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Research Article

A Bertrand Duopoly Game with Long-Memory Effects

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Reconsidering the Bertrand duopoly game based on the concept of long short-term memory, we construct a fractional-order Bertrand duopoly game by extending the integer-order game to its corresponding fractional-order form. We build such a Bertrand duopoly game, in which both players can make their decisions with long-memory effects. Then, we investigate its Nash equilibria, local stability, and numerical solutions. Using the bifurcation diagram, the phase portrait, time series, and the 0-1 test for chaos, we numerically validate these results and illustrate its complex phenomena, such as bifurcation and chaos.

1. Introduction

As we know, the game theory is one of hottest academic topics for a long time. There are many types of games, such as the Cournot game [1–7], the Bertrand game [8–14], the Stackelberg game [15–17], the evolutionary game [18], and the mixed game [19]. Many researchers have studied duopoly games with some interesting characteristics, such as bounded rationality [1, 15–17], technology licensing [3, 20], and R&D [6, 13]. Table 1 shows some recent research studies on duopoly games with integer-order difference equations. Generally speaking, the duopoly Bertrand game presents the optimal pricing strategies for two players. As a kind of pricing game, the Bertrand game is an alternative to the Cournot game. The duopoly Bertrand game describes two firms competitively providing a kind of homogeneous product, setting simultaneously prices rather than quantities, and reaching equilibria with the price equal to the marginal cost. In the industrial economics domain, long- or short-term memory effects are widespread with different forms. Classical dynamical Bertrand games mainly have short-term memory effects. The discrete-time dynamical Bertrand game represented by the integer-order difference equation can reveal the length of memory effect by its order number. For example, the first-order discrete dynamical game can only remember the decision-making behaviors during the previous period and the second-order discrete dynamical game can remember the decision-making behaviors during the previous two periods. Although the integer-order difference equation has a powerful expression ability and can represent its complexity evolution characteristics of the duopoly game, it is not very easy for the integer-order discrete-time dynamical Bertrand games to reflect long-term memory effects. Thus, it is meaningful to study the discrete-time dynamical Bertrand games with fractional-order calculus.

We can employ two forms of fractional-order calculus to represent long-term memory effects. The one is the continuous fractional-order calculus, which mainly corresponds to the integer-order differential calculus. The other is the discrete fractional-order calculus, which mainly corresponds to the integer-order difference calculus. Many scholars have utilized continuous fractional differential operators to represent memory effects in economics. As an extension of the integer-order difference equation, the fractional-order difference equation has the function of long-term memory effects, which can make up the deficiency of the integer-order difference equation. The fractional difference

Table 1: Recent studies on dynamical duopoly games with integer-order difference equations.

Study	Game type	Decision variables	Characteristics	Reference
Ueda; Tu and Wang	Cournot game	Quantity	Bounded rationality	[1, 2]
Hsu, et al.	Cournot game	Quantity	Technology licensing	[3]
Al-khedhairi	Cournot game	Quantity	Differentiated players	[4]
Fanti et al.; Gori and Sodini; Ahmed et al.	Bertrand game	Price	Product differentiation	[8, 12, 14]
Askar and Al-khedhairi	Bertrand game	Price	Limited information	[9]
Ma and Lou	Bertrand game	Price	Integer delay	[11]
Tu et al.	Bertrand game	Price	R&D	[13]
Yang, et al.	Stackelberg game	Quantity	Bounded rationality	[15]
Peng and Lu	Stackelberg game	Quantity	Bounded rationality	[16]
Shi; Le and Sheng	Stackelberg game	Quantity	Bounded rationality	[17]
Ma et al.	Mixed game	Price, quantity	Heterogeneity	[19]
Wu	Differential game	price	Technology licensing	[20]

equation, named the discrete fractional equation, can also be regarded as the discrete form of a fractional differential equation. Without a doubt, the fractional difference calculus also has a vast application space in the dynamical game. Using the discrete fractional calculus, Xin et al. [21] studied a Cournot game and revealed the evolution mechanism of two firms' output decisions. Different from Ref. [21], this article will employ the discrete fractional-order differential calculus to propose a Bertrand game, which can reveal the evolution mechanism of two firms' price decisions.

The remainder of this paper is organized as follows. In Section 2, we propose a discrete fractional-order Bertrand duopoly game with long-term memory effects. In Section 3, we study the game's Nash equilibria and local stability. In Section 4, we numerically verify the results by using bifurcation diagrams, phase portraits, maximal Lyapunov exponents, and the 0-1 test algorithm. This paper concludes in Section 5.

2. Modelling

In this following, we consider a simple Bertrand-type duopoly common market as mentioned in classic economics. Two firms, labelled by $i = 1, 2$, provide homogeneous products with perfect substitutes and decide their different product prices in the light of the same market rule. During period $t \in Z^+$, let $p_i(t)$ and $q_i(t)$ represent the product price and output of firm i, respectively. Following the classical linear inverse demand function, we can obtain the following relationship of q_i and p_i:

$$q_i(t) = 1 - p_i(t) + bp_j(t), \quad i, j = 1, 2, \text{ and } i \neq j, \tag{1}$$

where b is a positive constant and denotes a substitution effect between two products.

Let the marginal costs be linear, i.e., $C_i(t) = cq_i(t), \quad i = 1, 2$, where c is a positive constant and represents the marginal cost. Then, the production profit of firm i is

$$\Pi_i\big(p_i(t), p_j(t)\big) = \big(p_i(t) - c\big)q_i(t), \quad i, j = 1, 2, \text{ and } i \neq j. \tag{2}$$

Thus, the marginal profit of firm i with respect to p_i is

$$\Theta(t) = \frac{\partial \Pi_i\big(p_i(t), p_j(t)\big)}{\partial p_i(t)} = 1 + c - 2p_i(t) + bp_j(t), \quad i, j = 1, 2, \text{ and } i \neq j. \tag{3}$$

In the classical dynamical duopoly game, every firm naively think its rivals' price in period $t+1$ is equal to the same as in period t. Obviously, the kind of game has no long-memory effect. In the following, we will introduce the discrete fractional-order calculus to the classical game; then, both firms make their decisions under a novel dynamical adjustment mechanism with a long memory and local estimation of marginal profit.

Assume that the first firm has no complete knowledge of the market demand function and makes its price decision under a dynamical adjustment mechanism with the bounded rationality and a long-term estimation of marginal profit. If the long-term marginal profit is greater (less) than zero, the firm decides to raise (cut) its product price at next period. Thus, we describe the following adjustment process of the product price:

$$\Delta^{\alpha} p_1(t) = ap_1(t + \alpha - 1)\ \Theta\ (t + \alpha - 1), \tag{4}$$

where $a > 0$ represents the first firm's price adjustment speed and $\alpha \in (0, 1)$ represents fractional-order number, i.e., the long-term memory effect.

Assume that the second firm makes its decision under a dynamical adjustment mechanism with simple rationality and a long-term estimation of marginal profit. That is, its price decision mainly depends on two aspects. The one is its long-term optimal reaction function p_2^*, and the other is its long-term price p_2. Thus, we write the second firm's price adjustment mechanism as follows:

$$\Delta^{\alpha} p_2(t) = p_2^*(t + \alpha - 1) - p_2(t + \alpha - 1). \tag{5}$$

Thus, we write the following discrete-time fractional-order Bertrand duopoly game with a long memory:

$$\begin{cases} \Delta^{\alpha} p_1(t) = ap_1(t + \alpha - 1)\big(1 + c - 2p_1(t + \alpha - 1) + bp_2(t + \alpha - 1)\big), \\ \Delta^{\alpha} p_2(t) = \dfrac{1}{2}\big(1 + c + bp_1(t + \alpha - 1)\big) - p_2(t + \alpha - 1). \end{cases} \tag{6}$$

Remark 1. When $\alpha = 1$, equation (6) degenerates to the following equations:

$$\begin{cases} p_1(t+1) = p_1(t) + ap_1(t)\left(1 + c - 2p_1(t) + bp_2(t)\right), \\ p_2(t+1) = \dfrac{1}{2}\left(1 + c + bp_1(t)\right). \end{cases} \tag{7}$$

3. Nash Equilibrium and Local Stability

To obtain equilibria of system (6), we solve the following equations:

$$\begin{cases} kp_1(t)\left(1 + c - 2p_1(t) + bp_2(t)\right) = 0, \\ \dfrac{1}{2}\left(1 + c + bp_1\right) - p_2 = 0. \end{cases} \tag{8}$$

Their two Nash equilibria are $E_1 = (0, (1+c)/2)$ and $E_2 = ((bc+b+2)/(4-b^2), (b+2c+2)/(4-b^2))$. In economics, their equilibria mean the folllowing:

(i) The equilibrium E_1 means that the best price of the first firm is $p_1^* = 0$ if the second firm sets its optimal product price $p_2^* = (1+c)/2$. Similarly, the best price of the second firm is $p_2^* = (1+c)/2$ if the firm adopts zero price strategy. Obviously, E_1 is a bounded equilibrium [22].

(ii) The equilibrium E_2 means that both firms will maintain their equilibrium prices together because no firm can obtain any extra benefit by deviating unilaterally from its own equilibrium. Obviously, E_2 is a nonbounded equilibrium point.

Since the equilibrium E_1 has no real economic significance, we only analyze the complexity of the nonbounded equilibrium E_2.

The Jacobian matrix of system (6) computed at E_2 is

$$J(E_2) = \begin{pmatrix} \dfrac{2a(bc+b+2)}{b^2-4} & \dfrac{ab(bc+b+2)}{4-b^2} \\ \dfrac{b}{2} & -1 \end{pmatrix}. \tag{9}$$

From [21, 23], we can directly obtain Theorem 1.

Theorem 1. *System (5) is locally asymptotically stable at E_2 if $\det J > 0$ and either*

$$\frac{-trJ}{2} \geq \sqrt{\det J} \text{ and } \alpha > \log_2 \frac{\sqrt{A} - trJ}{2}, \tag{10}$$

$$\frac{|trJ|}{2} < \sqrt{\det J} < \left(2\cos\frac{B-\pi}{2-\alpha}\right)^{\alpha} \text{ and } \alpha < \frac{2B}{\pi}, \tag{11}$$

where $trJ = (2ab(1+c) + 4a - b^2 + 4)/(b^2 - 4)$, $\det J = a/2(bc+b+2)$, $A = |(trJ)^2 - 4\det J|$, and $B = trJ/\sqrt{A}$.

4. Numerical Simulation

From [24, 25], we can directly obtain Theorem 2.

Theorem 2. *For the following nonlinear system,*

$$\begin{cases} {}^{C}\Delta_0^{\alpha} X(t) = F(t+\alpha-1, X(t+\alpha-1)), \\ \Delta^j X(0) = X_j, \quad j = 0, \ldots, n-1, \quad n = [\alpha] + 1. \end{cases} \tag{12}$$

The equivalent form of system (12) is

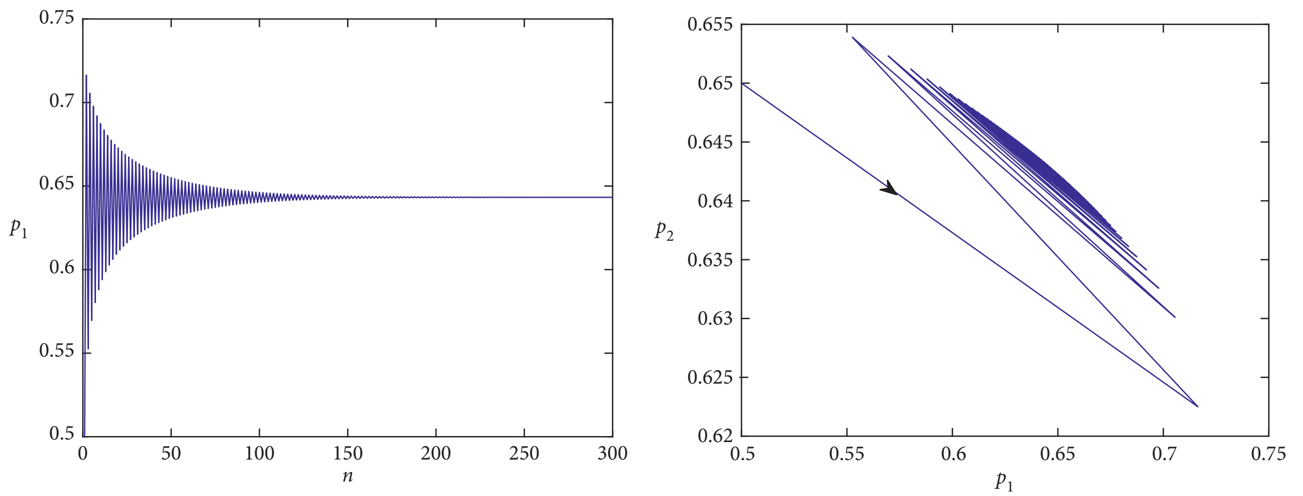
$$X(n) = X(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=1}^{n} \frac{\Gamma(n-j+\alpha)}{\Gamma(n-j+1)} F(X(j-1)), \quad n \in \mathbb{N}. \tag{13}$$

So we can rewrite system (5) as the following numerical form:

$$\begin{cases} p_1(n) = p_1(0) + \dfrac{1}{\Gamma(\alpha)} \displaystyle\sum_{j=1}^{n} \dfrac{\Gamma(n-j+\alpha)}{\Gamma(n-j+1)} ap_1(j-1)\left(1 + c - 2p_1(j-1) + bp_2(j-1)\right), \\ p_2(n) = p_2(0) + \dfrac{1}{\Gamma(\alpha)} \displaystyle\sum_{j=1}^{n} \dfrac{\Gamma(n-j+\alpha)}{\Gamma(n-j+1)} \dfrac{1}{2}\left(1 + c + bp_1(j-1)\right) - p_2(j-1). \end{cases} \tag{14}$$

Thus, we can illustrate the complexity of system (5) by using system (14). For equation (14), we set parameters $a = 1.5$, $b = 0.29$, $c = 0.1$, $\alpha = 0.999$, and the initial point $(p_1(0), p_2(0)) = (0.5, 0.65)$. From Theorem 1, we can obtain the following results: the nonbounded Nash equilibrium $E_2 = (0.5922, 0.6359)$, $\det J = 1.7393 > 0$, $trJ = -2.7766 < 0$, $A = 0.7525$, and $B = -3.2008$, so equation (10) holds. Thus, equation (5) is locally asymptotically stable at $E_2 = (0.5922, 0.6359)$. Figure 1 validates the results mentioned above.

In the following, we fix parameters $b = 0.2$ and $c = 0.5$, and the initial point $(p_1(0), p_2(0)) = (0.3, 0.3)$ in system (14). We will study chaos by using the bifurcation diagram, the phase portrait, the maximal Lyapunov exponent, and 0-1 test algorithm for chaos [26–28]. To reveal the complexity of



(a) (b)

Figure 1: Stabilized states of duopoly pricing in system (14) with $(p_1(0), p_2(0)) = (0.5, 0.65)$. (a) Price time series of firm 1. (b) Price phase portrait of two firms.

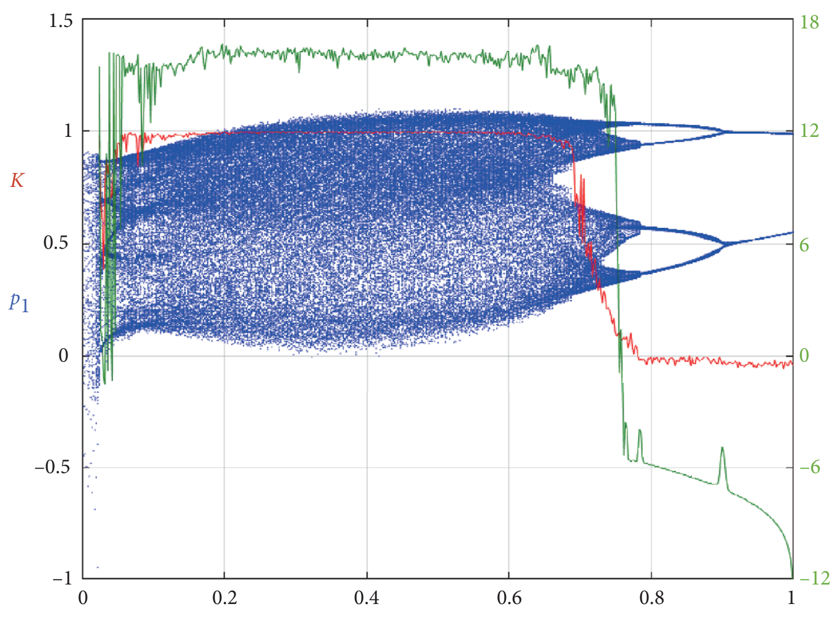


Figure 2: Firm 1's price diagrams of bifurcation (p_1, blue), maximal Lyapunov exponent (λ, dark green), and the median value of the correlation coefficient (K, red) in system (14) with varying $\alpha \in (0, 1)$.

system (14), we will study the influence of two main parameters: long-term memory effect and price adjustment speed.

4.1. Complexity with Varying the Order $\alpha \in (0, 1)$. To illustrate the complexity of system (14), such as bifurcation and chaos, we fix $a = 1.4$ and vary the order $\alpha \in (0, 1)$ with an increment of 0.002. We integrate three subfigures to Figure 2. Figure 2 has the y-axis divided into left and right (for K and λ) for better readability. The first subfigure is drawn for the bifurcation diagram of firm 1's price decision. The second subfigure is drawn for the median value K of the correlation coefficient. The last subfigure is drawn for the maximal Lyapunov exponent λ of firm 1's price decision. Figure 2 shows that the bifurcation diagram is in line with K and λ.

At $\alpha = 0.2$ in Figure 2, we note that $K = 0.9966 \approx 1$ and the maximal Lyapunov exponent $\lambda = 16.1760 > 0$, which indicate there exists chaos in system (14). To confirm the results in Figure 2, we draw Figure 3 with the above-mentioned parameter values. Figures 3(a) and 3(b) show that the time series of firms 1 and 2 are nonperiodic.

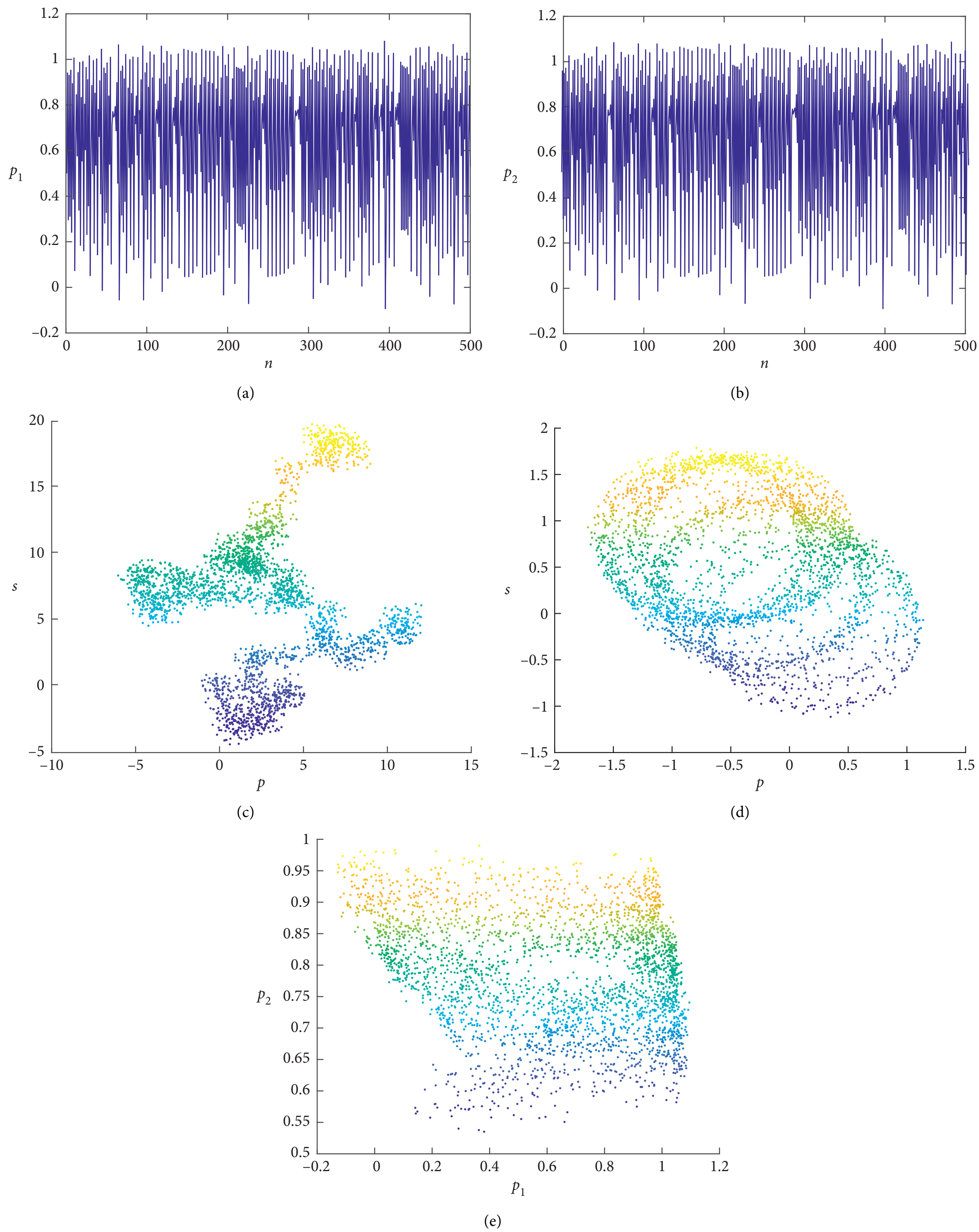


FIGURE 3: Chaos in system (14) with $(p_1(0), p_2(0)) = (0.3, 0.3)$. (a) Price time series of firm 1. (b) Price time series of firm 2. (c) Firm 1's pricing translation components (p, s). (d) Firm 2's pricing translation components (p, s). (e) Phase portrait of duopoly pricing.

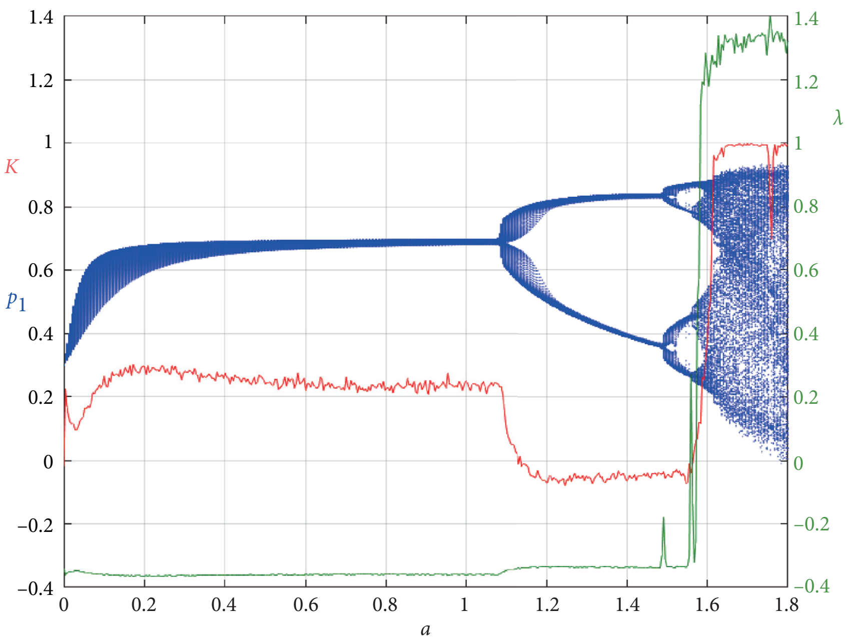


Figure 4: Firm 1's price diagrams of bifurcation (p_1, blue), maximal Lyapunov exponent (λ, dark green), and the median value of the correlation coefficient (K, red) in system (14) with varying $a \in (0, 1.8)$.

Figures 3(c) and 3(d) demonstrate that the pricing trajectories of K in the transformed coordinate (p, s) of firms 1 and 2 are Brownian-like. Figure 3(e) illustrates that the phase portrait of duopoly pricing is chaotic. They quite well coincide with Figure 2.

4.2. Complexity with Varying the Price Adjustment Speed $a \in (0, 1.8)$. To illustrate how the price adjustment speed a affects the complexity of system (14), we use above-mentioned parameter values, then set the order $\alpha = 0.6$, and vary the price adjustment speed $a \in (0, 1.8)$ with an increment of 0.004. We also integrate three subfigures to Figure 4. Figure 4 also has the y-axis divided into left and right (for K and λ) for better readability. The three subfigures are the bifurcation diagram, the median value K of the correlation coefficient, and the maximal Lyapunov exponent λ of firm 1's price decision, respectively. Figure 4 also shows that the bifurcation diagram is in line with K and λ.

The phase portrait and time series are similar to those in Section 4.1. For saving readers' time, we will not demonstrate the consistency between the phase portraits, the time series, the bifurcation diagram, the median value of the correlation coefficient, and the maximal Lyapunov exponent of firm 1's price decision.

5. Conclusion and Future Developments

Introducing the discrete fractional-order calculus into the integer-order nonlinear Bertrand game, we obtain a fractional-order Bertrand game with a long memory. Then, we qualitatively analyze the game's Nash equilibria and local stability. Finally, we validate these qualitative results and compare the consistency among bifurcation diagrams, phase portraits, time series, the maximal Lyapunov exponent, and the 0-1 test values.

These results illustrate the complexity of the discrete fractional-order nonlinear Bertrand game. When the long-term memory effect increases from 0 to 1, the game's complexity decreases gradually. When the price adjustment speed increases from 0 to 1.8, the game's complexity increases gradually. So we can design suitable game mechanisms according to actual demand by changing the size of the long-term memory effect or the price adjustment speed.

Also, we can extend some researches in the following potential directions. First, there are many types of games that can be extended by discrete fractional-order calculus, such as the Edgeworth game or the Bertrand–Edgeworth game or the Cournot–Bertrand game. Second, we can study the control and synchronization of discrete fractional-order games. Third, we can study the stochastic or fuzzy versions of a discrete fractional-order game. Fourth, we can construct data-driven models of a discrete fractional-order game. At last, we can construct discrete fractional-order models in many scientific fields [29–33].

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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