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Research Article

Performance Index Based Observer-Type Iterative Learning Control for Consensus Tracking of Uncertain Nonlinear Fractional-Order Multiagent Systems

Liming Wang [1,2] and Guoshan Zhang [1]

[1] *School of Electrical and Information Engineering, Tianjin University, Tianjin 300072, China*
[2] *Faculty of Physics and Electronic Information, Langfang Normal University, Langfang 065000, China*

Correspondence should be addressed to Liming Wang; wlm_shooker@163.com

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This paper is devoted to the perfect tracking problem of consensus and the monotonic convergence problem of input errors for the uncertain nonlinear fractional-order multiagent systems (FOMASs), where there exist the linear coupling relations between the fractional order, the perturbations of the system matrix, and input matrix. For the FOMASs including one leader agent and multiple follower agents, an observer-type fractional-order iterative learning consensus protocol is proposed. Based on the two-dimensional analysis for the FOMASs, a novel performance index, which can exhibit the monotonic convergence of the input errors, is constructed by using the definition of fractional integral. A Lyapunov-like method is applied to derive the sufficient conditions in terms of linear matrix inequalities, which can guarantee the perfect tracking of consensus and the monotonic convergence of input errors. Finally, the numerical simulation results including comparisons to traditional two-dimensional analysis are presented to demonstrate the effectiveness of the proposed methods.

1. Introduction

Over the past decades, due to the potential applications of the consensus for multiagent systems (MASs), their studies have attracted more and more attention [1–6]. Among most of the existing results, the MASs are assumed to have the integer-order dynamics, such as first-order dynamics [7, 8], second-order dynamics [9, 10], and higher-order dynamics [11, 12]. Recently, it has been found that many practical coordinate behaviors of agents in the complex environment, including vehicles moving on the top of macromolecule fluids and porous media [13] and aircrafts traveling at the high speeds in dust storm, rain/snow environment [14], often exhibit the fractional-order dynamics. Since the fractional-order multiagent systems (FOMASs) include the integer-order multiagent systems (IOMASs) as their special cases, it is significant to consider the consensus problem of FOMASs. The consensus problem of FOMASs has been widely investigated from various perspectives, such as heterogeneities [15], uncertainties [16] and information transmission delays [17] or information processing delays [18]. Furthermore, some control methods, including sliding mode control [19], event-triggered control [20], and impulsive control [21], are also developed for the consensus of FOMASs.

In the above literatures of FOMASs, the consensus is achieved in the one-dimensional system framework evolving along the time axis, and these studies are not suitable for arbitrary high precision tracking tasks of the FOMASs. The iterative learning control (ILC) method can create a two-dimensional process with time axis and iteration axis as two independent directions [22]. Thus, by iteratively adjusting the input signal from one iteration to the next, the ILC method can achieve the perfect consensus. Here the perfect consensus refers to the phenomena that the states or outputs of all follower agents converge to that of the leader agent in a finite time interval as the iteration step goes to infinity. The ILC method has been used for the perfect consensus of the IOMASs. [23, 24] have proposed the ILC protocols for the perfect consensus and the formation control of IOMASs, respectively. It is indicated in [25] that the iterative learning convergence rate of consensus can be enhanced by input sharing among agents with integer-order dynamics. Further, the perfect consensus problem of IOMASs with output saturation is solved by a distributed ILC algorithm [26]. Recently, to obtain the better performance of perfect consensus, the ILC scheme is combined with event-triggered control method [27].

In [23–27], the time-weighted norm based contraction mapping (CM) method is used to analyze the convergence of learning process. The method ignores the dynamics of agents and cannot derive the consensus convergence conditions guaranteeing the good transients of learning process. On the other hand, the bad learning transients, especially unacceptable overshoots, will be the obstacles to applying the ILC method into the practical problems, and should try to be overcome. The composite energy function (CEF) method is one of the remedies for the bad transient phenomena, because the CEF method is defined in $\mathscr{L}^2$ norm and can lead to point-wise convergence [28]. There have existed some instructive results of the CEF method in the perfect consensus problem for the linear IOMASs, including first-order agents [29], second-order agents [30] and high-order agents [31]. The similar methods are applied to the perfect consensus problems of the nonlinear IOMASs with unknown control direction [32]. In [33], the CEF method is used to design the ILC protocol for the perfect consensus of continuous and discrete IOMASs.

The above-mentioned literatures about the ILC consensus assume that the states of agents are measurable, while this is not always true in practice. There have been some literatures about the observer-based perfect consensus of IOMASs [34, 35] and the observer-based asymptotic consensus of the FOMASs [36–39]. It should be mentioned that the models in [34–39] are free from the perturbations, and the constructed observers are applicable to the systems without any uncertainty. Uncertainties are inevitable in the real physical systems, thus it should pay attention to the observer-based control for the consensus of uncertain FOMASs. There have been a few literatures about the observer-based control for the uncertain systems merely including one node [40–44]. From the perspectives of relations between the parameters of observers and those of corresponding systems, the existing results about observer-based control for the uncertain system can be generally classified into the following two cases. In the first case (see [40, 41]), the observers are constructed by using the parameters of nominal model of the corresponding systems, and the constructed observers are systems without any uncertainty. In the second case (see [42–44]), the time-varying structural uncertainties are assumed to be measurable, and the observers are constructed by using the uncertain time-varying system matrices same as the corresponding systems. Thus the constructed observers in fact are the time-varying uncertain systems. To the best of authors' knowledge, little work has been done about the observer-based control for consensus of uncertain MASs, especially uncertain FOMASs. Due to the invalidation of the well-known Leibniz rule in the fractional derivatives and the particularities of the perfect consensus problem, it is still a great challenge to design an observer-type ILC protocol for the good transients of learning process and the perfect tracking of consensus for the FOMASs.

Inspired by the above work, in this paper, we investigate the consensus perfect tracking problem with good transients for uncertain nonlinear FOMASs, where there exist the linear coupling relations among the fractional order, the perturbations of the system matrix and input matrix. Firstly, the perfect tracking problem of consensus and the monotonic convergence problem of input errors are formulated and are transformed into the control problem of the tracking error system, and a performance index reflecting the monotonic convergence of the input errors is constructed. Secondly, an observer-type fractional-order ILC consensus protocol is designed, and the consensus sufficient conditions are derived for the nonlinear FOMASs without any uncertainty. Then, the method is further developed for the consensus of the uncertain nonlinear FOMASs, and the related consensus criterions are derived. Finally, two numerical examples are presented to validate the proposed methods. The main contributions of this paper can be summarized as follows:

(1) Investigate the perfect tracking problem of consensus and the monotonic convergence problem of ILC law for a class of general models of uncertain nonlinear FOMASs with the parameter coupling. The proposed models can include the following existing models as their special cases: (a) linear IOMASs with (or without) uncertainties and parameter coupling [34, 35]; (b) nonlinear IOMASs with (or without) uncertainties and parameter coupling [25]; (c) linear FOMAS with (or without) uncertainties and parameter coupling [16, 20]. (d) Nonlinear FOMASs with (or without) uncertainties and parameter coupling [21]. In the existing studies about FOMASs, the asymptotic consensus or the finite time consensus has been considered, while there has not been any report about the perfect consensus of FOMASs. Moreover, in the existing literatures about the perfect consensus of IOMASs, the monotonic convergence problem of the learning law is seldom considered.

(2) Based on the observed information of agents' neighbors, an observer-type fractional-order ILC consensus protocol is designed. Based on the definition of the fractional integral, a novel performance index, which can reflect the monotonic convergence of the input errors, is constructed after the two-dimensional (2D) analysis for the nonlinear FOMASs. The stability and the monotonic convergence of iterative learning process are analyzed by a Lyapunov-like method, thereby the sufficient conditions are derived for the perfect consensus of the nonlinear FOMASs with or without uncertainties, respectively.

(3) By solving the linear matrix inequalities (LMIs) in the sufficient conditions, the appropriate learning gain matrices are obtained directly. Under the proposed consensus protocol with the obtained learning gain matrices, the perfect tracking of consensus and the monotonic convergence of the input errors are achieved simultaneously. That is, the consensus perfect tracking with good transients is achieved.

The rest of this paper is organized as follows. In Section 2, notations, basic terminologies in graph theory, and some existing results about fractional order calculus are introduced. Both the perfect tracking problem of consensus and the monotonic convergence problem of input errors for the uncertain nonlinear FOMASs with the parameter coupling relations are formulated. Subsequently, a Lyapunov-like method is applied to deal with the formulated problems and the related sufficient conditions in terms of LMIs are derived for the nonlinear FOMASs without any uncertainty, then the reasonable extension to the uncertain nonlinear FOMASs are presented in Section 3. Two numerical illustrative examples are presented in Section 4. Finally, the conclusions are drawn in Section 5.

2. Background and Preliminaries

In this section, some basic notations, useful lemmas and the algebraic graph theory as well as the model of FOMASs are presented.

2.1. Notations. Let R, $R^{m\times n}$, and Z_+ be the set of real numbers, the set of $m\times n$ real matrices and the set of nonnegative integers, respectively. The superscript T denotes the transposition of vector or matrix, and I_m (0_m) denotes the $m\times m$ identity (zero) matrix; $\mathrm{sym}\{M\}$ indicates the expression $M+M^{\mathrm{T}}$, and $\mathrm{diag}\{c_1,c_2,\ldots,c_n\}$ denotes the block diagonal matrix with c_i, $i=1,2,\ldots,n$ as its diagonal elements; $M>0$ $(M<0)$ implies a symmetric positive (negative) definite matrix, and the notation $*$ denotes the entries determined by symmetry; $\|\cdot\|$ refers to the Euclidean norm for vectors and to the spectrum norm for matrices, and the notation $\otimes$ denotes the Kronecker product of two matrices. Matrices, if not explicitly stated, are assumed to have appropriate dimensions. Moreover, $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ denotes a weighted directed graph with the set of vertices $\mathcal{V}=\{1,2,\ldots,N\}$, the set of edges $\mathcal{E}\in\mathcal{V}\times\mathcal{V}$, and the adjacency matrix $\mathcal{A}=(a_{kj})\in R^{N\times N}$. In particular, $a_{kk}=0$, and $a_{kj}=1$ if $(j,k)\in\mathcal{E}$, and $a_{kj}=0$ otherwise. The detailed description about graph theory can refer to [8, 23].

2.2. Some Definitions and Lemmas. The following definitions and lemmas will be used for the convergence analysis in the main results.

Definition 1 (see [45]). The definition of fractional integral is described by ${}_{t_0}D_t^{-\alpha}f(t)=(1/\Gamma(\alpha))\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)d\tau$, $\alpha>0$, where $\Gamma(\alpha)$ is the well-known Gamma function.

Definition 2 (see [45]). The Caputo derivative is defined as ${}_{t_0}^{C}D_t^{\alpha}f(t)={}_{t_0}D_t^{\alpha-m}D_t^{m}f(t)$, $\alpha\in(m-1,m)$, where $D_t^m(\cdot)$ denotes the classical m-order derivative, and m is a positive integer.

Note. In the following, ${}_{0}^{C}D_t^{\alpha}f(t)$ is denoted as $D_t^{\alpha}f(t)$ for simplicity.

Property 3 (see [45, 46]). Let $\alpha,\beta>0$. If $f_1(t)$ and $f_2(t)$ are continuous on the interval $t\in[0,T]$, then the following relations hold:

(1) $D_t^{\alpha}D_t^{-\beta}f_1(t)=D_t^{\alpha-\beta}f_1(t)$;

(2) $D_t^{\alpha}(f_1(t)\pm f_2(t))=D_t^{\alpha}f_1(t)\pm D_t^{\alpha}f_2(t)$;

(3) $D_t^{\alpha}D_t^{-\alpha}f_1(t)=f_1(t)$;

(4) $D_t^{-\alpha}D_t^{\alpha}f_1(t)=f_1(t)-f_1(0)$.

Property 4 (see [47]). Let $x(t)\in R^n$ be a vector of differentiable functions. Then, for any time instant, the following relation holds: $D_t^{\alpha}(x^{\mathrm{T}}(t)Px^{\mathrm{T}}(t))\le x^{\mathrm{T}}(t)PD_t^{\alpha}x(t)+(D_t^{\alpha}x^{\mathrm{T}}(t))Px(t)$, $\forall\alpha\in(0,1]$, $\forall t\ge 0$, where the equal sign corresponds to the case of $\alpha=1$, and $P\in R^{n\times n}$ is a constant, square, symmetric and positive definite matrix.

Lemma 5 (see [48]). *Given matrices $Q=Q^{\mathrm{T}}$, M, N of appropriate dimensions. The inequality $Q+M\sum(t)N+[M\sum(t)N]^{\mathrm{T}}<0$ holds for any $\sum(t)$ satisfying $\sum^{\mathrm{T}}(t)\sum(t)\le I$ if and only if there exists a scalar constant $\varepsilon>0$ such that $Q+\varepsilon N^{\mathrm{T}}N+\varepsilon^{-1}MM^{\mathrm{T}}<0$.*

2.3. Model Description and Problem Formulation. Consider the uncertain nonlinear FOMASs consisting of one leader agent and N follower agents, which work in a repeatable control environment. The leader agent, labeled as d, has nonlinear dynamics [49, 50] as

$$\begin{aligned} D_t^{\alpha}x_d(t)&=A(t)x_d(t)+f(t,x_d(t))+B(t)u_d(t),\\ y_d(t)&=Cx_d(t), \end{aligned}\tag{1}$$

where $\alpha\in(0,1)$ and $t\in[0,T]$. $x_d(t)\in R^n$, $y_d(t)\in R^m$ and $u_d(t)\in R^p$ are the state, output and control input of the leader agent, and $f(\cdot)\in R^n$ is a nonlinear function satisfying Assumption 6. $A(t)\in R^{n\times n}$, $B(t)\in R^{n\times p}$ and $C\in R^{m\times n}$ are the system matrix, input matrix and output matrix, and $A(t)$ and $B(t)$ satisfy Assumption 8.

Assumption 6 (see [51]). Assume that $f(\cdot)$ is a Lipschitz continuous nonlinear function, that is, there exists a positive constant l such that for any $z_1(t),z_2(t)\in R^n$, $\|f(t,z_1(t))-f(t,z_2(t))\|\le l\|z_1(t)-z_2(t)\|$, where l is called as the Lipschitz constant.

Remark 7. Assumption 6 is a classical Lipschitz condition for a continuous nonlinear function. It guarantees the existence and uniqueness of the solution of system (1) and will be used to prove Lemma 9 and to derive the consensus convergence conditions (See Theorems 20 and 23).

Assumption 8. Assume that there exist the simple linear coupling relations among the fractional order α, the perturbation $\Delta A(t)$ of system matrix $A(t)$ and the perturbation $\Delta B(t)$ of input matrix $B(t)$ [16]. More specifically, the linear coupling relations are described by

$$\begin{aligned} A(t)&=A_0+\mu\alpha\Delta A(t),\\ B(t)&=B_0+\mu\alpha\Delta B(t), \end{aligned}\tag{2}$$

where $\mu>0$ is a scalar constant, and A_0 and B_0 are the constant matrices, and A_0 is Hurwitz and (A_0,B_0,C) is stabilizable and detectable. The time-varying matrices $\Delta A(t)$ and $\Delta B(t)$ are measurable and satisfy [42–44]

$$\Delta A(t) = E\sum(t)F_1, \qquad \Delta B(t) = E\sum(t)F_2, \tag{3}$$

where E, F_1 and F_2 are the known real constant matrices of appropriate dimensions, while $\sum(t)$ is a time-varying matrix satisfying $\sum^{\mathrm{T}}(t)\sum(t) \le I$.

The follower agents are labeled by j, $j \in \{1, 2, \ldots, N\}$, and the interaction topology among all the follower agents is described by the fixed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$, where $\mathscr{V} = \{1, 2, \ldots, N\}$, $\mathscr{E}$ and $\mathscr{A}$ are the edge set and the weighted adjacency matrix of $\mathscr{G}$, respectively. Thus, the interaction topology among all agents can be described by the graph $\overline{\mathscr{G}} = \{\overline{\mathscr{V}}, \overline{\mathscr{E}}, \overline{\mathscr{A}}\}$, where $\overline{\mathscr{V}} = \{0, 1, 2, \ldots, N\}$, $\overline{\mathscr{E}}$ and $\overline{\mathscr{A}}$ are the edge set and the weighted adjacency matrix of $\overline{\mathscr{G}}$, respectively.

At the kth iteration, the dynamics of the follower agent j take the following form

$$\begin{aligned} D_t^{\alpha} x_{k,j}(t) &= A(t)x_{k,j}(t) + f\left(t, x_{k,j}(t)\right) \\ &\quad + B(t)u_{k,j}(t), \\ y_{k,j}(t) &= Cx_{k,j}(t), \end{aligned} \tag{4}$$

where $k \in Z_+$ denotes the iteration step. $x_{k,j}(t) \in R^n$, $y_{k,j}(t) \in R^m$ and $u_{k,j}(t) \in R^p$ are the state, output and control input of the system (4), and the other notations are the same as those in (1). Here the system matrix and input matrix of all agents are assumed to be identical. As pointed out in [52], this case is reasonable, because it can represent birds and school of fishes.

Lemma 9. *Let the scalar constant $\varepsilon_1 > 0$, the positive definite matrix $Q_1 = Q_1^{\mathrm{T}} \in R^{n\times n}$, and assume that Assumptions 6 and 8 hold. If A_0 is Hurwitz, and there exist a positive definite matrix $P_1 = P_1^{\mathrm{T}}$ satisfying*

$$\begin{bmatrix} A_0^{\mathrm{T}}P_1 + P_1A_0 + l^2I_n + \varepsilon_1\mu\alpha F_1^{\mathrm{T}}F_1 + Q_1 & P_1 & P_1E \\ P_1 & -I_n & 0 \\ * & * & -\varepsilon_1\mu^{-1}\alpha^{-1}I_n \end{bmatrix} < 0, \tag{5}$$

then the fractional-order nonlinear uncertain agent (4) under no control is stable.

Lemma 9 provides the sufficient condition guaranteeing that the fractional-order nonlinear uncertain agent (4) under no control is asymptotically stable, and its detailed proof is given in the Appendix.

Remark 10. (a) If $\mu = 0$ and $\Delta A(t) = \Delta B(t) = 0$, the model will degenerate into the nonlinear FOMASs without any uncertainty and parameter coupling relation. (b) If $f(\cdot) = 0$, the model will degenerate into the uncertain linear FOMASs, where there exist linear coupling relations among α, $\Delta A(t)$, and $\Delta B(t)$. (c) If $\mu = 1/\alpha$, the model will degenerate into the special uncertain nonlinear FOMASs, where there is no coupling relation among α, $\Delta A(t)$, and $\Delta B(t)$, but $A(t) = A_0 + \Delta A(t)$ and $B(t) = B_0 + \Delta B(t)$. (d) If $\mu \ne 1/\alpha$, $\Delta A(t) \ne 0$, $\Delta B(t) \ne 0$ and $f(\cdot) \ne 0$, the model will become the more general uncertain nonlinear FOMASs, where there exist linear coupling relations among α, $\Delta A(t)$ and $\Delta B(t)$. The asymptotic consensus of FOMASs in Case (a) and (b) has been studied [16, 20, 21], while there is no report about the perfect consensus of FOMASs in all of the above cases.

Assume that the state of each follower agent is not measurable. To estimate the states of the follower agent j, its state observer is designed as

$$\begin{aligned} D_t^{\alpha}\hat{x}_{k,j}(t) &= A(t)\hat{x}_{k,j}(t) + f\left(t, \hat{x}_{k,j}(t)\right) + B(t)u_{k,j}(t) \\ &\quad + L\left(y_{k,j}(t) - C\hat{x}_{k,j}(t)\right), \end{aligned} \tag{6}$$

where $\hat{x}_{k,j}(t) \in R^n$ denotes the observed state vector, $L \in R^{n\times m}$ is the gain matrix of observer, and the other notations are the same as those in (1).

Remark 11. Similar to [42–44], this paper assumes that the time-varying structural uncertainties $\Delta A(t)$ and $\Delta B(t)$ are measurable. This assumption holds in many applications. For example, the dynamics of an aero-engine can be described as a linear system subject to the time-varying measurable uncertainties [53]. By using the uncertain time-varying system matrices same as the corresponding agents, we construct the state observers. Thus the constructed observers in fact are the time-varying uncertain systems [42–44], which are different from the observer constructed by the nominal parameters [40, 41].

Let the observation error $\tilde{x}_{k,j}(t) = x_{k,j}(t) - \hat{x}_{k,j}(t)$. From (4) and (6), we have

$$\begin{aligned} D_t^{\alpha}\tilde{x}_{k,j}(t) &= (A(t) - LC)\tilde{x}_{k,j}(t) \\ &\quad + \left(f\left(t, x_{k,j}(t)\right) - f\left(t, \hat{x}_{k,j}(t)\right)\right). \end{aligned} \tag{7}$$

For the observation error system (7), we can obtain the following Lemma:

Lemma 12. *Let the scalar constant $\varepsilon_2 > 0$, the positive definite matrix $Q_2 = Q_2^{\mathrm{T}} \in R^{n\times n}$, and assume that Assumptions 6 and 8 hold. If L can be chosen such that $A_0 - LC$ is Hurwitz, and there exist a positive definite matrix $P_2 = P_2^{\mathrm{T}}$ satisfying*

$$\begin{bmatrix} (A_0 - LC)^{\mathrm{T}}P_2 + P_2(A_0 - LC) + l^2I_n + \varepsilon_2\mu\alpha F_1^{\mathrm{T}}F_1 + Q_2 & P_2 & P_2E \\ P_2 & -I_n & 0 \\ * & * & -\varepsilon_2\mu^{-1}\alpha^{-1}I_n \end{bmatrix} < 0, \tag{8}$$

then the observation error $\tilde{x}_{k,j}(t)$ is bounded and converges asymptotically to zero as time goes to infinity.

Lemma 12 provides the sufficient condition guaranteeing the boundedness and asymptotic convergence of the observation errors, and its proof is similar to that of Lemma 9, thus is omitted.

Remark 13. In this paper, the system matrix A_0 is assumed to be Hurwitz (i.e., the agent is assumed to be stable). It is necessary, because it ensures that (5) has positive definite solutions P_1, which will be needed in the proofs of Theorems 20 and 23. Meanwhile, it should be pointed out that it is meaningful to consider the observer-based perfect consensus of the FOMASs consisting of the stable agents. On the one hand, for the leader-following FOMASs such as (1) and (4), even though the system matrix A_0 is Hurwitz, when the state of agent is not measurable, to ensure the boundedness and convergence of the observation errors, the state observer and its gain matrices have to be designed (See Lemma 12 and Corollary 19). On the other hand, this paper focuses on the perfect consensus tracking problem. For the MASs consisting of the stable agents, the asymptotic consensus can be achieved without requiring any control. However, if no control is applied, even for the MASs consisting of the stable agents, the perfect consensus cannot be achieved, and especially the good transients such as the monotonic convergence of input errors cannot be achieved.

Let $\zeta_{k,j}(t)$ denote the information received by the follower agent j at the kth iteration. More specifically,

$$\zeta_{k,j}(t) = \sum_{i\in N_j} a_{ji}\left(\hat{x}_{k,i}(t) - \hat{x}_{k,j}(t)\right) + d_j\left(x_d(t) - \hat{x}_{k,j}(t)\right), \tag{9}$$

where a_{jk} is the $(j,k)th$ entry in the adjacency matrix $\mathscr{A}$, and N_j is the neighborhood set of the follower agent j. $d_j = 1$ if the follower agent j can receive directly the information of the leader agent, and $d_j = 0$ otherwise.

Let the consensus tracking error $e_{k,j}(t) = x_d(t) - \hat{x}_{k,j}(t)$. The compact format of (9) is rewritten as

$$\zeta_k(t) = \left(H\otimes I_n\right)e_k(t), \tag{10}$$

where $\zeta_k(t) = [\zeta_{k,1}{}^{\mathrm{T}}(t), \zeta_{k,2}{}^{\mathrm{T}}(t), \ldots, \zeta_{k,N}{}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $e_k(t) = [e_{k,1}{}^{\mathrm{T}}(t), e_{k,2}{}^{\mathrm{T}}(t), \ldots, e_{k,N}{}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $H = G+D$, G is the Laplacian matrix of graph $\mathscr{G}$, and $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$.

Based on (10), an observer-type ILC consensus protocol is designed as

$$u_{k+1,j}(t) = u_{k,j}(t) + K\zeta_{k,j}(t) + \Gamma D_t^{\alpha}\zeta_{k,j}(t), \tag{11}$$

where both $K \in R^{p\times n}$ and $\Gamma \in R^{p\times n}$ are the learning gain matrices to be designed later.

By (10), the compact format of (11) is rewritten as

$$u_{k+1}(t) = u_k(t) + (H\otimes K)e_k(t) + (H\otimes\Gamma)D_t^{\alpha}e_k(t), \tag{12}$$

where $u_k(t) = (u_{k,1}^{\mathrm{T}}(t), u_{k,2}^{\mathrm{T}}(t), \ldots, u_{k,N}^{\mathrm{T}}(t))^{\mathrm{T}}$.

Let $\delta u_{k,j}(t) = u_d(t) - u_{k,j}(t)$. From (1) and (6), we have

$$D_t^{\alpha}e_{k,j}(t) = A(t)e_{k,j}(t) + B(t)\delta u_{k,j}(t) - LC\tilde{x}_{k,j}(t) + \left(f(t,x_d(t)) - f\left(t,\hat{x}_{k,j}(t)\right)\right). \tag{13}$$

Let $\phi_{k,j}(t) = f(t,x_d(t)) - f(t,\hat{x}_{k,j}(t))$ and $\varphi_{k,j}(t) = f(t,x_{k,j}(t)) - f(t,\hat{x}_{k,j}(t))$. The compact formats of (13) and (7) are rewritten as

$$D_t^{\alpha}e_k(t) = \left(I_N\otimes A(t)\right)e_k(t) + \left(I_N\otimes B(t)\right)\delta u_k(t) - \left(I_N\otimes LC\right)\tilde{x}_k(t) + \phi_k(t), \tag{14}$$

$$D_t^{\alpha}\tilde{x}_k(t) = \left(I_N\otimes(A(t)-LC)\right)\tilde{x}_k(t) + \varphi_k(t), \tag{15}$$

where $\delta u_k(t) = (\delta u_{k,1}^{\mathrm{T}}(t), \delta u_{k,2}^{\mathrm{T}}(t), \ldots, \delta u_{k,N}^{\mathrm{T}}(t))^{\mathrm{T}}$, $\phi_k(t) = (\phi_{k,1}^{\mathrm{T}}(t), \phi_{k,2}^{\mathrm{T}}(t), \ldots, \phi_{k,N}^{\mathrm{T}}(t))^{\mathrm{T}}$, $\tilde{x}_k(t) = (\tilde{x}_{k,1}^{\mathrm{T}}(t), \tilde{x}_{k,2}^{\mathrm{T}}(t), \ldots, \tilde{x}_{k,N}^{\mathrm{T}}(t))^{\mathrm{T}}$ and $\varphi_k(t) = (\varphi_{k,1}^{\mathrm{T}}(t), \varphi_{k,2}^{\mathrm{T}}(t), \ldots, \varphi_{k,N}^{\mathrm{T}}(t))^{\mathrm{T}}$.

Combining (14) and (15) results in a system

$$D_t^{\alpha}\theta_k(t) = R_1\theta_k(t) + R_2\delta u_k(t) + \Psi_k(t), \tag{16}$$

where $\theta_k(t) = [e_k{}^{\mathrm{T}}(t), \tilde{x}_k{}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\Psi_k(t) = [\phi_k{}^{\mathrm{T}}(t), \varphi_k{}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $R_1 = \begin{pmatrix} I_N\otimes A(t) & -I_N\otimes(LC) \\ 0 & I_N\otimes(A(t)-LC)\end{pmatrix}$ and $R_2 = \begin{pmatrix} I_N\otimes B(t) \\ 0\end{pmatrix}$.

By (12), we have

$$\begin{aligned}\delta u_{k+1}(t) &= 1_N u_d(t) - u_{k+1}(t) = \delta u_k(t) - (H\otimes K) \\ &\cdot e_k(t) - (H\otimes\Gamma)D_t^{\alpha}e_k(t) = \delta u_k(t) - (H\otimes K) \\ &\cdot e_k(t) - (H\otimes\Gamma)\left(\left(I_N\otimes A(t)\right)e_k(t)\right. \\ &+ \left(I_N\otimes B(t)\right)\delta u_k(t) - \left(I_N\otimes(LC)\right)\tilde{x}_k(t) \\ &\left.+ \phi_k(t)\right) = R_3\theta_k(t) + R_4\delta u_k(t) - (H\otimes\Gamma)\phi_k(t),\end{aligned} \tag{17}$$

where $R_3 = [-H\otimes(K+\Gamma A(t)) \quad H\otimes(\Gamma LC)]$ and $R_4 = I_{(Np)} - H\otimes(\Gamma B(t))$.

Combining (16) and (17) results in a 2D Roesser system [54]

$$\begin{bmatrix} D_t^{\alpha}\theta_k(t) \\ \delta u_{k+1}(t)\end{bmatrix} = \begin{bmatrix} R_1 & R_2 \\ R_3 & R_4\end{bmatrix}\begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix} + \begin{bmatrix}\Psi_k(t) \\ -(H\otimes\Gamma)\phi_k(t)\end{bmatrix}. \tag{18}$$

Based on Assumption 6, we obtain that $\|\phi_{k,j}(t)\|_2 \le l\|e_{k,j}(t)\|_2$ and $\|\varphi_{k,j}(t)\|_2 \le l\|\tilde{x}_{k,j}(t)\|_2$. Thus, according to the 2D system theory [54], it is not difficult to know from (18) that $\lim_{k\longrightarrow+\infty}[\theta_k{}^{\mathrm{T}}(t), \delta u_k{}^{\mathrm{T}}(t)]^{\mathrm{T}} = 0$ holds if the condition $\|R_4\|_2 < 1$ is satisfied. However, the condition $\|R_4\|_2 < 1$ only indicates the asymptotic convergence of input errors, but cannot guarantee that the control input errors converge monotonically to zero as a function of iteration steps. Based on the observations, this paper mainly focuses on the following problems:

Problem Statement. Given the nonlinear FOMAS (1) and (4), let the observer (6) and the ILC consensus protocol (11) be applied. This paper will design the learning gain matrices K and Γ such that:

(i) The monotonic convergence of the input errors is achieved, i.e.,

$$\left\|\delta u_{k+1}(t)\right\|_2 < \gamma \left\|\delta u_k(t)\right\|_2 \tag{19}$$

holds for any $t \in [0,T]$, $k \in Z_+$, where $\gamma \in (0,1)$ is a prescribed scalar constant indicating the convergence rate of consensus.

(ii) The outputs of all follower agents converge to that of the leader agent in the finite time interval $t \in [0,T]$ as the iteration step k goes to infinity, that is,

$$\begin{gathered}\lim_{k\longrightarrow+\infty}\left\|y_d(t)-y_{k,j}(t)\right\|_2 = 0,\\ j=1,2,\ldots,N,\ t\in[0,T].\end{gathered} \tag{20}$$

To deal with the problem shown in (19), we define a novel performance index as

$$\begin{aligned}J(k,\gamma) &= \frac{1}{\Gamma(\alpha)}\int_0^T (T-s)^{\alpha-1}\\ &\cdot\left[\delta u_{k+1}{}^{\mathrm{T}}(s)\,\delta u_{k+1}(s)-\gamma^2\delta u_k{}^{\mathrm{T}}(s)\,\delta u_k(s)\right]ds.\end{aligned} \tag{21}$$

Since the inequality $(T-s)^{\alpha-1} > 0$ holds for all $s \in [0,T]$ when $\alpha \in (0,1)$, it is not difficult to see from (21) that the problem shown in (19) will be solved if $J(k,\gamma) < 0$ is satisfied. Moreover, the problem shown in (20) will be solved if $\lim_{k\longrightarrow+\infty}\theta_k(t) = 0$ is satisfied for all $t \in [0,T]$. Thus, for two problems shown in (19) and (20), the control objectives in this paper can be further summarized as follows:

$$J(k,\gamma) < 0,\quad \gamma\in(0,1),\ k\in Z_+; \tag{22}$$

$$\begin{gathered}\lim_{k\longrightarrow+\infty}\tilde{x}_k(t) = 0,\\ \lim_{k\longrightarrow+\infty}e_k(t) = 0,\\ k\in Z_+,\ t\in[0,T].\end{gathered} \tag{23}$$

Remark 14. In the exiting literatures [55, 56], the performance indexes are expressed in terms of integer-order integral, and the optimal controllers and the consensus conditions are obtained by the inverse optimal method. Since each agent in this paper is described by the fractional-order differential equation, it is difficult to extend the performance indexes in [55, 56] to the FOMASs in this paper. Moreover, due to the existence of structural uncertainties, the inverse optimal method is not applicable. Thus, in this paper, the performance index is designed in terms of fraction-order integral of linear quadratic forms of input errors, and the Lyapunov-like method instead of the inverse optimal method is applied to determine the learning gains and to derive the consensus convergence conditions (See Theorems 20 and 23).

Remark 15. If the fractional order $\alpha = 1$, then the performance index (21) will degenerate into

$$\begin{aligned}&J(k,\gamma)\\ &= \int_0^T\left[\delta u_{k+1}{}^{\mathrm{T}}(s)\,\delta u_{k+1}(s)-\gamma^2\delta u_k{}^{\mathrm{T}}(s)\,\delta u_k(s)\right]dt,\end{aligned} \tag{24}$$

which has been used to solve the robust ILC problem of uncertain time-delay integer-order systems [57]. Thus, the performance index (21) includes (24) as its special case.

To simplify the analysis, the following assumptions are needed.

Assumption 16. The initial states of each follower agent and its observer are reset to the same as that of the leader agent after each execution.

To better focus on the main problem, the strictly identical initial conditions [28] are assumed in Assumption 16. On the other hand, if Assumption 16 is not satisfied, the initial state learning law for each follower agent and its observer can be designed by the method in [29] before the methods in this paper are used.

Assumption 17. The communication graph $\overline{\mathscr{G}}$ at least contains a spanning tree with the leader agent being the root.

3. Main Results

In this section, we will consider the perfect tracking problem of consensus for the nonlinear FOMASs (1) and (4) by using Lyapunov-like method, and will present the sufficient conditions guaranteeing that two problems shown in (19) and (20) are solved simultaneously. First, the case of nonlinear FOMASs (1) and (4) without any uncertainty is considered.

3.1. Nonlinear FOMASs without Any Uncertainty

3.1. Nonlinear FOMASs without Any Uncertainty. In this subsection, both $A(t)$ and $B(t)$ are assumed to be the known matrices without any uncertainty, that is, $A(t) = A_0$ and $B(t) = B_0$. In this case, we can state the following two corollaries:

Corollary 18. *Let the positive definite matrix $Q_1 = Q_1^{\mathrm{T}} \in R^{n\times n}$, and assume that Assumption 6 holds. If A_0 is Hurwitz, and there exist a positive definite matrix $P_1 = P_1{}^{\mathrm{T}}$ satisfying*

$$\begin{pmatrix} A_0{}^{\mathrm{T}}P_1 + P_1A_0 + l^2I_n + Q_1 & P_1\\ P_1 & -I_n\end{pmatrix} < 0. \tag{25}$$

then the fractional-order nonlinear agent (4) without any uncertainty under no control is stable.

Corollary 19. *Let the positive definite matrix $Q_2 = Q_2^{\mathrm{T}} \in R^{n\times n}$, and assume that Assumption 6 holds. If the observer gain* *matrix L is chosen such that $A_0 - LC$ is Hurwitz, and there exists a positive definite matrix $P_2 = P_2^{\mathrm{T}} \in R^{n\times n}$ satisfying*

$$\begin{pmatrix} (A_0 - LC)^{\mathrm{T}} P_2 + P_2 (A_0 - LC) + l^2 I_n + Q_2 & P_2 \\ P_2 & -I_n \end{pmatrix} < 0, \quad (26)$$

then the observation error is bounded, and converges asymptotically to zero as time goes to infinity.

Corollaries 18 and 19 can be directly derived from Lemmas 9 and 12, respectively. Thus their proofs are omitted.

Based on the previous preparation, we can state the following results.

Theorem 20. *Assume that Assumptions 6, 16, and 17 and the conditions in Corollaries 18 and 19 are satisfied for the nonlinear FOMASs (1) and (4) without any uncertainty, and let the observer (6) and the consensus protocol (11) are applied. If, for a scalar $\gamma \in (0,1)$, there exist matrices X_i, $i = 1, 2$, satisfying*

$$\Xi_1 = \begin{bmatrix} -I_{(Np)} & 0 & 0 & 0 & \sqrt{3}H\otimes X_2 & 0 & 0 \\ * & -I_{(Np)} & 0 & 0 & 0 & 0 & I_{(Np)} - H\otimes (X_2B_0) \\ * & * & -I_{(Np)} & 0 & -H\otimes (X_1 + X_2A_0) & H\otimes (X_2LC) & 0 \\ * & * & * & -I_{(Np)} & -H\otimes (X_1 + X_2A_0) & H\otimes (X_2LC) & I_{(Np)} - H\otimes (X_2B_0) \\ * & * & * & * & I_N\otimes sym\{{A_0}^{\mathrm{T}}P_1\} + I_N\otimes Q_1 + I_N\otimes (P_1P_1) + l^2 I_{(Nn)} & -I_N\otimes (P_1LC) & I_N\otimes P_1B_0 \\ * & * & * & * & -I_N\otimes (C^{\mathrm{T}}L^{\mathrm{T}}P_1) & I_N\otimes sym\{(A_0 - LC)^{\mathrm{T}} P_2\} + I_N\otimes Q_2 + I_N\otimes (P_2P_2) + l^2 I_{(Nn)} & 0 \\ * & * & * & * & * & * & -\gamma^2 I_{(Np)} \end{bmatrix} < 0, \quad (27)$$

where P_1 and Q_1 (P_2 and Q_2) are given in Corollary 18 (Corollary 19). Then both the input errors and the output tracking errors converge to zero for all $t \in [0, T]$ as the iteration step goes to infinity and the monotonic convergence of input errors is obtained for all $k \in Z_+$. Moreover, if (27) holds, then the learning gain matrices K and Γ are determined by

$$K = X_1, \quad \Gamma = X_2. \quad (28)$$

Proof. (a) Proof of (19)

Let

$$P = \operatorname{diag}\{I_N\otimes P_1, I_N\otimes P_2\}, \quad Q = \operatorname{diag}\{I_N\otimes Q_1, I_N\otimes Q_2\}. \quad (29)$$

Note that both $P = P^{\mathrm{T}}$ and $Q = Q^{\mathrm{T}}$ are positive definite, we can define a Lyapunov function as

$$V(\theta_k(t)) = {\theta_k}^{\mathrm{T}}(t)\, P\theta_k(t) + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\, {\theta_k}^{\mathrm{T}}(s)\, Q\theta_k(s)\, ds. \quad (30)$$

It is easy to know that $V(\theta_k(t))$ is positive semidefinite. Moreover, according to Assumption 16, we obtain that $V(\theta_k(0)) = 0$.

Let $\xi_k(t) = \left[{\theta_k}^{\mathrm{T}}(t)\ \delta{u_k}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. By Definition 2 and Properties 3 and 4, we have

$$\begin{aligned} D_t^{\alpha}(V(\theta_k(t))) &\le (D_t^{\alpha}\theta_k(t))^{\mathrm{T}} P\theta_k(t) + \theta_k^{\mathrm{T}}(t) \\ &\cdot P(D_t^{\alpha}\theta_k(t)) + {\theta_k}^{T}(t)\, Q\theta_k(t) \\ &= (R_1\theta_k(t) + R_2\delta u_k(t) + \Psi_k(t))^{\mathrm{T}} P\theta_k(t) + \theta_k^{\mathrm{T}}(t) \\ &\cdot P(R_1\theta_k(t) + R_2\delta u_k(t) + \Psi_k(t)) + {\theta_k}^{\mathrm{T}}(t)\, Q\theta_k(t) \\ &= \theta_k^{T}(t)\left({R_1}^{\mathrm{T}}P + PR_1 + Q\right)\theta_k(t) + \theta_k^{T}(t)\, PR_2\delta u_k(t) \\ &+ \delta{u_k}^{\mathrm{T}}(t)\, {R_2}^{\mathrm{T}}P\theta_k(t) + 2\theta_k^{T}(t)\, P\Psi_k(t) \le \begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix}^{\mathrm{T}} \\ &\cdot \begin{bmatrix} {R_1}^{\mathrm{T}}P + PR_1 + Q & PR_2 \\ * & 0 \end{bmatrix} \begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix} + \theta_k^{\mathrm{T}}(t) \\ &\cdot PP\theta_k(t) + \begin{bmatrix}\phi_k(t) \\ \varphi_k(t)\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\phi_k(t) \\ \varphi_k(t)\end{bmatrix} \le \begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix}^{\mathrm{T}} \\ &\cdot \begin{bmatrix} {R_1}^{\mathrm{T}}P + PR_1 + Q + PP & PR_2 \\ * & 0 \end{bmatrix} \begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix} \\ &+ l^2 {e_k}^{\mathrm{T}}(t)\, e_k(t) + l^2 {\tilde{x}_k}^{\mathrm{T}}(t)\tilde{x}_k(t) \le \begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix}^{\mathrm{T}} \\ &\cdot \begin{bmatrix} {R_1}^{\mathrm{T}}P + PR_1 + Q + PP + l^2 I_{(2Nn)} & PR_2 \\ * & 0 \end{bmatrix} \begin{bmatrix}\theta_k(t) \\ \delta u_k(t)\end{bmatrix} \\ &= {\xi_k}^{\mathrm{T}}(t)\, \Pi_1 \xi_k(t), \end{aligned} \quad (31)$$

where

$$\Pi_1 = \begin{bmatrix} R_1{}^{\mathrm{T}}P + PR_1 + Q + PP + l^2 I_{(2Nn)} & PR_2 \\ * & 0 \end{bmatrix},$$

$$R_1 = \begin{pmatrix} I_N \otimes A_0 & -I_N \otimes (LC) \\ 0 & I_N \otimes (A_0 - LC) \end{pmatrix}, \quad (32)$$

$$R_2 = \begin{pmatrix} I_N \otimes B_0 \\ 0 \end{pmatrix}.$$

Based on (17), we have

$$\begin{aligned}
&\delta u_{k+1}{}^{\mathrm{T}}(t)\,\delta u_{k+1}(t) \\
&= \left(R_3\theta_k(t) + R_4\delta u_k(t) - (H\otimes\Gamma)\phi_k(t)\right)^{\mathrm{T}} \\
&\cdot \left(R_3\theta_k(t) + R_4\delta u_k(t) - (H\otimes\Gamma)\phi_k(t)\right) = \theta_k{}^{\mathrm{T}}(t) \\
&\cdot R_3{}^{\mathrm{T}}R_3\theta_k(t) + \delta u_k{}^{\mathrm{T}}(t)\,R_4{}^{\mathrm{T}}R_3\theta_k(t) - \phi_k{}^{\mathrm{T}}(t) \\
&\cdot \left(H^{\mathrm{T}}\otimes\Gamma^{\mathrm{T}}\right)R_3\theta_k(t) + \theta_k{}^{\mathrm{T}}(t)\,R_3{}^{\mathrm{T}}R_4\delta u_k(t) \\
&+ \delta u_k{}^{\mathrm{T}}(t)\,R_4{}^{\mathrm{T}}R_4\delta u_k(t) - \phi_k{}^{\mathrm{T}}(t)\left(H^{\mathrm{T}}\otimes\Gamma^{\mathrm{T}}\right) \\
&\cdot R_4\delta u_k(t) - \theta_k{}^{\mathrm{T}}(t)\,R_3{}^{\mathrm{T}}(H\otimes\Gamma)\phi_k(t) - \delta u_k{}^{\mathrm{T}}(t) \\
&\cdot R_4{}^{\mathrm{T}}(H\otimes\Gamma)\phi_k(t) + \phi_k{}^{\mathrm{T}}(t)\left(H^{\mathrm{T}}\otimes\Gamma^{\mathrm{T}}\right)(H\otimes\Gamma) \\
&\cdot \phi_k(t) = \xi_k{}^{\mathrm{T}}(t)\begin{bmatrix} R_3{}^{\mathrm{T}} \\ R_4{}^{\mathrm{T}} \end{bmatrix}[R_3 \;\; R_4]\,\xi_k(t) - 2\theta_k{}^{\mathrm{T}}(t) \\
&\cdot R_3{}^{\mathrm{T}}(H\otimes\Gamma)\phi_k(t) - 2\delta u_k{}^{\mathrm{T}}(t)\,R_4{}^{\mathrm{T}}(H\otimes\Gamma)\phi_k(t) \\
&+ \phi_k{}^{\mathrm{T}}(t)\left(H^{\mathrm{T}}\otimes\Gamma^{\mathrm{T}}\right)(H\otimes\Gamma)\phi_k(t) \le \xi_k{}^{\mathrm{T}}(t) \\
&\cdot \begin{bmatrix} R_3{}^{\mathrm{T}} \\ R_4{}^{\mathrm{T}} \end{bmatrix}[R_3 \;\; R_4]\,\xi_k(t) + \theta_k{}^{\mathrm{T}}(t)\,R_3{}^{\mathrm{T}}R_3\theta_k(t) \\
&+ \delta u_k{}^{\mathrm{T}}(t)\,R_4{}^{\mathrm{T}}R_4\delta u_k(t) + 3\phi_k{}^{\mathrm{T}}(t)\left(H^{\mathrm{T}}\otimes\Gamma^{\mathrm{T}}\right) \\
&\cdot (H\otimes\Gamma)\phi_k(t) = \xi_k{}^{\mathrm{T}}(t) \\
&\cdot \left(\begin{bmatrix} \Omega_2{}^{\mathrm{T}} & 0 & R_3{}^{\mathrm{T}} & R_3{}^{\mathrm{T}} \\ 0 & R_4{}^{\mathrm{T}} & 0 & R_4{}^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \Omega_2 & 0 \\ 0 & R_4 \\ R_3 & 0 \\ R_3 & R_4 \end{bmatrix}\right)\xi_k(t),
\end{aligned} \quad (33)$$

where

$$\begin{aligned}
\Omega_2 &= \left[\sqrt{3}H\otimes\Gamma \;\; 0\right], \\
R_3 &= \left[-H\otimes(K + \Gamma A_0) \;\; H\otimes(\Gamma LC)\right], \\
R_4 &= I_{(Np)} - H\otimes(\Gamma B_0).
\end{aligned} \quad (34)$$

By Property 3, (31) and (33), the index $J(k,\gamma)$ can be rewritten as

$$\begin{aligned}
J(k,\gamma) &= J(k,\gamma) + \frac{1}{\Gamma(\alpha)}\int_0^T (T-s)^{\alpha-1} \\
&\cdot D_s^{\alpha}\left(V(\theta_k(s))\right)ds - V(\theta_k(T)) + V(\theta_k(0)) \\
&\le \frac{1}{\Gamma(\alpha)}\int_0^T (T-s)^{\alpha-1}\left[\delta u_{k+1}{}^{\mathrm{T}}(s)\,\delta u_{k+1}(s)\right. \\
&\left. - \gamma^2\delta u_k{}^{\mathrm{T}}(s)\,\delta u_k(s) + D_s^{\alpha}\left(V(\theta_k(s))\right)\right]ds \\
&= \frac{1}{\Gamma(\alpha)}\int_0^T (T-s)^{\alpha-1}\,\xi_k{}^{\mathrm{T}}(s)\,\Pi_2\xi_k(s)\,ds,
\end{aligned} \quad (35)$$

where the matrix Π_2 satisfies

$$\begin{aligned}
\Pi_2 &= \begin{bmatrix} R_1{}^{\mathrm{T}}P + PR_1 + Q + PP + l^2 I_{(2Nn)} & PR_2 \\ * & -\gamma^2 I_{(Np)} \end{bmatrix} \\
&- \begin{bmatrix} \Omega_2{}^{\mathrm{T}} & 0 & R_3{}^{\mathrm{T}} & R_3{}^{\mathrm{T}} \\ 0 & R_4{}^{\mathrm{T}} & 0 & R_4{}^{\mathrm{T}} \end{bmatrix}\left(-I_{(4Np)}\right)\begin{bmatrix} \Omega_2 & 0 \\ 0 & R_4 \\ R_3 & 0 \\ R_3 & R_4 \end{bmatrix}.
\end{aligned} \quad (36)$$

By (29), (27) is rewritten as

$$\Xi_1 = \begin{bmatrix}
-I_{(Np)} & 0 & 0 & 0 & \left(\sqrt{3}H\otimes X_2 \;\; 0\right) & 0 \\
* & -I_{(Np)} & 0 & 0 & (0 \;\; 0) & I_{(Np)} - H\otimes(X_2B_0) \\
* & * & -I_{(Np)} & 0 & \left(-H\otimes(X_1 + X_2A_0) \;\; H\otimes(X_2LC)\right) & 0 \\
* & * & * & -I_{(Np)} & \left(-H\otimes(X_1 + X_2A_0) \;\; H\otimes(X_2LC)\right) & I_{(Np)} - H\otimes(X_2B_0) \\
* & * & * & * & sym\left\{\begin{pmatrix} I_N\otimes P_1A_0 & -I_N\otimes(P_1LC) \\ 0 & I_N\otimes(P_2A_0 - P_2LC) \end{pmatrix}\right\} + Q + \begin{pmatrix} I_N\otimes(P_1P_1) & 0 \\ 0 & I_N\otimes(P_2P_2) \end{pmatrix} + l^2 I_{(2Nn)} & \begin{pmatrix} I_N\otimes P_1B_0 \\ 0 \end{pmatrix} \\
* & * & * & * & * & -\gamma^2 I_{(Np)}
\end{bmatrix} < 0. \quad (37)$$

When (27) is satisfied, by selecting the learning gain matrices in (28) and substituting them into (37), we have

$$\Xi_1 = \begin{bmatrix} -I_{(Np)} & 0 & 0 & 0 & \Omega_2 & 0 \\ * & -I_{(Np)} & 0 & 0 & 0 & R_4 \\ * & * & -I_{(Np)} & 0 & R_3 & 0 \\ * & * & * & -I_{(Np)} & R_3 & R_4 \\ * & * & * & * & {R_1}^{\mathrm{T}}P + PR_1 + Q + PP + l^2 I_{(2Nn)} & PR_2 \\ * & * & * & * & * & -\gamma^2 I_{(Np)} \end{bmatrix} < 0. \quad (38)$$

Based on the Schur complement formula, it is easy to know from (38) that $\Pi_2 < 0$ is satisfied, which indicates that $J(k,\gamma) < 0$ in (22) holds for all non-zero vectors $\xi_k(t)$. That is, the monotonic convergence of input errors shown in (19) is achieved.

(b) The proof of (20)

Let $\Pi_3 = \begin{bmatrix} {R_1}^{\mathrm{T}}P+PR_1+Q+PP+l^2I_{(2Nn)} & PR_2 \\ * & -\gamma^2 I_{(Np)} \end{bmatrix}$. Applying the Schur complement formula to (38), we obtain that $\Pi_3 < 0$. Thus, based on (31) and (33), we have

$$\begin{aligned} & D_t^{\alpha}\left(V\left(\theta_k(t)\right)\right) - \gamma^2 \delta {u_k}^{\mathrm{T}}(t)\,\delta u_k(t) \\ & \quad \le {\xi_k}^{\mathrm{T}}(t)\,\Pi_1 \xi_k(t) + {\xi_k}^{\mathrm{T}}(t)\operatorname{diag}\left\{0, -\gamma^2 I_{(Np)}\right\}\xi_k(t) \\ & \quad = {\xi_k}^{\mathrm{T}}(t)\,\Pi_3 \xi_k(t) \le 0. \end{aligned} \quad (39)$$

From (30), (31), and (39), we have

$$\begin{aligned} {\theta_k}^{\mathrm{T}}(t)\,P\theta_k(t) &\le V\left(\theta_k(t)\right) \\ &\le \frac{1}{\Gamma(\alpha)}\int_0^t \gamma^2 (t-s)^{\alpha-1}\,\delta {u_k}^{\mathrm{T}}(s)\,\delta u_k(s)\,ds \\ &\quad + V\left(\theta_k(0)\right) \\ &= \frac{1}{\Gamma(\alpha)}\int_0^t \gamma^2 (t-s)^{\alpha-1}\,\delta {u_k}^{\mathrm{T}}(s)\,\delta u_k(s)\,ds. \end{aligned} \quad (40)$$

When (27) is satisfied, it is easy to know that $\lim_{k\longrightarrow+\infty}\|\delta u_k(t)\|_2 = 0$ holds. Thus, taking the limit of $k \longrightarrow +\infty$ on both sides of (40), we obtain that $\lim_{k\longrightarrow+\infty}\|\theta_k(t)\|_2 = 0$, which indicates that $\lim_{k\longrightarrow+\infty}\|y_d(t) - y_{k,j}(t)\|_2 = 0$ holds. That is, the perfect tracking of consensus shown in (20) is achieved.

This ends the proof of Theorem 20. □

Remark 21. Theorem 20 provides the sufficient condition in terms of LMIs, which belongs to the category of ILC where a consensus convergence condition is related to the learning gain matrices, the known plant knowledge and the structure of communication graph [33]. However, compared to [33], the obtained LMI condition not only provides a design method of learning gain matrices to achieve the perfect tracking of consensus, but also guarantees the monotonic convergence of the input errors and achieves the good learning transients.

Remark 22. Among the existing results about the consensus of FOMASs [15–21, 36–38], the controllers are merely related to time t, and the asymptotic consensus problem or the finite time consensus problem is addressed. In contrast, this paper has the following two distinct differences: (a) The controllers (11) is related to both time t and iterative learning step k. (b) The perfect tracking of consensus is obtained. That is, the consensus will be maintained perfectly in the finite time interval $t \in [0,T]$ as the iteration step k goes to infinity.

3.2. Extension to Uncertain Nonlinear FOMASs with Parameter Coupling Relations.

By using the knowledge of plant as the priors, Theorem 20 provides an LMI condition guaranteeing the perfect tracking of consensus and the monotonic convergence of the input errors. However, if the parameters of nonlinear FOMASs are subject to the perturbation, Theorem 20 is no longer suitable, thus should be further improved. Next, by studying the uncertain nonlinear FOMASs satisfying Assumptions 6, 8, 16, and 17, we obtain Theorem 23.

Theorem 23. *Assume that Assumptions 6, 8, 16, and 17 and the conditions in Lemmas 9 and 12 are satisfied for the nonlinear FOMASs (1) and (4), and let the observer (6) and the consensus protocol (11) are applied. If, for a scalar $\gamma \in (0,1)$, there exist matrices X_i, $i = 1, 2$ satisfying*

$$\begin{bmatrix} -I_{(6Nn)} & W \\ W^{\mathrm{T}} & \Pi_4 \end{bmatrix} < 0, \quad (41)$$

where W and Π_4 are given in (42) and (43), ε is a known positive scalar constant, P_1 and Q_1 (P_2 and Q_2) are given in Lemma 9 (Lemma 12). Then both the input errors and the output tracking errors converge to zero for all $t \in [0,T]$ as the iteration step goes to infinity and the monotonic convergence of input errors is obtained for all $k \in Z_+$. Moreover, if (41) holds, then the learning gain matrices K and Γ are determined by (28).

$$W = \begin{bmatrix} 0 & 0 & -\sqrt{\varepsilon}H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & -\sqrt{\varepsilon}H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & \sqrt{\varepsilon} I_N \otimes E^{\mathrm{T}} P_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\varepsilon} I_N \otimes E^{\mathrm{T}} P_2 & 0 \\ 0 & -\sqrt{\varepsilon}H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & 0 & -\sqrt{\varepsilon}H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & \sqrt{\varepsilon} I_N \otimes E^{\mathrm{T}} P_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\mu\alpha}{\sqrt{\varepsilon}} I_N \otimes F_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu\alpha}{\sqrt{\varepsilon}} I_N \otimes F_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu\alpha}{\sqrt{\varepsilon}} I_N \otimes F_2 \end{bmatrix}, \tag{42}$$

$$\Pi_4 = \begin{bmatrix} -I_{(Np)} & 0 & 0 & 0 & \sqrt{3} H \otimes X_2 & 0 & 0 \\ * & -I_{(Np)} & 0 & 0 & 0 & 0 & I_{(Np)} - H \otimes (X_2 B_0) \\ * & * & -I_{(Np)} & 0 & -H \otimes (X_1 + X_2 A_0) & H \otimes (X_2 LC) & 0 \\ * & * & * & -I_{(Np)} & -H \otimes (X_1 + X_2 A_0) & H \otimes (X_2 LC) & I_{(Np)} - H \otimes (X_2 B_0) \\ * & * & * & * & I_N \otimes sym\left\{{A_0}^{\mathrm{T}} P_1\right\} + I_N \otimes Q_1 + I_N \otimes (P_1 P_1) + l^2 I_{(Nn)} & -I_N \otimes (P_1 LC) & I_N \otimes P_1 B_0 \\ * & * & * & * & -I_N \otimes \left(C^{\mathrm{T}} L^{\mathrm{T}} P_1\right) & I_N \otimes sym\left\{(A_0 - LC)^{\mathrm{T}} P_2\right\} + I_N \otimes Q_2 + I_N \otimes (P_2 P_2) + l^2 I_{(Nn)} & 0 \\ * & * & * & * & * & * & -\gamma^2 I_{(Np)} \end{bmatrix}, \tag{43}$$

Proof. According to Theorem 20, we know that the proof of Theorem 23 will be accomplished if $\Xi_1 < 0$ is satisfied when A_0 and B_0 in (27) are replaced with $A(t) = A_0 + \mu\alpha\Delta A(t)$ and $B(t) = B_0 + \mu\alpha\Delta B(t)$, respectively. Thus, we should prove that after A_0 and B_0 in (27) are respectively replaced with $A(t) = A_0 + \mu\alpha\Delta A(t)$ and $B(t) = B_0 + \mu\alpha\Delta B(t)$, $\Xi_1 < 0$ holds for all $\sum(t)$ satisfying $\sum^{\mathrm{T}}(t)\sum(t) \leq I$ if (41) is satisfied.

By replacing respectively A_0 and B_0 in (27) with $A(t) = A_0 + \mu\alpha\Delta A(t)$ and $B(t) = B_0 + \mu\alpha\Delta B(t)$, we have

$$\Xi_1 = \Pi_4 + Y\Psi(t) Z + (Y\Psi(t) Z)^{\mathrm{T}}, \tag{44}$$

where Π_4 is given in (43), and Y, Z, and $\Psi(t)$ satisfy

$$Y = \mu\alpha \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ I_N \otimes F_1^{\mathrm{T}} & 0 & 0 \\ 0 & I_N \otimes F_1^{\mathrm{T}} & 0 \\ 0 & 0 & I_N \otimes F_2^{\mathrm{T}} \end{bmatrix}, \tag{45}$$

$$Z = \begin{bmatrix} 0 & 0 & -H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & -H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & I_N \otimes E^{\mathrm{T}} P_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_N \otimes E^{\mathrm{T}} P_2 & 0 \\ 0 & -H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & 0 & -H^{\mathrm{T}} \otimes \left(E^{\mathrm{T}} {X_2}^{\mathrm{T}}\right) & I_N \otimes E^{\mathrm{T}} P_1 & 0 & 0 \end{bmatrix}, \tag{46}$$

$$\Psi(t) = \begin{bmatrix} I_N \otimes \sum^{\mathrm{T}}(t) & 0 & 0 \\ 0 & I_N \otimes \sum^{\mathrm{T}}(t) & 0 \\ 0 & 0 & I_N \otimes \sum^{\mathrm{T}}(t) \end{bmatrix}. \tag{47}$$

Since Π_4 is symmetric, by using Lemma 5, we know that $\Xi_1 < 0$ holds for any $\sum(t)$ satisfying $\sum^{\mathrm{T}}(t)\sum(t) \leq I$ if there exists a scalar constant $\varepsilon > 0$ such that

$$\Pi_4 + \varepsilon Z^T Z + \varepsilon^{-1} Y Y^T < 0. \tag{48}$$

By using (45) and (46), W in (42) can be rewritten as $W = \left[\begin{smallmatrix} \sqrt{\varepsilon} Z \\ Y^{\mathrm{T}}/\sqrt{\varepsilon} \end{smallmatrix}\right]$. Thus, the inequality (48) is equivalent to

$$\Pi_4 + W^{\mathrm{T}} W < 0. \tag{49}$$

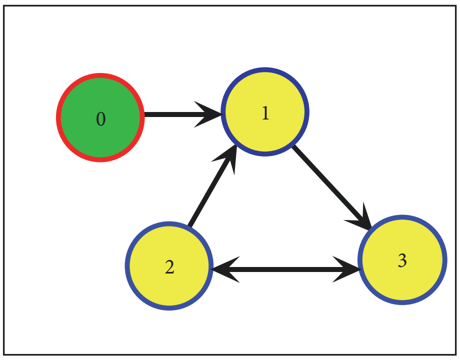


FIGURE 1: Communication graph among all agents. Leader: agent 0; Followers: agents 1, 2, and 3 (color online).

By using the Schur complement formula, (49) can be rewritten as (41). Thus, based on Theorem 20, for the uncertain nonlinear FOMASs (1) and (4) satisfying Assumptions 6, 8, 16, and 17 and the conditions in Lemmas 9 and 12, two problems shown in (19) and (20) can be solved simultaneously if (41) is satisfied.

This ends the proof of Theorem 23. □

Remark 24. [57] investigates the performance index based perfect synchronization of integer-order linear uncertain systems, where there is only a leader and a follower, and there is no coupling among parameters. [35] proposes the observer-type ILC protocol for the perfect consensus of linear IOMASs without any uncertainty and any parameter coupling. Compared to [35, 57], the results in this paper have the following distinct differences: (a) Each agent is described by the uncertain fractional-order nonlinear differential equation with linear coupling relation among parameters. (b) A novel performance index is designed as the form of the fractional integral of control input errors, and is used together with the Lyapunov-like method to derive the consensus sufficient conditions that can guarantee the perfect tracking of consensus and the better learning transients of ILC law. Moreover, if $\alpha = 1$, $\Delta A(t) = 0$, $\Delta B(t) = 0$ and $f(\cdot) = 0$, then the model in this paper will degenerate into the model of linear IOMAS in [35], Theorem 20 with $\alpha = 1$ and $l = 0$ can solve the consensus problem this kind of special IOMASs.

Remark 25. If $\mu = 1/\alpha$, then the FOMASs (1) and (4) will degenerate into the special uncertain nonlinear FOMASs without any parameter coupling. If $\alpha = 1$, then the FOMASs (1) and (4) will degenerate into the uncertain nonlinear IOMASs. Theorem 23 presents the sufficient conditions to guarantee that above two kinds of special uncertain nonlinear MASs can achieve the perfect tracking of consensus and the monotonic convergence of input errors simultaneously. The results show that Theorems 23 has broader applicability in the perfect consensus control of MASs. The FOMASs in this paper consist of N autonomous agents with the nonlinear dynamics satisfying the global Lipschitz condition. The consensus perfect tracking problem of T-S fuzzy systems [58] should be a further investigated topic, but it is still a challenging problem to design the distributed ILC protocol for the perfect consensus of MASs consisting of T-S fuzzy subsystems.

Remark 26. [40, 41, 59] design the observer-based repetitive-control systems for the systems with periodic time-varying uncertainties, with unknown aperiodic disturbances and without any uncertainties, respectively. The repetitive control (RC) process is also an iterative learning process (ILP). However, there exist the following differences between the ILPs of [40, 41, 59] and ILP of this paper. The ILPs of [40, 41, 59] are operating on an infinite time horizon, the state of the RC progresses continuously from one period to the next period (i.e., the state at the beginning of a period is the same as the final state of the system in the previous period). Thus the ILPs of [40, 41, 59] do not have time resetting or any iteration axis, and the RC controllers in [40, 41, 59] are merely related to time t. In contrast, in the ILP of this paper, the operation time is finite, that is $t \in [0, T]$. The identical initial condition (i.e., the initial state tracking error is zero in each iteration) is required (See Assumption 16). This requirement indicates that both time and state need to be reset after each iteration. Thus the ILP of this paper has a time axis and an iteration axis, and the controller is related to both time t and iterative learning step k.

4. Simulation Results

In this section, to illustrate the effectiveness of the proposed methods, we present two numerical simulation examples.

4.1. Example 1. Consider the leader-following FOMASs, which are described by (1) and (4) and consist of a leader agent and three follower agents over the fixed communication graph shown in Figure 1. It is easy to know that the communication graph in Figure 1 satisfies Assumption 17, and $H = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}$. Moreover, the parameters in (1), (4) and (6) satisfy

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & 1 \\ -0.5 & -0.6 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ f(t, x(t)) &= [0.1 \sin(x_1(t)), 0.1 \sin(x_2(t))]^{\mathrm{T}}, \\ C &= [1, 1], \\ \alpha &= 0.96, \\ \mu &= 0.05, \end{aligned} \tag{50}$$

and the uncertain matrices are expressed by $\sum(t) = \operatorname{diag}\{1 - 2\exp(-3t), \cos(5t)\}$ and

$$E = \begin{bmatrix} 0 & 1 \\ 0.8 & 0.8 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.2 \end{bmatrix},$$

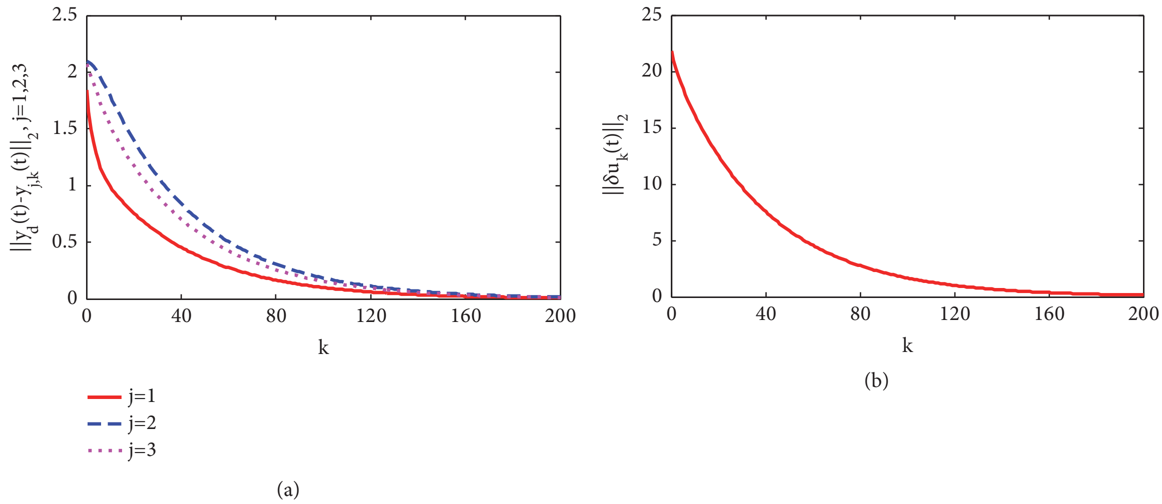


Figure 2: Under the observer (6) and the controller (11) with $K = [0.0748, 0.0931]$ and $\Gamma = [-0.0034, 0.1496]$: (a) Output tracking errors versus iteration numbers; (b) input errors $\|\delta u_k(t)\|_2$ versus iteration numbers (color online).

$$F_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}. \tag{51}$$

Thus the Lipschitz constant $l = 0.1$. Take $L = [0.09, 0.19]^{\mathrm{T}}$, it is easily check that both the system matrix A_0 and the observer matrix $A_0 - LC$ are Hurwitz. By solving the inequality (5) with $Q_1 = \left[\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.1 \end{smallmatrix}\right]$ and the inequality (8) with $Q_2 = \left[\begin{smallmatrix} 0.3 & 0 \\ 0 & 0.3 \end{smallmatrix}\right]$, we obtain that $P_1 = \left[\begin{smallmatrix} 0.0468 & -0.0284 \\ -0.0284 & 0.2213 \end{smallmatrix}\right]$ and $P_2 = \left[\begin{smallmatrix} 0.6283 & 0.2465 \\ 0.2465 & 0.9018 \end{smallmatrix}\right]$. To perform the simulation, the initial states of each follower agent and its observer are set the same as that of leader agent, the control input of leader agent is set as $u_d(t) = \sin(0.2\pi t)$, while the zero initial control input for each follower agent is adopted. Five scalars are prescribed as $T = 5$, $\gamma = 0.1$, $\varepsilon_1 = 1.5$, $\varepsilon_2 = 2$ and $\varepsilon = 4$.

By using the Matlab LMI toolbox to solve the LMI (41) with $Q_1 = \left[\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.1 \end{smallmatrix}\right]$, $Q_2 = \left[\begin{smallmatrix} 0.3 & 0 \\ 0 & 0.3 \end{smallmatrix}\right]$, $P_1 = \left[\begin{smallmatrix} 0.0468 & -0.0284 \\ -0.0284 & 0.2213 \end{smallmatrix}\right]$ and $P_2 = \left[\begin{smallmatrix} 0.6283 & 0.2465 \\ 0.2465 & 0.9018 \end{smallmatrix}\right]$, we obtain $K = [0.0748, 0.0931]$ and $\Gamma = [-0.0034, 0.1496]$, which indicates that the LMI (41) has feasible solutions. Thus, according to Theorem 23, we can conclude that under the observer (6) and the controller (11) with the obtained learning gain matrices, the perfect tracking of consensus and the monotonic convergence of input errors can be achieved simultaneously for uncertain nonlinear FOMASs (1) and (4) with the parameters satisfying (50) and (51). Next, we will further confirm the conclusion. By (2) and (3) satisfying (50) and (51), we randomly generate the system matrix $A(t)$ and input matrix $B(t)$, then obtain the numerical simulation results as shown in Figure 2. In Figure 2, both the output tracking errors and the control input errors converge monotonically to zero as the iteration step goes to infinity, which further confirms the previous conclusion.

The superiority of proposed method becomes clear in the following comparison. Figure 3 shows that the simulation results under the observer (6) and the controller (11) with the learning gain matrices $K = [0, 0]$ and $\Gamma = [-0.6908, 0.3945]$. It is easy to verify that the condition $\|I - H \otimes (\Gamma B)\|_2 < 1$ holds. According to [54], the observer-type ILC law (11) is stable, which also can be confirmed by Figure 3. However, this ILC process is not monotonously convergent but is asymptotically stable, and the high-overshoots (as much as 94) are generated even though the input errors approach zero as the iteration step tends to infinity. Thus, these results show that merely the 2D analysis of ILC cannot present the sufficient conditions guaranteeing the monotonic convergence of input errors.

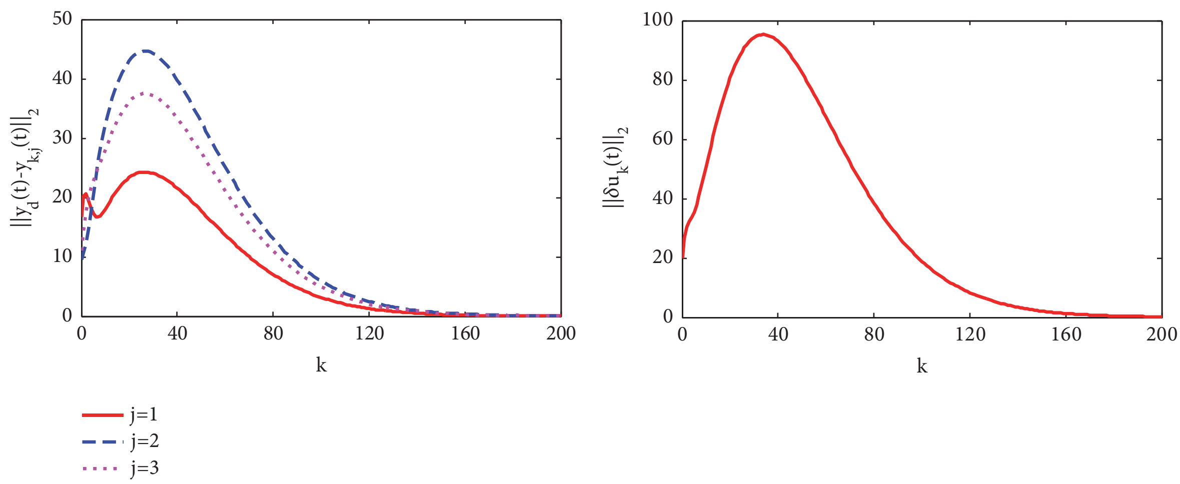
By comparing Figures 2 and 3, it is easily to know that the proposed method can achieve the perfect tracking of consensus and the good learning transients of control inputs simultaneously.

4.2. Example 2. Consider the leader-following multiple Newcastle robot systems [60] consisting of a leader robot and six follower robots. The communication graph among all robots is shown in Figure 4. It is easy to know that the communication graph in Figure 4 satisfies Assumption 17, and $H = \left[\begin{smallmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{smallmatrix}\right]$. The motion equations of leader robot and follower robots are respectively described by

$$m\ddot{q}_d(t) + c(t)\dot{q}_d(t) + \kappa(t) q_d(t) = \beta(t)\tau_d(t), \tag{52}$$

$$m\ddot{q}_{k,j}(t) + c(t)\dot{q}_{k,j}(t) + \kappa(t) q_{k,j}(t) = \beta(t)\tau_{k,j}(t), \tag{53}$$

where $k \in Z_+$, $j = 1, 2, \ldots, 6$. The mass $m = 2500\ kg$, the viscous damping factor $c(t) = c_0 + \Delta c(t)\ Ns/mm$, the linear stiffness of the force sensor $\kappa(t) = \kappa_0 + 0.5\Delta\kappa(t)\ N/mm$, and the coefficient of control inputs $\beta(t) = \beta_0 + \Delta\beta(t)$. $c_0 = 7\ Ns/mm$, $\kappa_0 = 44.5\ N/mm$ and $\beta_0 = 12500$ ($\max_{t\in[0,T]}|\Delta c(t)| = 0.8\ Ns/mm$, $\max_{t\in[0,T]}|\Delta\kappa(t)| = 5.5\ Ns/mm$ and $\max_{t\in[0,T]}|\Delta\beta(t)| = 100$) are the nominal values (the perturbation scopes) of uncertain viscous damping factor, uncertain linear stiffness and uncertain coefficient of control inputs, respectively. The control inputs of leader



(a) (b)

Figure 3: Under the observer (6) and the controller (11) with $K = [0, 0]$ and $\Gamma = [-0.6908, 0.3945]$: (a) output tracking errors versus iteration numbers; (b) Input errors $\|\delta u_k(t)\|_2$ versus iteration numbers (color online).

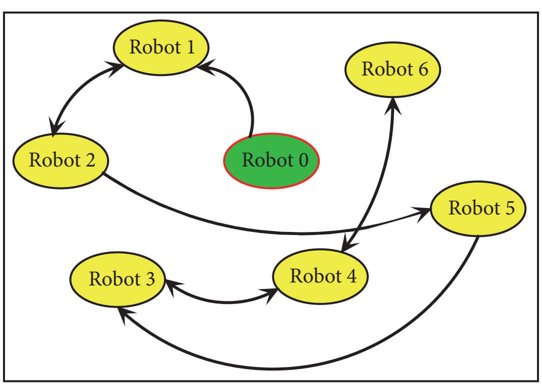


Figure 4: Communication graph among all robots. Leader: robot 0; Followers: robots 1, 2,. . .,6 (color online).

robot $\tau_d(t) = 1000\sin(0.5\pi t)\ N$. Let $x_d(t) = [{q_d}^{\mathrm{T}}(t), \dot{q}_d^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $x_{k,j}(t) = [{q_{k,j}}^{\mathrm{T}}(t), \dot{q}_{k,j}^{\mathrm{T}}(t)]^{\mathrm{T}}$. Then, (52) and (53) can be rewritten respectively as (1) and (4), where

$$
\begin{aligned}
A_0 &= \begin{bmatrix} 0 & 1 \\ -17.8 & -2.8 \end{bmatrix}, \\
B_0 &= \begin{bmatrix} 0 \\ 5 \end{bmatrix}, \\
C &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \\
f(t, x(t)) &= 0, \\
\alpha &= 1, \\
\mu &= 1, \\
E &= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \\
F_1 &= \begin{bmatrix} 2.2 & 0.32 \\ 0 & 0 \end{bmatrix}, \\
F_2 &= \begin{bmatrix} 0 \\ 0.04 \end{bmatrix}, \\
\sum(t) &= \operatorname{diag}\{\sin(0.1t), -\cos(0.5t)\}.
\end{aligned}
\tag{54}
$$

Thus the Lipschitz constant $l = 0$. Take $L = [0.3, 0.2]^{\mathrm{T}}$, it is easily check that both the system matrix A_0 and the observer matrix $A_0 - LC$ are Hurwitz. By solving the inequality (5) with $Q_1 = \left[\begin{smallmatrix} 0.2 & 0 \\ 0 & 0.2 \end{smallmatrix}\right]$ and the inequality (8) with $Q_2 = \left[\begin{smallmatrix} 0.4 & 0 \\ 0 & 0.4 \end{smallmatrix}\right]$, we obtain that $P_1 = \left[\begin{smallmatrix} 2.5302 & 0.2200 \\ 0.2200 & 0.1524 \end{smallmatrix}\right]$ and $P_2 = \left[\begin{smallmatrix} 3.1323 & 0.2491 \\ 0.2491 & 0.1842 \end{smallmatrix}\right]$. To perform the simulation, the initial states of each follower robot and its observer are set the same as that of leader robot, the zero initial control input for each follower robot is adopted. Five scalars are prescribed as $T = 3$, $\gamma = 0.01$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.15$ and $\varepsilon = 0.01$.

By using the Matlab LMI toolbox to solve the LMI (41) with $Q_1 = \left[\begin{smallmatrix} 0.2 & 0 \\ 0 & 0.2 \end{smallmatrix}\right]$, $Q_2 = \left[\begin{smallmatrix} 0.4 & 0 \\ 0 & 0.4 \end{smallmatrix}\right]$, $P_1 = \left[\begin{smallmatrix} 2.5302 & 0.2200 \\ 0.2200 & 0.1524 \end{smallmatrix}\right]$ and $P_2 = \left[\begin{smallmatrix} 3.1323 & 0.2491 \\ 0.2491 & 0.1842 \end{smallmatrix}\right]$, we obtain $K = [0.3113, 0.2105]$ and $\Gamma = [-0.0032, 0.0549]$, which indicates that the LMI (41) has feasible solutions. Under the observer (6) and the controller (11) with the obtained learning gain matrices, we obtain the simulation results as shown in Figure 5. The results illustrate that the proposed method is still suitable for the perfect tracking problem of consensus and the monotonic convergence problem of input errors for uncertain linear IOMASs.

5. Conclusions

In this paper, an observer-type fractional-order ILC protocol is designed to solve the perfect tracking problem of consensus and the monotonic convergence problem of input errors for the uncertain nonlinear fractional-order multiagent systems (FOMASs), where there exist the linear coupling relations

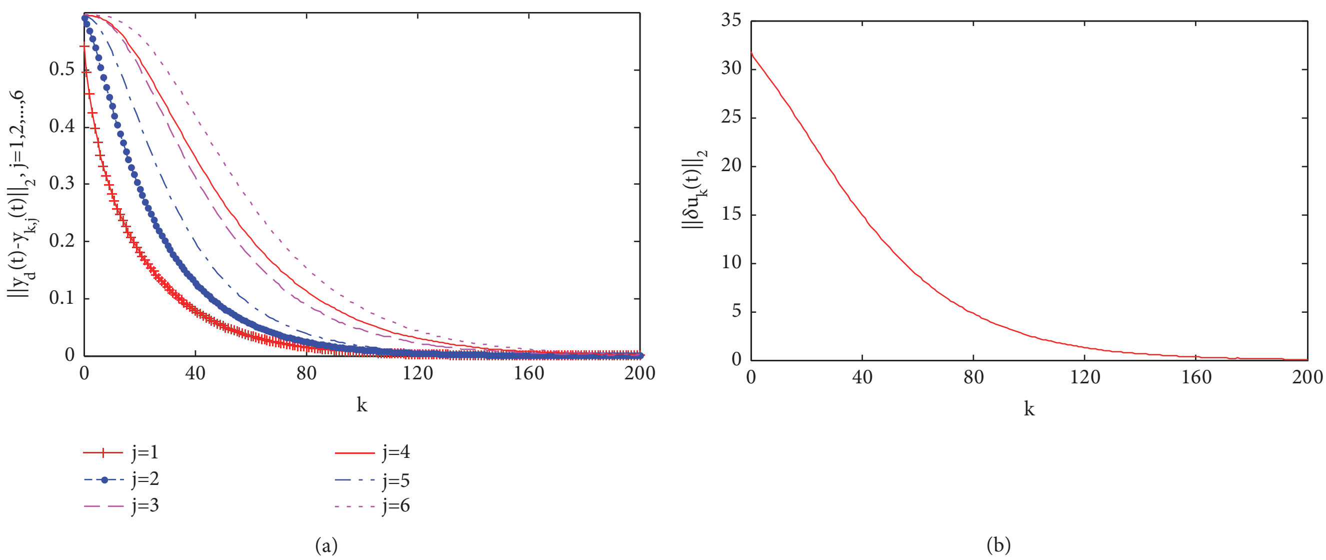


Figure 5: Under the observer (6) and the controller (11) with $K = [0.3113, 0.2105]$ and $\Gamma = [-0.0032, 0.0549]$: (a) output tracking errors versus iteration numbers; (b) input errors $\|\delta u_k(t)\|_2$ versus iteration numbers (color online).

among the fractional order, and the perturbation of the systems matrix and input matrix. By applying the 2D analysis method to analyzing the iterative learning process of such kind of FOMASs, the design problem of a robust monotonically convergent iterative learning law can be transformed into a robust control problem of the tracking error system. Based on the constructed performance index in terms of the fractional integral, the Lyapunov-like method is applied to derive the LMI conditions guaranteeing simultaneously the perfect tracking of consensus and the monotonic convergence of the input errors. The effectiveness of the proposed method is illustrated in the numerical simulation results including comparison to those by the traditional ILC method, which shows the superiority of the novel method over the traditional ILC method in terms of achieving good transients of the iterative learning process. A future work may be to consider an extension of the proposed methods to the heterogeneous FOMASs.

Appendix

Proof of Lemma 9

Proof. Consider the Lyapunov function candidate

$$V = x_{k,j}{}^{\mathrm{T}}(t) P_1 x_{k,j}(t). \tag{A.1}$$

Its derivative is

$$\begin{aligned}
\dot{V} &= x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot \left[(A_0 + \mu\alpha\Delta A(t))^{\mathrm{T}} P_1 + P_1 (A_0 + \mu\alpha\Delta A(t))\right] \\
&\cdot x_{k,j}(t) + 2x_{k,j}{}^{\mathrm{T}}(t) P_1 f\left(t, x_{k,j}(t)\right) = x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot \left[A_0{}^{\mathrm{T}} P_1 + P_1 A_0\right] x_{k,j}(t) + 2x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot P_1 f\left(t, x_{k,j}(t)\right) + \mu\alpha x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot \left[(\Delta A(t))^{\mathrm{T}} P_1 + P_1 \Delta A(t)\right] x_{k,j}(t) = x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot \left[A_0{}^{\mathrm{T}} P_1 + P_1 A_0\right] x_{k,j}(t) + 2x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot P_1 f\left(t, x_{k,j}(t)\right) + \mu\alpha x_{k,j}{}^{\mathrm{T}}(t) \\
&\cdot \left[\left(E\sum(t) F_1\right)^{\mathrm{T}} P_1 + P_1 \left(E\sum(t) F_1\right)\right] x_{k,j}(t).
\end{aligned} \tag{A.2}$$

Note that

$$\begin{aligned}
0 &\le \left(\sqrt{\varepsilon_1} F_1{}^{\mathrm{T}} \sum{}^{\mathrm{T}}(t) - \frac{1}{\sqrt{\varepsilon_1}} P_1 E\right) \\
&\cdot \left(\sqrt{\varepsilon_1} F_1{}^{\mathrm{T}} \sum{}^{\mathrm{T}}(t) - \frac{1}{\sqrt{\varepsilon_1}} P_1 E\right)^{\mathrm{T}} \\
&= \varepsilon_1 F_1{}^{\mathrm{T}} \sum{}^{\mathrm{T}}(t) \sum(t) F_1 + \varepsilon_1{}^{-1} P_1 E E^{\mathrm{T}} P_1 \\
&- F_1{}^{\mathrm{T}} \sum{}^{\mathrm{T}}(t) E^{\mathrm{T}} P_1{}^{\mathrm{T}} - P_1 E \sum(t) F_1 \le \varepsilon_1 F_1{}^{\mathrm{T}} F_1 \\
&+ \varepsilon_1{}^{-1} P_1 E E^{\mathrm{T}} P_1 - \left(P_1 E \sum(t) F_1\right)^{\mathrm{T}} \\
&- P_1 E \sum(t) F_1.
\end{aligned} \tag{A.3}$$

From (A.3), we have

$$\begin{aligned}
&P_1 E \sum(t) F_1 + \left(P_1 E \sum(t) F_1\right)^{\mathrm{T}} \\
&\le \varepsilon_1 F_1{}^{\mathrm{T}} F_1 + \varepsilon_1{}^{-1} P_1 E E^{\mathrm{T}} P_1.
\end{aligned} \tag{A.4}$$

Noticing Assumption 6, we have

$$\begin{aligned} 2x_{k,j}{}^{\mathrm{T}}(t)\, P_1 f\left(t, x_{k,j}(t)\right) &\le 2l \left\|P_1 x_{k,j}\right\| \left\|x_{k,j}\right\| \\ &\le x_{k,j}{}^{\mathrm{T}}(t)\, P_1 P_1 x_{k,j} + l^2 x_{k,j}{}^{\mathrm{T}}(t)\, x_{k,j}. \end{aligned} \tag{A.5}$$

By (A.4) and (A.5), we have

$$\begin{aligned} \dot{V} \le x_{k,j}{}^{\mathrm{T}}(t) \Big(& A_0{}^{\mathrm{T}} P_1 + P_1 A_0 + P_1 P_1 + l^2 I_n \\ & + \varepsilon_1 \mu\alpha F_1{}^{\mathrm{T}} F_1 + \varepsilon_1{}^{-1} \mu\alpha P_1 E E^{\mathrm{T}} P_1 \Big) x_{k,j}(t). \end{aligned} \tag{A.6}$$

Applying the Schur complement formula to (5), we obtain

$$\begin{aligned} & A_0{}^{\mathrm{T}} P_1 + P_1 A_0 + P_1 P_1 + l^2 I_n + \varepsilon_1 \mu\alpha F_1{}^{\mathrm{T}} F_1 \\ & \quad + \varepsilon_1{}^{-1} \mu\alpha P_1 E E^{\mathrm{T}} P_1 + Q_1 < 0. \end{aligned} \tag{A.7}$$

Noticing (A.7), we have

$$\dot{V} < -x_{k,j}{}^{\mathrm{T}}(t)\, Q_1 x_{k,j}(t) \le 0. \tag{A.8}$$

Thus, we can conclude that the state $x_{k,j}(t)$ is bounded and converges asymptotically to zero as time goes to infinity. That is, the fractional-order nonlinear uncertain agent (4) under no control is stable.

This ends the proof of Lemma 9. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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