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Research Article

Dynamical Analysis of Approximate Solutions of HIV-1 Model with an Arbitrary Order

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This article studies the dynamical behavior of the analytical solutions of the system of fraction order model of HIV-1 infection. For this purpose, first, the proposed integer order model is converted into fractional order model. Then, Laplace-Adomian decomposition method (L-ADM) is applied to solve this fractional order HIV model. Moreover, the convergence of this method is also discussed. It can be observed from the numerical solution that (L-ADM) is very simple and accurate to solve fraction order HIV model.

1. Introduction

AIDS (acquired immune deficiency syndrome) is a disease which is caused by the type of pathogen virus called human immune deficiency virus (HIV). This virus was introduced in 1981 in USA. AIDS is incurable disease that has high mortality rate (kills more than 25 million worldwide per year). Developing of AIDS takes about 6 months to 15 years. The virus attacks CD4+ T cells. The virus is transmitted through unprotected sexual contact and by sharing contaminated needle or transfusion of infected blood. It may also be transmitted from the mother to her child during pregnancy, lactation, or during birth.

Applied mathematicians have a great interest to study the HIV/AIDS dynamics. This research helps the biologists to find the appropriate treatment for infected humans. Mathematical models are important tools in analyzing the spread and control of HIV/AIDS as they provide short- and long-term prediction of HIV and AIDS incidences.

The dynamics of HIV epidemic models have been studied by several researchers [1–5]. For the numerical solutions of HIV-1 models, some methods have been introduced. Ghoreishi et al. [6] introduced homotopy analysis method for the solution of HIV-1 model. Further, Ongun [7] introduced the Laplace-Adomian decomposition method CLADMD for the solution of HIV-1 model. The HPM was introduced by Merdan in [8] to find the approximate solution of the model. But the most powerful method is LADM [7], which has gained much attention in the recent past.

In this paper, we will generalize AIDS/HIV model to a fractional order system of order ζ in sense of Caputo definition because it is equivalent to ordinary differential equation when $\zeta = 1$. We will study the nonlinear model with a fractional order $0 < \zeta \leq 1$. In recent years, the mathematician have taken interest in fractional calculus because it has many engineering and medical applications. Therefore, first, we consider the HIV infection integer order model which has been presented in [9]. This model consists of five compartments: $x(t)$ which stands for the uninfected CD4+ T cells, $x^{\star}(t)$ which represents the concentration of infected cells while the concentration of double cells is denoted by $x^{\star\star}$, and the densities of pathogen viruses and recombinant

viruses are denoted by $v_p(t)$ and $v_r(t)$, respectively. This model can be written as

$$\begin{aligned}\dot{x}(t) &= \lambda - d_1 x(t) - \beta_1 x(t) v_p(t),\\ \dot{x}^{\star}(t) &= \beta_1 x(t) v_p(t) - d_2 x^{\star}(t) - \alpha_1 v_r(t) x^{\star}(t),\\ \dot{x}^{\star\star}(t) &= \alpha_1 v_r(t) x^{\star}(t) - d_3 x^{\star\star}(t),\\ \dot{v}_p(t) &= k' x^{\star}(t) - d_4 v_p(t),\\ \dot{v}_r(t) &= c' x^{\star\star}(t) - d_5 v_r(t),\end{aligned} \tag{1}$$

with the initial condition $x^0 = r_1, x^{\star^{\circ}} = r_2, x^{\star\star^{\circ}} = r_3, v_p^0 = r_4$, and $v_r^0 = r_5$.

The parameters used in the system (1) can be defined as follows: the healthy cells are produced at the rate λ. Moreover, d_1, d_2, d_3, d_4, and d_5 are the rates of death of uninfected cells, infected cells, double infected cells, pathogen virus, and recombinant virus, respectively. β_1 is the rate of infection of healthy CD4+ T cells. The rate of production of pathogen virus is defined by k'. α_1 is infection rate of double infected cells. c' is the rate at which the recombinant viruses are produced.

$$\begin{aligned}{}^{c}D^{\varsigma} x(t) &= \lambda - d_1 x(t) - \beta_1 x(t) v_p(t),\\ {}^{c}D^{\varsigma} x^{\star}(t) &= \beta_1 x(t) v_p(t) - d_2 y(t) - \alpha_1 v_r(t) x^{\star}(t),\\ {}^{c}D^{\varsigma} x^{\star\star}(t) &= \alpha_1 v_r(t) x^{\star}(t) - d_3 x^{\star\star}(t),\\ {}^{c}D^{\varsigma} v_p(t) &= k' x^{\star}(t) - d_4 v_p(t),\\ {}^{c}D^{\varsigma} v_r(t) &= c' x^{\star\star}(t) - d_5 v_r(t),\end{aligned} \tag{2}$$

with the initial conditions $x^0 = r_1, x^{\star^0} = r_2, x^{\star\star 0} = r_3, v_p^0 = r_4$, and $v_r^0 = r_5$.

In many branches of sciences, fractional differential equations (FDEs) have shown their importance as these equations have much application in different sciences such as Physics, Chemistry, Mechanics, and Engineering [10, 11]. Currently (FDEs), systems have gained much attention. In the recent past, various integer order models have been converted into fractional order models. Fractional order operators have greater degree of freedom, and hence, using these in mathematical models produce best results as compared to integer order derivatives.

This research work will be organized as follows: the first section is related to some basic definitions from fractional calculus. In second section, Adomian decomposition method will be applied to the proposed system. Numerical simulation will be carried out in Section 3. Section 4 is dedicated to convergence analysis of the proposed fractional order model. In the last section, conclusion will be drawn.

2. Preliminaries

We will discuss some basic definitions in this section. For this, we will use [12]. Throughout the paper, we have used Caputo fractional order derivative.

Definition 1. The fractional Riemann-Liouville type integral of order $\kappa \in (0, 1)$ of a function $h \in L^1([0, \tilde{T}], \mathbb{R})$ is written as

$$I_{0+}^{\kappa} h(t) = \frac{1}{\Gamma(\kappa)} \int_0^t (t-s)^{\kappa-1} h(s) ds, \tag{3}$$

where ${}^{c}D^{\kappa}$ is Caputo derivative and $0 < \kappa < 1$. But κ is the order of fractional time derivative.

Definition 2. For the function h, the Caputo derivative for fractional order on the interval $[0, \tilde{T}]$ can be defined as

$${}^{c}D_{0+}^{\kappa} h(t) = \frac{1}{\Gamma(m-\kappa)} \int_0^t (t-s)^{m-\kappa-1} h^{(m)}(s) ds. \tag{4}$$

Here, $m = [\kappa] + 1$ and $[\kappa]$ denote the integral part of m.

Lemma 1. *The properties related to fractional order integral are satisfied.*

$$I^{\kappa}[{}^{c}D^{\kappa} g](t) = g(t) + u_0 + u_1 t + u_2 t^2 + \cdots + u_{m-1} t^{m-1}, \tag{5}$$

for $u_j \in \mathbb{R}, j = 0, 1, 2, \ldots, m-1$.

Definition 3. The Laplace transform of Caputo derivative is defined as follows:

$$\begin{gathered} L\{{}^{c}D^{\kappa} y(t)\} = s^{\kappa} h(s) - \sum_{k=0}^{m-1} s^{\kappa-k-1} y^{(k)}(0),\\ m-1 < \kappa < m, m \in N, \end{gathered} \tag{6}$$

where $m = [\kappa] + 1$ and $[\kappa]$ denote the integer part of m.

Theorem 1. *The initial value problem (2) with the given initial conditions (8) has unique solutions and will remain in* R_+^5.

Proof 1. It can be proved that the solution of the initial value problem exists on (0,1) and is unique as derived in [13]. Moreover, we will show that R_+^5 is positively invariant region. From system (2), we can obtain the following results:

$$\begin{aligned}\left.\frac{dx}{dt}\right|_{x=0} &= \lambda \geq 0, \left.\frac{dx^{\star}}{dt}\right|_{x^{\star}=0} = \beta x v_p \geq 0, \left.\frac{dx^{\star\star}}{dt}\right|_{x^{\star\star}=0}\\ &= \alpha v_r x^{\star} \geq 0, \left.\frac{dv_p}{dt}\right|_{v=0} = k x^{\star} \geq 0, \left.\frac{dv_r}{dt}\right|_{v_r=0} = c x^{\star\star}.\end{aligned} \tag{7}$$

Hence, the solution will remain in the region R_+^5.

3. The Laplace-Adomian Decomposition Method (L-ADM)

We will apply our considered method in the following steps. First, we will apply Laplace transform to both sides of (2) and obtain the equations as below

$$\begin{aligned}
s^{\varsigma}\mathscr{L}\{x(t)\} - s^{\varsigma-1}x(0) &= \mathscr{L}\{\lambda - d_1x(t) - \beta_1x(t)v_p(t)\},\\
s^{\varsigma}\mathscr{L}\{x^{\star}(t)\} - s^{\varsigma-1}x^{\star}(0) &= \mathscr{L}\{\beta_1x(t)v_p(t) - d_2x^{\star}(t)\\
&\quad - \alpha_1v_r(t)x^{\star}(t)\},\\
s^{\varsigma}\mathscr{L}\{x^{\star\star}(t)\} - s^{\varsigma-1}x^{\star\star}(0) &= \mathscr{L}\{\alpha_1v_r(t)x^{\star}(t) - d_3z(t)\},\\
s^{\varsigma}\mathscr{L}\{v_p(t)\} - s^{\varsigma-1}v(0) &= \mathscr{L}\left\{k' x^{\star}(t) - d_4v_p(t)\right\},\\
s^{\varsigma}\mathscr{L}\{v_r(t)\} - s^{\varsigma-1}v_r(0) &= \mathscr{L}\{c' x^{\star\star} - qv_r\}.
\end{aligned} \tag{8}$$

Now, using the initial conditions and after some rearrangement, we obtain

$$\begin{aligned}
\mathscr{L}\{x(t)\} &= \frac{r_1}{s} + \frac{1}{s^{\varsigma}}\mathscr{L}\{\lambda - d_1x(t) - \beta_1x(t)v_p(t)\}\\
\mathscr{L}\{x^{\star}\} &= \frac{r_2}{s} + \frac{1}{s^{\varsigma}}\mathscr{L}\{\beta_1x(t)v_p(t) - d_2x^{\star}(t)\\
&\quad - \alpha_1v_r(t)x^{\star}(t)\},\\
\mathscr{L}\{x^{\star\star}(t)\} &= \frac{r_3}{s} + \frac{1}{s^{\varsigma}}\mathscr{L}\{\alpha_1x^{\star}(t)v_r(t) - d_3x^{\star\star}(t)\}\\
\mathscr{L}\{v_p(t)\} &= \frac{r_4}{s} + \frac{1}{s^{\varsigma}}\mathscr{L}\left\{k' x^{\star}(t) - d_4v_p(t)\right\},\\
\mathscr{L}\{v_r(t)\} &= \frac{r_5}{s} + \frac{1}{s^{\varsigma}}\mathscr{L}\{c' x^{\star\star}(t) - d_5v_r(t)\}.
\end{aligned} \tag{9}$$

Assuming the solutions in the form of infinite series as

$$\begin{aligned}
x(t) &= \sum_{m=0}^{\infty} x^m,\\
x^{\star}(t) &= \sum_{m=0}^{\infty} x^{\star^m},\\
x^{\star\star}(t) &= \sum_{m=0}^{\infty} x^{\star\star^m},\\
v_p(t) &= \sum_{m=0}^{\infty} v_p^m,\\
v_r(t) &= \sum_{m=0}^{\infty} v_r^m,
\end{aligned} \tag{10}$$

and decomposing the nonlinear terms $x(t)v_p(t)$ and $v_r(t)x^{\star}$ by using Adomian polynomial as

$$\begin{aligned}
x(t)v_p(t) &= \sum_{m=0}^{\infty} K_m(t),\\
v_r(t)x^{\star}(t) &= \sum_{m=0}^{\infty} M_m(t),
\end{aligned} \tag{11}$$

where K_m and M_m can be defined by

$$\begin{aligned}
K_m(t) &= \frac{1}{\Gamma(m+1)}\frac{d^m}{d\eta^m}\left[\sum_{k=0}^{m}\eta^k x^k \sum_{k=0}^{m}\eta^k v_p^k\right]\Bigg|_{\eta=0},\\
M_m(t) &= \frac{1}{\Gamma(m+1)}\frac{d^m}{d\eta^m}\left[\sum_{k=0}^{m}\eta^k v_r^k \sum_{k=0}^{m}\eta^k x^{*k}\right]\Bigg|_{\eta=0}.
\end{aligned} \tag{12}$$

Here, K_m and M_m are Adomian polynomials. The use of system (11) and system (13) in model (9) gives the following systems.

$$\begin{aligned}
\mathscr{L}\left(x^0\right) &= \frac{r_1}{s},\\
\mathscr{L}\left(x^{\star^0}\right) &= \frac{r_2}{s},\\
\mathscr{L}\left(x^{\star\star^0}\right) &= \frac{r_3}{s},\\
\mathscr{L}\left(v_p^0\right) &= \frac{r_4}{s},\\
\mathscr{L}\left(v_r^0\right) &= \frac{r_5}{s},
\end{aligned} \tag{13}$$

$$\begin{aligned}
\mathscr{L}\left(x^1\right) &= \frac{1}{s^{\varsigma}}\left(\lambda - d_1x^0 - \beta K_0\right),\\
\mathscr{L}\left(x^{\star^1}\right) &= \frac{1}{s^{\varsigma}}\left(\beta_1K_0 - d_1x^{\star^0} - \alpha_1M_0\right),\\
\mathscr{L}\left(x^{\star\star^1}\right) &= \frac{1}{s^{\varsigma}}\left(\alpha_1M_0 - d_3x^{\star\star^0}\right),\\
\mathscr{L}\left(v^1\right) &= \frac{1}{s^{\varsigma}}\left(k' x^{\star^0} - d_4v_p^0\right),\\
\mathscr{L}\left(w^1\right) &= \frac{1}{s^{\varsigma}}\left(c' x^{\star\star^0} - d_5v_r^0\right),
\end{aligned} \tag{14}$$

and so on. The general terms for successive iterations taking $m \geq 1$ can be written as

$$\begin{aligned}
\mathscr{L}\left(x^{m+1}\right) &= \frac{1}{s^{\varsigma}}\left(\lambda - d_1x^m - \beta_1K_m\right),\\
\mathscr{L}\left(x^{\star^{m+1}}\right) &= \frac{1}{s^{\varsigma}}\left(\beta_1K_m - d_2x^{\star^m} - \alpha_1M_m\right),\\
\mathscr{L}\left(x^{\star\star^{m+1}}\right) &= \frac{1}{s^{\varsigma}}\left(\alpha_1M_m - d_3x^{\star\star^m}\right),\\
\mathscr{L}\left(v_p^{m+1}\right) &= \frac{1}{s^{\varsigma}}\left(k' x^{\star^m} - d_4v_p^m\right),\\
\mathscr{L}\left(v_r^{m+1}\right) &= \frac{1}{s^{\varsigma}}\left(c' x^{\star\star^m} - d_5v_r^m\right).
\end{aligned} \tag{15}$$

To proceed further, the behaviors of the solutions $x(t)$, $x^{\star}(t)$, $x^{\star\star}(t)$, $v_p(t)$, and $v_r(t)$ under different values of ζ will be discussed, and the differences of tendency of the behavior of the solutions between fractional order derivative and integer order derivative will be noted. To get the initial approximations x^0, $x^{\star^0}$, $x^{\star\star^0}$, v_p^0 and v_r^0, the inverse Laplace transform will be applied to the system (14). Then, using these values in the system (15), the second approximations, x^1, $x^{\star^1}$, $x^{\star\star^1}$, v_p^1,

and v_r^1, are determined. Similarly, x^2, $x^{\star^2}$, $x^{\star\star^2}$, v_p^2, and v_r^2 can be calculated. We write these solutions as

$$\begin{aligned} x(t) &= x^0 + x^1 + x^2 + x^3 + \cdots, \\ x^\star(t) &= x^{\star^0} + x^{\star^1} + x^{\star^2} + x^{\star^3} + \cdots, \\ x^{\star\star}(t) &= x^{\star\star^0} + x^{\star\star^1} + x^{\star\star^2} + x^{\star\star^3} + \cdots, \\ v_p(t) &= v_p^0 + v_p^1 + v_p^2 + v_p^3 + \cdots, \\ v_r(t) &= v_r^0 + v_r^1 + v_r^2 + v_r^3 + \cdots. \end{aligned} \tag{16}$$

In the coming section, the numerical behavior of the proposed system will be presented.

4. Multistage Adomian Decomposition Method with Numerical

The following analytical approximate solution can be obtained by using L-ADM. These solutions are represented in terms of an infinite power series. Different values of the parameters used in the proposed system are taken from [9]. Therefore, we can write

$$\begin{aligned} x^0 &= 3, \\ x^{\star^0} &= 6, \\ x^{\star\star^0} &= 3, \\ v_p^0 &= 149, \\ v_r^0 &= 1. \end{aligned} \tag{17}$$

The first approximations under the given initial values can be found as follows:

$$\begin{aligned} x^1 &= (1.4936)\frac{t^\zeta}{\Gamma(\zeta+1)}, \\ x^{\star^1} &= -(2.476)\frac{t^\zeta}{\Gamma(\zeta+1)}, \\ x^{\star\star^1} &= -(5.9928)\frac{t^\zeta}{\Gamma(\zeta+1)}, \\ v_p^1 &= (299.89)\frac{t^{\zeta_5}}{\Gamma(\zeta+1)}, \\ v_r^1 &= (117)\frac{t^\zeta}{\Gamma(\zeta+1)}, \end{aligned} \tag{18}$$

Next, to calculate the second approximation, we will use the initial and first approximate values.

Therefore, we get the following approximation:

$$\begin{aligned} x^2 &= 0.3\frac{t^{2\zeta+1}}{\Gamma(2\zeta+2)} - 014336\frac{t^{2\zeta}}{\Gamma(2\zeta+1)} - 0.72238\frac{t^\zeta}{\Gamma(\zeta+1)}, \\ x^{\star^2} &= 3\frac{t^\zeta}{\Gamma(\zeta+1)} - 0.3242\frac{t^{\zeta+1}}{\Gamma(\zeta+2)} - 0.508912\frac{t^{2\zeta+1}}{\Gamma(2\zeta+2)}, \\ x^{\star\star^2} &= 0.0144\frac{t^{\zeta+1}}{\Gamma(\zeta+2)} - 5.41856\frac{t^{2\zeta+1}}{\Gamma(2\zeta+2)}, \\ v_p^2 &= 299.8212\frac{t^\zeta}{\Gamma(\zeta+1)} + 296\frac{t^{2\zeta}}{\Gamma(2\zeta+1)}, \\ v_r^2 &= 117\frac{t^\zeta}{\Gamma(\zeta+1)} - 287.88\frac{t^{2\zeta}}{\Gamma(2\zeta+1)}. \end{aligned} \tag{19}$$

The other approximation can be found in similar way. We get the following solutions after three terms by using L-ADM at $\zeta = 1$ as

$$\begin{aligned} x(t) &= 3 + (4.5136)t - (1.0728)t^2 - 0.5600t^3, \\ x^\star(t) &= 6 + (8.9928)t - (34.42)t^2 - (0.5089121)t^3, \\ x^{\star\star}(t) &= 3 - (5.9928)t + (0.0014)t^2 - (5.41856)t^3, \\ v_p(t) &= 149 + (598.8212)t + (296)t^2, \\ v_r(t) &= 1 + (234)t - (287.88)t^2. \end{aligned} \tag{20}$$

The system of series solution up to three terms for $\zeta = 0.95$ is given below.

$$\begin{aligned} x(t) &= 3 + 0.7870550345t^{0.95} - 0.7845218562t^{1.9} \\ &\quad + 0.5661093291t^{2.90}, \\ x^\star(t) &= 6 + 1.773297592t^{0.95} - 0.7845218562t^{1.90} \\ &\quad + 0.5661093291t^{2.90}, \\ x^{\star\star}(t) &= 3 - 6.115846854t^{0.95} - 1.022499122t^{2.90} \\ &\quad + 0.7536239617t^{1.95}, \\ v_p(t) &= 149 + 612.0247390t^{0.95} + 162.4321420t^{2.90}, \\ v_r(t) &= 1 + 238.8045928t^{0.95} - 157.5391685t^{1.90}. \end{aligned} \tag{21}$$

Further, we obtain the following series after some simplification for $\zeta = 0.85$ up to three terms as

$$\begin{aligned} x(t) &= 3 + 0.8155783471t^{0.85} - 0.9280851532t^{1.70} \\ &\quad + 0.7193120298t^{2.70}, \\ x^\star(t) &= 6 + 1.837562884t^{0.85} - 0.9280851532t^{1.70} \\ &\quad + 0.7193120298t^{2.70}, \\ x^{\star\star}(t) &= 3 - 6.337488549t^{0.85} + 0.8231484547t^{1.85} \\ &\quad - 1.299211797t^{2.70}, \\ v_p(t) &= 149 + 634.2048563t^{0.85} + 192.1563538t^{1.70}, \\ v_r(t) &= 1 + 247.4590042t^{0.85} - 186.3679924t^{1.70}. \end{aligned} \tag{22}$$

The solutions for $\zeta = 0.75$ up to three terms is given by

$$\begin{aligned} x(t) &= 3 + 0.8391376836t^{0.75} - 0.1078429583t^{1.50} \\ &\quad + 0.9027033339et^{2.50}, \end{aligned}$$

$$x^{\star}(t) = 6 + 1.890643943t^{0.75} - 0.1078429583t^{1.50} + 0.9027033339t^{2.50},$$
$$x^{\star\star}(t) = 3 - 6.520557442t^{0.75} + 0.8953222645t^{1.75} - 1.630450726t^{2.50},$$
$$v_p(t) = 149 + 652.5249180t^{0.75} + 223.2845723t^{1.50},$$
$$v_r(t) = 1 + 254.6072690t^{0.75} - 216.5585298t^{1.50}. \quad (23)$$

5. Numerical Discussion

The numerical results shown in Figures 1–5 imply the effectiveness of numerical methods discussed here. These methods give highly accurate results in very few iterations. Some simulations results (21), (22), and (23) are given as illustrations.

The figures show the behavior of the solutions of the variables $x(t), x^{\star}(t), x^{\star}(t), v_p(t)$, and $v_r(t)$. It is observed from the Figures (1–5) that the Laplace-Adomian decomposition method is more accurate. The presented method can predict the behavior of these variables accurately for the region under consideration. Figure 2 shows the simulation of approximate solutions by assuming $\zeta = 1, 0.95, 0.85, 0.75$. It is clear from these figures that the behavior of the approximate solutions depends continuously on the time-fractional derivative. Moreover, we can see that the potential of this access can be rapidly increased if the step size is decreased.

Remark 1. Here, we assume that $w = (x, x^{\star}, x^{\star\star}, v_p, v_r), X = R_+{}^5$.

6. Convergence Analysis

To check the convergence of the series solutions given in (19), we give the following result.

Theorem 2 (see [14]). *Let $\tilde{T} : X \to X$ be a nonlinear operator which is also contractive, where X is Banach space, that is, for all $w, \bar{w} \in X$, $\|\tilde{T}(w) - \tilde{T}(\bar{w})\|, 0 \le \mu\|w - \bar{w}\|, 0 < \mu < 1$. Then, in view of Banach contraction theorem, $\tilde{T}$ has a unique point w^* such that $\tilde{T}w^* = w^*$. Further assume that the sequence of series generated by LADM can be written as the series (19) can be written as follows:*

$$w_n = \tilde{T}w_{n-1},$$
$$w_{n-1} = \sum_{j=1}^{m-1} w_{j-1}, \quad (24)$$
$$n = 1, 2, 3, \ldots,$$

and suppose that $w_0 = w_0 \in S_r(w)$ where $S_r(w) = \{w^* \in X : \|w^* - w\| < r\}$, then,

(i) $w_n \in S_r(w)$,

(ii) $\lim_{n\to\infty} w_n = w$.

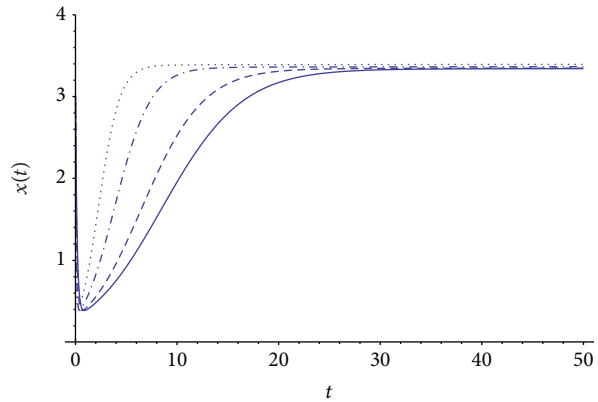

Figure 1: The graph represents the solution behavior for the variable $x(t)$ versus time t under different values of the order of derivative. The solid line, dashed line, dot-dashed line, and dotted line represent solution behavior at $\zeta = 1, \zeta = 0.95, \zeta = 0.85$, and $\zeta = 0.75$, respectively.

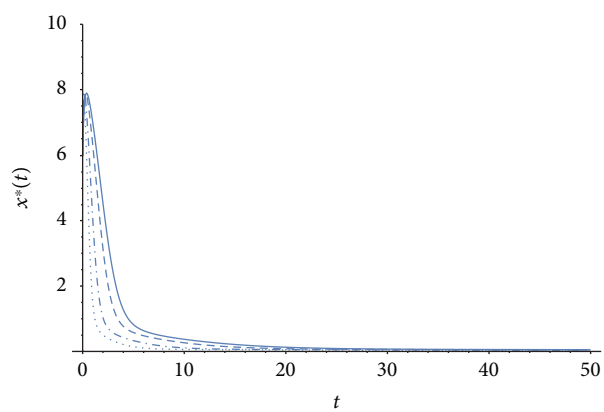

Figure 2: The graph represents the solution behavior for the variable $x^{\star}(t)$ (concentration of infected cells) versus time t under different values of the order of derivative. The solid line, dashed line, dot-dashed line, and dotted line represent solution behavior at $\zeta = 1, \zeta = 0.75, \zeta = 0.85$, and $\zeta = 0.95$, respectively.

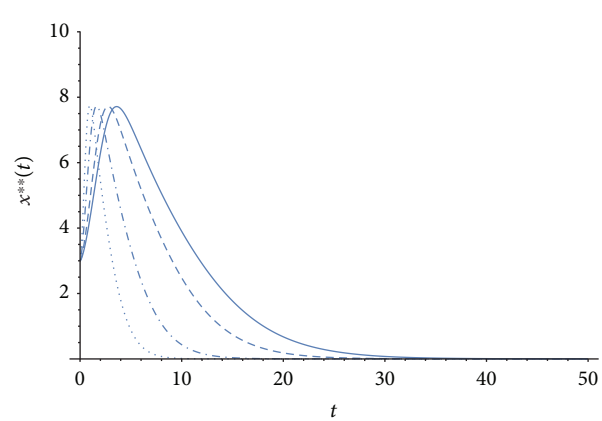

Figure 3: The graph represents the solution behavior the variable $x^{\star\star}(t)$ (concentration of double infected cells) versus time t under different values of the order of derivative. The solid line, dashed line, dot-dashed line, and dotted line represent solution behavior at $\zeta = 1, \zeta = 0.75, \zeta = 0.85$, and $\zeta = 0.95$, respectively.

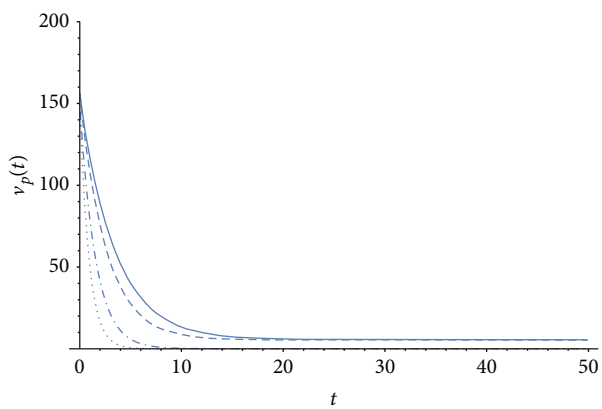

Figure 4: The graph represents the solution behavior for the variable $v_p(t)$ (concentration of pathogen virus) versus time t under different values of the order of derivative. The solid line, dashed line, dot-dashed line, and dotted line represent solution behavior at $\zeta = 1, \zeta = 0.75, \zeta = 0.85$, and $\zeta = 0.95$, respectively.

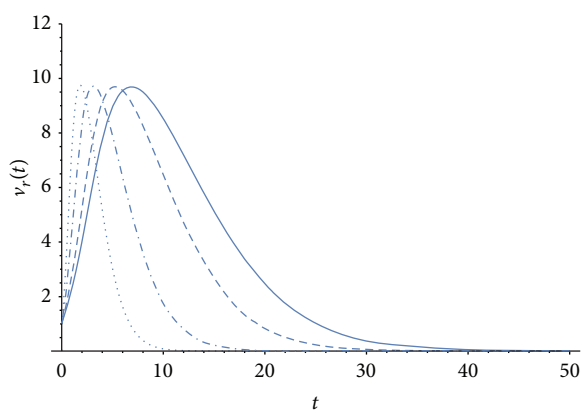

Figure 5: The graph represents the solution behavior for the variable $v_r(t)$ (concentration of recombinant virus) versus time t under different values of the order of the derivative. The solid line, dashed line, dot-dashed line, and dotted line represent solution behavior at $\zeta = 1, \zeta = 0.95, \zeta = 0.85$, and $\zeta = 0.75$, respectively.

Proof 2. Let $w = (x, x^{\star}, x^{\star\star}, v_p, v_r), X = R_+^5$ be the exact solution of the model $w = \tilde{T}w$. Also, $w_n = \tilde{T}w_{n-1}, w_1 = \tilde{T}w_0$.

Now

$$\|w_1 - w\| = \left\|\tilde{T}w_0 - \tilde{T}w\right\| \leq \mu^1 \|w_0 - w\|. \tag{25}$$

Assume that

$$\|w_{n-1} - w\| = \left\|\tilde{T}w_0 - \tilde{T}w\right\| \leq \mu^{n-1} \|w_0 - w\|. \tag{26}$$

We now prove for $n = m$,

$$\begin{aligned} \|w_n - w\| &= \left\|\tilde{T}w_{n-1} - \tilde{T}w\right\| \leq \mu^{n-1}\left\|\tilde{T}w_{n-2} - \tilde{T}w\right\| \\ &= \mu\|w_n - 1 = w\|/\leq\mu\left(\mu^{n-1}\|w_0 - w\|\right) \\ &\leq \mu^n \|w_0 - w\|, n \to \infty, \mu^n \to 0. \end{aligned} \tag{27}$$

Thus, we have

(i) $w_n \in S_r(w)$,

(ii) $\lim_{n\to\infty} w_n = w$.

7. Conclusion

In this research work, the numerical solution of fractional order HIV-1 model is discussed by using the method known as L-ADM. For this purpose, different values are given to ζ and numerical simulations for these values have been presented. It may be concluded from these figures that the concentration of virus decreases in the body as ζ increases from 0.75 to 0.85. But the viral load tends toward zero as ζ increases to 0.95 (see Figure 4). Moreover, the load of CD4+ T cells is increased by increasing the value of ζ (see Figure 1). One can conclude that increase in ζ can cause increase in the level of CD4+ T cells and decrease in viral level. Hence, when $\zeta \to 0$, then the solution of the fractional order model is reduced to the solution of integer order model. Moreover, the convergence of the method is also discussed. Thus, it is observed that LADM can be used to increase the interval of convergence for the derived series solution. It is also shown that this method is very efficient and accurate method as compared with RK4 method and it is better to use this method for the solution of the other nonlinear systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

There is no competing interest regarding this paper.

Authors' Contributions

All authors equally contributed this research work.

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