, the external investors are often afraid or reluctant to invest. This paper is dedicated to promoting SMEs’ innovation and establishing an ideal model of government support, external investor participation, and SMEs’ innovation. That is to say, the trilateral evolutionary game evolved into the ideal strategy selection state of government subsidizing, investor investing, and enterprise innovating ($x = 1$, $y = 1$, $z = 1$). Based on the above analysis, when $Q_1 + \Delta Q > Q_2 + P$, that is, under the condition that external investors invest in SMEs’ innovation, the gains of SMEs choosing the “Innovate” strategy are greater than those of SMEs choosing the “Not innovation” strategy, and all SMEs will choose “Innovate” strategy ($z \rightarrow 1$). When $\Delta V_{11} - S > \Delta V_{12}$, that is, under the condition that external investors invest in SMEs’ innovation,
5. Simulation Analysis

Numerical simulations of the trilateral game model are performed using Matlab software. In order to promote the model to achieve the “ideal state” [Subsidize, Invest, Innovate] and reach the ESS point \((x=1, y=1, z=1)\), the parameters need to satisfy the evolution conditions: \(Q_1 + \Delta Q > Q_2 + P\), \((\Delta V_{11} - \Delta V_{12}) > S\), and \(i > 1\). We set \(V_i = 100\), \(V_2 = 50\), \(\Delta V_{11} = 40\), \(\Delta V_{12} = 20\), \(S = 5\), \(Q_1 = 50\), \(Q_2 = 30\), \(\Delta Q = 5\), \(P = 10\), \(i = 0.15\), and \(C_1 = 0.3\). The time is set to \(t = 20\).

5.1. Phase 1: Effect of Initial Strategy Selection Differences on Evolution Results. The strategy selection results of participants are shown in Figure 6 when the initial values of the strategy combination \((x, y, z)\) are set as \(P_0 = (0.2, 0.4, 0.5)\). From the figure, we can see that the strategy ratios of participants increase with time. Ultimately, the government chooses the “Subsidize” strategy, the investors choose the “Invest” strategy, and the firms choose the “Innovate” strategy, thus reaching the ESS point \(P_1 = 1, 1, 1\). The evolution results are shown in Figure 7 when the initial values of the strategy combination \((x, y, z)\) are set as \(P_0 = (0.4, 0.6, 0.8)\). From the two-dimensional and three-dimensional simulation diagrams, we can see that the change of initial strategy ratio does not affect the results of system evolution. It can be seen from the comparison between Figures 6 and 7 that the higher the initial probability of participants is, the faster the evolutionary system converges to the ideal state.


The model involves government subsidies, external investors’ appraisal costs, external investors’ return rate, and the investment amount of external investors, which may have a significant effect on the strategy choices of the three players. We examine the above four antecedents on the evolutionary results.

5.2.1. Government Subsidies. In order to examine the effect of government subsidies, we set the parameter \(S\) at the interval \([5, 25]\), while fixing the value of the other parameters. The simulation results shown in Figure 8 suggest that the “Subsidize” strategy ratio of the government decreases when the government subsidy increases from 5 to 20. When the amount of government subsidies exceeds 20, the governments will change their strategy from “Subsidize” to “Not...
subsidize.” This result coincides with the explanation that market failure is the fundamental reason for the government to subsidize innovation activities, which creates a gap between private interests and social interests. The government can obtain more social benefits through supporting firms’ innovation. When the amount of government subsidies is greater than the social benefits obtained by the government, the government will select the “Not subsidize” strategy. But if it is profitable, the external investors will still choose the “Invest” strategy.

5.2.2. External Investors’ Return Rate. In order to examine the effect of the external investors’ return rate, we set the parameter $i$ at the interval $[0.05, 1]$, while fixing the other parameters. The simulation results shown in Figure 9 indicate that the “Invest” strategy ratio of external investors increases when the investment return rate increases from 0.05 to 1. The objective of external investors is to maximize their benefits. Therefore, the higher the benefits are, the more external investors will invest in SMEs’ innovation.

Figure 6: The dynamic evolution of the tripartite game model when $x = 0.2$, $y = 0.4$, and $z = 0.5$. (a) Time evolutions of $x$, $y$, and $z$. (b) Time evolution of $(x, y, z)$.

Figure 7: The dynamic evolution of the tripartite model when $x = 0.4$, $y = 0.6$, and $z = 0.8$. (a) Time evolutions of $x$, $y$, and $z$. (b) Time evolution of $(x, y, z)$.
5.2.3. External Investors’ Appraisal Costs. In order to examine the effect of external investors’ appraisal costs, we set the parameter $C_1$ at the interval $[0.3, 5]$, while keeping all other parameters constant. The simulation results shown in Figure 10 indicate that the increase of external investors’ appraisal costs can lead the government and external investors to change their strategy. When the external investors’ appraisal costs are less than or equal to the return of external investors’ innovation investment, the government and external investors will choose “Subsidize” and “Invest” strategies, respectively. When the external investors’ appraisal costs exceed 1.5, although it is greater than the investment return, the external investors will also invest in SMEs’ innovation directly as the government chooses the “Subsidize” strategy. This result coincides with the explanation that government subsidies have signaling effects, which can enhance external investors’ confidence in innovation enterprises and save external investors’ identification costs [13]. However, when the external investors’ appraisal costs exceed 9, the government and external investors will choose the “Not subsidize” and “Not invest” strategies, respectively. The results imply that the higher the cost of identification for external investors is, the greater the risk of innovation enterprises become. When the investors’

![Graphical representation of the effects of government subsidies on evolutionary strategies.](image1)

**Figure 8:** The effect of government subsidies on the evolutionary strategies.

![Graphical representation of the effects of external investors’ return rate on evolutionary strategies.](image2)

**Figure 9:** The effect of external investors’ return rate on the evolutionary strategies.
appraisal costs are too large, the willingness of the government to subsidize and external investors to invest will decline.

5.2.4. Investment Amount of External Investors. In order to examine the effect of the external investors’ investment amount, we set the parameter $P$ at the interval $[3, 30]$, while keeping all other parameters constant. The simulation results shown in Figure 11 indicate that the increase of external investors’ investment amount can lead the government and external investors to change their strategies. When the investors’ investment amounts increase from 3 to 25, the government and external investors choose “Subsidize” and “Invest” strategies, respectively. When the external investors’ investment amount exceeds 25, the government will still choose the “Subsidize” strategy in the long run, while the external investors and SMEs are in a difficult state of choice and cannot reach a balance in the long run. The results imply that when the amount of government subsidies remains unchanged, the higher the amount of external investor’s investment is, the more the government tends to choose the
"Subsidize" strategy, and the more the SMEs tend to choose the "Innovate" strategy. However, when the investors' investment amount is too high and the return of SMEs choosing "Not innovate" strategy is higher than that of SMEs choosing "Innovate" strategy, the external investor will choose not to invest and the government will choose not to subsidize in short time. However, in long time, external investors and SMEs will fall into a dilemma and cannot reach equilibrium.

6. Conclusions and Suggestions

Various aspects of SMEs' innovation financing have been investigated concerning the reasons for financing difficulties, how to obtain external financing, and government support policies. However, studies on the behavior of the multiple stakeholders in SMEs' innovation investment are limited. This paper fills gaps in existing theory by focusing on the collective strategies in SMEs' innovation investment. The tripartite evolutionary game model, including the government, external investors, and SMEs, is built to study how to choose strategies for the participants under limited relational conditions. We also analyze the antecedents on the strategy selection of the participants. Based on the game analysis and simulation, we can draw the following conclusions: (1) When the social benefits of the government by "Subsidize" strategy are more than those by "Not subsidize" strategy, the investors' return rate in SMEs' innovation is greater than 1, and when the SMEs' gains by "Innovate" strategy are more than those by "Not innovate" strategy, the ESS of the system is [Subsidize, Invest, Innovate]. (2) The game ESS changes into [Not subsidize, Invest, Innovate] when the social benefits of the government by "Subsidize" strategy are less than those by "Not subsidize" strategy. (3) The evolutionary system converges to desirable equilibrium stability faster, when government subsidies and external investors' appraisal costs decrease, and the investment amount and return rates of external investors increase.

The financing difficulty of SMEs' innovation is partly caused by the high risk characteristics of SMEs' innovation and information asymmetry between SMEs and external investors. For the broad masses of SMEs, insufficient funds prevent them from conducting normal production operations and innovation activities. It is very necessary for the government to provide innovation subsidies for SMEs. Government subsidies directly increase enterprises' R&D funds and turn SMEs' innovation ideas into reality. However, government subsidies mainly come from financial allocations, which are limited by the state and local government fiscal revenue. Therefore, it is difficult to fully compensate for the financing gap faced by innovating SMEs. Building up effective cooperation among SMEs, governments, and external investors is an inevitable path to promote SMEs' innovation and sustainable development.

Severe asymmetric information makes external investors afraid to participate in SMEs' innovation, so SMEs cannot obtain sufficient R&D funds and achieve satisfactory innovation returns. The government is required to implement a set of measures to ensure transparency and fairness in the process of supporting SMEs' innovation. In this paper, the simulation results of the trilateral game provide evidence that government subsidies have a signal transmission effect. The SMEs subsidized by the government can obtain evidence of good enterprises' qualifications. The external investors can invest in the SMEs subsidized by the government without having to spend on identifying costs. Therefore, on the one hand, the government should design a scientific and reasonable evaluation system and conduct rigorous screening procedures to select potential innovation SMEs. On the other hand, the government should timely publicize the information of the subsidized SMEs so that the external investors can receive the relevant information. The evolution paths of game strategies for the external investors and SMEs show that the higher the probability of SMEs choosing "Innovate" strategy, the higher the probability of the external investors choosing "Invest" strategy. Thus, the government should strictly supervise the subsidized SMEs to ensure that the public funds are used in innovation instead of other activities. Only in this way will more external investors be attracted to participate in SMEs' innovation and the equilibrium of [Subsidize, Invest, Innovate] be finally reached. This study has important theoretical and practical significance for solving the problem of SMEs' innovation financing difficulties. Yet, this study unavoidably has some limitations due to preset research objective and limited time. Future research should further analyze the role of the government, such as the effect of regulatory intensity, subsidy models, and subsidy levels, on the stakeholder behaviors.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the Tianjin Planning Leading Group Office of Philosophy and Social Sciences under grant no. TJYY17-017.

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