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Research Article

Research on Optimization of Production Decision Based on Payment Time and Price Coordination

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This paper focuses on the coordination and optimization between a manufacturer and multiple retailers in a supply chain. The manufacturer makes product quotes and delivery deadlines for all retailers, and each retailer selects product offers and delivery deadlines based on their own needs. Manufacturers maximize their own total profits by setting optimal quotes and delivery deadlines. This paper constructs the mathematical model of the optimal quotation and delivery deadline and proposes a scheduling algorithm that is different from the general M/M/1 and then studies the production scheduling problem and explores the effective implementation of quotation policy in management practice.

1. Introduction

For the order-based production model, delivery time (or order-to-delivery time) guarantees have been applied as an advanced strategic weapon to compete with other companies. In the market, some customers are willing to pay more for faster delivery. In addition, customers have different time sensitivity and price sensitivity in the market. A new strategy is to divide customers into different groups based on their sensitivity to price and time. In a group, customers have the same combination of delivery time guarantees and price quotes. Different quotations and delivery times for different customer groups are evident in the printing and packaging industry. In the literature, this issue is called customer segmentation and aggregation issues [1–3]. In the following example, Printing.com uses this split and merge strategy. Printing.com is a printing service provider. It offers customers a choice of menus with different delivery times and different prices. For short lead times, prices are naturally higher.

With regard to the ability to set up this strategy, there are two situations: the ability of all customers to share and the specific capabilities of each target customer. There are two reasons why dedicated capabilities are supported. First, the use of dedicated capabilities to provide different delivery time guarantees for each customer group is compatible [4]. Second, there is less interference from customers in different fields [1, 5].

In a dedicated capability setting, performing a split and merge strategy can be thought of as multiple single customer group issues, where the only difference is the ability of each group and the sensitivity range of each group customer to price and time. Therefore, the initial work of this strategy is to quote a common optimal price and a common delivery time for each customer group. For this reason, we consider the pricing and production issues of a single customer group in the context of small batch production. In a single group, customers' payment preferences and delay tolerance are different within a known range of price and time sensitivity.

One of our technical contributions is to propose a solution to the problem of joint pricing scheduling faced by manufacturers. We show the property quotes for the best delivery times and have developed an advanced scheduling algorithm to solve the problem optimally. This paper expands Chen and Hall's research on time and scheduling coordination issues by comprehensively considering time and price quotes [6].

2. Literature Review

This article mainly considers the issue of product pricing and production. The relevant literature is mainly elaborated in the following two aspects. On the one hand, the issue of pricing and production of the delivery deadline is not considered. On the other hand, the issue of price delivery time quotation is considered, which is more relevant to the content of this article.

Regarding pricing and production issues without considering delivery dates, in order to explore this issue, various models are mentioned in related articles [7–9]. These models assume that demand is independent of delivery time and sensitive to time quotes or other factors. In particular, Ata and Olsen reviewed the relevant due date quotes literature in detail and studied dynamic time quotes under different delay costs [10].

Since delivery time and price have a great influence on order acceptance, it is very important to integrate price quotations into delivery deadlines [11, 12]. Several papers consider different delivery times and prices for different customer groups [1, 5, 13–15], and some other literature that considers different customers groups has common price and delivery time quotes [2, 16, 17].

There is a large amount of literature to formulate relevant production policies with a stable queuing model. In these documents, a fixed scheduling rule such as First Come First Service (FCFS) is used. The customer's demand is a deterministic function of price, delivery time, and other attribute variables [17–20]. All these documents regard the production stage as an M/M/1 queue, and therefore, the FCFS rules are used therein. Our model is fundamentally different from those approaches by considering optimal production sequencing.

There are very few literatures that consider the price of production scheduling and the decision-making on the delivery date. Elhafsi studied how to determine the delivery time and price for an order in an order-type manufacturer [21]. Under the premise that the delivery time does not affect the demand, the main purpose is to quote an order of arrival based on FCFS rules for delivery time. Charnsirisakskul et al. also propose a decision model for comprehensive pricing and production decision-making using a single price model or multiple price models [22]. Their decision model is based on the inventory-based production scenario, and our research is based on order-based production. The research provided by Chen and Hall to solve this problem is the closest to our direction. In the context of detailed scheduling, Chen and Hall studied the quoting problem [6]. This article studied joint delivery times and quotations, rather than just studying quotations.

Chen and Hall analyzed the importance of adding detailed scheduling to the study. Compared with uncoordinated pricing decisions, the value of coordinated pricing and production decisions is more accurate. In our study, late fines were also included in the objective function of this article. Charnsirisakskul et al. have previously judged the importance of their participation in the study [22].

Based on the above discussion, what we have studied is the coordination pricing and delivery time quotation taking into account the detailed production scheduling decision mechanism under the order-based production environment. Specifically, the purpose of this study is to formulate a scheduling mechanism that handles delivery quotes, including prices. In the past, the scheduling study mainly solved the problem of the deadline of the quotation that did not take into account the price, so almost no researcher considered both the price and the delivery date quotation. Another major contribution of our research is that we consider the customer's heterogeneity of time and price requirements. In our study, the customer determines the order quantity. Charnsirisakskul et al. and Chen and Hall also made similar assumptions.

Table 1: Variable symbol table.

Variables	Definition
f	Total net profit
i, j	Order
c	Quoting constant for general price and delivery time
m	The number of orders completed on time at a given time t
n	Total order quantity
p	Quotation of all orders in quotation questions at regular prices and delivery times
$[p_c, p_{c+1}]$	The boundary of a particular price zone c
pt_i	The process time of order i in the quotation question of normal price and delivery time
p_i	The quotation of order i
s	The amount of a special price-time zone boundary in the quotation question for normal prices and delivery times
t	All orders in the quotation question of regular price and delivery time report delivery time
$[t_c, t_{c+1}]$	A specific time zone c
t_i	Delivery time for order i
w	General delay weight
C_i	Order i completion time
Q_i	Order quantity of order i
R	Total sales revenue
s_i	Initial process time of order i
T	Total delay penalty
T_i	Delay penalty for order i
Y_i	Process time for order i
α_i	Potential market size of order i
β_i	Price sensitivity of order i
θ_i	Time sensitivity of order i

3. Model Construction

Assuming that there are n retailers, denoted as $i = 1, 2, 3, \ldots, n$, the manufacturer introduces a price and delivery time for all retailers. When a retailer's order is completed, the manufacturer will deliver the order to the retailer. We assume that the scheduling time is not taken into account, and assuming that the production process time of each product is fixed and known, the capacity cost is also fixed and known. The model symbols are summarized in Table 1.

In order to reflect consumer sensitivity to price and delivery time, we assume the following demand function that reflects the number of consumer orders:

$$Q_i(p, t) = \alpha_i - \beta_i \cdot p - \theta_i \cdot t, \quad (1)$$

where $\alpha_i, \beta_i, \theta_i$ is the normal number, α_i indicates the potential market size of order i, β_i indicates the price sensitivity of order i, θ_i indicates the time sensitivity of order i.

We combine (β_i, θ_i) to describe consumer sensitivity to price and delivery time. For the possible zero demand or negative demand quantity, we assume that consumers will not place orders. We assume that a valid order placed by the consumer must be produced and not delivered separately. Assume that the manufacturer is a pure production machine and u denotes the priority given per unit product process time, so the production process time of the order is described as:

$$pt_i = u \cdot Q_i. \tag{2}$$

At the beginning of each planned production cycle, the manufacturer will set product price and delivery time quotation, so the process time of all orders has been determined at the initial moment, so the initial release time standard for all orders is 0.

If the order is completed beyond the delivery time, the manufacturer will bear the delay penalty. Since the consumers we consider come from a consumer group, we have already discussed in the introduction section that it is reasonable to determine a weight penalty for delay for all consumers, that is to say:

$$T_i(p,t) = w \cdot \max(0, C_i - t), \tag{3}$$

where C_i is the completion time of order i and w is the general delay weight for all consumers.

Our goal is to maximize the manufacturer's net profit. Since fixed facility costs do not affect the optimization decision on product prices and delivery time quotes, our profit function will not include fixed facility costs. The specific profit function is expressed as follows:

$$\begin{aligned} &\text{Maximize} f(p,t) = p\sum_{i=1}^{n} \max(Q_i(p,t),0) - \sum_{i=1}^{n} (g \cdot T_i(p,t)), \\ &\text{Subject to } g = \begin{cases} 0, & Q_i(p,t) \le 0 \\ 1, & Q_i(p,t) < 0 \end{cases}, \quad p, t \ge 0. \end{aligned} \tag{4}$$

Let p^* and t^* denote the optimal solutions for p and t, respectively.

According to the objective function (4), the manufacturer's production plan is introduced into the model, and we will study the optimal production sequence problem for this scheduling problem. The following model will be divided into two types of general models to study. One is fixed quotes, and the other is fixed delivery time.

3.1. Optimal Production Sequence. According to the objective function (4), the optimal net profit is not only related to delivery time and price, but also related to the production sequence of the order. Xia et al. demonstrated that the shortest processing time (SPT) has a gradual optimality [23], and because of the existence of delayed punishment, we have Lemma 1.

Lemma 1. *Shortest processing time (SPT) is the optimal sequence rule for maximizing profit in product production.*

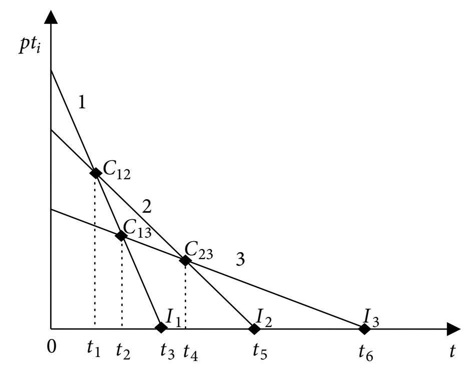

Figure 1: Order processing times with respect to one decision variable.

Table 2: SPT sequences in different time zones.

Time zones	$0-t_1$	t_1-t_2	t_2-t_3	t_3-t_4	t_4-t_5	t_5-t_6
SPT sequences	3–2–1	3–1–2	1–3–2	3–2	2–3	3

This article will apply SPT rules in production scheduling problems. First, we will explain how to combine sequence constraints in the problem of profit maximization.

When the price is assumed to be fixed, the order processing time is a linear function of the delivery time:

$$pt_i = (\alpha_i - \beta_i \cdot p) - \theta_i \cdot t. \tag{5}$$

Figure 1 shows the processing time of three consumer orders. Three lines represent three consumers 1, 2, 3, respectively. The three lines intersect at the intersections of C_{12}, C_{13}, and C_{23}. Each intersection indicates that the processing time of the two orders is equal. As can be seen in Figure 1, the intersection point between the three straight lines and the intersection point of each straight line with the t-axis divide the t-axis into six segments from t_1 to t_6, that is, the SPT sequence of the three orders is uniquely determined in the interval $[0,t_1], [t_1,t_2], \ldots, [t_5,t_6]$. Table 2 shows all SPT production sequences for each time interval or region three orders.

If C_{ij} is the point of intersection of two lines L_i, L_j, then there is an equation $pt_i = pt_j$. Bring Equation (1) into this equation to get the t coordinate of C_{ij} as:

$$t_{ij} = \frac{\alpha_i - \alpha_j + p(\beta_j - \beta_i)}{\theta_i - \theta_j}. \tag{6}$$

According to $pt_i = 0$, the intercept point I_i on the t-axis can be expressed as:

$$t_i = \frac{\alpha_i - p\beta_i}{\theta_i}. \tag{7}$$

With the above formulae (6) and (7), all the time zone boundaries can be calculated and then they are sorted in ascending order. When the price is fixed, the delivery time is variable, and we constrain the time. Use $t_c \le t < t_{c+1}$ to represent the general time zone, where t_c and t_{c+1} represent any two adjacent

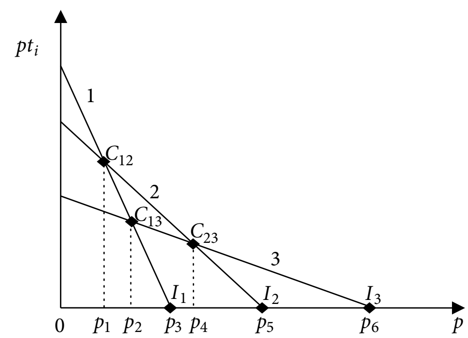

FIGURE 2: Order price with respect to one decision variable.

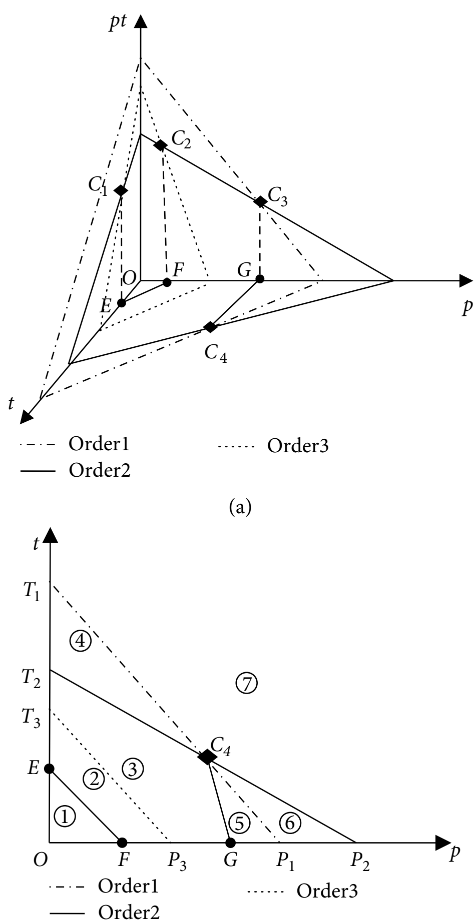

FIGURE 3: Order processing time function in general problem.

boundaries of a time zone. Under this time limit, the optimal sequence of orders is uniquely determined.

When the delivery time quotation is fixed, similar to the previous analysis, the price range constraint is $p_c \leq p < p_{c+1}$, where p_c and p_{c+1} represent any two adjacent boundaries of the price region. As shown in Figure 2, the p coordinate of C_{ij} is expressed as:

$$p_{ij} = \frac{\alpha_j - \alpha_i + t\left(\theta_i - \theta_j\right)}{\beta_j - \beta_i}. \tag{8}$$

The intercept point I_i on the p-axis is expressed as:

$$p_i = \frac{\alpha_i - t\theta_i}{\beta_i}. \tag{9}$$

The boundary values of the price region can be obtained by the above formulas (8) and (9). In this one of the price regions, the unique production sequence can be uniquely determined.

In the general case, the quotation and delivery deadline are two decision variables. For example, the price and time sensitivity of three orders are different. The order processing time is a linear function of t and p; that is:

$$pt_i = \alpha_i - \beta_i \cdot p - \theta_i \cdot t. \tag{10}$$

Figure 3(a) illustrates the processing time for three orders in a three-dimensional Cartesian coordinate system, in which three orders are divided into three planes, (pt, t), (pt, p) and (p, t) plane. Each plane has three orders intersecting at four intersection points C_1, C_2, C_3, and C_4. Each projection point $\left(p_{C_i}, t_{C_i}\right)$ corresponding to C_i can be obtained by solving the following two equations in the (p, t) plane:

$$\alpha_i - \beta_i \cdot p - \theta_i \cdot t = \alpha_i - \beta_j \cdot p - \theta_j \cdot t. \tag{11}$$

$$p = 0, t = 0 \quad \text{or} \quad pt = 0. \tag{12}$$

According to Figure 3(a), it can be seen that the points E, F, and G are projections of the points C_1, C_2, and C_3 on the plane (p, t), respectively. Line C_1C_2 consists of points on $pt_2 = pt_3$. Since EF is a projection of C_1C_2 on the plane (p, t), then a point (p, t) on EF also satisfies $pt_2 = pt_3$.

Figure 3(b) depicts the linear projection in Figure 3(a) projected onto the (p, t) plane, showing its two-dimensional planar graphic region. In Figure 3(b), the straight line is the intersection of the plane i and the plane (p, t) and represents the set of points at the time $pt_i = 0$; that is, the points (p, t) and the straight line satisfy the following equation:

$$\alpha_i - \beta_i \cdot p - \theta_i \cdot t = 0. \tag{13}$$

From the above analysis, we can see that the methods for exploring fixed order price and delivery time are similar. Under normal circumstances, there are only two cases of optimal production sequence change. One is that the two orders' processing time is equal; the other is that it is an order processing time equal to 0. This means that the order of the optimal production of the order is transformed into straight line P_iT_i and lines EF, GC_4 in Figure 3(b).

Therefore, in the first quadrant of Figure 3(b), the optimal production sequence is uniquely determined by the concave regions segmented by the straight lines P_iT_i, EF, GC_4, and t/p axes, and we define these concave regions as price-time regions. In Figure 3(b), each concave area is represented by ① to ⑦. The decision variable is a price-time zone with a unique optimal production order, and each optimal production order zone is shown in Table 3.

In order to obtain the SPT sequences for all price-time regions, all regions in the first quadrant of Figure 3(b) need to be obtained, so we have designed an algorithm to create a detailed price-time region, recorded as a sequence listing algorithm (SLA); the main idea of this algorithm is as follows:

Table 3: All price-time zone SPT ordering for three orders.

Zone index	1	2	3	4	5	6	7
Sequence	2-3-1	3-2-1	2-1	1	1-2	2	None

Step 1. Think of the first quadrant as the entire price-time region, dividing the first quadrant into a series of price-time regions by a single $P_i T_i$.

Step 2. Check each price-time area to ensure that it is a projection of the intersection of each two order surfaces on the (p, t) plane.

Step 3. Get all price-time zones.

3.2. Optimal Delivery Deadline. In Section 3.1, we studied the optimal SPT sequence for fixing each time zone. This subsection will discuss the change of delivery time when fixing the price of a product. If the special time zone is discussed in Section 3.1, the resulting sequence decision will not be important. Therefore, according to the number of time zones, the problem is decomposed into several simple subproblems. We divide the objective function (4) into two parts. one part is sales revenue; that is:

$$R(t) = p \sum_{i=1}^{n} (\alpha_i - p \cdot \beta_i - t \cdot \theta_i). \tag{14}$$

The other part is the delay penalty:

$$T(t) = w \sum_{i=m+1}^{n} (\alpha_i - p \cdot \beta_i - t \cdot \theta_i) - w(n-m)t, \tag{15}$$

where m is the number of orders completed on time in a given time t.

If m is independent of time t, then the gradient of profit return with respect to time t is:

$$\frac{\partial f}{\partial t} = \frac{\partial R}{\partial t} - \frac{\partial T}{\partial t} = -p \sum_{i=1}^{n} \theta_i - w \sum_{i=m+1}^{n} (-\theta_i) + w(n-m) \tag{16}$$

Theorem 1. *Assume that the profit function (4) is a concave function, then:*

(i) *Small segments between two time nodes are linear functions.*
(ii) *The derivative of each subparagraph with respect to time is decremental.*
(iii) *The first part of the derivative of time is positive, and the latter part of the derivative of time is negative.*
(iv) *The point of maximum return can be obtained between two small segments.*

The conclusion for Theorem 1 can be explained from Figure 4. For a concave profit function with respect to time, a small piecewise linear function is formed between every two time points. Each small fragment is slowly increased before t^*. Its derivative is positive, but the growth rate of its derivative is monotonically decreasing with respect to t. After

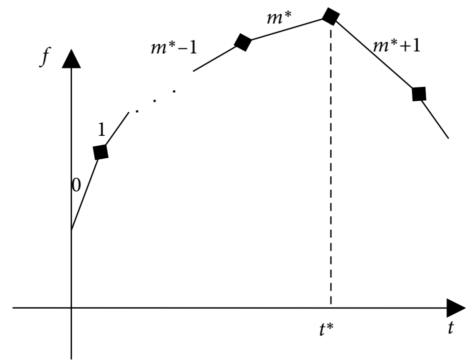

Figure 4: The profit function with respect to delivery time quotation.

t^*, it can be seen that each small segment linear function is declining; that is, the derivative of each small segment linear function is negative and can be seen from the figure. It can be seen that when $t = t^*$, the profit function takes the maximum value.

Corollary 1. *Assume that given a time zone, you can find two possible conditions for obtaining the optimal delivery time*:

(i) *The optimal delivery time is the same as the completion time of an order.*
(ii) *The optimal delivery time is any boundary of the time zone.*

According to Theorem 1 and Figure 4, it can be concluded that the number of small segments represents the number of orders completed on time in that situation. As time t increases, m is also an intermittent increase. The convergence time t of two segments on the t-axis is equal to the completion time of an order. The point of maximum profit is the point of convergence of two small segments. The value of the derivative of the small linear function on the left side of this point with respect to t is positive when $m = m^*$. The derivative of the small linear function on the right side of this point with respect to t is negative when $m = m^* + 1$; that is, it satisfies the following inequality:

$$\frac{\partial f}{\partial t} \Big| (m = m^*) > 0, \tag{17}$$

$$\frac{\partial f}{\partial t} \Big| (m = m^* + 1) \le 0. \tag{18}$$

Based on the above two inequalities, we bring the values of $m = m^*$ and $m = m^* + 1$ into (16), which yields:

$$w(n - m^*) + w \sum_{i=m+1}^{n} \theta_i \le p \sum_{i=1}^{n} \theta_i, \tag{19}$$

$$w(n - m^* + 1) + w \sum_{i=m+1}^{n} \theta_i < p \sum_{i=1}^{n} \theta_i. \tag{20}$$

The value of m^* can be obtained by (19) and (20).

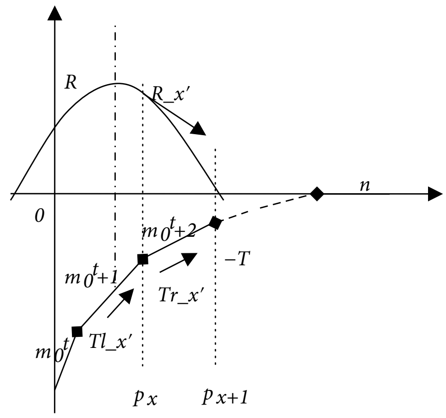

FIGURE 5: The revenue function and the tardiness function.

From Figure 4, we can see that when $t = t^*$, the profit reaches the maximum value. t^* is the completion time of the order in the $(m^*)_{\text{th}}$ sequence, so the optimal delivery time satisfies:

$$t^* = \sum_{i=1}^{m^*} (\alpha_i - p \cdot \beta_i - t^* \cdot \theta_i). \tag{21}$$

Then, t^* is calculated as:

$$t^* = \frac{\sum_{i=1}^{m^*}\alpha_i - p\sum_{i=1}^{m^*}\beta_i}{1 + \sum_{i=1}^{m^*}\theta_i}. \tag{22}$$

Since the obtaining of the optimal production sequence is based on the special time zone $(t_c \le t \le t_{c+1})$, we should verify that t^* is located in each specific time zone. As a result, we need to compare the values of t^* at the boundary of the time zone. If t^* is in the time zone, the optimal delivery time equals t^*. If t^* is outside the time zone $(t^* < t_c)$, the optimal delivery time is t_c or t_{c+1}; that is:

$$t = \begin{cases} t_{c+1}, & t_{c+1} < \dfrac{\sum_{i=1}^{m^*}\alpha_i - p\sum_{i=1}^{m^*}\beta_i}{1 + \sum_{i=1}^{m^*}\theta_i}, \\ \dfrac{\sum_{i=1}^{m^*}\alpha_i - p\sum_{i=1}^{m^*}\beta_i}{1 + \sum_{i=1}^{m^*}\theta_i}, & t_c \le \dfrac{\sum_{i=1}^{m^*}\alpha_i - p\sum_{i=1}^{m^*}\beta_i}{1 + \sum_{i=1}^{m^*}\theta_i} \le t_{c+1}, \\ t_c, & t_c < \dfrac{\sum_{i=1}^{m^*}\alpha_i - p\sum_{i=1}^{m^*}\beta_i}{1 + \sum_{i=1}^{m^*}\theta_i}. \end{cases} \tag{23}$$

The interpretation of Equation (23) is the same as Corollary 1.

3.3. Fixed-Quote Pricing Decision. This section will examine the effect of adjusting the price on optimal profit when the delivery deadline is given. By a method similar to Theorem 1, we can find the corresponding price region $p_c \le p < p_{c+1}$, so that we can get the optimal ordering for each order price area.

Similar to the previous section, we decompose the problem into several subproblems based on the number of price regions, dividing the objective function into two parts: the income function and the delay penalty function. The derivative of the income function is:

$$\frac{\partial R}{\partial p} = -2p\sum_{i=1}^{n}\beta_i + \sum_{i=1}^{n}(\alpha_i - t\theta_i). \tag{24}$$

When the assumption of m does not depend on the price p, the derivative of the price delay penalty function is:

$$\frac{\partial T}{\partial p} = \begin{cases} -w\displaystyle\sum_{i=m+1}^{n}\beta & m = [m_0^t, n-1], \\ 0, & m = n. \end{cases} \tag{25}$$

Theorem 2. *Negative delay penalty $(-T)$ is a piecewise linear function of price p. The function has the following properties:*

(i) *It is a nondecreasing continuous function of the price p.*
(ii) *The absolute gradient of each segment is decreasing relative to the price p.*
(iii) *When the price p is large enough to reach a certain value, the function gradient is 0.*

Corollary 2. *Obtain the optimal price through 3 possible conditions.*

(i) *In the case of optimal prices, the completion time of an order is the same as the delivery time.*
(ii) *The best price is located on the border of the price area.*
(iii) *The optimal price is located in the derivative of the income function and is equal to the gradient of the penalty function.*

In Figure 5, the penalty function $(-T)$ is located below the p-axis, and each small segment on $-T$ indicates that the order was completed on time in the price state at this time. Assuming that when $p = 0$, the number of orders completed in time at a given time t is m_0^t, and then, the connection point between each two-stage function on $-T$ represents the time of completion of an order at a given time t. When the m orders are completed, the time of completion is satisfied:

$$t = \sum_{i=1}^{m}(\alpha_i - p \cdot \beta_i - t \cdot \theta_i). \tag{26}$$

Then,

$$p = \frac{m\alpha_i - \left(1 + \sum_{i=1}^{m}\theta_i\right)t}{\sum_{i=1}^{m}\beta_i}. \tag{27}$$

As shown in Figure 5, as the p increases, the order processing time decreases gradually until the delay penalty is reduced to 0 when p increases to p_n^t, i.e., when $p \ge p_n^t$, $T = 0$.

From (24), we can know that the income function is represented by a concave quadratic function. In the function image of the upper half of the p-axis in Figure 5, the maximum point of the profit function is obtained when $\partial R/\partial p = 0$. We can get the maximum point of the income function, which is the highest point in Figure 6:

$$p_0^t = \frac{\sum_{i=1}^{n}\alpha_i - t\sum_{i=1}^{n}\theta_i}{2\sum_{i=1}^{n}\beta_i}. \tag{28}$$

Now, we discuss the interval. If $p_n^t \le p_0^t$, when $p \ge p_n^t$, since the delay penalty is 0, the maximum profit price is $p^* = p_0^t$.

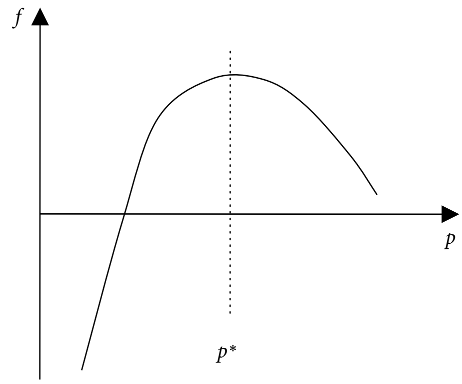

FIGURE 6: The net profit function.

When $p_n^t > p_0^t$, the situation is more complicated, as discussed in detail below.

In Figure 5, when $p = p_0^t$, the derivative of the income function is 0, while the derivative of $-T$ is positive. When $p > p_0^t$, the derivative of R decreases continuously from 0 to $-\infty$ as p increases. At the same time, $-T$ decreases from a positive value to zero. Therefore, when the derivative of $-T$ is added to the derivative of R, the optimal solution of the objective function is obtained, while $\partial f/\partial p$ reaches the minimum nonnegative value.

If $\partial f/\partial p = 0$, the best quote is:

$$p = \frac{\sum_{i=1}^{n}(\alpha_i - t\theta_i) + w\sum_{i=m+1}^{n}\beta_i}{2\sum_{i=1}^{n}\beta_i}. \tag{29}$$

If $\partial f/\partial p \neq 0$, (29) does not apply to calculate the optimal solution. Based on Figure 5, we have developed a search algorithm to find the best price (optimal price searching algorithm, OPS algorithm). The main idea of this algorithm is to find the value of p when $(\partial R/\partial p - \partial T/\partial p)$ reaches the minimum nonnegative value.

According to the OPS algorithm, since the iteration number of the algorithm only depends on the size range of x, we find that the computational complexity of this algorithm is $O(n)$, so the solution of this problem is established on the entire price axis. However, the entire price axis is divided into price ranges. We assume that the price range is $p_c \leq p^* \leq p_{c+1}$. Then, we need to compare the size of the boundary between p^* and the price region. The specific section is as follows:

$$p = \begin{cases} p_{c+1}, & p_{c+1} < p^*; \\ p^*, & p_c \leq p^* \leq p_{c+1}; \\ p_c, & p_c < p^*, \end{cases} \tag{30}$$

where

$$p^* = \begin{cases} \dfrac{\sum_{i=1}^{m}\alpha_i - t\sum_{i=1}^{m}\theta_i}{2\sum_{i=1}^{m}\beta_i}, & \dfrac{\partial R}{\partial p} \neq \dfrac{\partial T}{\partial p}; \\ \dfrac{\sum_{i=1}^{m}\alpha_i - t + w\sum_{i=1}^{m}\beta_i}{\sum_{i=1}^{m}\beta_i}, & \dfrac{\partial R}{\partial p} = \dfrac{\partial T}{\partial p}. \end{cases} \tag{31}$$

3.4. Simultaneous Decision of Quotation and Delivery Time. From Section 3.1, we know that the (p, t) price-time region can be fixed by the SPT production order, and all price-time regions of the corresponding production order can be obtained by the SLA. The solution to the general problem can be solved in each price-time zone, and time series decision-making is not important. Therefore, the entire problem is divided into two simple questions based on the number of price-time regions.

According to the SLA, the price-time region is a concave region consisting of two straight-line boundaries, such as the EF and P_1T_1 lines in Figure 3(b). The mathematical plan for the price-time region has been given in Equation (13). When we study the general time-price region problem, the constraint (13) must be satisfied.

Theorem 3. *The obtaining of the optimal solution requires that any one of the following two conditions be satisfied:*

(i) *The delivery time is consistent with the order completion time.*

(ii) *(p^*, t^*) is the sequence change point, which means that (p^*, t^*) allows both the processing time of two orders to equal zero and the processing time of one order equal to zero.*

Next, we will discuss how the two forms in Theorem 3 can get the optimal solution. First, suppose the profit is maximized when the delivery time of one order is the same as the completion time, and the decision variable (p, t) satisfies (27), where m is an integer in $[1, n]$, each possible m value in (27) is computable and replaceable, and a straight line $L_m(p = (\sum_{i=1}^{m}\alpha_i - (1 + \sum_{i=1}^{m}\theta_i)t)/\sum_{i=1}^{m}\beta_i)$is formed in the $p - t$ Cartesian coordinate system. If there are two intersections between the straight line L_m and the price-time zone boundary, then the value of m is valid and stored in an M concentration, $M = m, m+1, \ldots, m+c$, where c is an integer. Then, in order to find the optimal profit, every element in M is computable.

Bringing formula (27) into formula (13), all price-time zone constraints can be reduced to only constraints on time t; that is, all the inequalities in (13) will become the form of $t_{l1} \leq t \leq t_{r1}$, where t_{l1} and t_{r1} are integrated by the cluster of inequalities.

Bringing (27) into the objective function (4), the objective function will become a quadratic function only with respect to time t; that is:

$$f(t) = \frac{\sum_{i=1}^{m}\alpha_i - (1 + \sum_{i=1}^{m}\theta_i)t}{\sum_{i=1}^{m}\beta_i}\sum_{i=1}^{n}Q_i(t) - w\sum_{i=m+1}^{n}(C_i(t) - t). \tag{32}$$

By comparing the sizes of $f(t_{l1})$, $f(t_{r1})$, and $f(t|(\partial f/\partial t) = 0)$, the maximum profit is filtered out and the problem is directly solved. We find a solution to each of M's candidates and get the maximum profit at the same time.

Next, we discuss the second condition of Theorem 3 and find the optimal solution for (p^*, t^*) at the point of sequence change. As discussed in Section 3.1, the sequence change point for a time-price region is the boundary point of the $p - t$ Cartesian coordinate system. Each boundary value can be expressed by Equation (14). In general, the boundary of a special time-price region can be represented by $c_{j1} \cdot p + c_{j2} \cdot t = c_{j3}$, where $j = 1, 2, \ldots, s$, so the decision variable p is a function of the decision variable t; that is:

$$p = \frac{\left(c_{j3} - c_{j2} \cdot t\right)}{c_{j1}}. \tag{33}$$

Bringing (32) to the objective function (4), the objective function becomes a quadratic function with respect to the delivery time t:

$$f(t) = \left(\frac{c_{j3} - c_{j2} \cdot t}{c_{j1}}\right)\sum_{i=1}^{n}\left(\alpha_i - \frac{\left(c_{j3} - c_{j2} \cdot t\right)\beta_i}{c_{j1}} - t\theta_i\right) - w\sum_{i=1}^{n}\max\left(0, C_i - t\right). \tag{34}$$

Bringing (33) to (14), all constraints on the price-time region become constraints only on time t; that is, all inequalities in (13) become $t_{l2} \le t \le t_{r2}$, where t is determined by the family of inequality sets. As a result, this problem can be solved by directly solving $f(t_{l2})$, $f(t_{r2})$, and $f(t\partial f/\partial t = 0)$, and filtering out the maximum profit. We find a solution to the boundary of the price region at each time and filter out one of the largest profits.

The optimal questions about (31) and (33) can be resolved in a clear time-price region. By solving (31) and (33), a detailed price-time region can be filtered out to obtain the maximum profit and optimal solution. For general problems, in order to find the optimal delivery time, we developed a price-time problem algorithm (PTA). The main idea of the PTA is: first, form the SPT sequences for all price-time regions; secondly, based on the formulae (31) and (33), we find the optimal solution for each time-price region; finally, the optimal solution is filtered out of the maximum values in all time-price regions obtained.

Theorem 4. *In the case of $O(n^3)$, for general problems, the algorithm PTA can obtain the optimal solution (p^*, t^*).*

4. Conclusion

In the manufacturer's delivery pricing problem for retailers, the retailer's demand is a determinant function of the commodity price, delivery time, and other attribute variables. In order to maximize production profits, manufacturers publish a common optimal product price and a common delivery time for each customer group. Typically, the manufacturer formulates the relevant production policy through a stable queuing model, which uses a fixed scheduling rule, such as FCFS, which is considered as the M/M/1 queue.

Delivery pricing problems based on the M/M/1 scheduling algorithm apply to quotes for mass production for mass customers. For a single customer group, the pricing, and production problems in the context of small batch production cannot be maximized by the manufacturer's profits through the M/M/1 scheduling algorithm. In a single group of customers, the customer's payment preference and delay tolerance are different within a known range of price and time sensitivity. Based on the optimization of production sequencing, we mainly study the coordination and optimization between a manufacturer and multiple retailers in a supply chain.

Under the scenario of order-based production, we studied the issue of coordination pricing and delivery time quotation taking into account the detailed production scheduling decision mechanism and then formulated a scheduling mechanism that deals with delivery dates, including a price. The mechanism takes into account the customer's heterogeneity of time and price requirements and also includes penalties for late delivery to the manufacturer. In order to maximize the manufacturer's own profit, we constructed a mathematical model of the best quotation and delivery deadline and created a price-time problem algorithm (PTA) to obtain the optimal solution of the model. We proposed a solution to the problem of joint pricing scheduling faced by manufacturers.

Data Availability

The article is about the research of model methods. No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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