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Research Article

Stability Analysis of Systems with Interval Time-Varying Delays via a New Integral Inequality

Zerong Ren and Jun-kang Tian

School of Mathematics, Zunyi Normal College, Zunyi 563002, China

Correspondence should be addressed to Jun-kang Tian; zunytian@163.com

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This paper focuses on delay-dependent stability analysis for systems with interval time-varying delays. Based on a new integral inequality and a generalized reciprocally convex combination matrix inequality, a new delay-dependent stability criterion is obtained in terms of a linear matrix inequality (LMI). Finally, the merits of the proposed criterion are shown by two numerical examples.

1. Introduction

Consider the following systems with interval time-varying delays:

$$\dot{x}(t) = Ax(t) + Bx(t - h(t)), \tag{1}$$

$$x(t) = \phi(t), \quad t \in [-h_2, 0], \tag{2}$$

where $x(t) \in \mathscr{R}^n$ is the state vector and $A, B, \in \mathscr{R}^{n\times n}$ are constant matrices. The time-varying delay $h(t)$ is continuous and satisfies

$$0 \le h_1 \le h(t) \le h_2, \quad h_{12} \triangleq h_2 - h_1. \tag{3}$$

Over the past decades, providing less conservative stability conditions for linear systems with time-varying delays has attracted considerable attention. The difficulty relies on the handling of the integral terms arising in the derivative of the LKF. The free-weighting-matrix approach [1, 2] was applied to handle the integral terms in the early literature. In recent years, estimating integral terms directly via integral inequalities gradually becomes more popular. Various integral inequalities have been proposed, such as Jensen inequality [3–5], Wirtinger-based inequality [6–8], auxiliary function based inequalities [9], free-matrix-based inequalities [10], and relaxed integral inequalities [11]. Very recently, Bessel–Legendre inequality [12] is proposed to handle the stability of system (1), and some less conservative criteria are obtained. However, the relationship between $\int_a^b (s - (a + b/2))^k x(s)\mathrm{d}s$ and $\int_a^b x(s)\mathrm{d}s$, $\int_a^b \int_{u_1}^b x(s)\mathrm{d}s\,\mathrm{d}u_1$, $\ldots$, $\int_a^b \int_{u_1}^b \ldots \int_{u_k}^b x(s)\mathrm{d}s\,\mathrm{d}u_k \ldots \mathrm{d}u_1$ was not considered in [12], which may yield conservative results. Then, a new integral inequality was proposed in [13] to consider the relationship fully. But the integral inequality was only used to handle the constant time delay. A new integral inequality for dealing with delays is introduced in [14]. A less conservative stability criterion for linear systems with a time-varying delay is proposed by using the new integral inequality. However, there are two aspects which need to be improved. (1) When estimating the derivative of $V(x_t)$, the term $\int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s$ is only estimated as $\int_a^b \dot{x}^T(s) R\dot{x}(s)\mathrm{d}s \ge (1/(b-a))\Omega_1^T R\Omega_1 + (3/b-a)\Omega_2^T R\Omega_2 + (5/b-a) \Omega_3^T R\Omega_3$, which may yield conservative results. (2) The assumption on the derivative of time-varying delay $h(t)$ is included in the stability criterion [14]. Thus, there is still some room for further investigation.

In this paper, a new delay-dependent stability criterion for linear systems with interval time-varying delays is developed by using a new integral inequality and a generalized reciprocally convex combination matrix inequality. The

highlights of our paper are as follows. (1) The features of the new integral inequality in [13] are fully integrated into the construction of the LKF. (2) A less conservative stability criterion is proposed in terms of an LMI without the assumption on the derivative of time-varying delay $h(t)$. (3) Upper bound of h_2 in our paper is quite close to the analytical bound. The advantage of the proposed criterion has been illustrated via two numerical examples.

Throughout the paper, the set $\mathbb{S}_+^n$ denotes the set of $n \times n$ symmetric positive definite matrices and the set $\mathbb{S}^n$ denotes the set of $n \times n$ symmetric matrices. For any square matrix A, define $He(A) = A + A^T$.

2. Main Result

Based on the following lemmas, a less conservative stability criterion for systems with interval time-varying delays is established.

Lemma 1 (see [8]). For any matrices $Q \in \mathbb{S}_+^n$, $N_1, N_2 \in \mathscr{R}^{m\times n}$, $\Gamma \in \mathscr{R}^{2n\times m}$, $\forall \alpha \in (0,1)$, the following inequality holds:

$$-\Gamma^T \begin{bmatrix} \frac{1}{\alpha}Q & 0 \\ 0 & \frac{1}{1-\alpha}Q \end{bmatrix} \Gamma \leq -\Gamma^T \mathscr{R}(\alpha)\Gamma - He\left(\Gamma^T \begin{bmatrix} (1-\alpha)N_1^T \\ \alpha N_2^T \end{bmatrix}\right) + \alpha N_1 Q^{-1} N_1^T + (1-\alpha) N_2 Q^{-1} N_2^T, \tag{4}$$

where

$$\mathscr{R}(\alpha) = \begin{bmatrix} (2-\alpha)Q & 0 \\ 0 & (1+\alpha)Q \end{bmatrix}. \tag{5}$$

Lemma 2 (see [8]). Consider a parameter dependent matrix $\Phi(\alpha) \in \mathbb{S}^m$, such that the convex inequality

$$\Phi(\alpha) \leq (1-\alpha)\Phi(0) + \alpha\Phi(1), \tag{6}$$

holds for all $\alpha \in [0,1]$. If there exist a matrix $R \in \mathbb{S}_+^n$ and two matrices $N_1, N_2 \in \mathscr{R}^{m\times n}$, such that the inequality

$$\Psi(\alpha) = \begin{bmatrix} \Phi(\alpha) - \Gamma^T\mathscr{R}(\alpha)\Gamma - He\left(\Gamma^T \begin{bmatrix} (1-\alpha)N_1^T \\ \alpha N_2^T \end{bmatrix}\right) & * \\ \alpha N_1^T + (1-\alpha)N_2^T & -R \end{bmatrix} < 0, \tag{7}$$

holds for $\alpha = \{0,1\}$, then the following inequality holds:

$$\Phi(\alpha) - \Gamma^T \begin{bmatrix} \frac{1}{\alpha}R & 0 \\ 0 & \frac{1}{1-\alpha}R \end{bmatrix} \Gamma < 0, \quad \forall \alpha \in (0,1). \tag{8}$$

Lemma 3 (see [13]). For a matrix $R \in \mathbb{S}_+^n$ and any continuously differentiable function x: $[a,b] \longrightarrow \mathscr{R}^n$, the following inequality holds:

$$\begin{aligned} \int_a^b \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \geq{}& \frac{1}{b-a}\Omega_1^T R\Omega_1 + \frac{3}{b-a}\Omega_2^T R\Omega_2 \\ &+ \frac{5}{b-a}\Omega_3^T R\Omega_3 + \frac{7}{b-a}\Omega_4^T R\Omega_4, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \Omega_1 &= x(b) - x(a), \\ \Omega_2 &= x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)\mathrm{d}s, \\ \Omega_3 &= x(b) - x(a) + \frac{6}{b-a}\int_a^b x(s)\mathrm{d}s \\ &\quad - \frac{12}{(b-a)^2}\int_a^b\int_u^b x(s)\mathrm{d}s\,\mathrm{d}u, \\ \Omega_4 &= x(b) + x(a) - \frac{12}{b-a}\int_a^b x(s)\mathrm{d}s \\ &\quad + \frac{60}{(b-a)^2}\int_a^b\int_u^b x(s)\mathrm{d}s\,\mathrm{d}u \\ &\quad - \frac{120}{(b-a)^3}\int_a^b\int_u^b\int_v^b x(s)\mathrm{d}s\,\mathrm{d}v\,\mathrm{d}u. \end{aligned} \tag{10}$$

Theorem 1. *For given scalars h_1 and h_2, system (1) with time-varying delays satisfying (2) is asymptotically stable if there exist matrices $P \in \mathbb{S}_+^{5n}$, $Q_1, Q_2, Q_3, Q_4 \in \mathbb{S}_+^n$, $N_1, N_2 \in \mathscr{R}^{13n\times 4n}$, such that the LMI*

$$\Psi(\alpha) = \begin{bmatrix} \Phi(\alpha) - \Gamma^T\mathscr{R}(\alpha)\Gamma - He\left(\Gamma^T \begin{bmatrix} (1-\alpha)N_1^T \\ \alpha N_2^T \end{bmatrix}\right) & * \\ \alpha N_1^T + (1-\alpha)N_2^T & -Q \end{bmatrix} < 0, \tag{11}$$

holds for $\alpha = \{0,1\}$, where

$$\Phi(\alpha) = He\left(\Sigma_1^T P \Sigma_2\right) + \varepsilon_1^T Q_1 \varepsilon_1 - \varepsilon_2^T Q_1 \varepsilon_2 + \varepsilon_2^T Q_2 \varepsilon_2 - \varepsilon_4^T Q_2 \varepsilon_4 + h_1^2 \varepsilon_0^T Q_3 \varepsilon_0 + h_{12}^2 \varepsilon_0^T Q_4 \varepsilon_0 - \Sigma_3^T Q_3 \Sigma_3 - 3\Sigma_4^T Q_3 \Sigma_4 - 5\Sigma_5^T Q_3 \Sigma_5 - 7\Sigma_6^T Q_3 \Sigma_6,$$

$$\Sigma_1 = \left[\ \varepsilon_1^T \quad h_1 \varepsilon_5^T \quad \alpha h_{12} \varepsilon_6^T + (1-\alpha) h_{12} \varepsilon_7^T \quad h_1^2 \varepsilon_8^T \quad h_1^3 \varepsilon_{11}^T\ \right]^T,$$

$$\Sigma_2 = \left[\ \varepsilon_0^T \quad \varepsilon_1^T - \varepsilon_2^T \quad \varepsilon_2^T - \varepsilon_4^T \quad h_1 \varepsilon_1^T - h_1 \varepsilon_5^T \quad \tfrac{h_1^2}{2} \varepsilon_1^T - h_1^2 \varepsilon_8^T\ \right]^T,$$

$$\Sigma_3 = \varepsilon_1 - \varepsilon_2,$$

$$\Sigma_4 = \varepsilon_1 + \varepsilon_2 - 2\varepsilon_5,$$

$$\Sigma_5 = \varepsilon_1 - \varepsilon_2 + 6\varepsilon_5 - 12\varepsilon_8,$$

$$\Sigma_6 = \varepsilon_1 + \varepsilon_2 - 12\varepsilon_5 + 60\varepsilon_8 - 120\varepsilon_{11},$$

$$\Sigma_7 = \varepsilon_2 - \varepsilon_3,$$

$$\Sigma_8 = \varepsilon_2 + \varepsilon_3 - 2\varepsilon_6,$$

$$\Sigma_9 = \varepsilon_2 - \varepsilon_3 + 6\varepsilon_6 - 12\varepsilon_9,$$

$$\Sigma_{10} = \varepsilon_2 + \varepsilon_3 - 12\varepsilon_6 + 60\varepsilon_9 - 120\varepsilon_{12},$$

$$\Sigma_{11} = \varepsilon_3 - \varepsilon_4,$$

$$\Sigma_{12} = \varepsilon_3 + \varepsilon_4 - 2\varepsilon_7,$$

$$\Sigma_{13} = \varepsilon_3 - \varepsilon_4 + 6\varepsilon_7 - 12\varepsilon_{10},$$

$$\Sigma_{14} = \varepsilon_3 + \varepsilon_4 - 12\varepsilon_7 + 60\varepsilon_{10} - 120\varepsilon_{13},$$

$$\varepsilon_0 = A\varepsilon_1 + B\varepsilon_3,$$

$$\Gamma = \left[\ \Sigma_7^T \quad \Sigma_8^T \quad \Sigma_9^T \quad \Sigma_{10}^T \quad \Sigma_{11}^T \quad \Sigma_{12}^T \quad \Sigma_{13}^T \quad \Sigma_{14}^T\ \right]^T,$$

$$Q = \operatorname{diag}\left(Q_4, 3Q_4, 5Q_4, 7Q_4\right), \tag{12}$$

and $\varepsilon_i \in \mathscr{R}^{n\times 13n}$ *is defined as* $\varepsilon_i = \left[\ 0_{n\times(i-1)n} \quad I_n \quad 0_{n\times(13-i)n}\ \right]$ *for* $i = 1, 2, \ldots, 13$.

Proof. Consider a LKF candidate given by

$$V(x_t) = \eta^T(t) P \eta(t) + \int_{t-h_1}^{t} x^T(s) Q_1 x(s) \mathrm{d}s + \int_{t-h_2}^{t-h_1} x^T(s) Q_2 x(s) \mathrm{d}s + h_1 \int_{t-h_1}^{t} \int_{u}^{t} \dot{x}^T(s) Q_3 \dot{x}(s) \mathrm{d}s\, \mathrm{d}u + h_{12} \int_{t-h_2}^{t-h_1} \int_{u}^{t} \dot{x}^T(s) Q_4 \dot{x}(s) \mathrm{d}s\, \mathrm{d}u, \tag{13}$$

where

$$\eta(t)=\left[\begin{array}{ccccc} x^T(t) & \int_{t-h_2}^{t} x^T(s)\mathrm{d}s & \int_{t-h_2}^{t-h_1} x^T(s)\mathrm{d}s & \int_{t-h_1}^{t}\int_{u}^{t} x^T(s)\mathrm{d}s\,\mathrm{d}u & \int_{t-h_1}^{t}\int_{u}^{t}\int_{s}^{t} x^T(r)\mathrm{d}r\,\mathrm{d}s\,\mathrm{d}u \end{array}\right]^T. \tag{14}$$

Calculate the derivative of $V(x_t)$ along the solution of system (1) as follows:

$$\begin{aligned}\dot{V}(x_t)&=2\eta^T(t)P\dot{\eta}(t)+x^T(t)Q_1x(t)-x^T(t-h_1)Q_1x(t-h_1)\\&\quad+x^T(t-h_1)Q_2x(t-h_1)-x^T(t-h_2)Q_2x(t-h_2)+h_1^2\dot{x}^T(t)Q_3\dot{x}(t)\\&\quad+h_{12}^2\dot{x}^T(t)Q_4\dot{x}(t)-h_1\int_{t-h_1}^{t}\dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)Q_4\dot{x}(s)\mathrm{d}s.\end{aligned} \tag{15}$$

Then, it can be rewritten as

$$\begin{aligned}\dot{V}(x)_t&=\xi^T(t)\left\{He\left(\Sigma_1^TP\Sigma_2\right)+\varepsilon_1^TQ_1\varepsilon_1-\varepsilon_2^TQ_1\varepsilon_2+\varepsilon_2^TQ_2\varepsilon_2-\varepsilon_4^TQ_2\varepsilon_4+h_1^2\varepsilon_0^TQ_3\varepsilon_0+h_{12}^2\varepsilon_0^TQ_4\varepsilon_0\right\}\xi(t)\\&\quad-h_1\int_{t-h_1}^{t}\dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s-h_{12}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)Q_4\dot{x}(s)\mathrm{d}s,\end{aligned} \tag{16}$$

where

$$\begin{aligned}&\xi(t)=\left[\begin{array}{cccccc} x^T(t) & x^T(t-h_1) & x^T(t-h(t)) & x^T(t-h_2) & \varphi_1^T(t) & \varphi_2^T(t) \quad \varphi_3^T(t)\end{array}\right]^T,\\&\varphi_1(t)=\left[\begin{array}{ccc}\frac{1}{h_1}\int_{t-h_1}^{t}x^T(s)\mathrm{d}s & \frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1}x^T(s)\mathrm{d}s & \frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)}x^T(s)\mathrm{d}s\end{array}\right]^T,\\&\varphi_2(t)=\left[\begin{array}{ccc}\frac{1}{h_1^2}\int_{t-h_1}^{t}\int_{u}^{t}x^T(s)\mathrm{d}s\,\mathrm{d}u & \frac{1}{(h(t)-h_1)^2}\int_{t-h(t)}^{t-h_1}\int_{u}^{t-h_1}x^T(s)\mathrm{d}s\,\mathrm{d}u & \frac{1}{(h_2-h(t))^2}\int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)}x^T(s)\mathrm{d}s\end{array}\right]^T,\\&\varphi_3(t)=\left[\begin{array}{ccc}\frac{1}{h_1^3}\int_{t-h_1}^{t}\int_{u}^{t}\int_{v}^{t}x^T(s)\mathrm{d}s\,\mathrm{d}v\,\mathrm{d}u & \frac{1}{(h(t)-h_1)^3}\int_{t-h(t)}^{t-h_1}\int_{u}^{t-h_1}\int_{v}^{t-h_1}x^T(s)\mathrm{d}s\,\mathrm{d}v\,\mathrm{d}u & \frac{1}{(h_2-h(t))^3}\int_{t-h_2}^{t-h(t)}\int_{u}^{t-h(t)}\int_{v}^{t-h(t)}x^T(s)\mathrm{d}s\,\mathrm{d}v\,\mathrm{d}u\end{array}\right]^T.\end{aligned} \tag{17}$$

Let $\alpha = (h(t) - h_1/h_{12})$, and applying Lemma 3 yields

$$-h_1 \int_{t-h_1}^{t} \dot{x}^T(s) Q_3 \dot{x}(s) \mathrm{d}s \le \xi^T(t)\left(-\Sigma_3^T Q_3 \Sigma_3 - 3\Sigma_4^T Q_3 \Sigma_4 - 5\Sigma_5^T Q_3 \Sigma_5 - 7\Sigma_6^T Q_3 \Sigma_6\right)\xi(t), \tag{18}$$

$$\begin{aligned}
&-h_{12} \int_{t-h_2}^{t-h_1} \dot{x}^T(s) Q_4 \dot{x}(s) \mathrm{d}s \\
&\quad = -h_{12} \int_{t-h(t)}^{t-h_1} \dot{x}^T(s) Q_4 \dot{x}(s) \mathrm{d}s - h_{12} \int_{t-h_2}^{t-h(t)} \dot{x}^T(s) Q_4 \dot{x}(s) \mathrm{d}s \\
&\le -\frac{h_{12}}{h(t) - h_1} \xi^T(t)\left(\Sigma_7^T Q_4 \Sigma_7 + 3\Sigma_8^T Q_4 \Sigma_8 + 5\Sigma_9^T Q_4 \Sigma_9 + 7\Sigma_{10}^T Q_4 \Sigma_{10}\right)\xi(t) \\
&\quad - \frac{h_{12}}{h_2 - h(t)} \xi^T(t)\left(\Sigma_{11}^T Q_4 \Sigma_{11} + 3\Sigma_{12}^T Q_4 \Sigma_{12} + 5\Sigma_{13}^T Q_4 \Sigma_{13} + 7\Sigma_{14}^T Q_4 \Sigma_{14}\right)\xi(t) \\
&= -\frac{1}{\alpha} \xi^T(t)\left(\Sigma_7^T Q_4 \Sigma_7 + 3\Sigma_8^T Q_4 \Sigma_8 + 5\Sigma_9^T Q_4 \Sigma_9 + 7\Sigma_{10}^T Q_4 \Sigma_{10}\right)\xi(t) \\
&\quad - \frac{1}{1-\alpha} \xi^T(t)\left(\Sigma_{11}^T Q_4 \Sigma_{11} + 3\Sigma_{12}^T Q_4 \Sigma_{12} + 5\Sigma_{13}^T Q_4 \Sigma_{13} + 7\Sigma_{14}^T Q_4 \Sigma_{14}\right)\xi(t) \\
&= -\xi^T(t) \Gamma^T \begin{bmatrix} \frac{1}{\alpha} Q & 0 \\ 0 & \frac{1}{1-\alpha} Q \end{bmatrix} \Gamma \xi(t).
\end{aligned} \tag{19}$$

Let $\alpha = (h(t) - h_1/h_{12})$, then $1 - \alpha = (h_2 - h(t)/h_{12})$, for any matrices $N_1, N_2 \in \mathscr{R}^{13n \times 4n}$, and applying Lemma 1 yields

$$\begin{aligned}
&-\Gamma^T \begin{bmatrix} \frac{1}{\alpha} Q & 0 \\ 0 & \frac{1}{1-\alpha} Q \end{bmatrix} \Gamma \le -\Gamma^T \mathscr{R}(\alpha) \Gamma - He\left(\Gamma^T \begin{bmatrix} (1-\alpha) N_1^T \\ \alpha N_2^T \end{bmatrix}\right) \\
&\quad + \alpha N_1 Q^{-1} N_1^T + (1-\alpha) N_2 Q^{-1} N_2^T = \Xi(\alpha),
\end{aligned} \tag{20}$$

where

$$\mathscr{R}(\alpha) = \begin{bmatrix} (2-\alpha) Q & 0 \\ 0 & (1+\alpha) Q \end{bmatrix}. \tag{21}$$

From (16)–(20), the $\dot{V}(x_t)$ can be estimated as

$$\dot{V}(x_t) \le \xi^T(t)(\Phi(\alpha) + \Xi(\alpha))\xi(t). \tag{22}$$

According to Lemma 2, if LMI (11) is verified for $\alpha = \{0, 1\}$, then the inequality $\Phi(\alpha) + \Xi(\alpha) < 0$ holds for all $\alpha \in (0, 1)$. This completes the proof. □

Remark 1. The integral inequality in Lemma 3 is proposed to derive less conservative results because of the term of a triple integral being included. In order to fully consider the features of the integral inequality in Lemma 3, the LKF of our paper includes the terms of a triple integral $\int_{t-h_1}^{t} \int_{u}^{t} \int_{s}^{t} x^T(r) \mathrm{d}r \, \mathrm{d}s \, \mathrm{d}u$, which may yield less conservative results.

3. Numerical Examples

In this section, two numerical examples are introduced to illustrate the advantage of the proposed criterion.

Example 1. Consider system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{23}$$

Table 1 presents the maximum upper bounds of h_2 with respect to different h_1 calculated by Theorem 1 and several criteria from [4, 6, 9, 10, 12]. Table 1 shows that our method is more effective than those in [4, 6, 9, 10, 12]. It should be pointed out that our result is quite close to the analytical bound. For $h_2 = 3.45$, initial state $(-0.02, 0.02)^T$, the simulation of the state trajectories of system (1) is given in Figure 1.

Example 2. Consider system (1) with

$$A = \begin{bmatrix} 0.0 & 1.0 \\ -10.0 & -1.0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}. \tag{24}$$

Table 2 presents the maximum upper bounds of h_2 with respect to different h_1 calculated by Theorem 1 and several criteria from [6, 9, 10, 12]. Table 2 shows that our method is

Table 1: Upper bound of h_2 for different h_1.

h_1	0.0	0.4	0.7	1.0	$h_1 = h_2$
Theorem 2 [4]	1.86	1.88	1.95	2.06	4.47
Theorem 7 [6]	2.11	2.17	2.23	2.31	6.05
Theorem 1 [9]	2.14	2.19	2.24	2.31	6.1641
Theorem 1 [10]	2.18	2.21	2.25	2.32	6.05
Theorem 2 [12]	2.26	2.29	2.34	2.40	6.1664
Theorem 1	2.92	3.27	3.38	3.45	6.1686
Analytical bound	—	—	—	—	6.1725

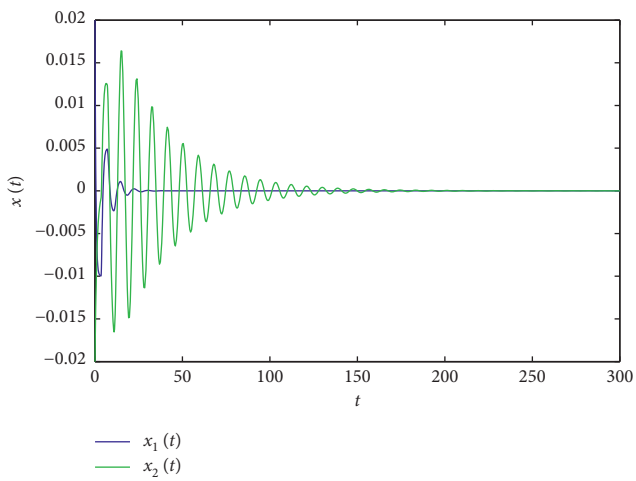

Figure 1: The state trajectories of system (1) for Example 1.

Table 2: Upper bound of h_2 for different h_1.

h_1	0.0	0.3	0.7	1.0	2.0
Theorem 7 [6]	1.59	2.01	2.41	2.62	3.59
Theorem 1 [9]	1.64	2.13	2.70	2.96	3.63
Theorem 1 [10]	1.80	2.19	2.58	2.79	3.68
Theorem 2 [12]	2.39	2.76	3.15	3.41	4.04
Theorem 1	2.76	3.13	3.48	3.53	4.16

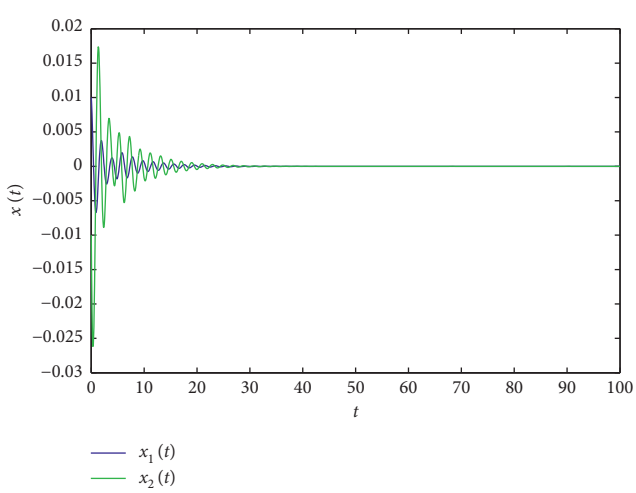

Figure 2: The state trajectories of system (1) for Example 2.

more effective than those in [6, 9, 10, 12]. For $h_2 = 4.16$, initial state $(-0.01, 0.01)^T$, the simulation of the state trajectories of system (1) is given in Figure 2.

4. Conclusions

An improved stability criterion of systems with interval time-varying delays has been proposed in our paper. The features of the new integral inequality are fully integrated into the construction of the LKF. Finally, the merits of the proposed criterion are shown by two numerical examples.

Data Availability

No additional data are available for this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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