Research Article

A Chimera Oscillatory State in a Globally Delay-Coupled Oscillator Network

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Abstract

Oscillatory behavior is absolutely necessary for the normal functioning of various organisms and their performance. Therefore, it is necessary to protect the oscillatory behavior in an aging network which consists of oscillatory and nonoscillatory nodes. In this work, we investigate numerically and theoretically the effect of time delay on oscillatory behaviors in a network which includes active and inactive Stuart–Landau oscillators. Interestingly, we find a chimera oscillatory state where a part of oscillators is a steady state while other oscillators preserve oscillatory motion; such dynamical behaviors are considered generally to be impossible for globally coupled systems when the coupling strength is sufficiently large. Furthermore, our results reveal that time delay can effectively inhibit aging transition and recover the oscillatory behavior from the aging network.

1. Introduction

The dynamics of complex systems has been a hot topic in nonlinear science due to its wide application in biology, ecology, physics, and chemistry. To investigate the collective behavior, the model of coupled nonlinear oscillators is simple but powerful [1–8]. Aging transition (AT), one of the significant collective behaviors, has been concerned by physical researchers and studied in different dynamical models [9–13]. The AT phenomenon appears usually in a damaged network, in which some active oscillators lose their activity and become a mixed network made up with self-oscillatory and non-self-oscillatory oscillators. The interesting question is how the oscillatory behavior of networks terminates with increasing the ratio of the inactive oscillators. Motivated by the problem, Daido and Nakanishi studied the collective dynamical behavior in a globally coupled network of periodic and steady elements [14]. They found the whole oscillatory network which undergoes the transition from the oscillation state to the steady state as the increasing of the ratio of inactive oscillators over a critical value with a sufficiently large coupling weight, and the universal scaling function for order parameter is revealed theoretically and numerically. Some similar works were performed for a globally coupled network with oscillatory and excitable elements or two kinds of oscillatory elements with different periods [15, 16]. To avoid the aging effect, many strategies are presented. For example, the aging effect can be eliminated by regulating a control parameter in the normal diffusive coupling, and the dynamical robustness of damaged networks can be efficiently enhanced [11]. A scheme based on external feedback has been put forward, and the method can efficiently protect oscillatory behavior in the aging networks of active and inactive elements [17]. To revive rhythm in the damaged networks, Bera designed the low-pass filtering technique and found that the oscillatory behavior can be restored in an aging network when the cutoff frequency decreases over a critical value, and the network survivability can be enhanced [18]. On the contrary, many researchers focus on the robustness of the network dynamics from the perspective of rhythm revival in which quenching of oscillation can be annihilated. This concept has been presented in [19]. Since then, the restoration of rhythmicity and revival of oscillation have been investigated...
widely with the numerical, theoretical, and experimental method [20–25]. For example, a scheme for reviving the oscillation states from the quenching states in the coupled dynamical networks has been proposed, and an experiment of chemical reaction was performed to verify this method [26]. Gosh et al. elaborated theoretically and experimentally that an appropriate mean-field density parameter can revoke oscillation of networks [27].

Time delay, original from finite speeds of signal propagation in the medium, plays a positive or negative effect in dynamical behavior of complex systems, and rich dynamical phenomena have been observed in oscillatory networks with time-delay coupling. For the positive role, the multistable state can be induced by time delay in the coupled periodic systems [28, 29]. The phenomenon of slow switching was first discovered in the globally coupled phase oscillators with time delay [30]. Processing delay, another type of time delay, raises from a finite response time in the processing of information. The processing delay is not only universal, as the response time is necessary in any realistic systems (e.g., relaxation time in reaction systems, latency time in lasers, synapse, and epidemics) but also important in the dynamical system. For example, time delay can recover the oscillation state from oscillation death in the coupled oscillator networks [19, 31]. Zhang et al. revealed that time delay in the control channel can improve effectively the control performance in an offshore steel jacket platform [32, 33].

The negative role of time delay has been studied and discussed since time delay can deteriorate oscillation behaviors of a complex system [34], for example, oscillation death can be induced by time delay in the coupled periodic systems [35–38]. Time-delay effect in aging transition has been discussed; Rahman et al. and Thakur et al. found that time delay can contribute to the aging transition by decreasing the threshold of coupling strength [39, 40].

Inspired by the above studies, we investigate the effect of time delay on the oscillatory behaviors in a globally delayed-coupled oscillator network which consists of oscillatory and nonoscillatory nodes. Our results reveal that the oscillatory state and the steady state can coexist in the globally delayed-coupled networks. This is a different chimera state which is characterized usually by the synchronization between a part of oscillators while other oscillators preserve incoherent motion [41–47]. Furthermore, the rich dynamical phenomena are observed with the different parameter settings of coupling strength and time delay. Previous work revealed that the oscillators in each cluster fall usually in complete synchronization in globally coupled networks with active and inactive oscillators [14]. However, we find that time delay can drive oscillators in each cluster transition from complete synchronization to asynchronous state, while two clusters are in antiphase synchronization. Interestingly, we find that the time delay can effectively prevent aging transition even for the globally-coupled oscillator network with all inactive oscillators. We organize the paper as follows: in Section 2, the phenomenon of chimera oscillatory state is investigated and discussed. The numerical results and theoretical analysis for the aging transition are shown in Section 3. Finally, we give a conclusion in Section 4.

2. The Phenomenon of Chimera Oscillatory State

We consider a globally-coupled network with the Stuart–Landau oscillators, and the equation is described as follows:

$$\dot{z}_j = \left(\alpha_j + i \omega - |z_j|^2 z_j \right) + \frac{\kappa}{N} \sum_{k=1}^{N} (z_k(t-\delta) - z_j(t-\delta)), \tag{1}$$

for $j = 1, \ldots, N$, where $z_j = x_j + iy_j$ is the complex variable, $\kappa$ represents the coupling strength, $\delta$ stands for time delay which comes from a finite response time due to the processing of information, $\omega$ is the characteristic frequency of the oscillator, and $\alpha_j$ determines the dynamical behavior of uncoupled oscillators, and $\alpha_j > 0$ and $\alpha_j < 0$ correspond to oscillator’s active (periodic) and inactive (steady) state, respectively. In our paper, we set $\alpha_j = -1, j = 1, 2, \ldots, pN$ for the inactive oscillators and $\alpha_j = 2, j = pN + 1, pN + 2, \ldots, N$ for the active oscillators. We split the populations into two groups: group A ($G_A$) with $i \in \{1, 2, 3, \ldots, pN\}$ and group B ($G_B$) with $i \in \{pN + 1, pN + 2, \ldots, N\}$. The parameter $p$ denotes the ratio of the active oscillator for the oscillatory network. Throughout the paper, the network size $N = 50$ and the natural frequency $\omega = 3.0$ are chosen.

When these oscillators are decoupled, there are 40 oscillators with steady state and 10 oscillators with oscillatory state for $p = 0.8$. The dynamical behaviors in the globally coupled oscillators without delay have been investigated in detail [14]: depending on the coupling strength $\kappa$ and the ratio $p$, the whole network behaves either in the oscillatory state or the aging state. For checking the dynamical behaviors of the globally coupled oscillators with delay, we show firstly the average radius $|Z_A|$ (|Z_A| = (1/pN) |Z^\text{pN}\_A|) and $|Z_B|$ (|Z_B| = (1/(1-p)N) |Z^\text{pN}\_B|) for two groups against the coupling strength $\kappa$ for $\delta = 0.1$ and 0.4 in Figure 1. For the small delay, the familiar aging transition has been observed with increasing of coupling strength (Figure 1(a)); we will discuss it in Section 3. For the large delay, one can see that $|Z_B|$ decreases with increasing coupling strength $\kappa$ and then increases after reaching to a minimum value (the minimum values are larger than zero), indicating clearly that the oscillators in group B are always in oscillatory state. Interestingly, we find that the values of $|Z_A|$ become zero for a sufficiently large interval of $\kappa$, which means that the oscillators in groups A are in steady state (Figure 1(b)). Thus, the new oscillatory behaviors in which a part of oscillators is steady state while other oscillators preserve oscillatory motion occur in the globally coupled oscillator networks.

Furthermore, Figures 2(a)–2(c) show the radius ($|Z_A|$) of every oscillator. From these figures, we find the networks have been split into two parts for all parameter settings, and two common features are observed: two oscillatory groups (Figures 1(a) and 1) and aging phenomenon (Figure 1(c)).
Figure 1: (a) and (b) The average radius $|Z_A|$ ($|Z_A| = (1/pN) \sum_{j=1}^{pN} |Z_j|$) and $|Z_B|$ ($|Z_B| = (1/(1-p)N) \sum_{j=pN+1}^{N} |Z_j|$) for two groups against the coupling strength $\kappa$ for $\delta = 0.1$ and 0.4, respectively. $p = 0.8$.

Figure 2: The radius $|Z_i| = \sqrt{x_i^2 + y_i^2}$ vs. $i$ for the parameters $\kappa = 2$ and $\delta = 0.1$ (a), $\kappa = 2$ and $\delta = 0.4$ (b), $\kappa = 6$ and $\delta = 0.1$ (c), and $\kappa = 6$ and $\delta = 0.4$ (d). $p = 0.8$. 
From Figure 1(b), however, we find interestingly the radius of 40 oscillators for group A is zero, while the radius of 10 oscillators for group B is about 1.6. Apparently, 10 oscillators with oscillatory state coexist with 40 oscillators with steady state in the globally coupled networks, two groups seem to be uncorrelated. Based on these observations, we suggest it a chimera oscillatory state.

To make the dynamical behaviors of two groups clearer, the time series of \( x_A = (1/pN) \sum_{i=1}^{pN} x_{j}, \) \( x_B = (1/(1 - p)) \sum_{j=pN+1}^{N} x_{j}, \) and \( x_N(t) \) are shown in Figures 3(a)–3(d); the black solid line, pink dot line, blue dash line, and green dash-dot-dot line in Figures 3(a)–3(d) stand for \( x_A(t), x_B(t), x_1(t), \) and \( x_N(t), \) respectively. The phase diagrams are given in Figures 3(e)–3(h). The parameter settings are the same as Figure 2. In Figure 3(a), the whole network is divided into two groups, and the oscillators in each group are completely synchronized from the phase diagram of Figure 3(e). The chimera oscillatory state is shown clearly in Figures 3(b) and 3(f), where the oscillators in group A are at the origin, and the oscillators in group B are at a circle with radius 1.19. Figures 3(c) and 3(g) are devoted to examples of the aging effect, while Figure 3(d) shows antiphase synchronized solutions between two groups from the observation of \( x_A \) and \( x_B \), and the oscillators in each group are not synchronous and distributed at two circles (Figure 3(h)).

Figure 4 gives phase diagrams on the \((\kappa, \delta)\) plane based on the observation of dynamical behaviors of two groups. The system presents four primary features: aging effect (AE), in-phase synchronization (IPS), antiphase synchronization (APS), and the chimera oscillatory state (COS). In Figure 4, the blue region corresponds to AE, and here, all oscillators are in the steady state; the dark yellow region where the two clusters are in antiphase synchronous corresponds to APS; and the pink region corresponds to COS in which a part of oscillators is steady state, while other oscillators preserve oscillatory motion. IPS is marked with the light gray region in which the oscillators in each group are synchronous, and two groups are in-phase synchronized.

3. The Phenomenon of Aging Transition

3.1. The Numerical Results. Next, we will investigate the effect of time delay on the aging transition. For case \( \delta = 0 \), the aging transition has been investigated in detail in [14]; they found the aging phenomenon with increasing of the ratio of inactive oscillators when the coupling strength is sufficiently large. The similar results are presented in Figure 5(a), where the order parameter \( |Z| = (1/N) \sum_{i=1}^{N} |z_i| \), \( |z_i| = \sqrt{x_i^2 + y_i^2} \) as a function of the ratio of the inactive oscillators \( p \) with different values of coupling strength \( \kappa \) for \( \delta = 0.1 \). However, we observe clearly that the aging phenomenon of the network can be prevented by a proper tuning of \( \delta \) with a fixed coupling strength \( \kappa \). The order parameter \( |Z| \) versus \( p \) is shown in Figure 5(b) with different values of \( \delta \) for coupling strength \( \kappa = 8 \). For \( \delta = 0 \), the critical value of \( p \) for aging transition is 0.75. With increasing \( \delta \) over a critical value, the order parameter \( |Z| \) is large than zero for arbitrary \( p \), indicating that the aging effect disappears, and the inactive oscillators regain activation.

Figures 6(a)–6(d) show the dynamical diagrams on the \((\kappa, \delta)\) space for \( \delta = 0.0, 0.05, 0.12, \) and 0.15, respectively. In these figures, the gray regions which are determined by \( |Z| = 0.0 \) stand for the aging transition region, and the white regions represent the oscillatory region. Comparing four figures, we found that the size of the gray region decreases with increasing of \( \delta \), while the oscillatory region becomes larger, indicating that the aging transition is inhibited by the processing delay.

To show further the inhibitory effect on the aging transition, Figures 7(a)–7(d) show the order parameter \( |Z| \) as a function of \( \delta \) with \( \kappa = 2.0, 4.0, 6.0, \) and 8.0, respectively. In each figure, black points, green squares, blue circles, and pink triangles correspond to \( p = 0.4, 0.6, 0.8, \) and 1.0, respectively. From these figures, we find that the order parameter \( |Z| \) depends nonmonotonously on time delay, and the order parameter \( |Z| \) is almost invariable before a critical value and increases with increasing of \( \delta \). The aging effect is inhibited by the time delay, even when all coupled oscillators are inactive.

So far, we have shown the oscillation robustness can be preserved by the time delay in the globally coupled network with active and inactive oscillators. Next, we show that such robustness can be easily observed in other complex networks. As a paradigmatic example, the small-world networks with \( N = 200 \) and connecting probability \( p_s = 0.3 \) are tested. Figure 8(a) shows the order parameter \( |Z| \) as a function of \( \delta \) with \( \kappa = 1.2 \) for \( p = 0.7, 0.8, \) and 0.9. The order parameter \( |Z| \) also increases with increasing of \( \delta \) for the arbitrary \( p \), especially for aging networks \( (p = 0.9) \), indicating that the oscillation behavior is also protected by time delay in the small-world networks. For checking generality, the order parameter \( |Z| \) versus the ratio of the inactive oscillators \( p \) with different values of \( \delta \) for coupling strength \( \kappa = 12 \) and different values of \( \kappa \) for coupling strength \( \delta = 0.1 \) are shown in Figures 8(b) and 8(c), respectively. As a result, the process-delay-eliminated aging is verified again. Our result revealed that the processing delay can inhibit the aging transition and preserve oscillation robustness, whether in the regular network or complex network.

3.2. Theoretical Analysis. To obtain the conditions of the stability of fixed points in delay-coupled oscillators, in the following, we apply the method of linear stability analysis. By setting \( z_j = 0 + \xi_j \), where \( \xi_j \) is a small perturbation at the origin, then we can obtain the linearization equation

\[
\dot{\xi}_j(t) = (a_j + i\omega)\xi_j + \kappa \sum_{i=1}^{N} (\xi_i(t - \delta) - \xi_j(t - \delta)).
\]

Denote \( \xi = (\xi_1, \xi_2, \ldots, \xi_N) \) and \( \Gamma = (1/N) \sum_{j=1}^{N} \xi_j \) as a function of the ratio of the inactive oscillators \( p \) with different values of coupling strength \( \kappa \). The order parameter \( |Z| \) versus \( p \) is shown in Figure 5(b) with different values of \( \delta \) for coupling strength \( \kappa = 8 \). For \( \delta = 0 \), the critical value of \( p \) for aging transition is 0.75. With increasing \( \delta \) over a critical value, the order parameter \( |Z| \) is large than zero for arbitrary \( p \), indicating that the aging effect disappears, and the inactive oscillators regain activation.

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Figure 3: (a)–(d) Time series for $x_A = (1/pN) \sum_{j=1}^{pN} x_j$ (black solid line), $x_B = (1/(1-p)N) \sum_{j=pN+1}^{N} x_j$ (pink dot line), $x_1$ (t) (blue dash line), and $x_N$ (t) (green dash-dot-dot line). (e)–(h) Phase diagrams for $(x_i, y_i)$. $\kappa = 2$ and $\delta = 0.1$ (a, c), $\kappa = 2$ and $\delta = 0.4$ (b, f), $\kappa = 6$ and $\delta = 0.1$ (c, g), and $\kappa = 6$ and $\delta = 0.4$ (d, h). $p = 0.8$.

Figure 4: Continued.
and we further get the characteristic equation under $N \to \infty$ limit:

$$\lambda = -1 + i\omega - \kappa e^{-\lambda \delta},$$

there are $pN - 1$ roots,

$$\lambda = 2 + i\omega - \kappa e^{-\lambda \delta},$$

there are $(1 - p)N - 1$ roots,

$$\lambda = \frac{1 + 2i\omega - \kappa e^{-\lambda \delta} \pm \sqrt{(ke^{-\lambda \delta} - 3)^2 + 12ke^{-\lambda \delta}(1-p)}}{2},$$

there are two roots.
Figure 6: (a)–(d) Phase diagrams on the \((\kappa, \delta)\) plane for aging transition and oscillatory regions with \(\delta = 0.0, 0.05, 0.12,\) and 0.15, respectively. The word "aging" represents the aging transition region, and the white region is the oscillatory region.

Figure 7: Continued.
For $\delta = 0$, we obtain

$$
\begin{align*}
\lambda & = -1 - \kappa + i\omega, \\
\lambda & = 2 - \kappa + i\omega, \\
\lambda & = \frac{1 + 2i\omega - \kappa \pm \sqrt{(\kappa - 3)^2 + 12\kappa(1 - p)}}{2},
\end{align*}
$$

there are $pN - 1$ roots,
there are $(1 - p)N - 1$ roots,
there are two roots.

According to the second formula of equation (6), we have $\kappa_c = 2$, and we can also get the critical value of ratio $p$ according to the third formula of equation (6):

$$
P_c = \frac{2(\kappa + 1)}{3\kappa}
$$

The above two critical values are the same as the result with [14]. We cannot get the roots of equation (5) since it is very complex; we consider the case $p = 1.0$, and the characteristic equation can be written:

$$
\begin{align*}
\lambda & = -1 + i\omega - \kappa e^{-i\beta}, \\
\lambda & = -1 + i\omega,
\end{align*}
$$

where $\beta$ is real. Furthermore, we have

$$
\begin{align*}
\beta & = \omega + \kappa \sin(\delta\beta), \\
0 & = -1 - \kappa \cos(\delta\beta).
\end{align*}
$$

Therefore, we arrive at the critical conditions for the new rhythm:

$$
\begin{align*}
\delta_1 & = \frac{(2m + 1)\pi - \cos^{-1}(1/\kappa)}{\omega + \sqrt{\kappa^2 - 1}}, \\
\delta_2 & = \frac{(2m + 1)\pi + \cos^{-1}(1/\kappa)}{\omega - \sqrt{\kappa^2 - 1}},
\end{align*}
$$

where $m = 1, 2, 3, \ldots, \infty$. From Figure 5(d), we can see that the plane is split into two parts by two critical curves $\delta_1$ and $\delta_2$ (solid line), which shows a good agreement with the numerical results.
4. Conclusion

As a summary, the roles of time delay on the collective dynamical behavior of the globally coupled network with inactive and active oscillators are investigated systematically. Numerical results, supplemented by a theoretical analysis from the linear stability analysis, reveal that the time delay can eliminate aging effect and enhance the oscillatory robustness of the oscillator network. This positive effect of time delay on oscillation of the network has been tested with other complex networks. Furthermore, the rich dynamical phenomena are observed in the globally coupled network: aging effect (AE), in-phase synchronization (IPS), antiphase synchronization (APS), and the chimera oscillatory state (COS) which is performed with the steady state between a part of oscillators while other oscillators preserve oscillatory motion are observed, respectively.

Due to the destruction of complex systems’ structure and the defect of oscillatory properties of the individual units in real systems, the degradation of oscillation behavior is very common. Such an oscillation deterioration which may be triggered by aging transition can affect the functioning and performance of the organism. Our research could be an effective step to control the aging process for reviving the oscillation behavior of the damaged network. Finally, we expect that all these results can be extended to the coupled damaged networks with different oscillatory properties and provide a valuable clue to design various experiments in future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.
Conflicts of Interest
The authors declare that they have no conflicts of interest.

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